If we use the values we have been assuming throughout this gastronomic feast ($V = 2$, $p_y = 4$), then

$$\text{CV} = 2 \cdot 2 \cdot 2 \cdot (4)^{0.5} - 2 \cdot 2 \cdot 2 \cdot (1)^{0.5} = 8. \tag{5.62}$$

This figure would be cut in half (to 4) if we believed that the utility level after the price increase ($V = 1$) were the more appropriate utility target for measuring compensation. If instead we had used the Marshallian demand function

$$x(p_x, p_y, I) = 0.5Ip_x^{-1}, \tag{5.63}$$

the loss would be calculated as

$$\text{loss} = \int_1^4 x(p_x, p_y, I)\, dp_x = \int_1^4 0.5Ip_x^{-1} dp_x = 0.5I \ln p_x \Big|_1^4. \tag{5.64}$$

Thus, with $I = 8$, this loss is

$$\text{loss} = 4 \ln(4) - 4 \ln(1) = 4 \ln(4) = 4(1.39) = 5.55, \tag{5.65}$$

which seems a reasonable compromise between the two alternative measures based on the compensated demand functions.

QUERY: In this problem, none of the demand curves has a finite price at which demand goes to precisely zero. How does this affect the computation of total consumer surplus? Does this affect the types of welfare calculations made here?

REVEALED PREFERENCE AND THE SUBSTITUTION EFFECT

The principal unambiguous prediction that can be derived from the utility-maximation model is that the slope (or price elasticity) of the compensated demand curve is negative. We have shown this result in two ways. The first proof was based on the quasi-concavity of utility functions, that is, because any indifference curve must exhibit a diminishing MRS, any change in a price will induce a quantity change in the opposite direction when moving along that indifference curve. A second proof derives from Shephard's lemma—because the expenditure function is concave in prices, the compensated demand function (which is the derivative of the expenditure function) must have a negative slope. Again utility is held constant in this calculation as one argument in the expenditure function. To some economists, the reliance on a hypothesis about an unobservable utility function represented a weak foundation on which to base a theory of demand. An alternative approach, which leads to the same result, was first proposed by Paul Samuelson in the late 1940s.[10] This approach, which Samuelson termed the *theory of revealed preference*, defines a principle of rationality that is based on observed behavior and then uses this principle to approximate an individual's utility function. In this sense, a person who follows Samuelson's principle of rationality behaves *as if* he or she were maximizing a proper utility function and exhibits a negative substitution effect. Because Samuelson's approach provides additional insights into our model of consumer choice, we will briefly examine it here.

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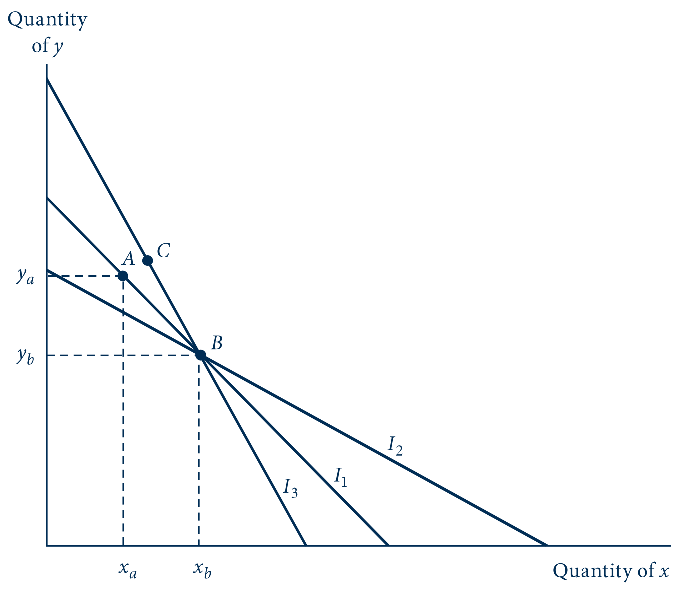
[10]Paul A. Samuelson, *Foundations of Economic Analysis* (Cambridge, MA: Harvard University Press, 1947).

FIGURE 5.10

Demonstration of the Principle of Rationality in the Theory of Revealed Preference

With income I_1 the individual can afford both points *A* and *B*. If *A* is selected, then *A* is revealed preferred to *B*. It would be irrational for *B* to be revealed preferred to *A* in some other price-income configuration.



Graphical approach

The principle of rationality in the theory of revealed preference is as follows: Consider two bundles of goods, A and B. If, at some prices and income level, the individual can afford both A and B but chooses A, we say that A has been "revealed preferred" to B. The principle of rationality states that under any different price–income arrangement, B can never be revealed preferred to A. If B is in fact chosen at another price–income configuration, it must be because the individual could not afford A. The principle is illustrated in Figure 5.10. Suppose that, when the budget constraint is given by I_1, point A is chosen even though B also could have been purchased. Then A has been revealed preferred to B. If, for some other budget constraint, B is in fact chosen, then it must be a case such as that represented by I_2, where A could not have been bought. If B were chosen when the budget constraint is I_3, this would be a violation of the principle of rationality because, with I_3, both A and B can be bought. With budget constraint I_3, it is likely that some point other than either A or B (say, C) will be bought. Notice how this principle uses observable reactions to alternative budget constraints to rank commodities rather than assuming the existence of a utility function itself. Also notice how the principle offers a glimpse of why indifference curves are convex. Now we turn to a formal proof.

Negativity of the substitution effect

Suppose that an individual is *indifferent* between two bundles, C (composed of x_C and y_C) and D (composed of x_D and y_D). Let p_x^C, p_y^C be the prices at which bundle C is chosen and p_x^D, p_y^D the prices at which bundle D is chosen.

Because the individual is indifferent between C and D, it must be the case that when C was chosen, D cost at least as much as C:

$$p_x^C x_C + p_y^C y_C \le p_x^C x_D + p_y^C y_D. \tag{5.66}$$

A similar statement holds when D is chosen:

$$p_x^D x_D + p_y^D y_D \le p_x^D x_C + p_y^D y_C. \tag{5.67}$$

Rewriting these equations gives

$$p_x^C (x_C - x_D) + p_y^C (y_C - y_D) \le 0, \tag{5.68}$$

$$p_x^D (x_D - x_C) + p_y^D (y_D - y_C) \le 0. \tag{5.69}$$

Adding these together yields

$$(p_x^C - p_x^D)(x_C - x_D) + (p_y^C - p_y^D)(y_C - y_D) \le 0. \tag{5.70}$$

Now suppose that only the price of x changes; assume that $p_y^C = p_y^D$. Then

$$(p_x^C - p_x^D)(x_C - x_D) \le 0. \tag{5.71}$$

But Equation 5.71 says that price and quantity move in the opposite direction when utility is held constant (remember, bundles C and D are equally attractive). This is precisely a statement about the nonpositive nature of the substitution effect:

$$\partial x^c(p_x, p_y, V) \qquad \partial x \,\big|$$

$$\frac{\partial x^c(p_x, p_y, V)}{\partial p_x} = \frac{\partial x}{\partial p_x}\bigg|_{U=\text{constant}} \leq 0. \quad \textbf{(5.72)}$$

We have arrived at the result by an approach that does not require the existence of a quasi-concave utility function.

SUMMARY

In this chapter, we used the utility-maximization model to study how the quantity of a good that an individual chooses responds to changes in income or to changes in that good's price. The final result of this examination is the derivation of the familiar downward-sloping demand curve. In arriving at that result, however, we have drawn a wide variety of insights from the general economic theory of choice.

- Proportional changes in all prices and income do not shift the individual's budget constraint and therefore do not change the quantities of goods chosen. In formal terms, demand functions are homogeneous of degree 0 in all prices and income.
- When purchasing power changes (i.e., when income increases with prices remaining unchanged), budget constraints shift and individuals will choose new commodity bundles. For normal goods, an increase in purchasing power causes more to be chosen. In the case of inferior goods, however, an increase in purchasing power causes less to be purchased. Hence the sign of $\partial x_i/\partial I$ could be either positive or negative, although $\partial x_i/\partial I \geq 0$ is the most common case.
- A decrease in the price of a good causes substitution and income effects that, for a normal good, cause more of the good to be purchased. For inferior goods, however, substitution and income effects work in opposite directions, and no unambiguous prediction is possible.
- Similarly, an increase in price induces both substitution and income effects that, in the normal case, cause less to be demanded. For inferior goods the net result is again ambiguous.
- Marshallian demand curves represent two-dimensional depictions of demand functions for which only the own-price varies—other prices and income are held constant. Changes in these other variables will usually shift the position of the demand curve. The sign of the slope of the Marshallian demand curve $\left(\frac{\partial x(p_x, p_y, I)}{\partial p_x}\right)$ is theoretically ambiguous because substitution and income effects may work in opposite directions. The Slutsky equation permits a formal study of this ambiguity.