CHAMAS TURBULENTAS PRÉ-MISTURADAS

INTROD.

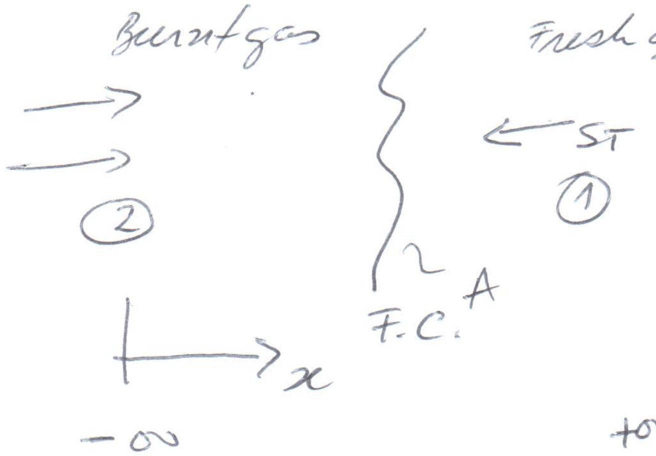
- Interação Turbulência $\leftrightarrow$ chama laminar

S_L : 20 - 10 cm/s

$\delta_L \simeq 0.1$ mm

- Estabilização : aumento de S_T

- CHAMA ESTACIONÁRIA



~ p/ observador sentado na F.C.

$$\overline{\rho}\,\vec{\tilde{u}}_1 = -\rho_1 \vec{S_T}$$

$$\cancel{\frac{\partial}{\partial t}\left(\overline{\rho}\tilde{Y}_F\right)} + \rho_1 S_T \frac{\partial \tilde{Y}_F}{\partial x_i} = -\frac{\partial}{\partial x_i}\left(\overline{V_{F,i} Y_F} + \overline{\rho}\,\widetilde{u_i'' Y_F''}\right) + \tilde{\dot{\omega}}_F \qquad (1)$$

$= 0$ (Estacionário)

$$\int_{x_1=-\infty}^{x_1=+\infty} -\rho_1 S_T \frac{\partial \tilde{Y}_F}{\partial x_i}\, dx_i\, A = \underbrace{\int_{-\infty}^{+\infty} -\frac{\partial}{\partial x_i}\left(\overline{V_{F,i} Y_F} + \overline{\rho}\,\widetilde{u_i'' Y_F''}\right) dx\, A}_{\text{DIFUSÃO}} + \int_{-\infty}^{+\infty} \tilde{\dot{\omega}}_F\, dx\, A$$

- em $x = \pm\infty$, não há gradientes e portanto só há difusão.

Assim:

$$-\rho_1 S_T A \Big|_{Y_F^2}^{Y_F^1} = \int \tilde{\dot{w}}_F \, dx \, A$$

mas $Y_F^2 = 0$ (lean mixture)

$$-\rho_1 S_T A Y_F^1 = \int_{\forall} \tilde{\dot{\omega}}_F \, d\forall \qquad (2)$$

$$\boxed{\rho_1 S_T A Y_F^1 = -\int_{\forall} \tilde{\dot{\omega}}_F \, d\forall}$$

- CORRELAÇÕES

$$S_T / S_L = 1 + u' / S_L$$

- Região Linear - Flamelet:

$$-\int_{\forall} \dot{\omega}_F \, d\forall = A_T \rho_1 Y_F^1 S_L^{\circ}$$

S_L° → LAMINAR

A_T → AREA TOTAL DA F.C.

Subst. (2) em (3)

$$\frac{S_T}{S_L^o} = \frac{A_T}{A} = \Sigma \quad (4)$$

↳ FATOR DE AMARROTAMENTO DA F.C.

→ QUENCHING LIMIT

→ DIFICIL VALIDAÇÃO DA CORRELAÇÃO

$$S_T/S_L = 1 + u'/S_L$$

- INFLUÊNCIA DA TEMPERATURA NA F.C.

$T \rightarrow \nu \rightarrow \quad \nu \propto T^{1.7} \qquad T_2/T_1 \cong 8$

$Re_2/Re_1 \approx 1/40$ → Relaminarização nos gases queimados

↕

Diferença de velocid. em 1 e 2 produz turbulência e intermitência

- LIMITE DE F.C. INFINITAMENTE FINA

(DESCONTINUIDADE!!!)

- BRAY - MOSS - LIBBY - MODEL

- TEMPERATURA REDUZIDA (θ)

$\rightarrow \theta = 0 \rightarrow$ mistura fresca

ou

$\rightarrow \theta = 1 \rightarrow$ completamente queimado

- PDF:

$$P(\theta) = \alpha\,\delta(\theta) + \beta\,\delta(1-\theta) \qquad (5)$$

α ↓ Probabilidade de ter gás fresco

β ↓ Probabilidade de ter gás completamente queimado

$$\alpha + \beta = 1 \qquad (6)$$

- p/ QUALQUER QUANTIDADE FÍSICA:

$$\bar{f} = \int_0^1 f(\theta)\,P(\theta)\,d\theta = \alpha\,\bar{f}^u + \beta\,\bar{f}^b \qquad (7)$$

$\bar{f}^b$ ↳ nos gases queimados

$\bar{f}^u$ ↓ Média condicionada de f nos gases frescos

– MÉDIA DE REYNOLDS DE θ:

$$\overline{\theta} = \int_0^1 \theta\, p(\theta)\, d\theta = \alpha(\theta=0) + \beta(\theta=1) = \beta$$

Probabilidade de estar na mistura queimada

– FAVRE DE θ:

$$\overline{\rho\theta} = \bar{\rho}\,\tilde{\theta} = \int_0^1 \rho\,\theta\, p(\theta)\, d\theta = \rho_u(\theta=0)\,\alpha + \rho_b(\theta=1)\,\beta$$

$$\overline{\rho\theta} = \bar{\rho}\,\tilde{\theta} = \rho_b\,\beta \qquad (9)$$

$$\bar{\rho} = \alpha\rho_u + \beta\rho_b = (1-\beta)\rho_u + \beta\rho_b \qquad (10)$$

fazendo $\tau = \dfrac{\rho_u}{\rho_b} - 1$

$$\bar{\rho} + \tau\bar{\rho}\tilde{\theta} = (1-\beta)\rho_u + \beta\rho_b + \tau\underbrace{\bar{\rho}\tilde{\theta}}_{\rho_b\beta}$$

$$\bar{\rho} + \tau\bar{\rho}\tilde{\theta} = (1-\beta)\rho_u + \beta(\rho_b + \tau\rho_b)$$

$$= \cancel{\rho_u}$$

$$= \rho_u + \beta\underbrace{\left(-\rho_u + \rho_b + \left(\frac{\rho_u}{\rho_b} - 1\right)\rho_b\right)}_{=0} \quad (*)$$

$$\bar{\rho} + \tau\bar{\rho}\tilde{\theta} = \rho_u \longrightarrow \boxed{\bar{\rho}(1+\tau\tilde{\theta}) = \rho_u} \quad (10a)$$

ainda partindo de (*)

$$\bar\rho + \tilde\tau\bar\rho\tilde\theta = \rho_b\left(\frac{\rho_\mu}{\rho_b} - \cancel{\frac{\beta\rho_\mu}{\rho_b}} + \cancel{\beta} + \cancel{\frac{\beta\rho_\mu}{\rho_b}} - \cancel{\beta}\right)$$

$$\boxed{\bar\rho + \tilde\tau\bar\rho\tilde\theta = \rho_b\left(\frac{\rho_\mu}{\rho_b}\right) = \rho_b(1+\tilde\tau)}$$

$$\bar\rho(1+\tilde\tau\tilde\theta) = \rho_b(1+\tilde\tau)$$

$$\boxed{\bar\rho = \rho_b\frac{(1+\tilde\tau)}{(1+\tilde\tau\tilde\theta)}} \qquad (10b)$$

- OBTENÇÃO DE α e β

por (10a) $\bar\rho = \dfrac{\rho_\mu}{(1+\tilde\tau\tilde\theta)}$ e por (10):

$$\frac{\rho_\mu}{(1+\tilde\tau\tilde\theta)} = \alpha\rho_\mu + \underbrace{\beta\rho_b}_{=\bar\rho\tilde\theta\ \text{(por 9)}}$$

$$\frac{\rho_\mu}{(1+\tilde\tau\tilde\theta)} = \alpha\rho_\mu + \bar\rho\tilde\theta$$

$$\alpha = \frac{1}{(1+\tilde\tau\tilde\theta)} - \frac{\bar\rho\tilde\theta}{\rho_\mu} \Rightarrow \alpha = \frac{1}{(1+\tilde\tau\tilde\theta)} - \frac{\tilde\theta}{\rho_\mu}\times\frac{\rho_\mu}{(1+\tilde\tau\tilde\theta)}$$

$$\boxed{\alpha = \frac{1-\tilde\theta}{(1+\tilde\tau\tilde\theta)}} \qquad (10c)$$

β: por (10b):

$$\bar{\rho} = \rho_b \frac{(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})} = \rho_\mu \overbrace{(1-\beta)}^{\alpha} + \overbrace{\beta\rho_b}^{\bar{\rho}\tilde{\theta}} = \rho_\mu(1-\beta) + \bar{\rho}\tilde{\theta}$$

$$\rho_b \frac{(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})} = \rho_\mu(1-\beta) + \rho_b \frac{(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})}\tilde{\theta}$$

$\div \rho_b$

$$\frac{(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})} = \frac{\rho_\mu}{\rho_b}(1-\beta) + \frac{(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})}\tilde{\theta}$$

$$\frac{\cancel{(1+\tilde{\tau})}}{(1+\tilde{\tau}\tilde{\theta})}(1-\tilde{\theta}) = \underbrace{\frac{\rho_\mu}{\rho_b}}_{=\tilde{\tau}+1}(1-\beta) = \cancel{(\tilde{\tau}+1)}(1-\beta)$$

$$\frac{(1-\tilde{\theta})}{(1+\tilde{\tau}\tilde{\theta})} = 1-\beta \rightarrow \beta = 1 - \frac{(1-\tilde{\theta})}{(1+\tilde{\tau}\tilde{\theta})} = \frac{(1+\tilde{\tau}\tilde{\theta})-1+\tilde{\theta}}{(1+\tilde{\tau}\tilde{\theta})}$$

$$\boxed{\beta = \frac{\tilde{\theta}(1+\tilde{\tau})}{(1+\tilde{\tau}\tilde{\theta})}} \qquad (10d)$$

$$\tilde{Q} = \frac{\overline{\rho Q}}{\bar{\rho}}$$

$$\overline{\rho Q} = \int_0^1 \rho(\theta)\, Q(\theta)\, P(\theta)\, d\theta = \alpha\, \overline{\rho_\mu Q^\mu} + \beta\, \overline{\rho_b Q^b}$$

$$\overline{\rho_b Q^b} = \rho_b \overline{Q^b}$$

$$\overline{\rho_\mu Q^\mu} = \rho_\mu \overline{Q^\mu}$$

média de $\overline{Q^\mu}$ condicionada a estar na região "μ"

fica só a flutuação da turbulência e não ~~a~~ a intermitência

então:

$$\overline{\rho Q} = \alpha \rho_\mu \overline{Q^\mu} + \beta \rho_b \overline{Q^b}$$

ou

(10a)

$$\tilde{Q} = (1-\tilde{\theta}) \frac{\cancel{\bar{\rho}(1+\tilde{\tau}\tilde{\theta})}}{\cancel{(1+\tilde{\tau}\tilde{\theta})\bar{\rho}}}\, \overline{Q^\mu} + \frac{\cancel{(1+\tilde{\tau})}\tilde{\theta}}{\cancel{(1+\tilde{\tau}\tilde{\theta})}} \frac{\cancel{\bar{\rho}(1+\tilde{\tau}\tilde{\theta})}}{\cancel{(1+\tilde{\tau})\bar{\rho}}}\, \overline{Q^b}$$

$$\boxed{\tilde{Q} = (1-\tilde{\theta})\, \overline{Q^\mu} + \tilde{\theta}\, \overline{\theta^b}} \qquad (11)$$

– TURBULENT FLUXES AND COUNTERGRADIENT TURB. TRANSP.

$$\bar{\rho}\,\widetilde{u''\phi''} = \bar{\rho}\left(\widetilde{u\theta} - \tilde{u}\tilde{\theta}\right)$$

$$= \bar{\rho}\Big[(1-\tilde{\theta})\,\overline{u\underset{=0}{\theta}}^{u} + \tilde{\theta}\,\overline{u\underset{=1}{\theta}}^{b} - \big[(1-\tilde{\theta})\bar{u}^{u} + \tilde{\theta}\bar{u}^{b}\big]\big[\tilde{\theta}\big]\Big]$$

$$= \bar{\rho}\left[\tilde{\theta}\,\bar{u}^{b} - (1-\tilde{\theta})\,\bar{u}^{u}\tilde{\theta} - \tilde{\theta}\,\bar{u}^{b}\tilde{\theta}\right]$$

$$= \bar{\rho}\,\tilde{\theta}(1-\tilde{\theta})(\bar{u}_b - \bar{u}_u) \qquad (12)$$

- RANS

- Química de 1 step - Irreversível

$$R \longrightarrow P$$

Fuel: $$\dot{\omega}_F = B \rho Y_F \exp\left(-T_a/T\right) \qquad (13)$$

$$\Theta = \frac{T - T_1}{T_2 - T_1} = \frac{C_p (T - T_1)}{Q Y_F^1} \qquad (14)$$

T_2 burned, T_1 fresh, Q → Heat of Reaction

$$Q_1 = \frac{\dot{\omega}_K}{W_K \nu_{K,1}} \qquad (15)$$

$\dot{\omega}_K$ → espécie K

- Termo "fonte" na eq. da Energia (Temperatura)

$$\dot{\omega}_T = -\sum_{K=1}^{N} \Delta h^{o}_{f,K} \dot{\omega}_K = -Q_1 \sum \Delta h^{o}_{f,K} W_K \nu_{K1}$$

$$= |\nu_{F1}| Q^{m} Q_1 \qquad (16)$$

$$Q^{m} = -\sum \Delta h^{o}_{f,K} W_K \frac{\nu_{K1}}{|\nu_{F1}|} = -\sum \Delta h^{o,m}_{f,K} \frac{\nu_{K1}}{|\nu_{F1}|}$$

massa

mas $\nu_{F1} = \nu''_{F1} - \nu'_{F1}$ é sempre negativo

$(\nu''_{F1} = 0 \;,\; \nu'_{F} = 1, 2, 3 \text{ etc})$

então:

$$Q''' = \sum \Delta h^{o,m}_{f,k} \frac{\nu_{k1}}{\nu_{F1}} \qquad kJ/kg \text{ p/ 1 mol de combust.}$$

mas por (15) $\quad \dot{\omega}_F = Q_1 W_F \nu_{F1} \quad (17)$

Definindo-se $\quad Q = \dfrac{Q'''}{W_F} = \sum \Delta h^o_{f,k} \dfrac{W_k}{W_F} \dfrac{\nu_{k1}}{\nu_{F1}}$

e com (16) e (17)

$$\dot{\omega}_T = -\frac{\dot{\omega}_F}{W_F} Q''' = -\dot{\omega}_F Q \qquad (18)$$

– EQ. DE CONSERVAÇÃO NO REFERENCIAL DA CHAMA

$$\dot{m}'' = \rho u = cte = \rho_1 u_1 = \rho_1 S_L = cte \qquad (19)$$

$$\left[kg/m^3 \times \frac{m}{s} \right] = \left[\frac{kg}{m^2 s} \right]$$

$$\rho_1 S_L \frac{dY_F}{dx} = \frac{d}{dx}\left(\rho D \frac{dY_F}{dx} \right) + \dot{\omega}_F \qquad (20)$$

$$\rho_1 c_p S_L \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx} \right) - Q \dot{\omega}_F \qquad (21)$$

Definindo-se as Variáveis Reduzidas

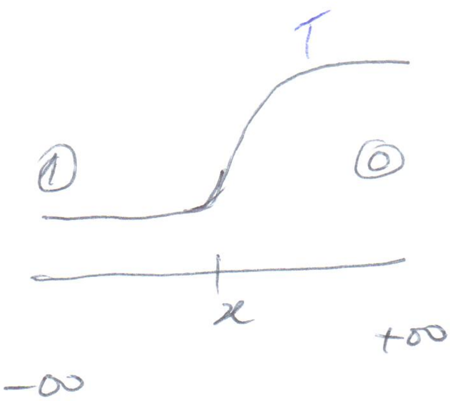
$$Y = Y_F / \underset{\text{fresh gas}}{Y_F^1} \qquad (22a)$$

1 - fresh gás

0 - burnt gás

$$\theta = \frac{c_p (T - T_1)}{Q Y_F^1} \qquad (22b)$$

Integrando-se (20) e (21) de $x=-\infty$ até $x=+\infty$, lembrando que os termos difusivos são nulos nestes extremos:



$$\rho_1 S_L \int_1^0 dY_F = \underbrace{\int_{x=-\infty}^{x=+\infty} \rho D \frac{dY_F}{dx}}_{=0} + \int_{-\infty}^{+\infty} \dot{\omega}_F \, dx$$

$$\rho_1 S_L (\overset{=0}{Y_F^0} - Y_F^1) = \int_{-\infty}^{+\infty} \dot{\omega}_F \, dx = -\Omega_F \qquad (23)$$

Taxa de Consumo Total de Comb. no domínio

$$\rho_1 S_L Y_F^1 = \Omega_F \qquad (23b)$$

e

$$\rho_1 c_p S_L (T^0 - T^1) = -Q \int_{-\infty}^{+\infty} \dot{\omega}_F \, dx = Q \Omega_F \qquad (23c)$$

então eliminando-se Ω_F em (23b) e (23c):

$$c_p (T^0 - T^1) = \frac{Q Y_F^1}{\cancel{c_p}}$$

então $$T^0 - T^1 = \frac{Q Y_F^1}{c_p} \qquad (24)$$

Voltando à eq. (22b): $$\theta = \frac{c_p (T - T_1)}{c_p (\underbrace{T^0}_{= T_2} - T_1)} = \frac{T - T_1}{T_2 - T_1}$$

Re-escrevendo-se (20) com (22a):

$$\rho_1 S_L Y_F^1 \frac{dY}{dx} = Y_F^1 \frac{d}{dx}\left(\rho D \frac{dY}{dx}\right) + \dot{\omega}_F$$

então:

$$\rho_1 S_L \frac{dY}{dx} = \frac{d}{dx}\left(\rho D \frac{dY}{dx}\right) + \dot{\omega}_F / Y_F^1 \qquad (25)$$

e (21):

$$c_p \rho_1 S_L (T_2 - T_1) \frac{d\theta}{dx} = (T_2 - T_1) \frac{d}{dx}\left(\lambda \frac{d\theta}{dx}\right) - Q \dot{\omega}_F$$

$$\rho_1 S_L \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{\lambda}{c_p}\frac{d\theta}{dx}\right) - \frac{Q\dot{\omega}_F}{c_p(T_2 - T_1)}$$

por (22b)

$$= \frac{1}{Y_F^1}$$

$$\rho_1 S_L \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{\lambda}{c_p}\frac{d\theta}{dx}\right) - \frac{\dot{\omega}_F}{Y_F^1} \qquad (26)$$

Somando-se (25) e (26); com $Le = \frac{\lambda}{\rho c_p D} = 1$,

$$\rho_1 S_L \frac{d}{dx}(Y + \theta) = \frac{d}{dx}\left(\rho D \frac{d}{dx}(\theta + Y)\right) \qquad (27)$$

ESCALAR CONSERVATIVO

Uma vez que $\theta + Y = 1$ nos gases frescos e 1 nos gases queimados, a única solução de (27) é $\boxed{\theta + Y = 1}$

Basta resolver θ – Reduced Temperature or Progress Variable

$\theta = 0 \rightarrow$ fresh gas

$\theta = 1 \rightarrow$ burnt gas

Eq. de Transporte p/ $\tilde{\theta} = 1 - \tilde{Y}$

$\hookrightarrow \tilde{Y} = \tilde{Y}_F / Y_F^1$

$$\frac{\partial(\bar{\rho}\tilde{\theta})}{\partial t} + \frac{\partial}{\partial x_i}\left(\bar{\rho}\,\widetilde{\theta u_i}\right) = \frac{\partial}{\partial x_i}\left(\bar{\rho}\,D\frac{\partial\tilde{\theta}}{\partial x_i} - \bar{\rho}\,\widetilde{u_i''\theta''}\right) + \overline{\dot{\omega}_\theta}$$

onde $\overline{\dot{\omega}_\theta} = -\dfrac{\overline{\dot{\omega}_F}}{Y_F^1}$

$\longrightarrow$ Termo de fonte p/ a Temperatura Reduzida

$\longrightarrow$ FORMULAÇÃO DO TERMO DE FONTE MÉDIO:

1) "No-model"

$\longrightarrow$ Valido para $Da = \dfrac{\tilde{\tau}_t}{\tilde{\tau}_c} \simeq 0$ ou

se $\tilde{\tau}_c \gg \tilde{\tau}_t \longrightarrow \underline{\underline{PSR}}$

$$\overline{\dot{\omega}_\theta} = \dot{\omega}_\theta(\tilde{\theta}) = -B\,\bar{\rho}\,(1-\tilde{\theta})\exp\left(-\frac{T_a}{T_1 + (T_2 - T_1)\tilde{\theta}}\right)$$