An Introduction to Cognitive Diagnostic Models using different R packages

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Grupo de Pesquisa em variáveis latentes
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Research and Evaluation Methodology
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USA
Gainesville, November 4, 2019

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In this presentation we will talk about the use of Classical and Bayesian approach to the estimation of parameters of the Cognitive Diagnostic models (CDM) using different R packages.

Specifically we showed the codes to reproduce the fit of the application to a Depression data set from the paper da Silva, de Oliveira, Davier and Bazán (2018) and give some comments about the use of this type of models in the Educational Assessment.

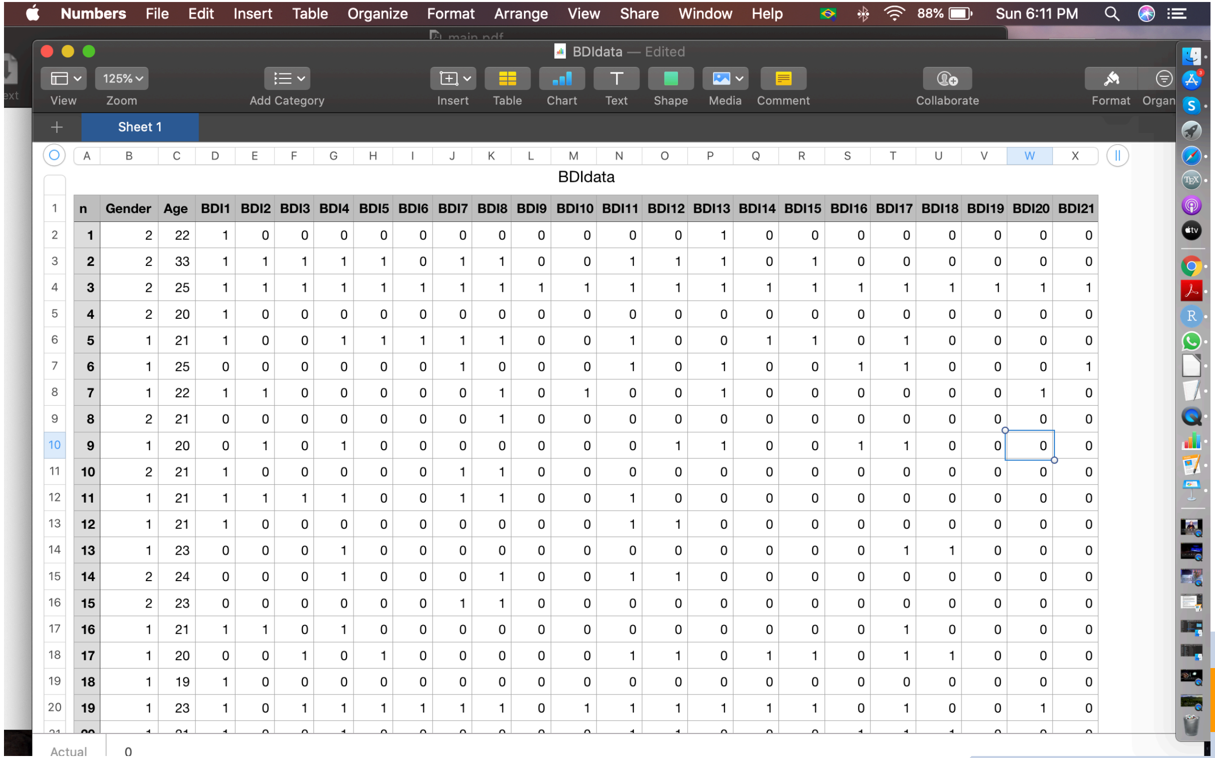
da Silva, M. A., de Oliveira, E. S., Davier, A. A., Bazán, J. L. (2018). Estimating the DINA model parameters using the No-U-Turn Sampler. *Biometrical Journal*, 60(2), 352-368.

1.Motivation: An practical example
2.A short literature review
3.Modeling
4. Estimation Methods and R packages
5. Results for BDI data
6.Comments
7.References
8.(Appendix)

1

MOTIVATION: A PRACTICAL EXAMPLE

We use a dataset from Fragoso and Curi (2013). Improving psychometric assessment of the beck depression inventory using multidimensional item response theory. *Biometrical Journal*, 55, 527 540.



BDIdata

n	Gender	Age	BDI1	BDI2	BDI3	BDI4	BDI5	BDI6	BDI7	BDI8	BDI9	BDI10	BDI11	BDI12	BDI13	BDI14	BDI15	BDI16	BDI17	BDI18	BDI19	BDI20	BDI21
1	2	22	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
2	2	33	1	1	1	1	1	0	1	1	0	0	1	1	1	0	1	0	0	0	0	0	0
3	2	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	2	20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	21	1	0	0	1	1	1	1	1	0	0	1	0	0	1	1	0	1	0	0	0	0
6	1	25	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1	1	0	0	0	1
7	1	22	1	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0
8	2	21	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	20	0	1	0	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0
10	2	21	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	21	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0
12	1	21	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
13	1	23	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
14	2	24	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0
15	2	23	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	21	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
17	1	20	0	0	1	0	1	0	0	0	0	0	1	1	0	1	1	0	1	1	0	0	0
18	1	19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	23	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	1	0

Beck Depression Inventory (BDI)

- We consider a data set of the BDI (Beck et al, 1981)

- The Test have 21 questions of four alternatives (0 to 3 points) about depression.

- The test was completed for 1111 students e was applied for the Dr Teng Chei-Tung in the Clinical Hospital from University of São Paulo.

- BDI is probably the most used questionnaire about depression and has been translated to many idioms and validated in several countries.

- In the traditional approach (See Kendall et al, 1987), a score is obtained adding the responses of the questiones. The minimal score is 0 in the maximum is 63. Then, individuals are classified as being non depressed (BDI total score 0–15), dysphoric (BDI total score 16–20), and depressed subjects (BDI total score 21–63).

- Fragoso and Curi (2013) dichotomize the data, such that the value 0 was attributed to the answers equal to zero, and a value of 1 was attributed to positive answers (1, 2, or 3). Then, they proposed a Multidimensional Item Response Model including discrimination and difficulty parameters and two dimensions.

- Fragoso and Curi (2013) identify the following distribution of the item in the two dimensions identified in the BDI Test

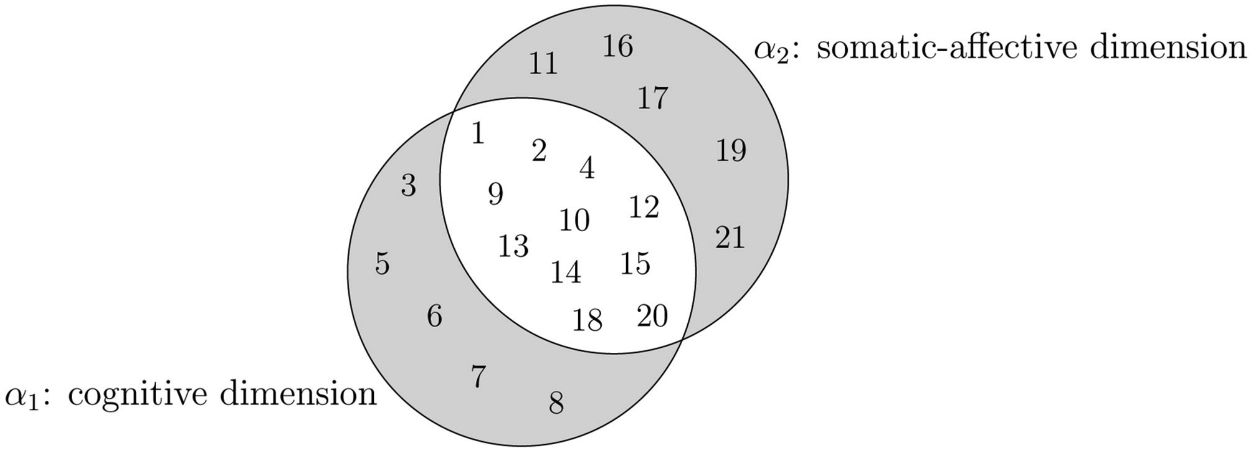


FIGURE 5 Items 3 and 5 through 8 primarily evaluate the cognitive aspect of depression, while items 11, 16, 17, 19, 21 primarily evaluate the somatic-affective aspect. Notes: The remaining items evaluate both dimensions in a balanced manner.

- Cognitive diagnosis models (CDMs) are useful psychometric tools for identifying test-takers' profile or level of possession of a set of latent attributes underlying a latent variable; the latent variable may be a cognitive skill (say, mathematics achievement), a psychological trait, or an attitude.

- We want propose a classification method for questionnaire respondent's in a clinical context using a CDM model.

- We will use different estimation methods using R packages

2

A SHORT LITERATURE REVIEW

In general, CDMs or diagnostic classification models allows the classification o examinees in multiple skills or cognitive attributes.

These models are relatively newer psychometric framework for collecting, analyzing, and reporting diagnostic data. They are a third generation of models in Psychometric after Theory Classical of the Test (TCT) and Item Response Theory (IRT).

CDMs have received increasing attention in many disciplines, such as educational, psychological, and psychiatric measurement and different models are being proposal attending the different formats of response of the Test how dichotomous, polytomous, count and continuous response.

This models are important because there is a real interest in developed formative assessments to provide examinees (students) and evaluators (teachers) with detailed feedback on what examinees (students) what skills the have (are able to do) yielding information that can optimize counseling (instruction) and improvement (learning).

In other words, a formative assessment should identify individual strengths and weaknesses in a particular content, which results in enhanced teaching and learning environment (DiBello & Stout, 2007).

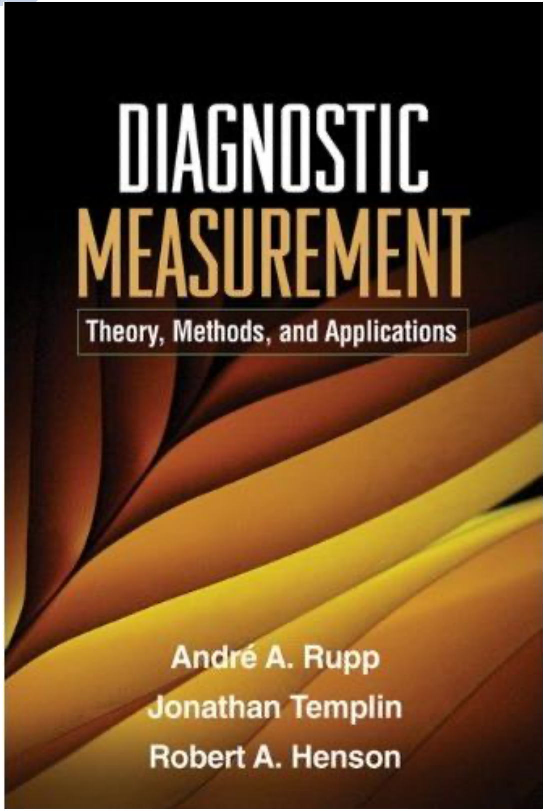
- George and Robitzsch (2015) say that CDMs are a class of discrete latent variable models that trace a respondent's answer to an item back to his possession of basic characteristics underlying the domain or latent trait covered by the items.

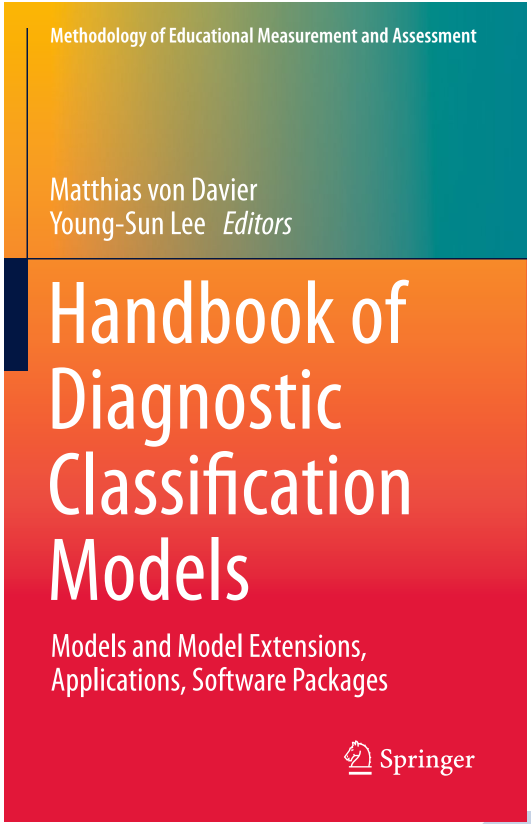
- For example, in educational assessment of cognitive skills, the test developers map the attributes necessary for responding correctly to each question on a test; this map is called the **Q**-matrix.

- A CDM analysis provides an individual attribute profile for each student in addition to the percentage of students who possess the attributes evaluated; these profiles could be useful to teachers for designing classroom materials and developing pedagogy.

- CDMs can provide test takers with specific feedback on their strengths and weaknesses, and hence CDM applications go beyond a simple ranking or locating individuals in relation to an underlying latent trait.

- This model is commonly estimated under a frequentist approach using Maximum Likelihood (ML) estimation methods since that Bayesian estimation considering MCMC methods are usually slow for large data sets.


DIAGNOSTIC
MEASUREMENT
Theory, Methods, and Applications
André A. Rupp
Jonathan Templin
Robert A. Henson


Methodology of Educational Measurement and Assessment
Matthias von Davier
Young-Sun Lee Editors
Handbook of Diagnostic Classification Models
Models and Model Extensions, Applications, Software Packages
Springer

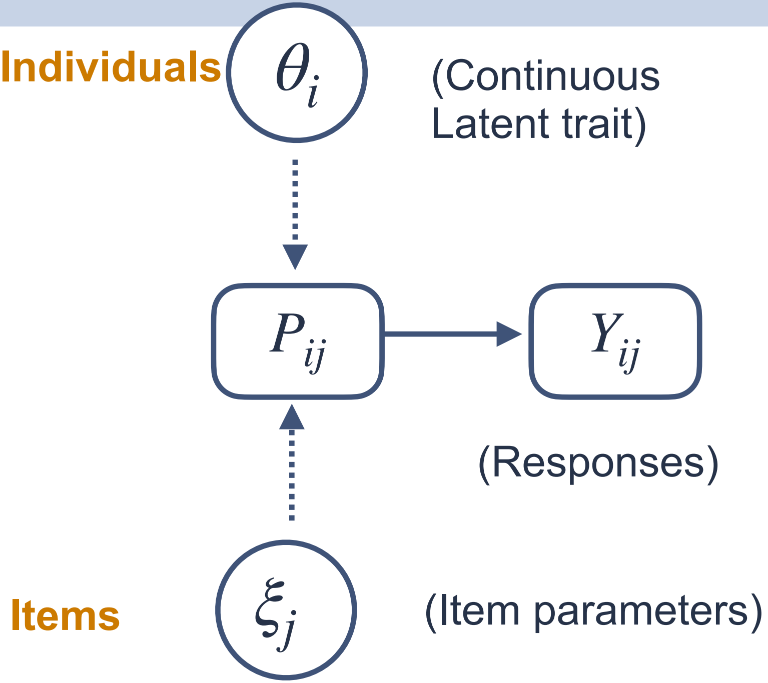
3

MODELING

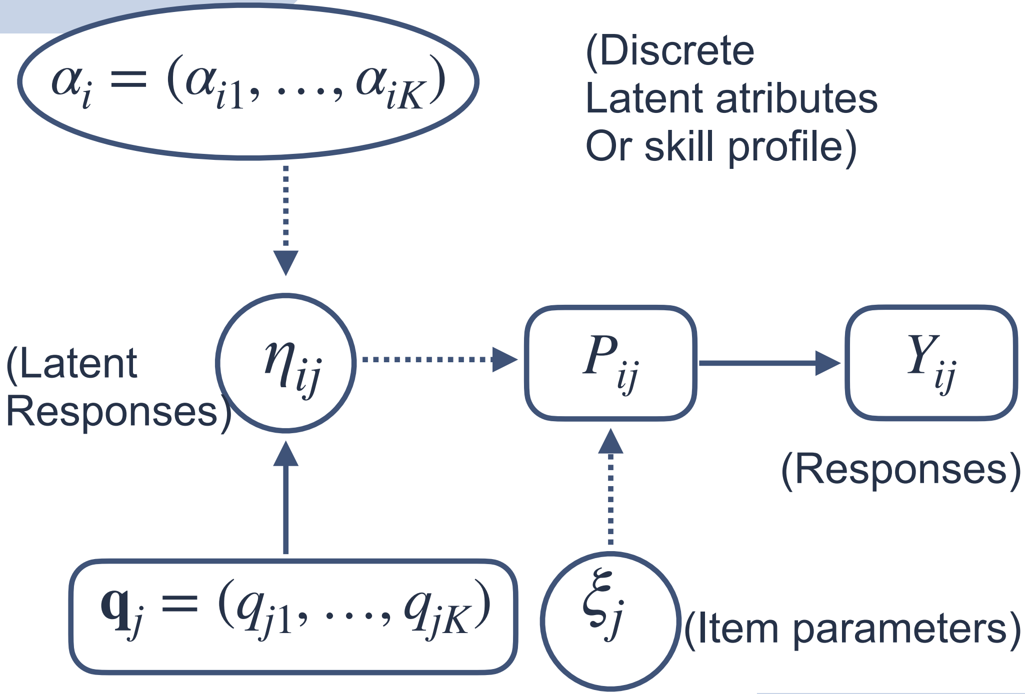
- For BDI data we can use an Item Response Theory (IRT) models used for identify latent trait and item parameters. Concerning to the respondents, in IRT models, the primarily intent is ranking individuals; We want rank the Depression’s individuals?

- Other possibility is use Cognitive Diagnostic Models (CDM) where the intent is classifying individuals as possessing or not a skill or characteristic of the Depression;

3.1. IRT vs CDM



(a) IRT model



(b) CDM model

- In IRT models the performance of the individual is based in a continuous latent trait. Then, individual with higher latent trait have higher probability to answer correctly the item.

- In CDM models the perfomance of the individual is based in discrete latent trait (atributes). Then, individual which has all skills defined in one item have higher probability to answer correctly the item.

- In IRT, the probability of correct response is affected for two kind of latent factors. The first is associated with the individual (Trait latent) and the other is associated wit the item (item parameters).
- In CDM, the probability of correct response is affected for the latent response of the individual for the item and the item parameters. The latent response is affected for two kind of factors. The first is a latent factor associated with the skill of the individual and the other is the specification of skills in the item.

3.2. CDMs

- There are several different approaches to the modeling using CDM. A good initial revision can be seen in George and Robitzsch (2015), but since then more models are being developed each year;

- The non compensatory deterministic input noisy-and gate (DINA; Haertel 1989; Junker and Sijtsma 2001) model.
- The compensatory deterministic input noisy-or-gate (DINO; Junker and Sijtsma 2001) model,
- The generalized version (G-DINA; de la Torre 2011)

- Others “ACDM”, “LLM”, “RRUM”, and “MSDINA”.

3.3. DINA model

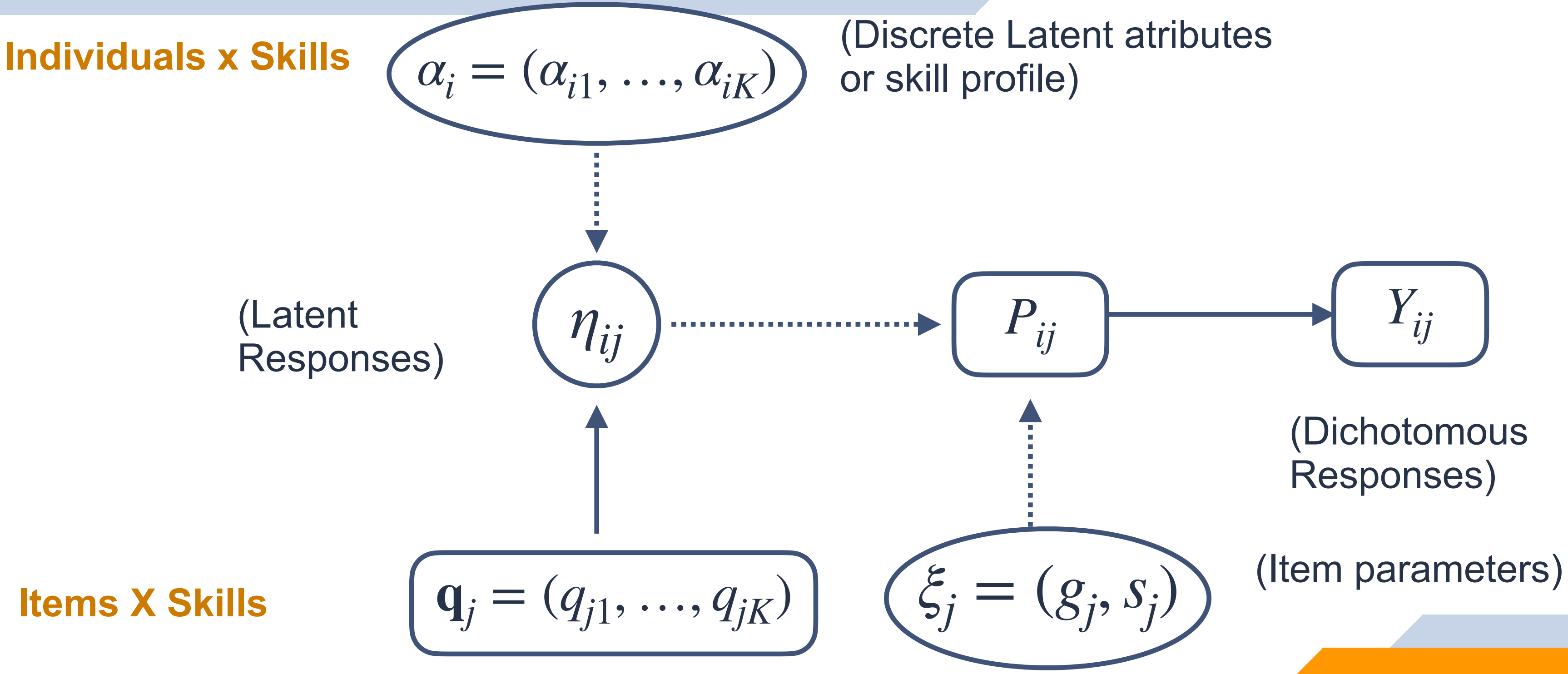
- One of the most popular models in the CDM class is the Deterministic Input Noisy ``and'' gate, due to its good performance and easiness of interpretation.

- To understand the model, it is important to define some quantities for the input. We have:

  $i = 1,\ldots,N$ respondents to a questionnaire;

  $j = 1,\ldots,J$ items to be responded;

  $k = 1,\ldots,K$ skills (or dimensions) to be evaluated.


Individuals x Skills
$\alpha_i = (\alpha_{i1}, \ldots, \alpha_{iK})$
(Discrete Latent atributes or skill profile)
(Latent Responses)
η_{ij}
P_{ij}
Y_{ij}
(Dichotomous Responses)
Items X Skills
$\mathbf{q}_j = (q_{j1}, \ldots, q_{jK})$
$\xi_j = (g_j, s_j)$
(Item parameters)

The skills for the DINA model

- Take by example a Grade Level Assessment Test. End of 6th grade. This test can evaluate different aspects concerning to the knowledge of Math;

- The test to evaluates three different skills that the students will had: 1) Reading, 2) English and 3) Math; It is $K = 3$

- Each item $j = 1, \ldots, J$ of the test can evaluate only one of the atribute (skills) or more than one simultaneously.

- It is considered what each individual will have a *skill profile,* which is the vector containing the possessing of skills of that individual $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{iK})$ which is considered latent.

- By example, if the individual evaluated had the ability in Reading but not in English and Math, his latent skill profile will be $\alpha_i = (0,0,1)$

- (This construct is vey important but it is latent!)

- An important part of adjusting a CDM is defining the **Q-**matrix;
- This matrix contains, in each row, information about which skills are evaluated by which item;
- In the Test example, if an item j evaluates the possessing of the two first skills but no the last (Reading, English but no Math), the row of the that item in the **Q**-matrix will be $\mathbf{q}_j = (1,1,0)$;
- The **Q**-matrix can be defined by a group of experts in the field of the assessment or using automated procedures. However, recently there is contributions for made proposing different algorithms.

Specification of Q-matrix is very important!! Here some works

- Chen, Y., Liu, J., Xu, G., and Ying, Z. (2015). Statistical analysis of q-matrix based diagnostic classification models. *J. Am. Statist. Assoc.* 110, 850–866

- de la Torre, J., and Chiu, C.-Y. (2016). A general method of empirical Q-matrix validation. *Psychometrika* 81, 253–273.

- Liu, R., Huggins-Manley, A. C., and Bradshaw, L. (2016). The impact of q-matrix designs on diagnostic classification accuracy in the presence of attribute hierarchies. *Educ. Psychol. Meas.* 76, 220–240.

- Köhn, HF. & Chiu, CY. (2018) How to Build a Complete Q-Matrix for a Cognitively Diagnostic Test. *Journal of Classification* 35(2): 273-299.

By considering α_i and $\mathbf{q}_j$ above, we can define a latent response variable η_{ij} for the
jth item in the ith individual as

$$\eta_{ij} = \prod_{k=1}^{K} \alpha_{ik}^{q_{jk}} = \mathbb{1}(\boldsymbol{\alpha}_i'\mathbf{q}_j = \mathbf{q}_j'\mathbf{q}_j),$$

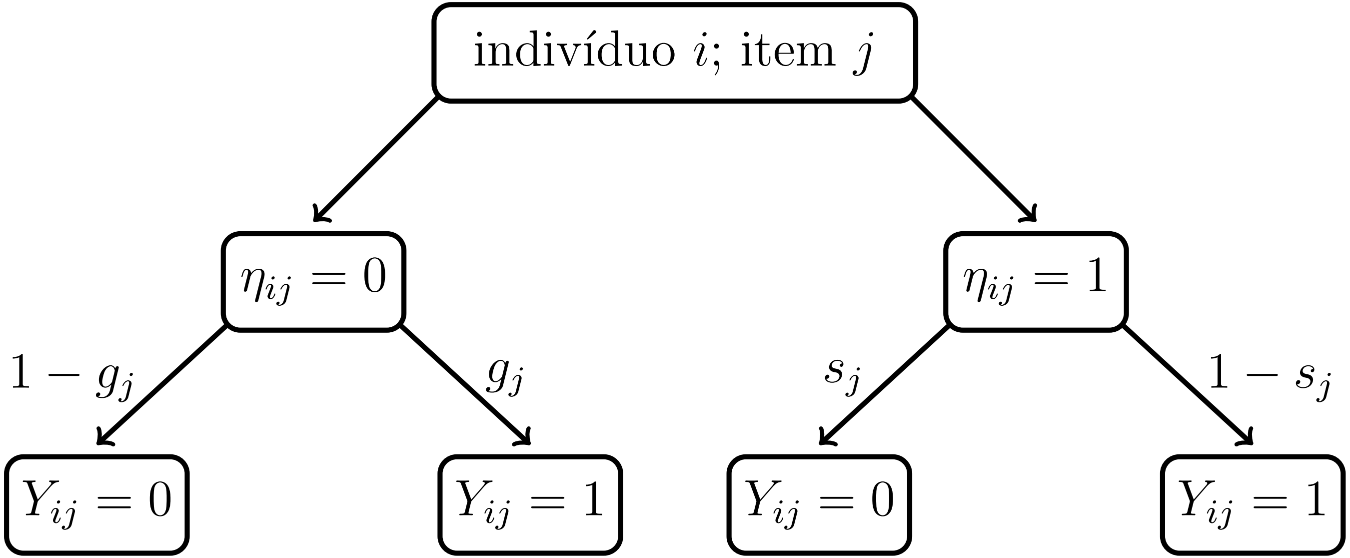
where $\mathbb{1}(\cdot)$ denoting the indicator function. Here, η_{ij} indicates if the ith individual has the skills demanded by the j th item or not.

- In the Test example, consider the individual with the following latent profile $\alpha_i = (0{,}0{,}1)$ (only has Math skills). which answer the item j with the following information $\mathbf{q}_j = (1{,}1{,}0)$ indicating that this item measure the skills of Reading and English. Then

$\eta_{ij} = \alpha_{i1}^{q_{j1}} \times \alpha_{i2}^{q_{j2}} \times \alpha_{i3}^{q_{j3}} = (0)^1 \times (0)^1 \times (1)^0 = 0$ indicate what the **individual i has not the skills required in the item j.**

The student have not the skills of Reading and English measured on the test.

- Another important thing is to define the format of the answers;
- In usual DINA Model the answers need to be dichotomous, that is, correct or incorrect, yes or no, agree or disagree, etc.
- There is also a DINA Model for polytomous answers (Tu et. al., 2017), which is useful for agreement tests, allowing the researcher to evaluate the degree of agreement;
- Recently a DINA Model for continuous responses was proposed (Minchen et. al, 2017), allowing the researcher to use questionaries with this kind of answers or latent traits such as the time to respond to an item;
- **Our study is based in the dichotomous case**



indivíduo i; item j
$\eta_{ij} = 0$
$\eta_{ij} = 1$
$1 - g_j$
g_j
s_j
$1 - s_j$
$Y_{ij} = 0$
$Y_{ij} = 1$
$Y_{ij} = 0$
$Y_{ij} = 1$

For dichotomous answers we will have the following item parameters for the item j :

- The probability of ``guessing'', that is, getting a right answer to an item the individual does not possess the skills to answer correctly

$$g_j = P(Y_{ij} = 1 | \eta_{ij} = 0)$$

- The probability of ``slipping'', that is, answering wrongly an item the individual possess the skills demanded by it;

$$s_j = P(Y_{ij} = 0 | \eta_{ij} = 1)$$

4

ESTIMATION METHODS AND R PACKAGES

- For DINA models is possible use Frequentist and Bayesian approach.
- R packages are available for both estimation methods (CDM, GDINA, dina)
- Additionally is possible use R with interface for other Bayesian software as WinBUGS, JAGS or STAN (R2wingbugs,R2jags,Rstan)

Approah	R package	Method	Referencia	Models	**Home page**
Classical or Frequentist	CDM	EM Algorithm	Robitzsch, Kiefer, George, & Uenlue, (2016)	Several	https://cran.r-project.org/web/packages/CDM/index.html
	GDINA	MMLE/EM algorithm	Ma and de la Torre (2019)	Several	https://cran.r-project.org/web/packages/GDINA/index.html
Bayesian	Dina	Gibbs Sampling	Culpepper (2015) ,Culpepper and Balamuta (2019)	DINA	https://cran.r-project.org/web/packages/dina/index.html
	R2BUGS; R2JAGS (WINBUS, JAGS)	Metropolis Hastin g	Zhan et al (2019)	Several	
	RSTAN (STAN)	NUTS	Silva et al (2018) submitted 2016, Lee (2017)	DINA	https://mc-stan.org/documentation/case-studies/dina_independent.html

- **Frequentist CDM**

Practical Assessment, Research & Evaluation

A peer-reviewed electronic journal.

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Cognitive Diagnostic Modeling Using R

Hamdollah Ravand, *Vali-e-Asr University of Rafsanjan, Iran*
Alexander Robitzsch, *Federal Institute for Education Research, Innovation & Development of the Austrian School*

Journal of Statistical Software

October 2016, Volume 74, Issue 2. doi:10.18637/jss.v074.i02

The R Package CDM for Cognitive Diagnosis Models

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Ali Ünlü, TU München

2015 Vol. 11 no. 3 DOI: 10.20982/tqmp.11.3.p189

Cognitive Diagnosis Models in R: A Didactic

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Journal of Statistical Software
May 2020, Volume 93, Issue 14. doi: 10.18637/jss.v093.i14

GDINA: An R Package for Cognitive Diagnosis Modeling

Wenchao Ma
The University of Alabama

Jimmy de la Torre
The University of Hong Kong

Eurasian Journal of Educational Research 80 (2019) 171-192
Eurasian Journal of Educational Research
www.ejer.com.tr
EJER

Implementation of Cognitive Diagnosis Modeling using the GDINA R Package

Jimmy de la TORRE[1], Lokman AKBAY[2]

MEASUREMENT: INTERDISCIPLINARY RESEARCH AND PERSPECTIVES
2018, VOL. 16, NO. 1, 71–77
https://doi.org/10.1080/15366367.2018.1437243
Routledge Taylor & Francis Group

SOFTWARE REVIEW

Check for updates

GDINA and CDM Packages in R

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- **Bayesian Estimation**

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Journal of Educational and Behavioral Statistics
Vol. XX, No. X, pp. 1–23
DOI: 10.3102/1076998615595403

Bayesian Estimation of the DINA Model With Gibbs Sampling

Tutorial

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Received: 31 October 2016 | Revised: 18 August 2017 | Accepted: 21 August 2017
DOI: 10.1002/bimj.201600225

RESEARCH PAPER

Biometrical Journal

Estimating the DINA model parameters using the No-U-Turn Sampler

Marcelo A. da Silva[1,2] | Eduardo S. B. de Oliveira[1,2] | Alina A. von Davier[3] | Jorge L. Bazán[1]

Journal of Educational and Behavioral Statistics
Vol. XX, No. X, pp. 1–31
DOI: 10.3102/1076998619826040
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Using JAGS for Bayesian Cognitive Diagnosis Modeling: A Tutorial

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- If you want to apply the methodology of CDM the best recommendation is use the frequentist approach, CDM and GDINA are recommendable packages and many models could be fitted using them quickly.

- If you have more interest in methodological research and then propose new models or explore variants of the previous models a good recommendation is use bayesian approach, specially using JAGS or STAN where both could be implemented in R and Python,

- There is some important advantages when used a Bayesian approach and when a intermediary program is used as JAGS (BUGS) or STAN:

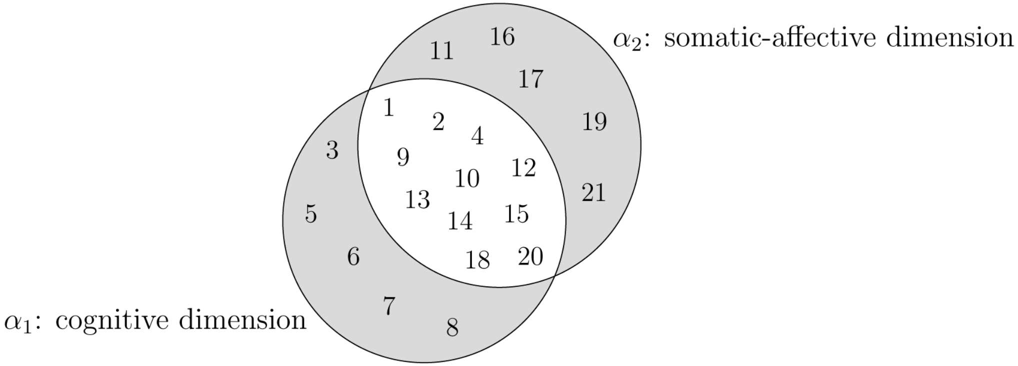
A. Distribution of the parameters of the model and not only a pontual estimation and standard deviation assuming Asymptotic normality, It is specially relevant since that parameters in the model are in the (0,1) interval
B. Possibility of implement easily new models,
C. Restrictions in the model are substituted by priors and priors can include historic information and then the model is identified.

- **How fit a bayesian approach computationally (reasonably) for CDM and which is the perfomance of this estimation in comparison with frequents approach?**

5

RESULTS FOR BDI DATA

- In order to adjust the DINA model for BDI data, we used a dichotomization of the answers, as proposed first by Fragoso and Curi (2013);
- The Q-matrix was constructed based on $K = 2$ skills, which we call dimensions in this work, for interpretation facility;
- These dimensions are based in IRT and are the cognitive (α_1) and somatic-affective (α_2) dimensions.
-



Estimation of item parameters

Item	Q (dimensions)		$\widehat{g}$		$\widehat{s}$	
	α_1	α_2	Mean	sd	Mean	sd
1. Sadness	1	1	0.468	0.020	0.101	0.017
2. Pessimism	1	1	0.189	0.017	0.334	0.026
3. Sense of failure	1	0	0.045	0.012	0.444	0.024
4. Lack of satisfaction	1	1	0.309	0.020	0.163	0.020
5. Guilty feelings	1	0	0.039	0.011	0.423	0.024
6. Sense of punishment	1	0	0.115	0.017	0.453	0.023
7. Self-dislike	1	0	0.242	0.022	0.158	0.019
8. Self-accusation	1	0	0.422	0.023	0.163	0.017
9. Suicidal wishes	1	1	0.032	0.008	0.694	0.023
10. Crying spells	1	1	0.142	0.014	0.502	0.026
11. Irritability	0	1	0.283	0.024	0.279	0.023
12. Social withdrawal	1	1	0.210	0.017	0.394	0.025
13. Indecisiveness	1	1	0.205	0.016	0.320	0.025
14. Distortion of body image	1	1	0.222	0.017	0.458	0.025
15. Work inhibition	1	1	0.259	0.018	0.206	0.023
16. Sleep disturbance	0	1	0.262	0.028	0.288	0.022
17. Fatigability	0	1	0.348	0.030	0.162	0.019
18. Loss of appetite	1	1	0.178	0.016	0.560	0.026
19. Weight loss	0	1	0.062	0.012	0.851	0.016
20. Somatic preoccupation	1	1	0.223	0.016	0.518	0.026
21. Loss of libido	0	1	0.109	0.017	0.645	0.022

sd: standard deviation.

Profile estimate and comparison with usual classification

c		Dimensions		$\widehat{\pi}$	
		α_1	α_2	Mean	sd
1	(non-depressive)	0	0	0.363	0.024
2	(symptomatic of cognitive dimension)	1	0	0.124	0.016
3	(symptomatic of somatic-affective dimension)	0	1	0.124	0.021
4	(both symptoms)	1	1	0.389	0.019

sd: standard deviation.

Diagnosis proposed by DINA	Groups according to usual classification		
	Depressed	Dysphoric	Non Depressed
Non-depressive	0(0%)	0(0%)	442(51.64%)
Symptomatic to cognitive	0(0%)	5(4.39%)	116(13.55%)
Symptomatic to somatic-affective	0(0%)	0(0%)	106(12.38%)
Both symptoms	141(100%)	109(95.61%)	192(22.43%)

- The DINA model approach in this application, consider two skills which characterize the Depression: cognitive and somatic-affective dimensions
- This dimensions were obtained using previous literature (Fragoso and Curi, 2013) considering IRT approach which was used to define a Q matrix.
- The results obtained using DINA model permit classify the examinees in four groups defining the probability of each examinee is in each group.
- The results obtained can be interpreted similarly to traditional classification using BDI scores but had some interesting different results which is useful in classifying individuals as part of diagnostic of depression.

- However, it is notable that using this approach may overestimate depression, mainly because the dichotomization used causes all positive responses to an item to have the same weight in final diagnostics.

- Our example with BDI items is not a direct proposal to clinical use, but has the intention of showing the kind of data DINA model fits and to motivate further studies with the possibilities brought by this methodology.

- Similar exemplos can be use in Education identifying the skills that the students can do offering a best interpretation of the results of Assessment.

6

COMMENTS

- With the already existent models and the one to be proposed, it is possible to evaluate many kinds of questionnaires;
- The outputs are interesting both for evaluating the items and the respondents;
- To run applications using CDM to an assessment it is important to define skills (or dimensions) evaluated by each item of a test and use Q matrix well defined;
- Possible applications can be done in many study fields such as education, psychology, sociology and others.

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Thank you for your attention!

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Is possible use Q-matrix in Item Response Theory models?

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