

Eletrromagnetismo Avançado

18 de setembro
Ondas Eletromagnéticas

Meios lineares

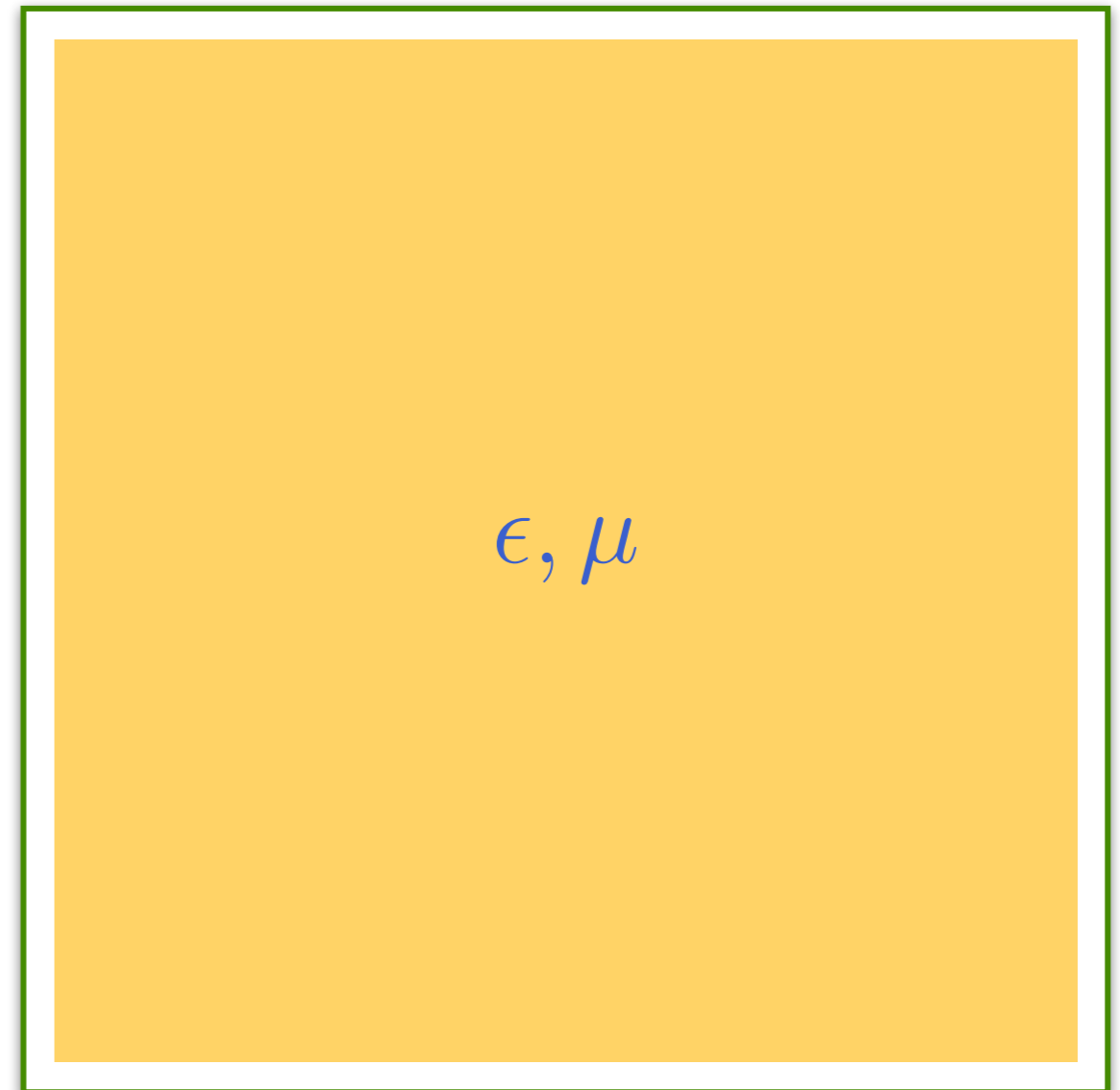
$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\partial_t \vec{\mathbf{B}}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{B}} = \mu(\vec{\mathbf{J}} + \epsilon \partial_t \vec{\mathbf{E}})$$

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{v^2} \partial_t^2 \vec{\mathbf{E}} \quad (v \equiv \frac{c}{n})$$



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{E}}_0 \exp\left(i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\right)$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{B}}_0 \exp\left(i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\right)$$

$$\omega = \frac{kc}{n}$$

$$\hat{\mathbf{k}} \times \vec{\mathbf{E}}_0 = \frac{c}{n} \vec{\mathbf{B}}_0$$

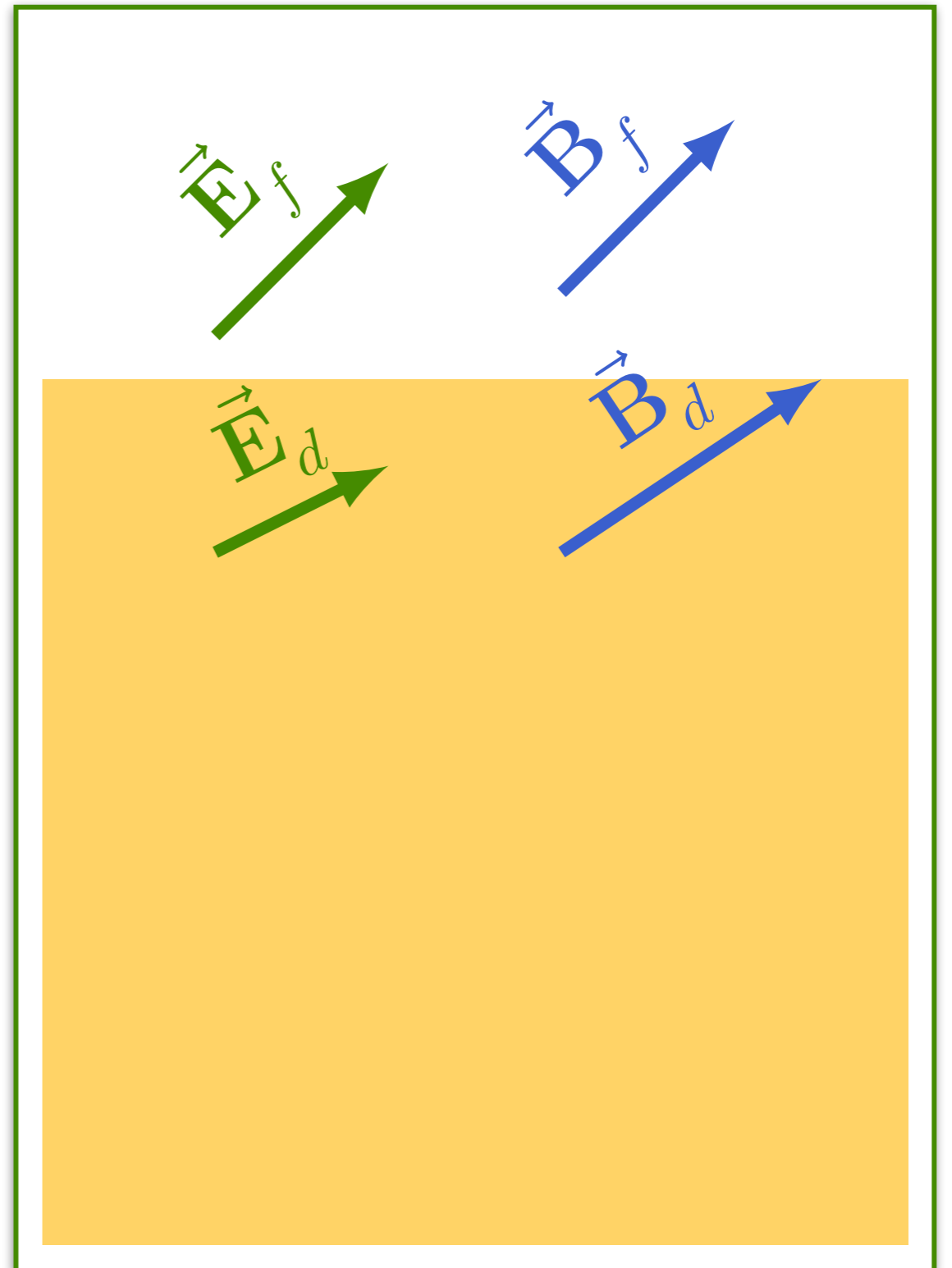
Meios lineares - fronteiras

$$E_{d\parallel} = E_{f\parallel}$$

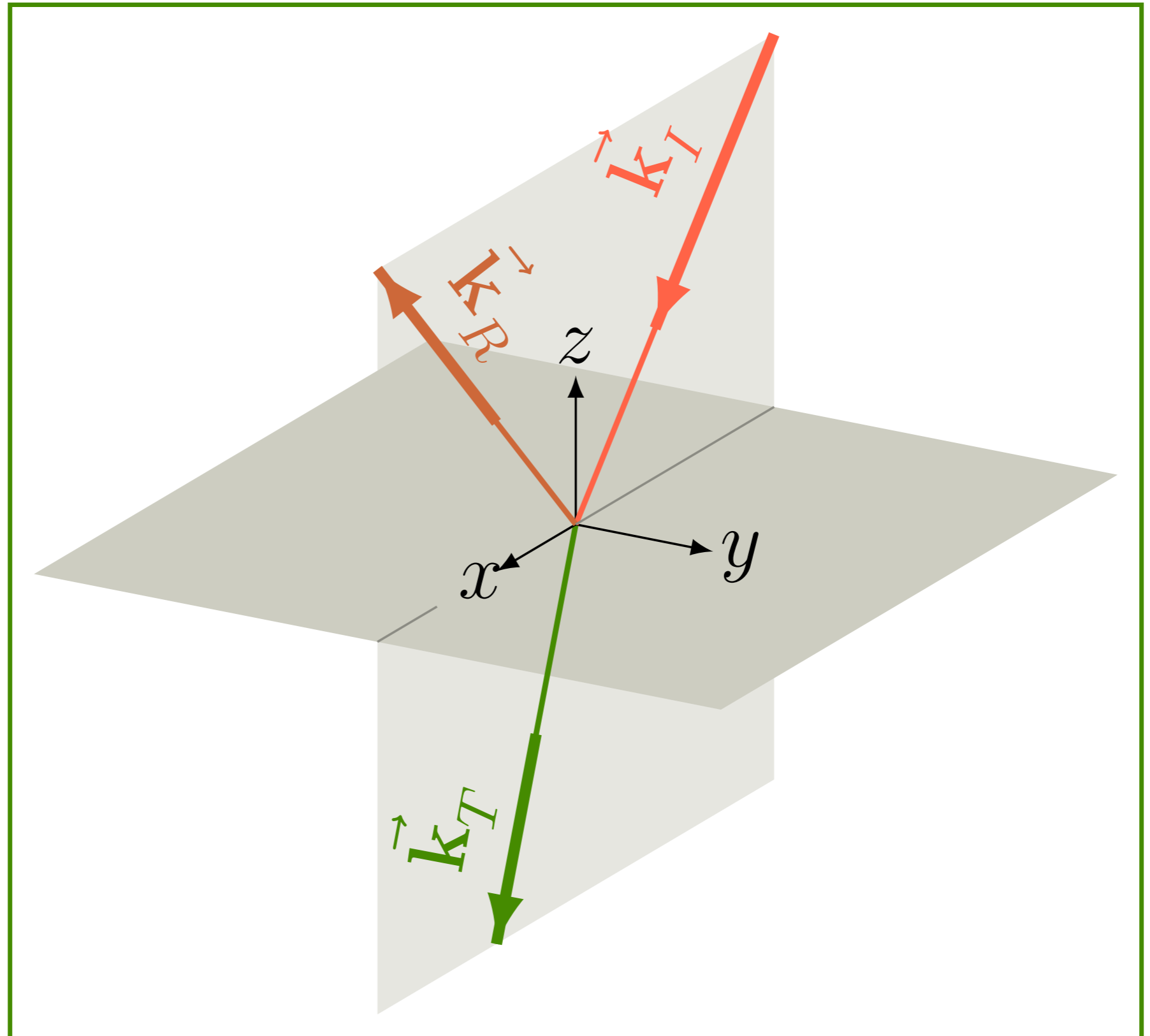
$$\epsilon E_{d\perp} = \epsilon_0 E_{f\perp}$$

$$\frac{1}{\mu} B_{d\parallel} = \frac{1}{\mu_0} B_{f\parallel}$$

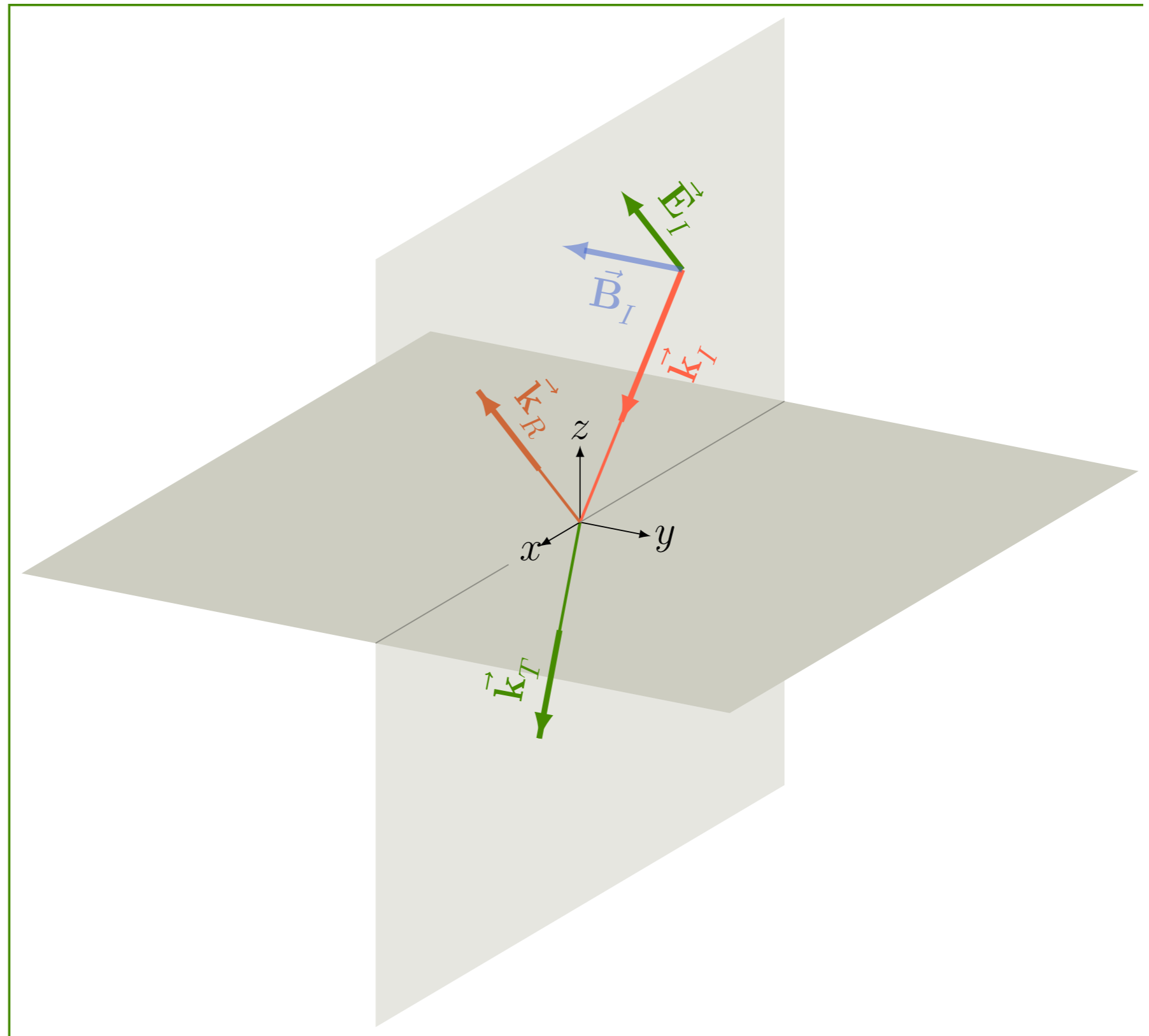
$$B_{d\perp} = B_{f\perp}$$



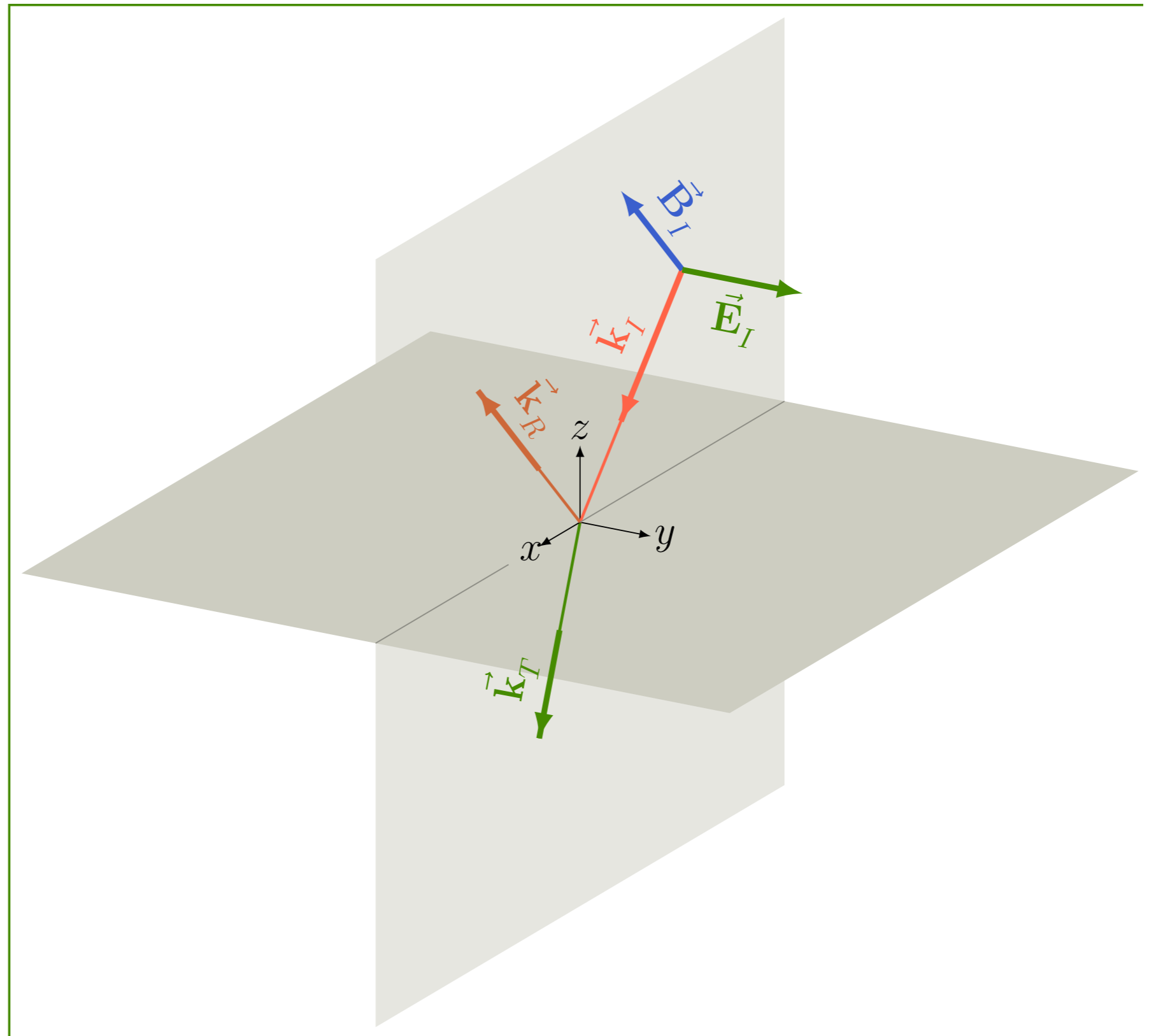
Equações de Fresnel



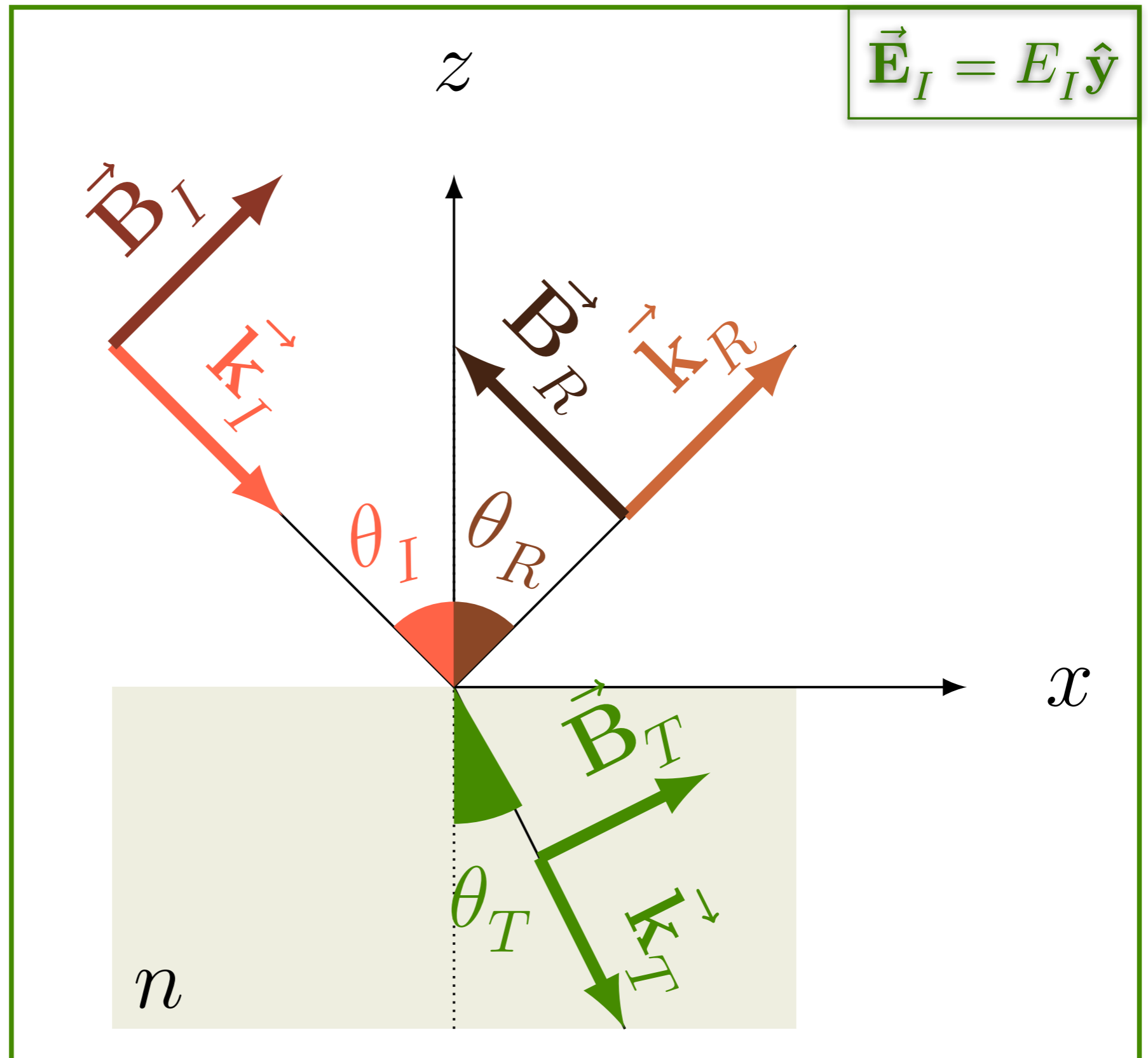
Equações de Fresnel



Equações de Fresnel



Equações de Fresnel



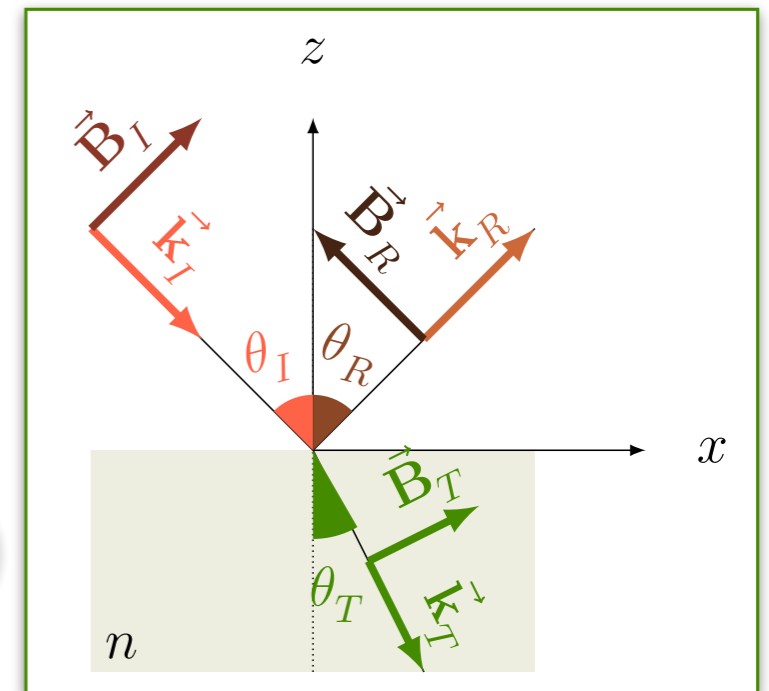
Condições de contorno

$$\vec{\mathbf{E}}_I(0, x, y, t) + \vec{\mathbf{E}}_R(0, x, y, t) = \vec{\mathbf{E}}_T(0, x, y, t)$$

$$B_{Iz}(0, x, y, t) + B_{Rz}(0, x, y, t) = B_{Tz}(0, x, y, t)$$

$$B_{Ix}(0, x, y, t) + B_{Rx}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Tx}(0, x, y, t)$$

$$B_{Iy}(0, x, y, t) + B_{Ry}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Ty}(0, x, y, t)$$



$$\vec{\mathbf{E}}_I = E_I \hat{\mathbf{y}}$$

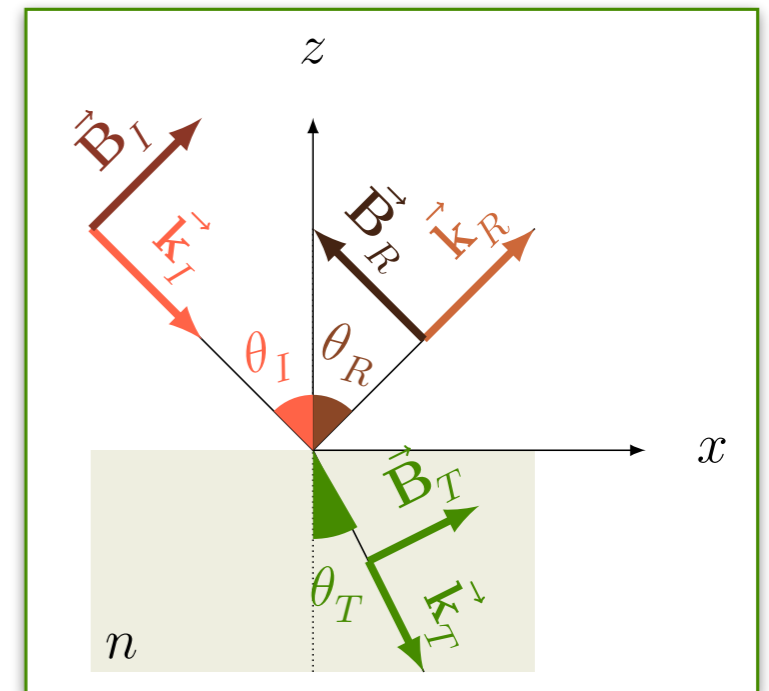
Condições de contorno

$$E_{Iy}(0, x, y, t) + E_{Ry}(0, x, y, t) = E_{Ty}(0, x, y, t)$$

$$B_{Iz}(0, x, y, t) + B_{Rz}(0, x, y, t) = B_{Tz}(0, x, y, t)$$

$$B_{Ix}(0, x, y, t) + B_{Rx}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Tx}(0, x, y, t)$$

$$\cancel{B_{Iy}(0, x, y, t)} + B_{Ry}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Ty}(0, x, y, t)$$



$$\vec{E}_I = E_I \hat{y}$$

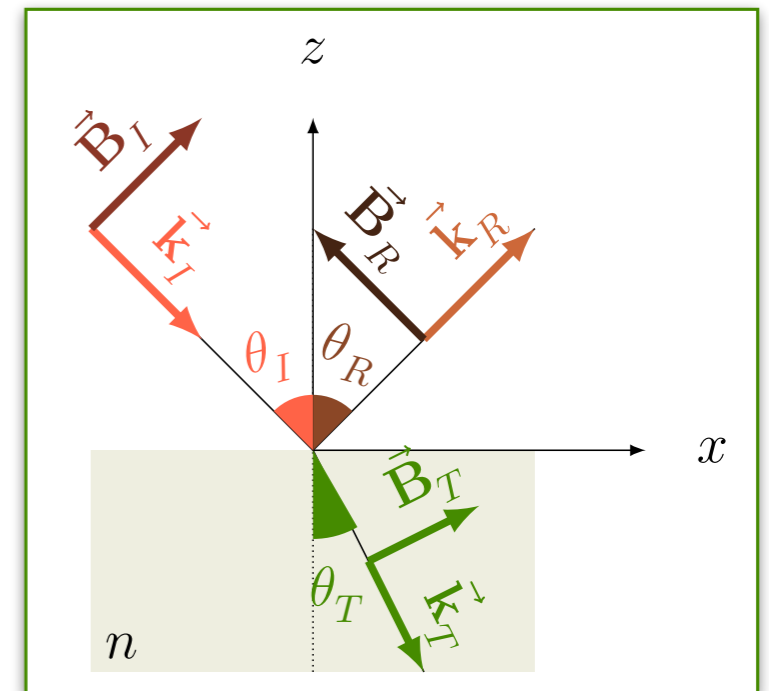
Condições de contorno

$$E_{Iy}(0, x, y, t) + E_{Ry}(0, x, y, t) = E_{Ty}(0, x, y, t)$$

$$B_{Iz}(0, x, y, t) + B_{Rz}(0, x, y, t) = B_{Tz}(0, x, y, t)$$

$$B_{Ix}(0, x, y, t) + B_{Rx}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Tx}(0, x, y, t)$$

~~$$B_{Iy}(0, x, y, t) + B_{Ry}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Ty}(0, x, y, t)$$~~

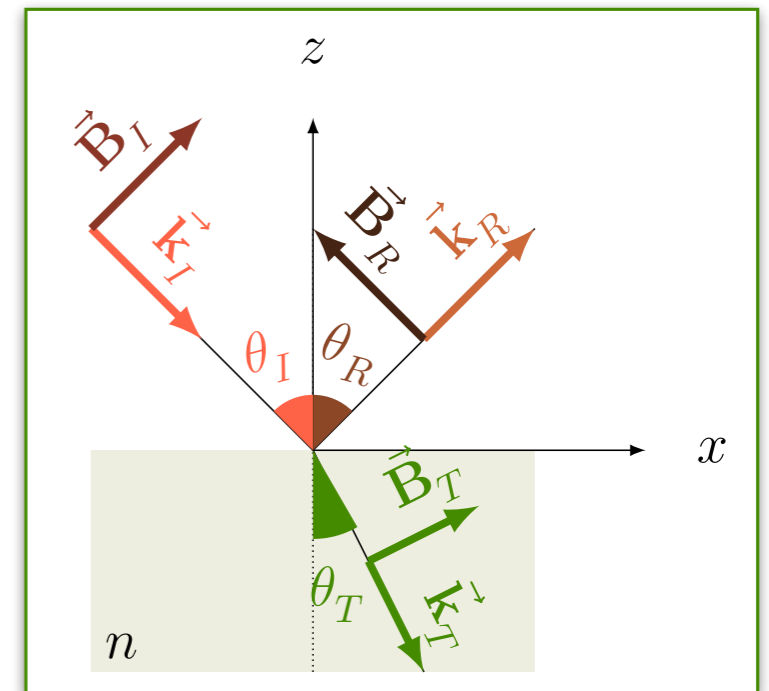


$$\vec{E}_I = E_I \hat{y}$$

Condições de contorno

$$\Rightarrow \vec{k}_{Iy} = \vec{k}_{Ry} = \vec{k}_{Ty} = 0$$

$$\Rightarrow \vec{k}_{Ix} = \vec{k}_{Rx} = \vec{k}_{Tx}$$



$$\theta_R = \theta_I \equiv \theta$$

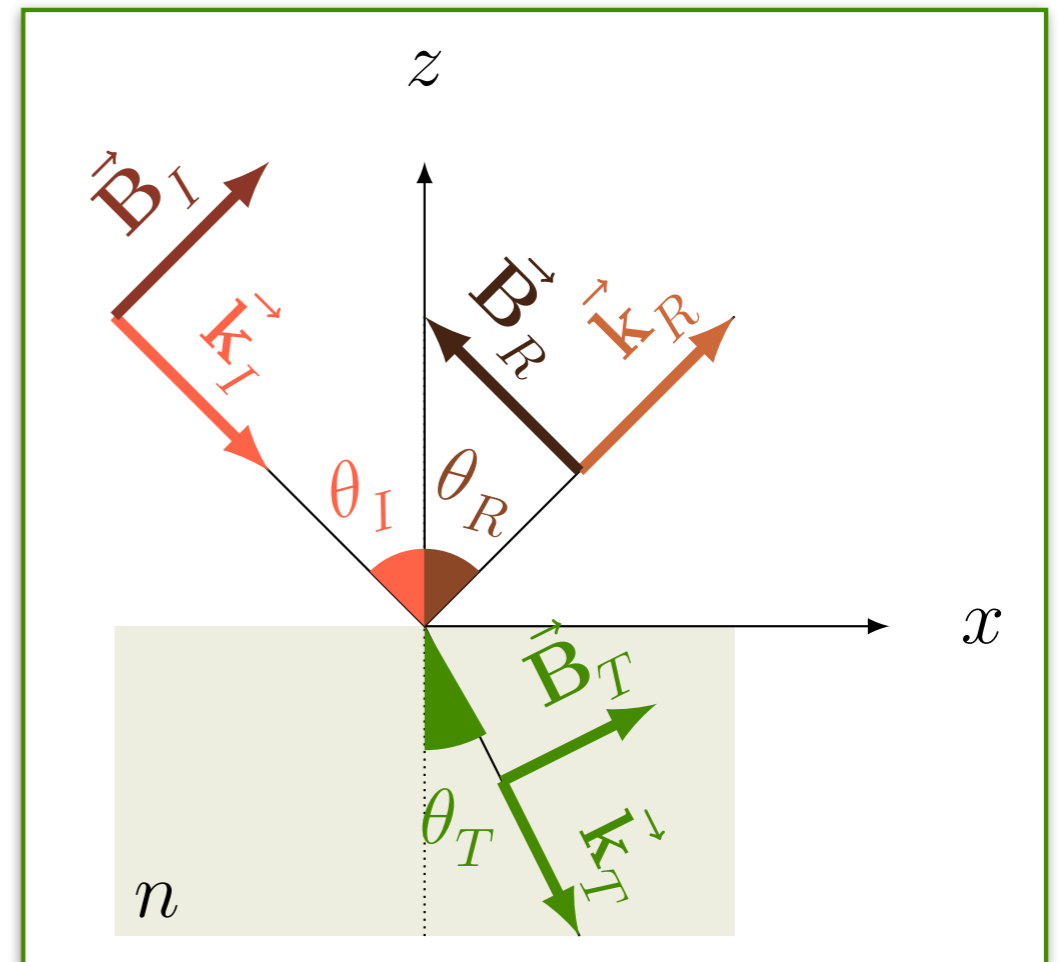
Condições de contorno

$$\Rightarrow \vec{k}_{Iy} = \vec{k}_{Ry} = \vec{k}_{Ty} = 0$$

$$\Rightarrow \vec{k}_{Ix} = \vec{k}_{Rx} = \vec{k}_{Tx}$$

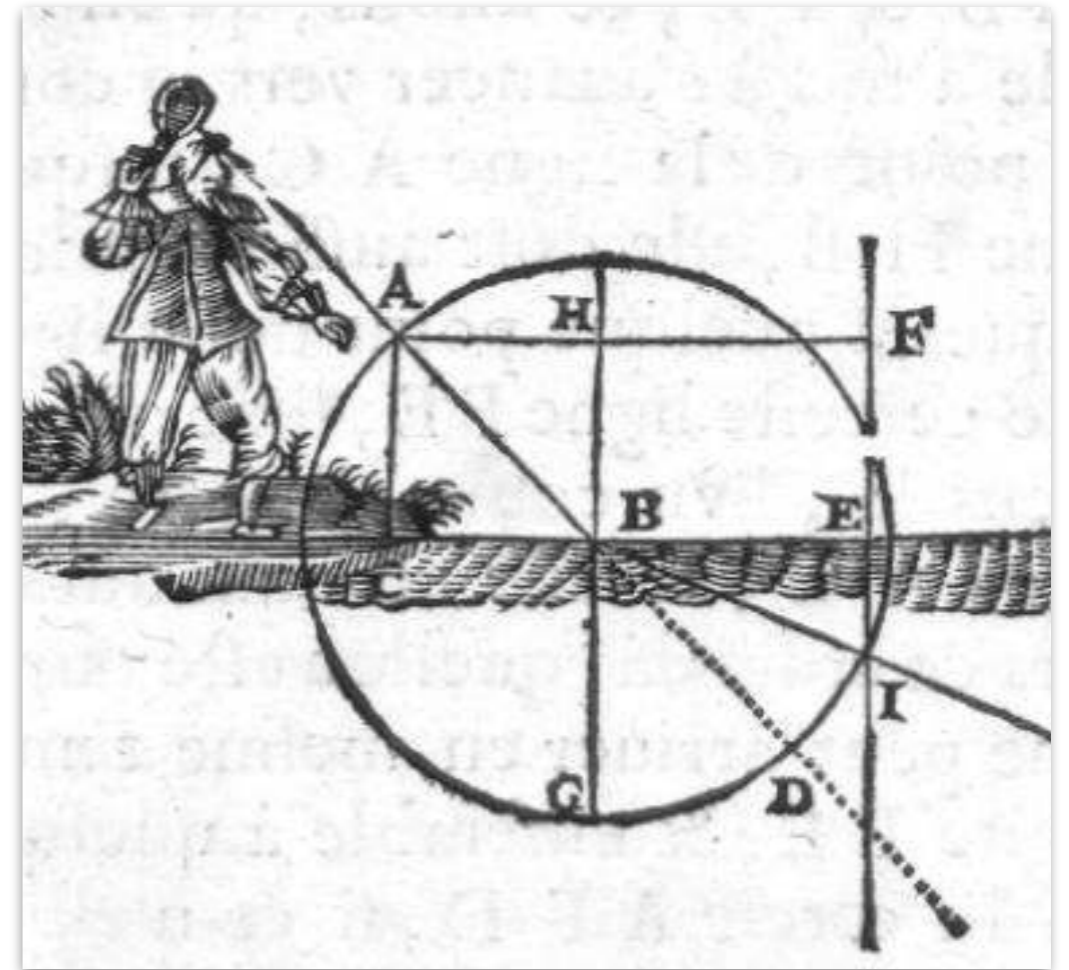
$$\theta_R = \theta_I \equiv \theta$$

$$n \sin \theta_T = \sin \theta$$



Evolução histórica

- Euclides
- Ptolomeu
- Ibn Sahl
- Al Haythem
- Snell
- Descartes
- Huygens



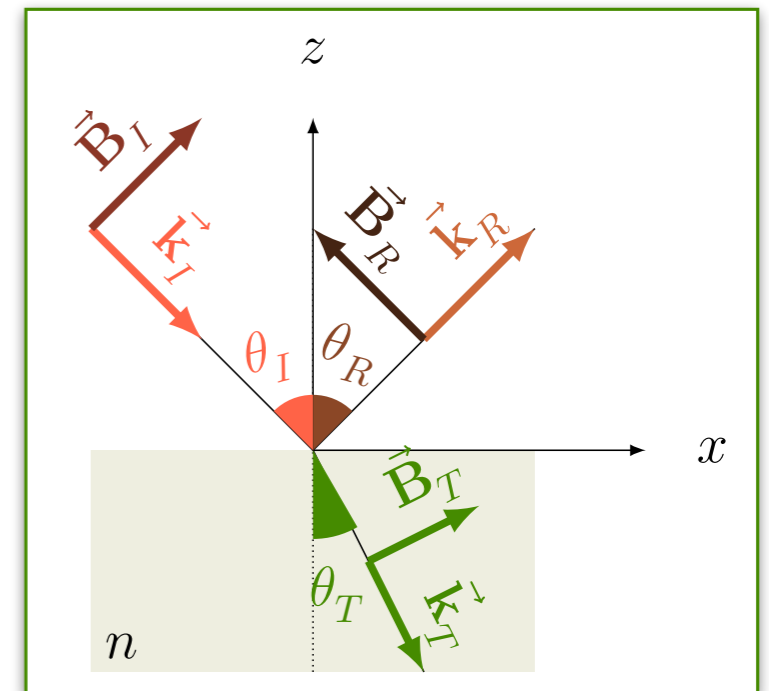
Condições de contorno

$$E_{Iy}(0, x, y, t) + E_{Ry}(0, x, y, t) = E_{Ty}(0, x, y, t)$$

$$B_{Iz}(0, x, y, t) + B_{Rz}(0, x, y, t) = B_{Tz}(0, x, y, t)$$

$$B_{Ix}(0, x, y, t) + B_{Rx}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Tx}(0, x, y, t)$$

~~$$B_{Iy}(0, x, y, t) + B_{Ry}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Ty}(0, x, y, t)$$~~



$$\vec{E}_I = E_I \hat{y}$$

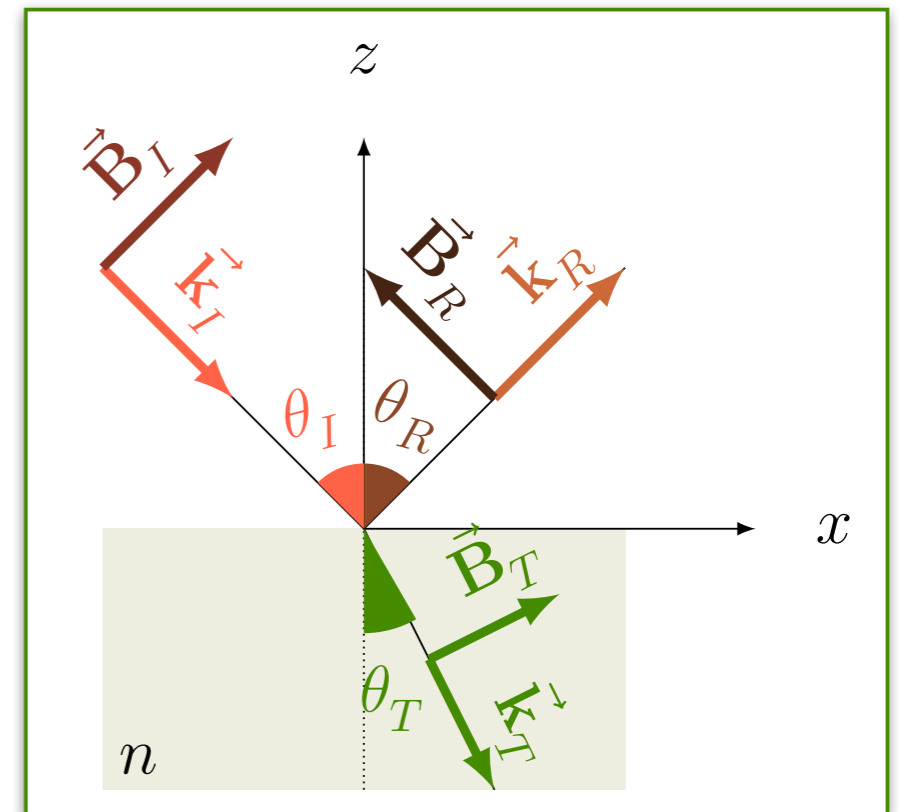
Condições de contorno

$$B_{Iz}(0, x, y, t) + B_{Rz}(0, x, y, t) = B_{Tz}(0, x, y, t)$$

$$B_{Ix}(0, x, y, t) + B_{Rx}(0, x, y, t) = \frac{\mu_0}{\mu} B_{Tx}(0, x, y, t)$$

$$B_{Iz} + B_{Rz} = B_{Tz}$$

$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$



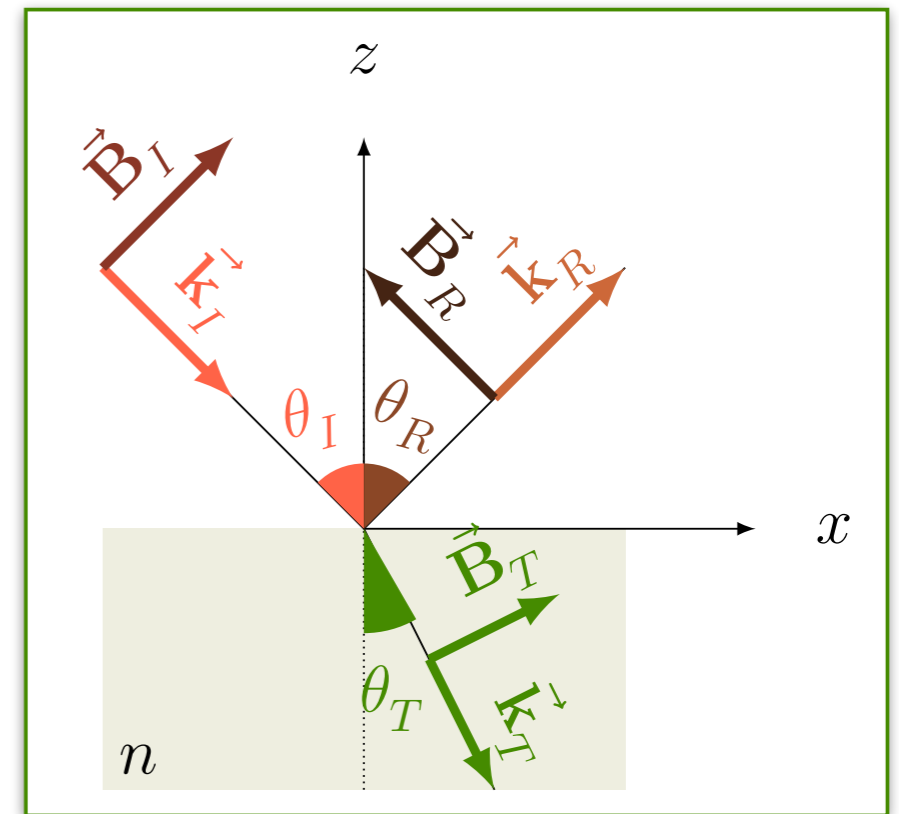
Condições de contorno

$$B_{Iz} + B_{Rz} = B_{Tz}$$

$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$

$$(B_I + B_R) \sin \theta = B_T \sin \theta_T$$

$$(B_I - B_R) \cos \theta = \frac{\mu_0}{\mu} B_T \cos \theta_T$$



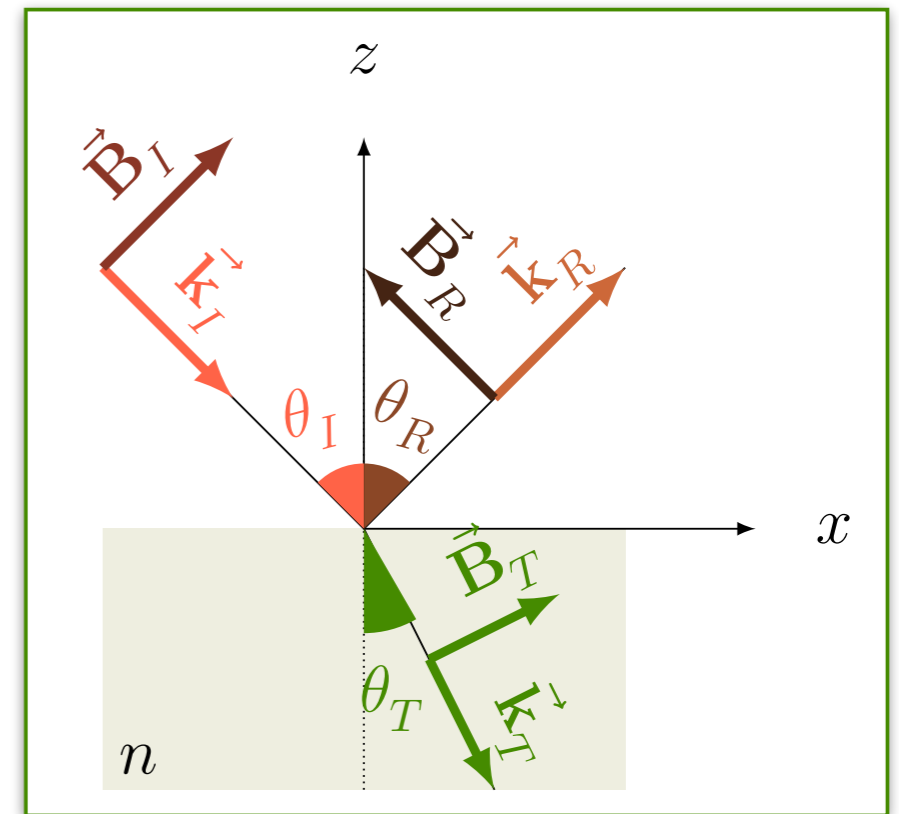
Condições de contorno

$$B_{Iz} + B_{Rz} = B_{Tz}$$

$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$

$$(B_I + B_R) \sin \theta = B_T \sin \theta_T$$

$$(B_I - B_R) \cos \theta = \frac{\mu_0}{\mu} B_T \cos \theta_T$$



$$n \sin \theta_T = \sin \theta$$

Condições de contorno

$$B_{Iz} + B_{Rz} = B_{Tz}$$

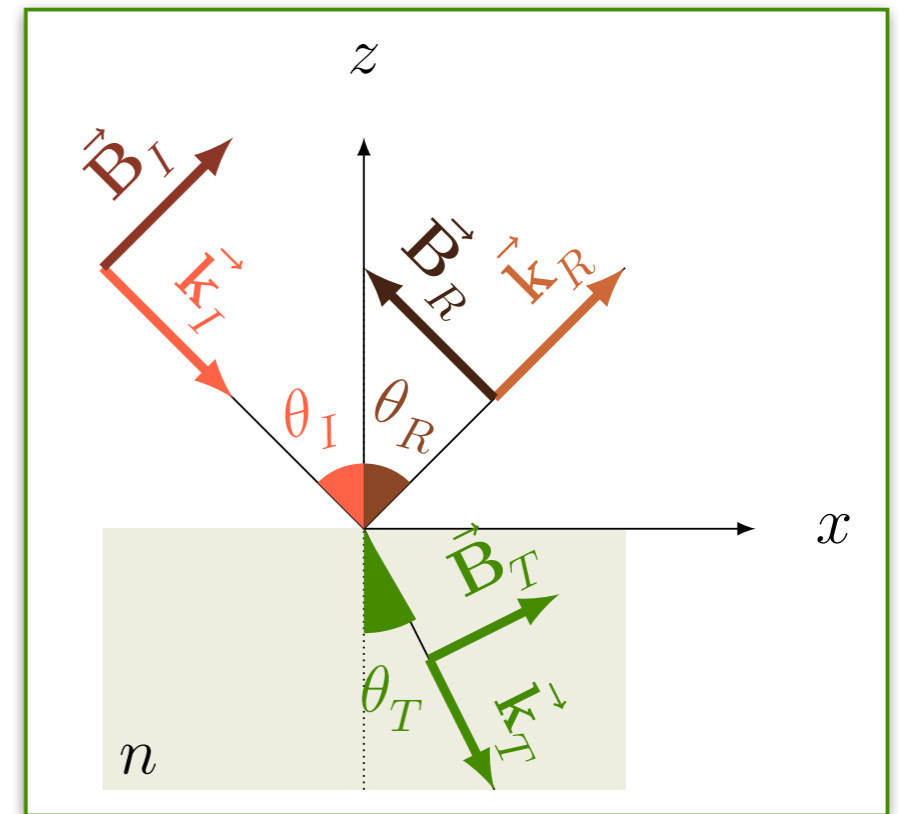
$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$

$$(B_I + B_R) \sin \theta = B_T \sin \theta_T$$

$$(B_I - B_R) \cos \theta = \frac{\mu_0}{\mu} B_T \cos \theta_T$$

$$n(B_I + B_R) = B_T$$

$$B_I - B_R = \underbrace{\frac{\mu_0 \cos \theta_T}{\mu \cos \theta}}_{\gamma} B_T$$



$$n \sin \theta_T = \sin \theta$$

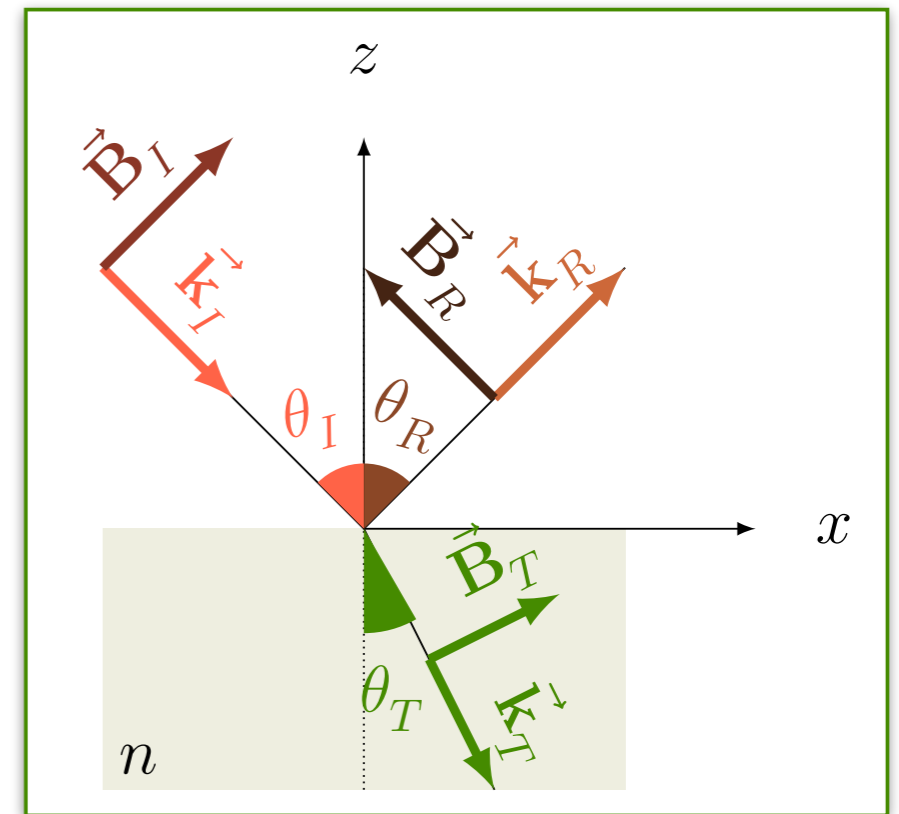
Condições de contorno

$$B_{Iz} + B_{Rz} = B_{Tz}$$

$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$

$$B_R = \frac{1 - n\gamma}{1 + n\gamma} B_I$$

$$B_T = \frac{2n}{1 + n\gamma} B_I$$



Condições de contorno

$$B_{Iz} + B_{Rz} = B_{Tz}$$

$$B_{Ix} + B_{Rx} = \frac{\mu_0}{\mu} B_{Tx}$$

$$B_R = \frac{1 - n\gamma}{1 + n\gamma} B_I$$

$$B_T = \frac{2n}{1 + n\gamma} B_I$$

