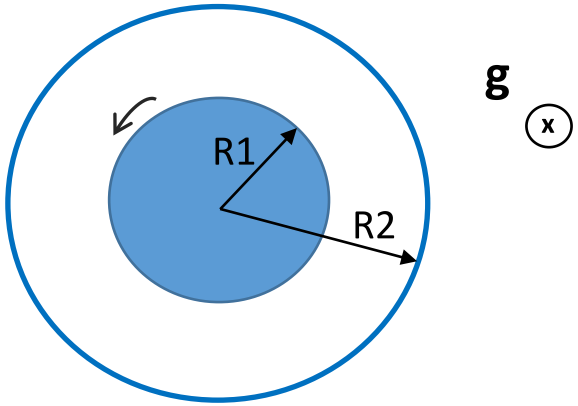
PQI 3203: FENÔMENOS DE TRANSPORTE I

Tensão em fluidos – Navier Stokes

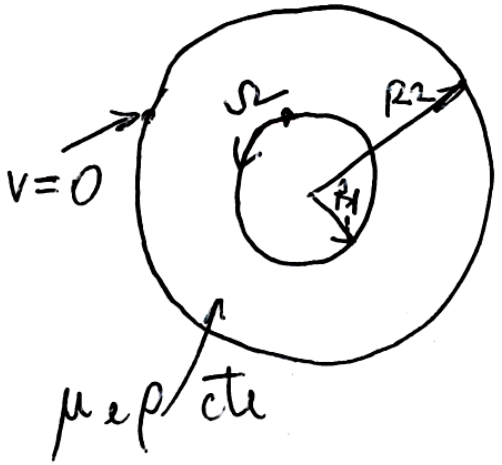
Exercício - 4

Exercício

Um líquido newtoniano incompressível (densidade ρ, viscosidade dinâmica μ) está contido na região anular entre um longo recipiente cilíndrico (raio R2) e um longo bastão cilíndrico (raio R1). O bastão e o recipiente são coaxiais, sendo o bastão mantido com velocidade angular constante e o recipiente fixo. Pode-se desprezar os efeitos de extremidade. A partir dos balanços diferenciais pertinentes, deduzir as expressões: (a) do perfil de velocidades do escoamento. (b) da tensão de cisalhamento no líquido (c) da força tangencial do líquido sobre as paredes (d) do torque no bastão. Justificar sucintamente todas as passagens da solução.



VISCOSÍMETRO COUETTE



$\otimes \ \vec{g}$

Hipóteses:

- regime permanente
- incompressível
- bidimensional (r,θ)
- $\vec{g}$ em $\vec{e}_z$

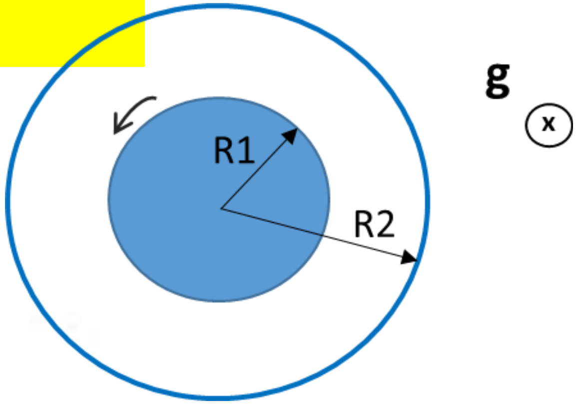
Continuidade e incompressível:

$$\frac{1}{r}\frac{\partial}{\partial r}(\cancel{v_r}\cdot r) + \frac{1}{r}\cancel{\frac{\partial}{\partial \theta}} v_\theta + \frac{\partial v_z}{\partial z} = 0$$

desenvolvido   simetria

NAVIER – STOKES : Coord. Cilíndricas

Escoamento incompressível, newtoniano e viscosidade constante



$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}}\boxed{-\frac{v_\theta^2}{r}}+\cancel{v_z\frac{\partial v_r}{\partial z}}\right)$$

$$=\boxed{-\frac{\partial p}{\partial r}}+\cancel{\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]}+\rho g_r,$$

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}}+\cancel{v_r}\frac{\partial v_\theta}{\partial r}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}+\frac{\cancel{v_r}v_\theta}{r}+\cancel{v_z\frac{\partial v_\theta}{\partial z}}\right)$$

$$=\cancel{-\frac{1}{r}\frac{\partial p}{\partial \theta}}+\boxed{\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)\right.}\cancel{+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}+\left.\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right]+\rho\cancel{g_\theta},$$

$$\cancel{\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)}$$

$$\cancel{=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z.}$$

Quantidade de Movimento: Naviet-Stokes

direção r $\boxed{-\rho \frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r}}$

direção θ: $\boxed{\mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) = 0}$

$$\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r} = C_1 \Rightarrow \frac{\partial (r v_\theta)}{\partial r} = C_1 . r$$

$$\frac{\partial (r v_\theta)}{\partial r} = C_1 r \Longrightarrow r v_\theta = \frac{C_1 r^2}{2} + C_2$$

$$\boxed{v_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}}$$

$v_\theta(r)$

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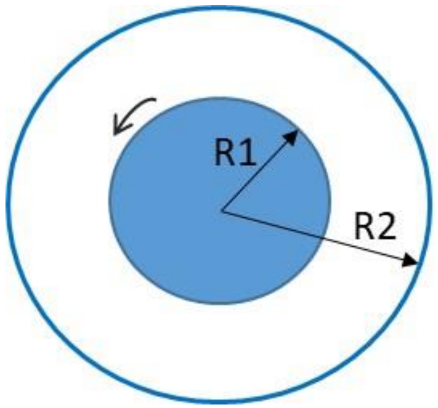


Condições de contorno:

$$\begin{cases} r = R_1 \rightarrow v_\theta = \Omega R_1 \\ r = R_2 \rightarrow v_\theta = 0 \end{cases}$$

$$r = R_2 \Rightarrow v_\theta = 0 = \frac{C_1 R_2}{2} + \frac{C_2}{R_2} \Rightarrow C_2 = -\frac{C_1 R_2^2}{2}$$

$$r = R_1 \Rightarrow \Omega R_1 = \frac{C_1 R_1}{2} + \frac{C_2}{R_1} = \frac{C_1 R_1}{2} - \frac{C_1 R_2^2}{2R_1}$$

$$\boxed{C_1 = \frac{2\Omega R_1^2}{R_1^2 - R_2^2}} \quad e \quad \boxed{C_2 = \frac{-\Omega R_1^2 R_2^2}{R_1^2 - R_2^2}}$$

$$\boxed{v_\theta = \frac{\Omega R_1^2}{(R_1^2 - R_2^2)}\left[r - \frac{R_2^2}{r}\right]}$$

//

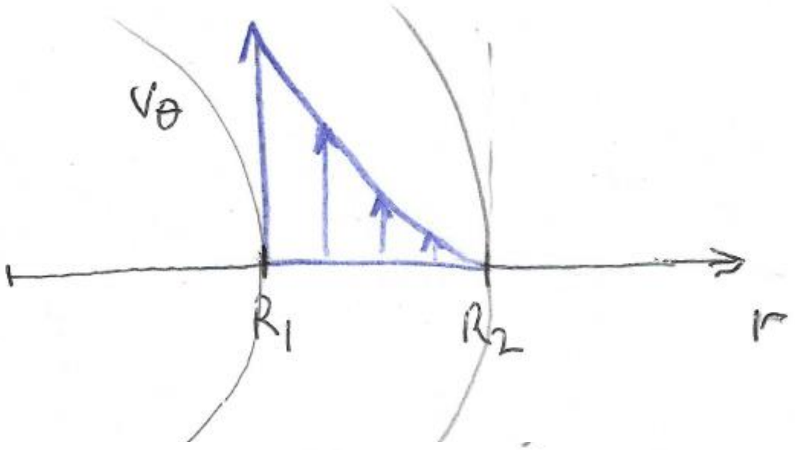
$$\boxed{v_\theta = \frac{\Omega R_1^2}{(R_1^2 - R_2^2)}\left[r - \frac{R_2^2}{r}\right]}$$

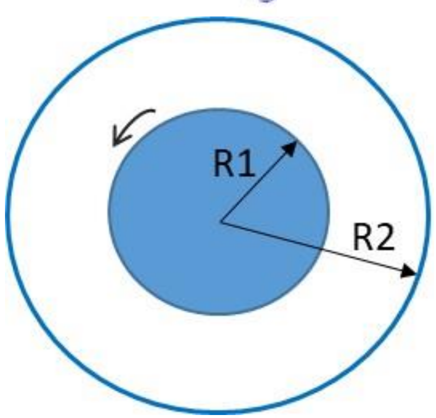
$$\mathrm{rot}\,\vec{v} = \frac{1}{r}\left[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right]\vec{e_3} =$$

$$= \frac{1}{r}\left\{\frac{\partial}{\partial r}\left(\frac{\Omega R_1^2}{R_1^2 - R_2^2}\left[r^2 - R_2^2\right]\right)\right\} =$$

$$= \frac{1}{r}\left[\frac{\Omega R_1^2}{(R_1^2 - R_2^2)} \cdot 2r\right] = \boxed{\frac{\Omega R_1^2}{(R_1^2 - R_2^2)} = \text{vorticidade cte!}}$$

$$\boxed{v_\theta = \frac{\Omega R_1^2}{(R_1^2 - R_2^2)}\left[r - \frac{R_2^2}{r}\right]}$$





$$\tau_{r\theta} = \tau_{\theta r} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$$

$$\tau_{z\theta} = \tau_{\theta z} = \mu\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)$$

$$\tau_{rz} = \tau_{zr} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \cancel{\frac{1}{r}\frac{\partial v_r}{\partial \theta}}\right] \quad 0$$

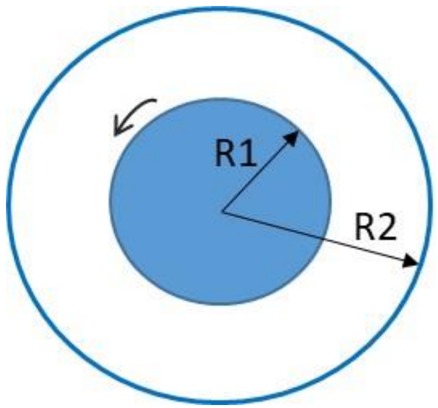
$$\tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{\Omega R_1^2}{(R_1^2 - R_2^2)}\left(1 - \frac{R_2^2}{r^2}\right)\right)\right]$$

$$\boxed{\tau_{r\theta} = \frac{\mu\Omega R_1^2}{(R_1^2 - R_2^2)} \cdot \frac{2R_2^2}{r^2}}$$

$$\tau_{r\theta} \propto \frac{1}{r^2}$$

$$\boxed{\tau_{r\theta} = \frac{\mu\Omega R_1^2}{(R_1^2-R_2^2)}\cdot\frac{2R_2^2}{r^2}} \qquad \tau_{r\theta} \propto \frac{1}{r^2}$$

$$\begin{cases} r = R_1 \Rightarrow \tau_{r\theta} = \dfrac{2\mu\Omega R_2^2}{(R_1^2-R_2^2)} \\[2ex] r = R_2 \Rightarrow \tau_{r\theta} = \dfrac{2\mu\Omega R_1^2}{(R_1^2-R_2^2)} \end{cases}$$



$$F_{r\theta} = 2\pi r L \tau_{r\theta} = \boxed{\frac{4\pi L\mu\Omega R_1^2 R_2^2}{(R_1^2-R_2^2)\, r} = F_{r\theta}}$$

$$F_{r\theta} \propto \frac{1}{r}$$

Torque: $M_1 = F_{r\theta}\cdot r$

$$\boxed{M_1 = \frac{4\pi L\mu\Omega R_1^2 R_2^2}{R_1^2-R_2^2}}$$

cte, p/ Newtoniano