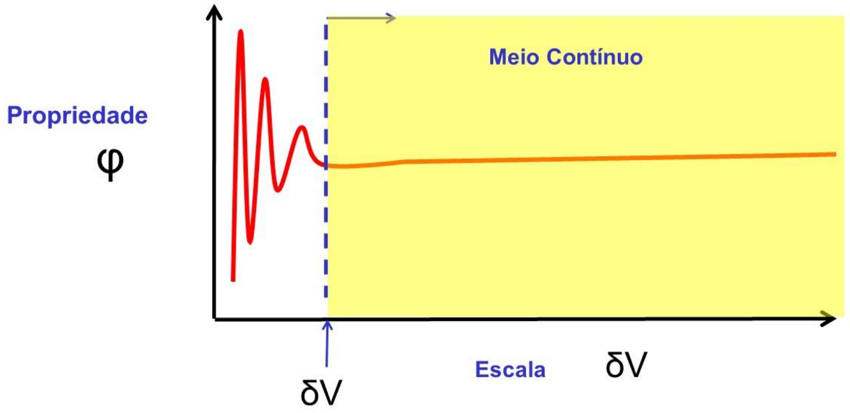
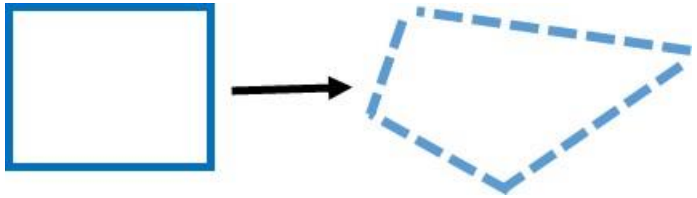
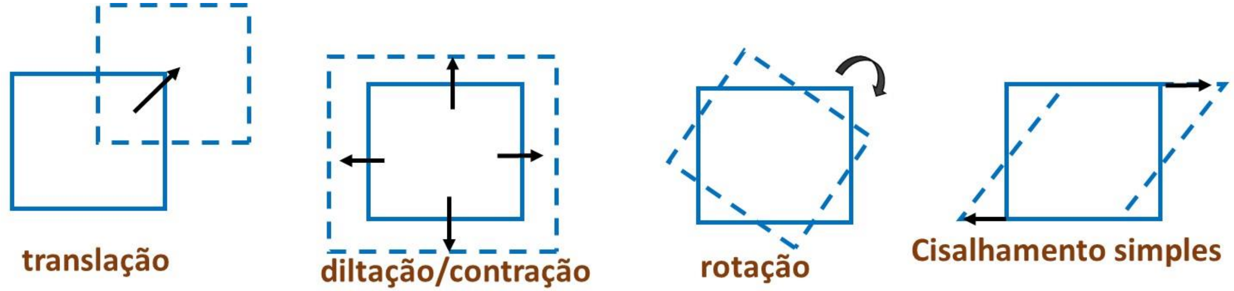
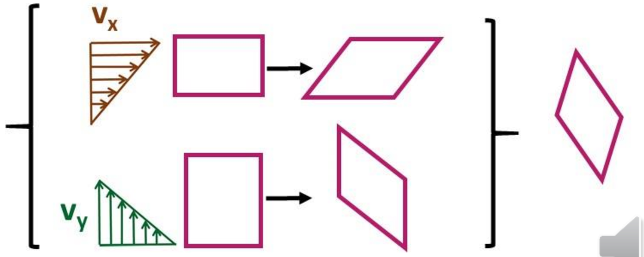
FENÔMENOS DE TRANSPORTE
Campo de velocidades





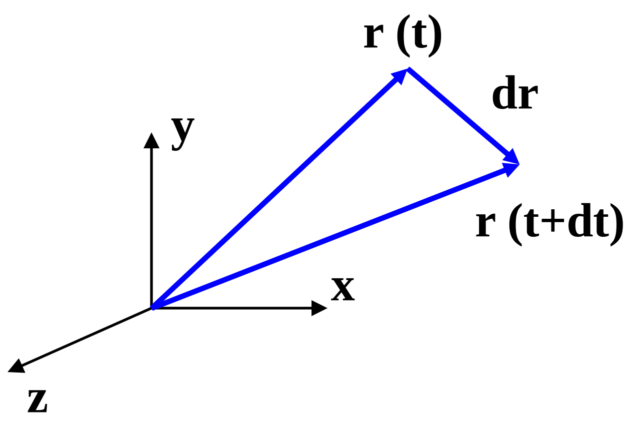




Derivadas no tempo- Observações

- **MEIO CONTÍNUO – CAMPO**

PROPRIEDADE : $\varphi\left(\vec{r}(t), t\right)$



$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \left(\frac{d\varphi}{d\vec{r}}\right)_{t} d\vec{r}$$

$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \vec{\text{grad}}\,\varphi \bullet d\vec{r}$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right) + \vec{\text{grad}}\,\varphi \bullet \vec{w}$$

$$\vec{w} = \frac{d\vec{r}}{dt}$$

Velocidade de observação

instantâneo

Derivadas no tempo- Observações Euler e Lagrange

$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \left(\frac{d\varphi}{d\vec{r}}\right)_t d\vec{r} \longrightarrow \qquad \frac{d\varphi}{dt} = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} \frac{dt}{dt} + \left(\frac{d\varphi}{d\vec{r}}\right)_t \frac{d\vec{r}}{dt}$$

$$\varphi(x, y, z, t) = \varphi[x(t), y(t), z(t), t]$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right)_{x,y,z} + \left(\frac{\partial\varphi}{\partial x}\right)_t \left(\frac{dx}{dt}\right) + \left(\frac{\partial\varphi}{\partial y}\right)_t \left(\frac{dy}{dt}\right) + \left(\frac{\partial\varphi}{\partial z}\right)_t \left(\frac{dz}{dt}\right)$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right)_{x,y,z} + \left[v_x\vec{i} + v_y\vec{j} + v_z\vec{k}\right]. \left[\left(\frac{\partial\varphi}{\partial x}\right) \vec{i} + \left(\frac{\partial\varphi}{\partial y}\right) \vec{j} + \left(\frac{\partial\varphi}{\partial z}\right) \vec{k}\right]$$

OPERADOR

$$\frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + \vec{v}.\,\vec{grad}\,\varphi \qquad \frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + \vec{v}.\vec{\nabla}\varphi$$

instantâneo

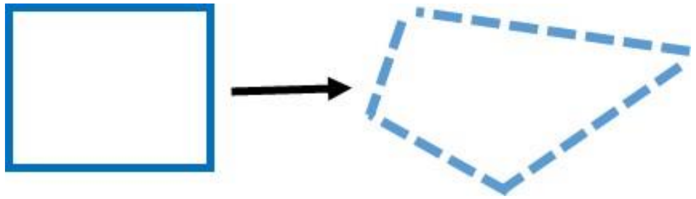
Derivadas no tempo- Observações Euler e Lagrange

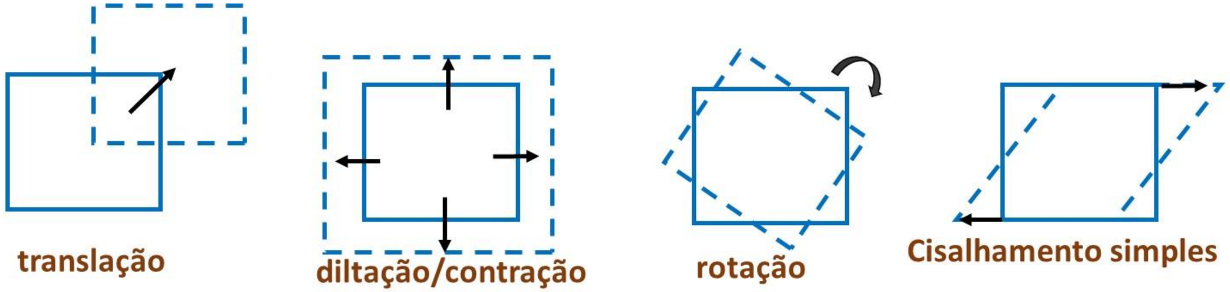
Observação	Observação	$\left(\frac{d\varphi}{dt}\right)$
$\vec{w} = 0$	**EULER** VARIAÇÃO LOCAL	$\left(\frac{\partial\varphi}{\partial t}\right)_{\vec{r}}$
$\vec{w} = \vec{v}$	**LAGRANGE** MATERIAL OU SUBSTANTIVA	$\left(\frac{D\varphi}{Dt}\right) = \frac{\partial\varphi}{\partial t} + \vec{v}.\text{gr}\vec{\text{a}}\text{d}\varphi$
$\vec{w}$	TOTAL	$\left(\frac{d\varphi}{dt}\right) = \frac{\partial\varphi}{\partial t} + \vec{w}.\text{gr}\vec{\text{a}}\text{d}\varphi$

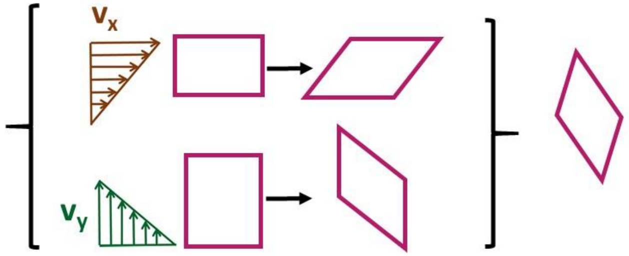
OPERADOR

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}.\text{gr}\vec{\text{a}}\text{d}$$

FENÔMENOS DE TRANSPORTE
Campo de velocidades







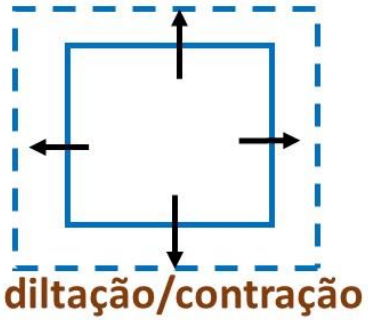
$$\vec{a} = \frac{\mathrm{D}\vec{v}}{\mathrm{Dt}} = \frac{\partial \vec{v}}{\partial \mathrm{t}} + \vec{v}.\,\mathrm{gr\vec{a}d}\vec{v}$$

$$\frac{\mathrm{d}v_x}{\mathrm{dt}} = \left(\frac{\partial v_x}{\partial \mathrm{t}}\right)_{x,y,z} + \left(\frac{\partial v_x}{\partial x}\right)_{\mathrm{t}} v_x + \left(\frac{\partial v_x}{\partial y}\right)_{\mathrm{t}} v_y + \left(\frac{\partial v_x}{\partial z}\right)_{\mathrm{t}} v_z$$

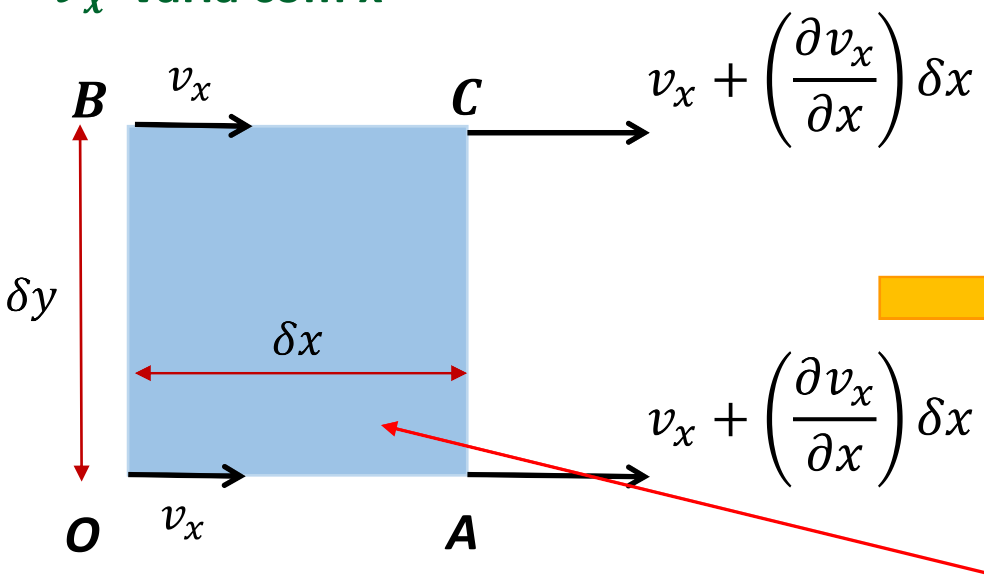
$$\frac{\mathrm{d}v_y}{\mathrm{dt}} = \left(\frac{\partial v_y}{\partial \mathrm{t}}\right)_{x,y,z} + \left(\frac{\partial v_y}{\partial x}\right)_{\mathrm{t}} v_x + \left(\frac{\partial v_y}{\partial y}\right)_{\mathrm{t}} v_y + \left(\frac{\partial v_y}{\partial z}\right)_{\mathrm{t}} v_z$$

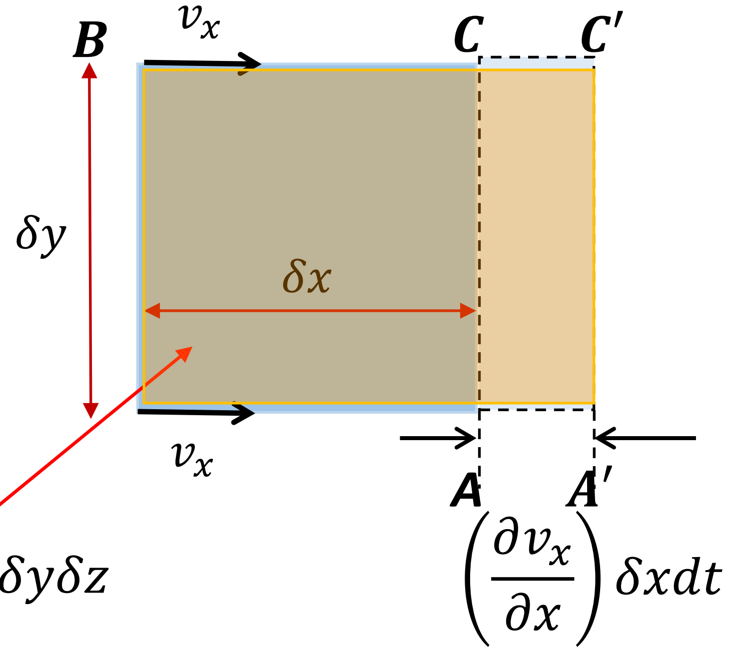
$$\frac{\mathrm{d}v_z}{\mathrm{dt}} = \left(\frac{\partial v_z}{\partial \mathrm{t}}\right)_{x,y,z} + \left(\frac{\partial v_z}{\partial x}\right)_{\mathrm{t}} v_x + \left(\frac{\partial v_z}{\partial y}\right)_{\mathrm{t}} v_y + \left(\frac{\partial v_z}{\partial z}\right)_{\mathrm{t}} v_z$$

FENÔMENOS DE TRANSPORTE
Campo de velocidades



v_x **varia com x**





$$\delta V_x = \delta x \delta y \delta z$$

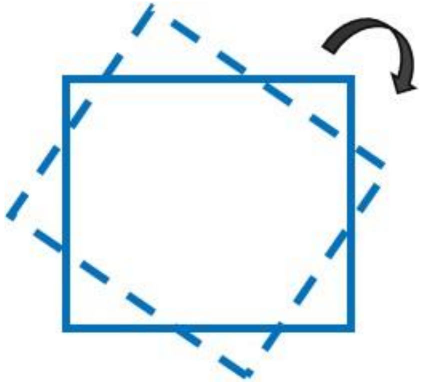
$$\mathrm{d}(\delta V_x) = \left(\frac{\partial v_x}{\partial x}\right)\delta x dt.\delta y \delta z$$

$$\frac{1}{\delta V_x}\frac{\mathrm{d}(\delta V_x)}{\mathrm{dt}} = \left(\frac{\partial v_x}{\partial x}\right) \qquad \frac{1}{\delta V_y}\frac{\mathrm{d}(\delta V_y)}{\mathrm{dt}} = \left(\frac{\partial v_y}{\partial y}\right) \qquad \frac{1}{\delta V_z}\frac{\mathrm{d}(\delta V_z)}{\mathrm{dt}} = \left(\frac{\partial v_z}{\partial z}\right)$$

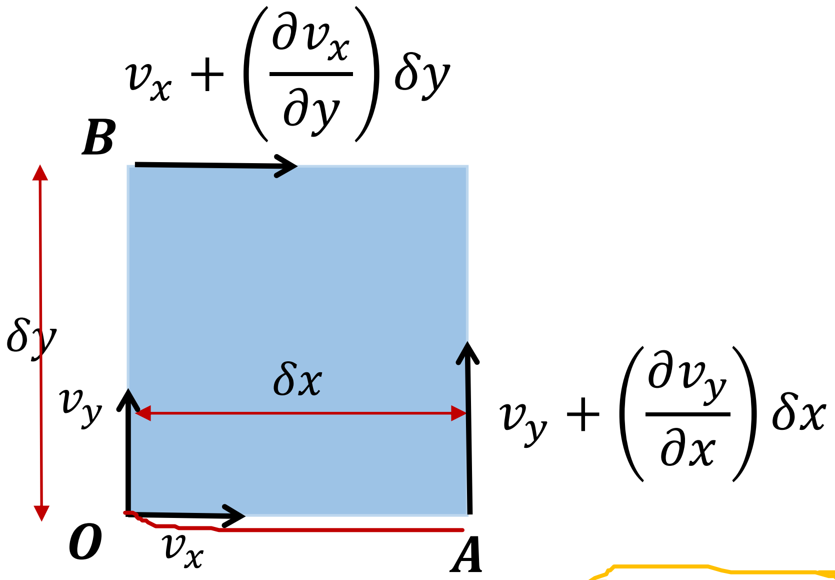
$$\frac{1}{\delta V}\frac{\mathrm{d}(\delta V)}{\mathrm{dt}} = \left(\frac{\partial v_x}{\partial x}\right) + \left(\frac{\partial v_y}{\partial y}\right) + \left(\frac{\partial v_z}{\partial z}\right) = div\,\vec{v}$$

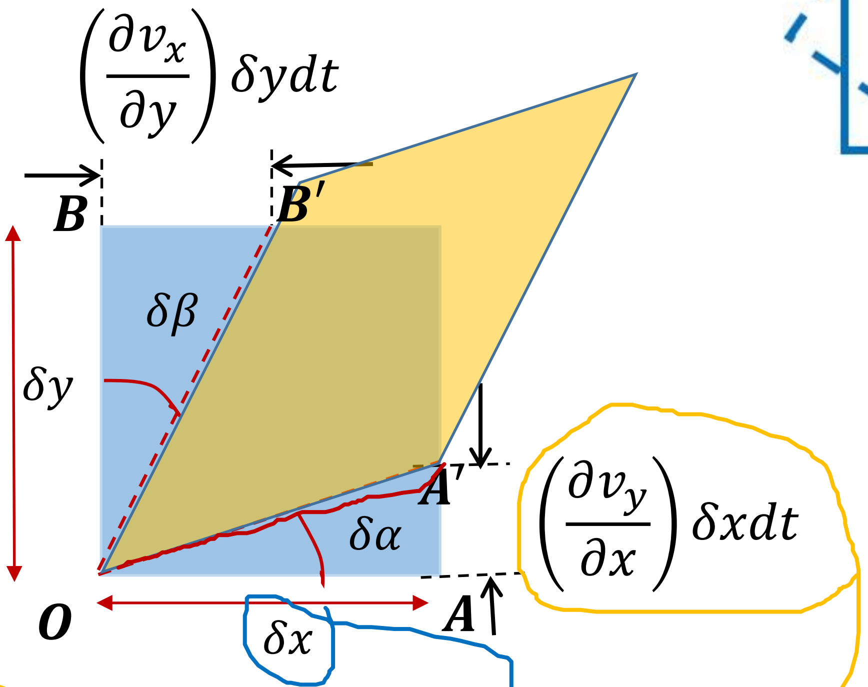
Escoamento incompressível => $div\,\vec{v} = 0$

FENÔMENOS DE TRANSPORTE
Rotação



v_x **varia com y e** v_y **varia com x**





$$\tan(\delta\alpha) \cong \delta\alpha \cong \left(\frac{\partial v_y}{\partial x}\right)\frac{\delta x dt}{\delta x} = \left(\frac{\partial v_y}{\partial x}\right)dt$$

$$\omega_{OA} = \lim_{dt\to 0}\frac{\delta\alpha}{dt} = \left(\frac{\partial v_y}{\partial x}\right)$$

$$\omega_{OB} = \lim_{dt\to 0}\frac{\delta\beta}{dt} = \left(\frac{\partial v_x}{\partial y}\right)$$

Se $\left(\frac{\partial v_y}{\partial x}\right) > 0$ anti horário

Se $\left(\frac{\partial v_x}{\partial y}\right) > 0$ horário

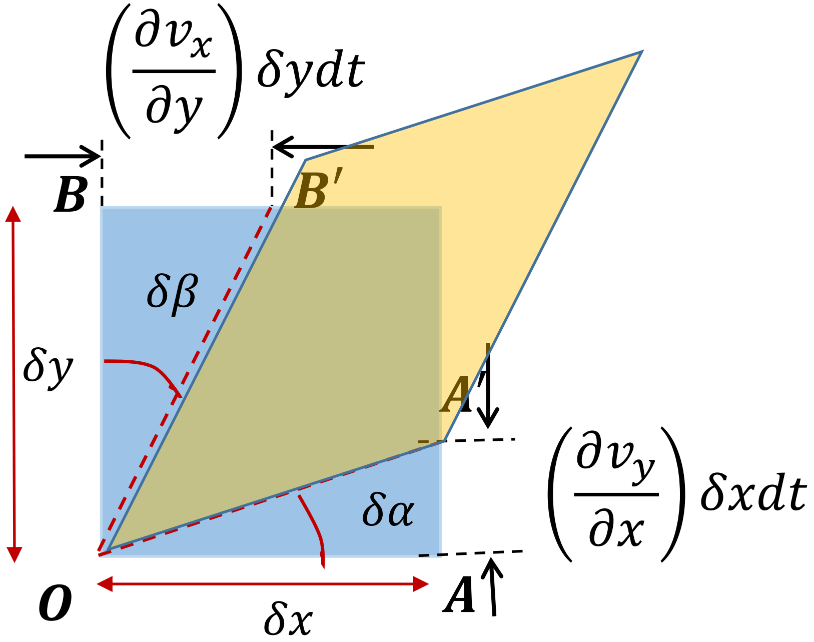
FENÔMENOS DE TRANSPORTE
Rotação

$$\omega_{OA} = \lim_{dt\to 0} \frac{\delta\alpha}{dt} = \left(\frac{\partial v_y}{\partial x}\right)$$

Se $\left(\frac{\partial v_y}{\partial x}\right) > 0$ anti horário

$$\omega_{OB} = \lim_{dt\to 0} \frac{\delta\beta}{dt} = \left(\frac{\partial v_x}{\partial y}\right)$$

Se $\left(\frac{\partial v_x}{\partial y}\right) > 0$ horário



Velocidade angular média , ω_z

$$\omega_z = \frac{\omega_{OA} - \omega_{OB}}{2} = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Velocidade angular média , ω

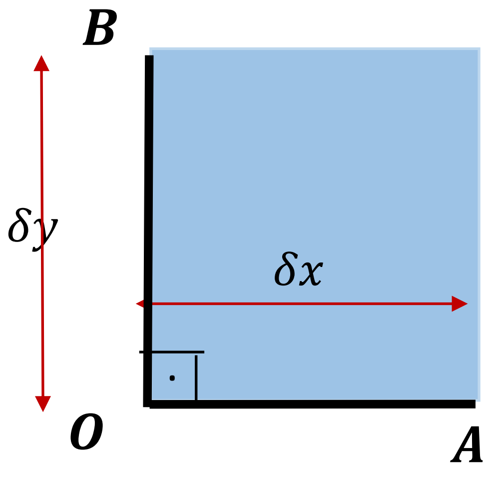
$$\omega_x = \frac{1}{2}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \qquad \omega_y = \frac{1}{2}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \qquad \omega_z = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

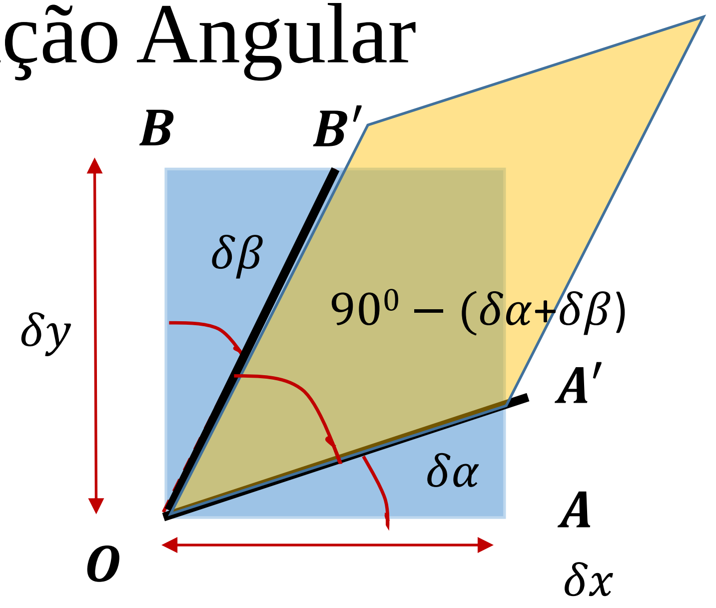
$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k} = \frac{1}{2}\text{rot } \vec{v} = \frac{1}{2}\nabla \times \vec{v}$$

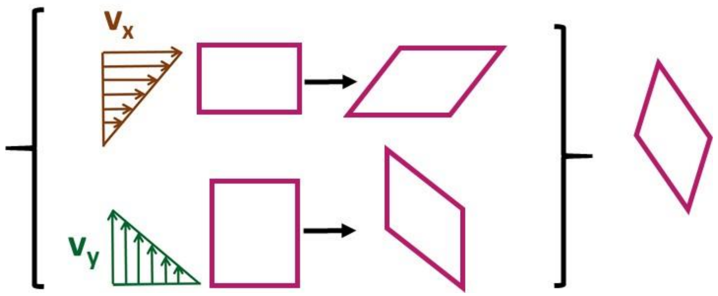
Vorticidade: $\vec{\zeta} = 2\,\vec{\omega} = \text{rot } \vec{v}$

Escoamento irrotacional => rot $\vec{v} = 0$

FENÔMENOS DE TRANSPORTE
Deformação Angular







$$\delta\gamma = \delta\alpha + \delta\beta \quad ; \quad \delta\alpha \cong \left(\frac{\partial v_y}{\partial x}\right) dt \quad ; \delta\beta \cong \left(\frac{\partial v_x}{\partial y}\right) dt$$

Taxa de deformação angular

$$\dot{\gamma} = \lim_{dt \to 0} \frac{\delta\gamma}{dt} = \lim_{dt = 0} \left[\frac{\left(\frac{\partial v_y}{\partial x}\right) dt + \left(\frac{\partial v_x}{\partial y}\right) dt}{dt} \right]$$

$$\dot{\gamma} = \lim_{dt \to 0} \frac{\delta\gamma}{dt} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}$$

$$\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}$$

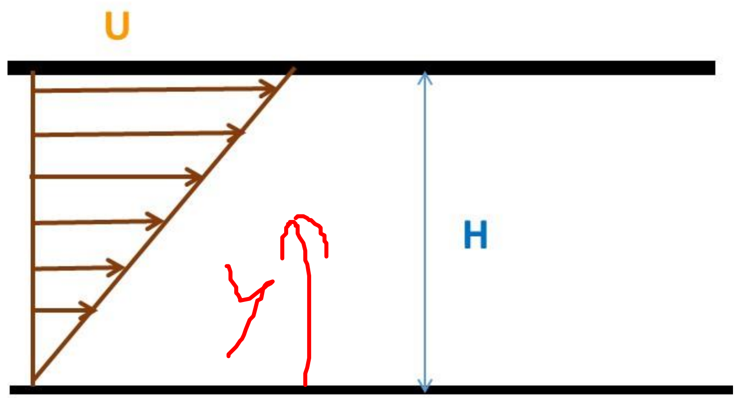
$$\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}$$

Relacionado à Tensão de Cisalhamento

Exercício

Considere o escoamento de um fluido entre duas placas paralelas , no qual

$v_x = U(y/H)$



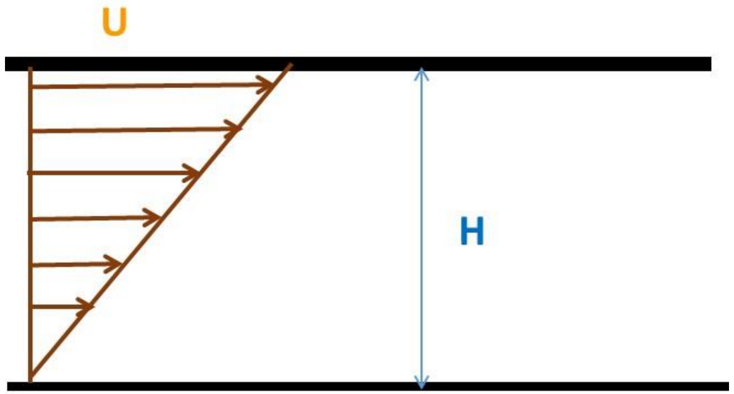
Calcule:

a) Divergente da velocidade. O escoamento é compressível?

b) A vorticidade

c) A taxa de deformação

Exercício



Divergente da velocidade

$$v_x = \frac{y}{H}U \qquad v_y = 0 \qquad v_z = 0$$

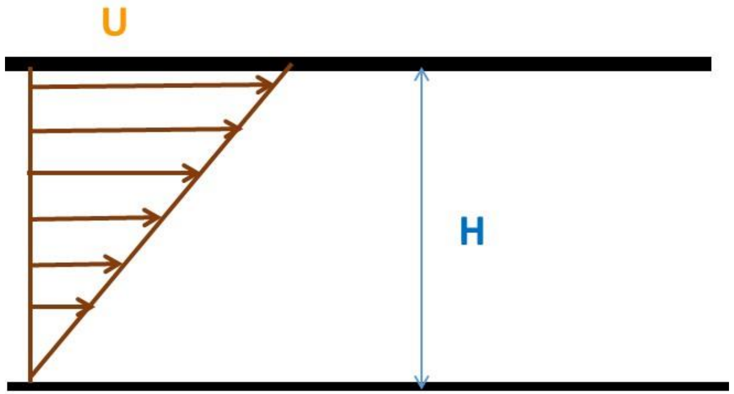
$$div\,\vec{v} = \left(\frac{\partial v_x}{\partial x}\right) + \left(\frac{\partial v_y}{\partial y}\right) + \left(\frac{\partial v_z}{\partial z}\right) = 0$$

Vorticidade

$$\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) = 0 \qquad \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) = 0 \qquad \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) = 0 - \frac{U}{H} = -\frac{U}{H}$$

$$\vec{\zeta} = \text{rot}\,\vec{v} = -\frac{\partial v_x}{\partial y}\vec{k} = -\frac{U}{H}\vec{k}$$

Exercício



Taxa de deformação angular

$$\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} = 0$$

$$\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0 + \frac{U}{H} = \frac{U}{H}$$

$$\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} = 0$$