PEA3390 - Eletricidade Aplicada I (2023)
Prof. Giovanni

Conteúdo da aula:

Começamos em breve

5.3. Representação fasorial de bipolos elétricos
- 5.3.1. Bipolos ativos
  - a) Fontes de tensão
  - b) Fontes de corrente
- 5.3.2. Bipolos passivos
  - a) Resistores
  - b) Indutores
  - c) Capacitores
  - d) Impedância e admitância

5.4. Técnicas de análise de circuitos em regime permanente senoidal
- 5.4.1. Análise de malhas
- 5.4.2. Análise nodal

13/09

L·PROT

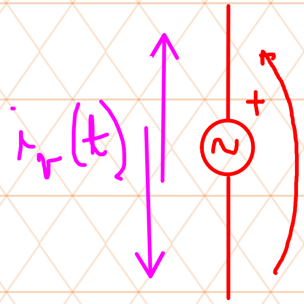
5.3. Representação fasorial de bipolos elétricos

5.3.1. Bipolos ativos

ATENÇÃO:

$$\cos\theta = \operatorname{sen}\left(\frac{\pi}{2}+\theta\right)$$

a) Fontes de tensão

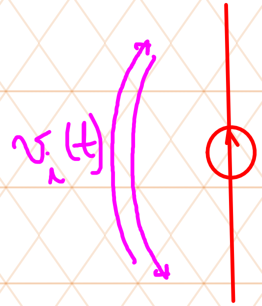


$$v(t) = V_p \cdot \cos(\omega t + \theta_v) \rightsquigarrow \underbrace{\dot{V} = V \angle \theta_v}_{\text{polar}}, \text{ onde } V = \frac{V_p}{\sqrt{2}} \text{ (valor eficaz)}$$

θ_v (fase)

$$\Rightarrow \underbrace{\dot{V} = V\cos\theta_v + j\,V\operatorname{sen}\theta_v}_{\text{retangular}}$$

b) Fontes de corrente



$$i(t) = I_p \cdot \cos(\omega t + \theta_i) \rightsquigarrow \underbrace{\dot{I} = I \angle \theta_i}_{\text{polar}}, \text{ onde } I = \frac{I_p}{\sqrt{2}} \text{ (valor eficaz)}$$

θ_i (fase)

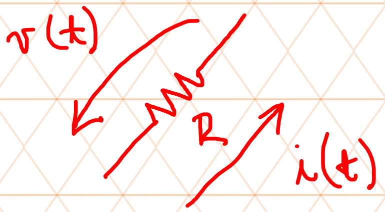
$$\Rightarrow \underbrace{\dot{I} = I\cos\theta_i + j \cdot I\operatorname{sen}\theta_i}_{\text{retangular}}$$

L·PROT

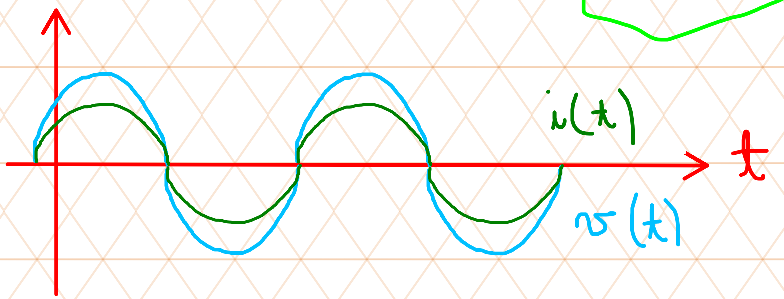
5.3. Representação fasorial de bipolos elétricos

5.3.2. Bipolos passivos

a) Resistor



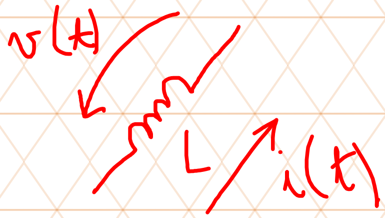
$$v(t) = R\cdot i(t) \xrightarrow{Fas} \dot{V} = R\cdot\dot{I} \quad \therefore \quad R = \frac{\dot{V}}{\dot{I}}$$



$$\left.\begin{array}{l} v(t) = V_p\cos(\omega t+\theta_v) \\ i(t) = I_p\cos(\omega t+\theta_i) \end{array}\right] \Rightarrow \frac{V_p}{I_p} = R;\ \theta_v = \theta_i$$

$$\theta_v - \theta_i = 0$$

b) Indutor



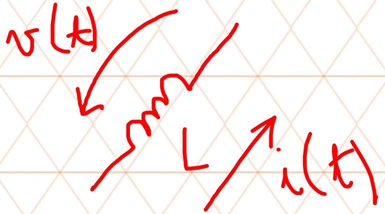
$$v(t) = L\,\frac{di(t)}{dt} \rightsquigarrow Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot\underbrace{e^{j\theta_v}\cdot\frac{V_p}{\sqrt{2}}}_{fasor}\right\} = Re\left\{L\cdot\frac{d}{dt}\left[e^{j\omega t}\cdot\sqrt{2}\cdot\underbrace{e^{j\theta_i}\cdot\frac{I_p}{\sqrt{2}}}_{fasor}\right]\right\}$$

Fas

$$\dot{V} = j\omega L\cdot\dot{I}$$

$$Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot e^{j\theta_v}\cdot V\right\} = Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot j\omega L\cdot e^{j\theta_i}\cdot I\right\}$$

$X_L > 0$ (reatância indutiva)



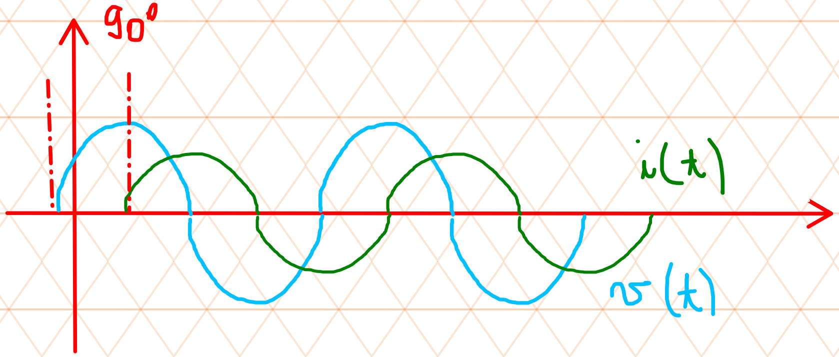
$$v(t) = L\,\frac{di(t)}{dt} \rightsquigarrow Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot \underbrace{e^{j\theta_v}\cdot\frac{V_p}{\sqrt{2}}}_{\text{fasor}}\right\} = Re\left\{L\cdot\frac{d}{dt}\left[e^{j\omega t}\cdot\sqrt{2}\cdot \underbrace{e^{j\theta_i}\cdot\frac{I_p}{\sqrt{2}}}_{\text{fusor}}\right]\right\}$$

Fas

$$\dot{V} = j\omega L\cdot\dot{I}$$

$$Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot e^{j\theta_v}\cdot V\right\} = Re\left\{e^{j\omega t}\cdot\sqrt{2}\cdot j\omega L\cdot e^{j\theta_i}\cdot I\right\}$$

$$\omega L = \frac{V_p}{I_p} = \frac{V}{I} \quad e \quad \theta_v = \theta_i + 90^\circ \rightsquigarrow \underbrace{\theta_v - \theta_i}_{\text{defasagem}} = 90^\circ$$

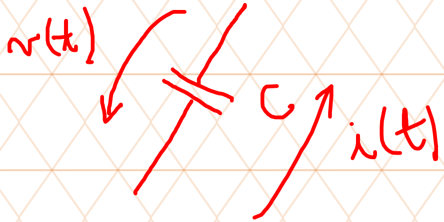


No capacitor, a corrente está ATRASADA da tensão de exatos 90 graus

5.3. Representação fasorial de bipolos elétricos

5.3.2. Bipolos passivos

c) Capacitor



$$i(t) = C \cdot \frac{dv(t)}{dt} \xrightarrow{\text{Fas}} \dot{I} = j\omega C \cdot \dot{V}$$

$$\frac{1}{\omega C} = \frac{V_p}{I_p} = \frac{V}{I}$$

$$\theta_v = \theta_i - 90^\circ$$

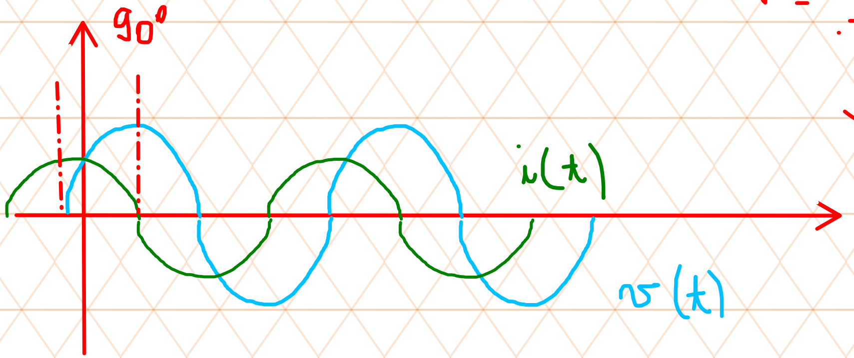
$$\theta_v - \theta_i = -90^\circ$$

$$\dot{V} = \frac{1}{j\omega C} \cdot \dot{I}$$

$$\hookrightarrow \dot{V} = -j\frac{1}{\omega C} \cdot \dot{I}$$

$x_c < 0$

(reatância capacitiva)



No capacitor, a corrente está ADIANTADA da tensão de exatos 90 graus

5.3. Representação fasorial de bipolos elétricos

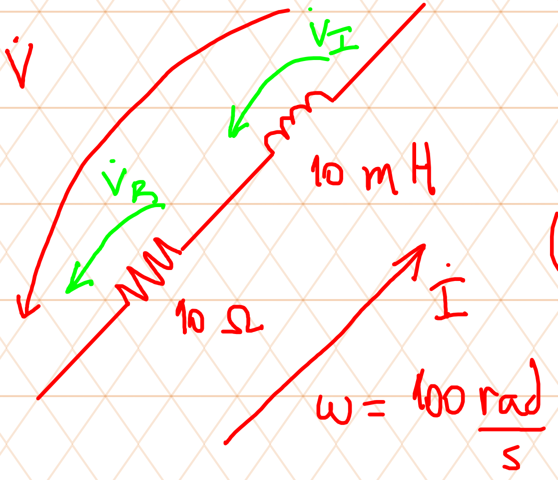
5.3.2. Bipolos passivos

d) Impedância (e admitância)

$$\bar{Z} = \frac{\dot{V}}{\dot{I}} = R + jX$$

$X \rightarrow$ Reatância [ohms]

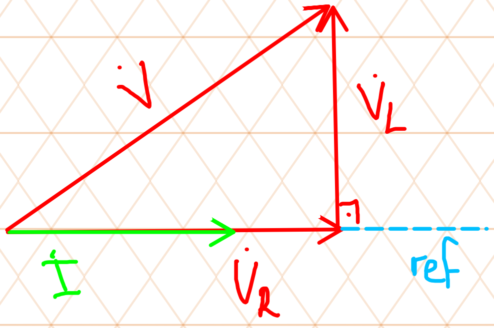
$R \rightarrow$ Resistência [ohms]



$$\dot{V}_R + \dot{V}_I = \dot{V}$$

$$R.\dot{I} + j\omega L.\dot{I} = \dot{V}$$

$$\underbrace{(R + j\omega L)}_{\bar{Z}}.\dot{I} = \dot{V}$$

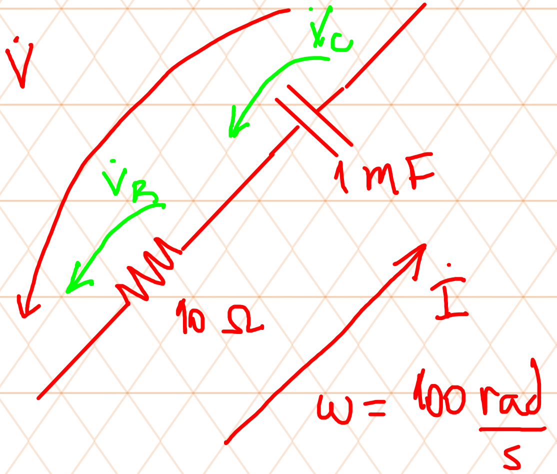


$$\bar{Y} = \frac{\dot{I}}{\dot{V}} = \frac{1}{\bar{Z}} = G + jB$$

$B \rightarrow$ Susceptância [siemens] ou [mho]

$G \rightarrow$ Condutância [siemens] ou [mho]

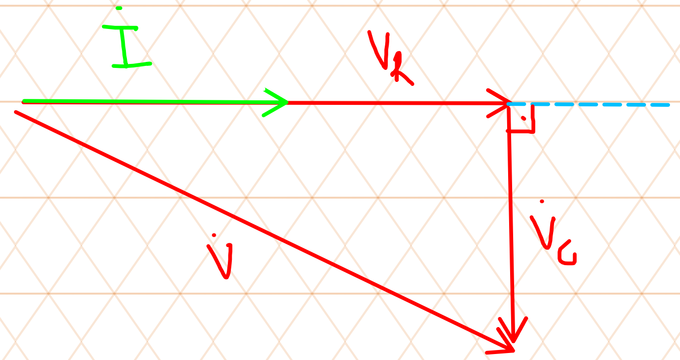
$$= \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R}{R^2 + X^2} + j\frac{-X}{R^2 + X^2}$$

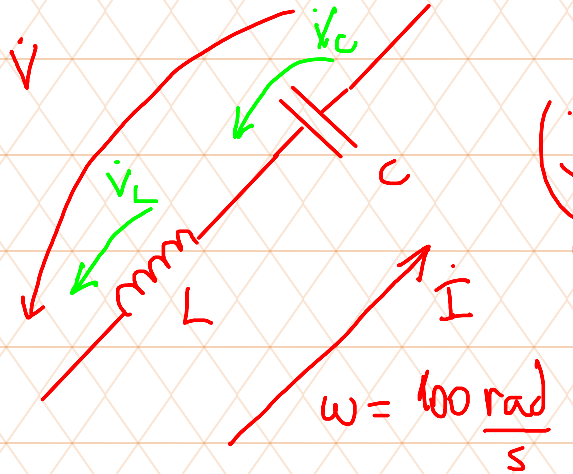


$$\dot V_R + \dot V_C = \dot V$$

$$R\cdot\dot I + \frac{1}{j\omega C}\cdot\dot I = \dot V$$

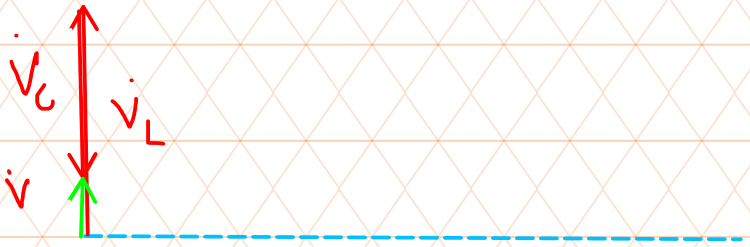
$$\underbrace{\left(R - \frac{j}{\omega C}\right)}_{\bar Z}\cdot\dot I = \dot V$$





$$\dot V_L + \dot V_C = \dot V$$

$$\underbrace{\left(j\omega L - \frac{j}{\omega C}\right)}_{\bar Z}\cdot\dot I = \dot V$$



5.3. Técnicas de análise de circuitos em regime permanente senoidal

5.3.2. Análise de malhas

Dado o circuito abaixo, calculo tensões e correntes

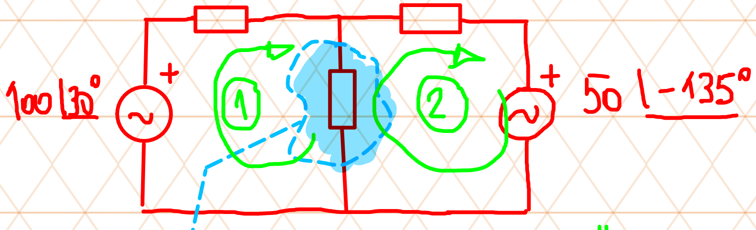
$$\cos\theta = \text{sen}\left(\frac{\pi}{2}+\theta\right)$$

$$100t - 45 = 100t + \frac{\pi}{2} + \theta$$

$$\theta = -45 - 90^\circ$$

$t \to f$

$$\frac{1}{j\cdot 100\cdot 1000\cdot 10^{-6}} = \frac{10}{j} = -j10$$



$$v_1 = 100\sqrt{2}\cos(\overset{\omega}{100}t + 30^\circ)$$

$$v_2 = 50\sqrt{2}\,\text{sen}(100t - 45^\circ)$$

$$R_1 = 5\,\Omega \;;\; R_2 = 10\,\Omega$$

$$L = 10\,[mH]$$

$$C = 1000\,[\mu F]$$

$$R + j\omega L$$

$$10 + j\cdot 100\cdot\frac{10}{1000}$$

$$= 10 + j1$$

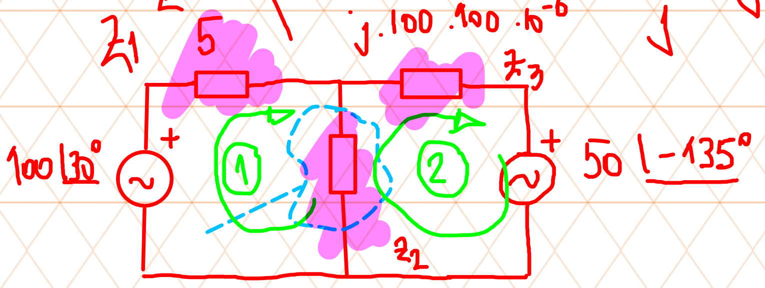
Malha 1:

$$100\angle 30^\circ - 5\cdot\dot{I}_1 - (10+j1)\cdot(\dot{I}_1 - \dot{I}_2) = 0$$

Malha 2:

$$-50\angle -135^\circ - (10+j1)(\dot{I}_2 - \dot{I}_1) - (-j10) = 0$$

$$\frac{1}{j \cdot 100 \cdot 100 \cdot 10^{-6}} = \frac{10}{j} = -j10$$



$10 + j1$

$$\underbrace{\begin{array}{c|cc} & ① & ② \\ & Z_1+Z_2 & -Z_2 \\ \hline ① & 5+10+j1 & -10-j1 \\ ② & -10-j1 & 10+j1-j10 \\ & -Z_2 & Z_2+Z_3 \end{array}}_{[A]} \times \underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}}_{\underset{\sim}{x}} = \underbrace{\begin{bmatrix} 100\angle 30^\circ \\ -50\angle -135^\circ \end{bmatrix}}_{\underset{\sim}{b}}$$

$$\dot{I}_1 = 10{,}437 \angle 71{,}353^\circ \ [A] \quad \sim \quad i_1(t) = 10{,}437 \cdot \sqrt{2} \cos(100t + 71{,}353^\circ) \ [A]$$

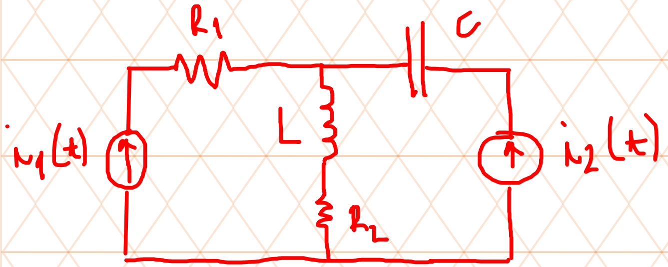
$$\dot{I}_2 = 11{,}123 \angle 108{,}83^\circ \ [A] \qquad\quad i_2(t) = 11{,}123 \cdot \sqrt{2} \cos(100t + 108{,}83^\circ) \ [A]$$

```
%
% No MATLAB (ou OCTAVE): sqrt(-1) = 1i;
%
% Dados do problema
clear all;
V1 = 100*exp(1i*30*pi/180);     % V1 = 100*(cos(30*pi/180) + 1i*sin(30*pi/180))
V2 = 50*exp(1i*(-135)*pi/180);  % V2 = 50*(cos(-135*pi/180) + 1i*sin(-135*pi/180))
R1 = 5;
R2 = 10;
C  = 1E-3;
L  = 10E-3;
w  = 100;
%
% Solucao do problema
Z1 = R1;
Z2 = R2 + 1i*w*L;
Z3 = 1/(1i*w*C);
A  = [Z1+Z2 -Z2; -Z2 Z2+Z3]
b  = [V1; -V2];
x  = A\b; % x = inv(A)*b
I1 = x(1)
I2 = x(2)
```

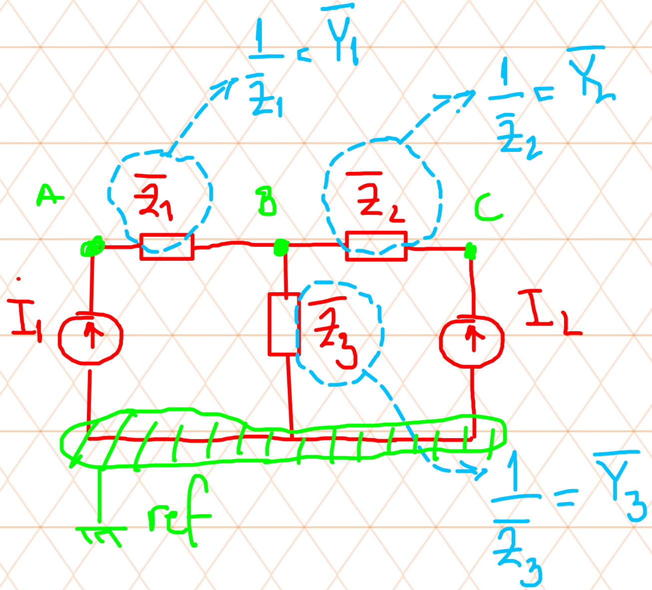
5.3. Técnicas de análise de circuitos em regime permanente senoidal

5.3.2. Análise nodal

Dado o circuito abaixo, calculo tensões e correntes



$t \to f$



A) $-\dot{I}_1 + \bar{Y}_1(\dot{E}_A - \dot{E}_B) = 0$

B) $\bar{Y}_1(\dot{E}_B - \dot{E}_A) + \bar{Y}_2(\dot{E}_B - \dot{E}_C) + \bar{Y}_3\dot{E}_B$

C) $-\dot{I}_2 + \bar{Y}_2(\dot{E}_C - \dot{E}_B) = 0$

$\to$ Por inspeção:

$$\underbrace{\begin{bmatrix} \bar{Y}_1 & -\bar{Y}_1 & 0 \\ -\bar{Y}_1 & \bar{Y}_1+\bar{Y}_2+\bar{Y}_3 & -\bar{Y}_2 \\ 0 & -\bar{Y}_2 & \bar{Y}_2 \end{bmatrix}}_{[A]} \times \underbrace{\begin{bmatrix} \dot{E}_A \\ \dot{E}_B \\ \dot{E}_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \dot{I}_1 \\ 0 \\ \dot{I}_2 \end{bmatrix}}_{b}$$

$$\underset{\sim}{x} = \underset{\sim}{b}$$