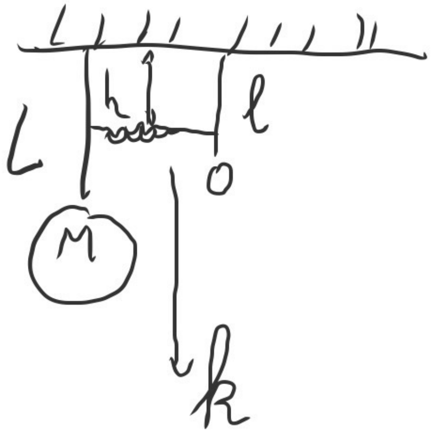
Pêndulos Acoplados

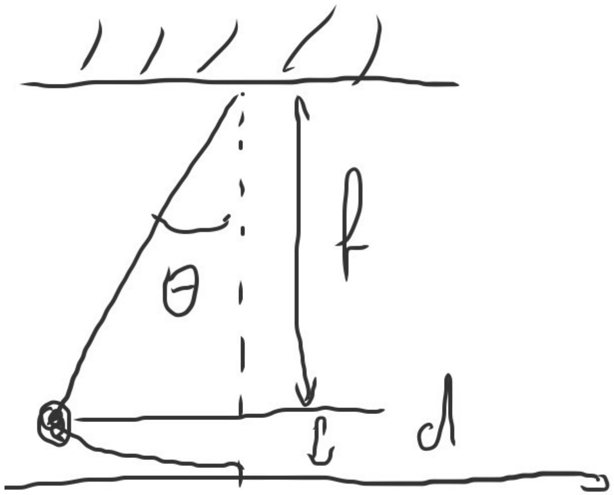


$$U_{gM} = MgL\,\frac{\theta_M^2}{2} \; ; \; U_{gm} = mg\ell\,\frac{\theta_m^2}{2}$$

$$U_k = \frac{k}{2}\Delta x^2 = \frac{k}{2}\left[h(\theta_m - \theta_M)\right]^2$$

$$E = Mg\frac{L}{2}\theta_M^2 + \overset{K}{\frac{ML^2}{2}\dot{\theta}_M^2} + mg\frac{\ell}{2}\theta_m^2 + \overset{K}{m\frac{\ell^2}{2}\dot{\theta}_m^2}$$

$$+ \frac{k}{2}h^2\left[\theta_m^2 + \theta_M^2 - 2\theta_m\theta_M\right]$$



$$V_g = M \cdot g\, d$$

$$d + f = L$$

$$f = L \cos\theta$$

$$|\theta| << \pi$$

$$\cos\theta \simeq \cos 0 - \cancel{\text{sen}\, 0}^{\,0} \cdot \theta - \frac{(\cos 0)}{2!}\theta^2 + \frac{\cancel{(\text{sen}\, 0)}^{\,0}{}^3}{3!} \cdot \theta^3$$

$$\cos\theta \simeq 1 - \frac{\theta^2}{2}$$

$$d = L - f$$

$$= \cancel{L} - \left(\cancel{L} - \frac{\theta^2}{2} L\right)$$

$$d = L\frac{\theta^2}{2}$$ → quadrático com a perturbação

$$V = Mg\frac{L}{2}\theta^2$$

Simplificando : $M = m$, $h = l = L$

$$E = ML^2\left[\left(\frac{g}{L}\right)\frac{\theta_M^2}{2} + \left(\frac{g}{L}\right)\frac{\theta_m^2}{2} + \frac{k}{2M}\left(\theta_m - \theta_M\right)^2 + \frac{\dot{\theta}_M^2}{2} + \frac{\dot{\theta}_m^2}{2}\right]$$

Mudança de variáveis

$$\Delta\theta = \frac{(\theta_m - \theta_M)}{2} \quad ; \quad \bar{\theta} = \frac{(\theta_m + \theta_M)}{2}$$

$$\theta_M = \bar{\theta} - \Delta\theta \quad ; \quad \theta_m = \bar{\theta} + \Delta\theta$$

$$E = ML^2\left[\frac{1}{2}\frac{g}{L}\cdot 2\bar{\theta}^2 + \frac{1}{2}\cdot 2\dot{\bar{\theta}}^2 + \frac{1}{2}\frac{g}{L}\cdot 2\Delta\theta^2 + \frac{k}{2M}\cdot 4\Delta\theta^2 + \frac{1}{2}\cdot 2\Delta\dot{\theta}^2\right]$$

$$\theta_M^2 = \bar{\theta}^2 + \Delta\theta^2 - 2\bar{\theta}\cdot\Delta\theta$$

$$\theta_m^2 = \bar{\theta}^2 + \Delta\theta^2 + 2\bar{\theta}\cdot\Delta\theta$$

$$E = E_{\bar{\theta}}(\bar{\theta}, \dot{\bar{\theta}}) + E_{\Delta\theta}(\Delta\theta, \Delta\dot{\theta})$$

A mudança de referencial dá duas eqs. desacopladas!

1ª Parte: $E_{\bar{\Theta}} = ML^2 \frac{g}{L} \cdot \bar{\Theta}^2 + ML^2 \dot{\bar{\Theta}}^2$

$$= \frac{M}{2} \cdot (2Lg)\, \bar{\Theta}^2 + \frac{1}{2} (2ML^2)\, \dot{\bar{\Theta}}^2$$

$$\frac{2M}{2} \left(\frac{g}{L}\right) (L \cdot \bar{\Theta})^2 + \frac{1}{2} (2M) (L \cdot \dot{\bar{\Theta}})^2$$

$$\frac{m}{2}\, \omega_0^2\, x^2 + \frac{1}{2}\, m\, \dot{x}^2$$

$$\omega_0^2 = g/L$$

$$\bar{\Theta} = A \cos(\omega_0 t + \varphi)$$

2^a parte: $E_{\Delta\theta} = \frac{1}{2}(2M)\left(\frac{g}{L}\right)\cdot(L\Delta\theta)^2 + ML^2\cdot\frac{k}{M}\cdot 2\,\Delta\theta^2$

$$+\frac{2M}{2}\cdot(L\Delta\dot{\theta})^2$$

$$= \underbrace{\frac{(2M)}{2}}_{\frac{m}{2}}\cdot\underbrace{\left[\left(\frac{g}{L}\right)+2\frac{k}{M}\right]}_{\omega_2^2}\underbrace{(L\Delta\theta)^2}_{x^2} + \underbrace{\frac{(2M)}{2}}_{\frac{m}{2}}\underbrace{(L\cdot\Delta\dot{\theta})^2}_{\dot{x}^2}$$

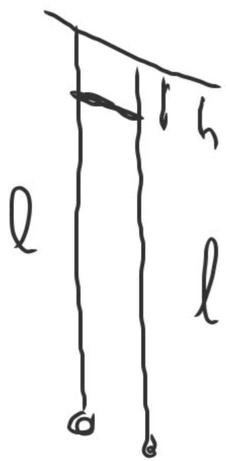
$$\omega_2^2 = \omega_0^2 + 2\frac{k}{M}$$

$$\Theta_m = \bar{\Theta} + \Delta\theta = A\cos(\omega_0 t + \varphi_0) + B\cos(\omega_2 t + \varphi_2)$$

$$\Theta_M = \bar{\Theta} - \Delta\theta = A\cos(\omega_0 t + \varphi_0) - B\cos(\omega_2 t + \varphi_2)$$

$$\omega_0 = \sqrt{\frac{g}{L}} \quad ; \quad \omega_2 = \sqrt{\omega_0^2 + 2\frac{k}{M}}$$

Pêndulos acoplados



$$\omega_0 = \sqrt{\frac{g}{l}} \qquad \omega_2 = \sqrt{\frac{g}{l-h}}$$

$$T_2 = \frac{2\pi}{\omega_2} = \sqrt{\frac{l-h}{g}} = 0$$

Batimento: $t=0$ $\quad \theta_m(0)=\alpha$; $\Theta_M(0)=0$; $\dot{\theta}_m=\dot{\theta}_M=0$

$$\Theta_M(t)=A\cos(\omega_0 t+\varphi_0)-B\cos(\omega_2 t+\varphi_2)$$

$$\dot{\Theta}_M(t)=-\omega_0 A\,\mathrm{sen}(\omega_0 t+\varphi_0)+B\cdot\omega_2\,\mathrm{sen}(\omega_2 t+\varphi_2)$$

$\dot{\Theta}_M(0)=-\omega_0 A\,\mathrm{sen}\,\varphi_0+B\,\omega_2\,\mathrm{sen}\,\varphi_2$; $\dot{\theta}_m(0)=-\omega_0 A\,\mathrm{sen}\,\varphi_0-\omega_2 B\,\mathrm{sen}\,\varphi_2$

$$\Theta_M(0)=A\underbrace{\cos\varphi_0}_{=1}-B\underbrace{\cos\varphi_2}_{=1}=A-B=0$$

$$\theta_m(t)=A\cos(\omega_0 t+\varphi_0)+B\cos(\omega_2 t+\varphi_2)$$

$$\dot{\theta}_m(t)=-\omega_0 A\,\mathrm{sen}(\omega_0 t+\varphi_0)-B\,\omega_2\,\mathrm{sen}(\omega_2 t+\varphi_2)$$

$-\omega_0 A\,\mathrm{sen}\,\varphi_0=0$

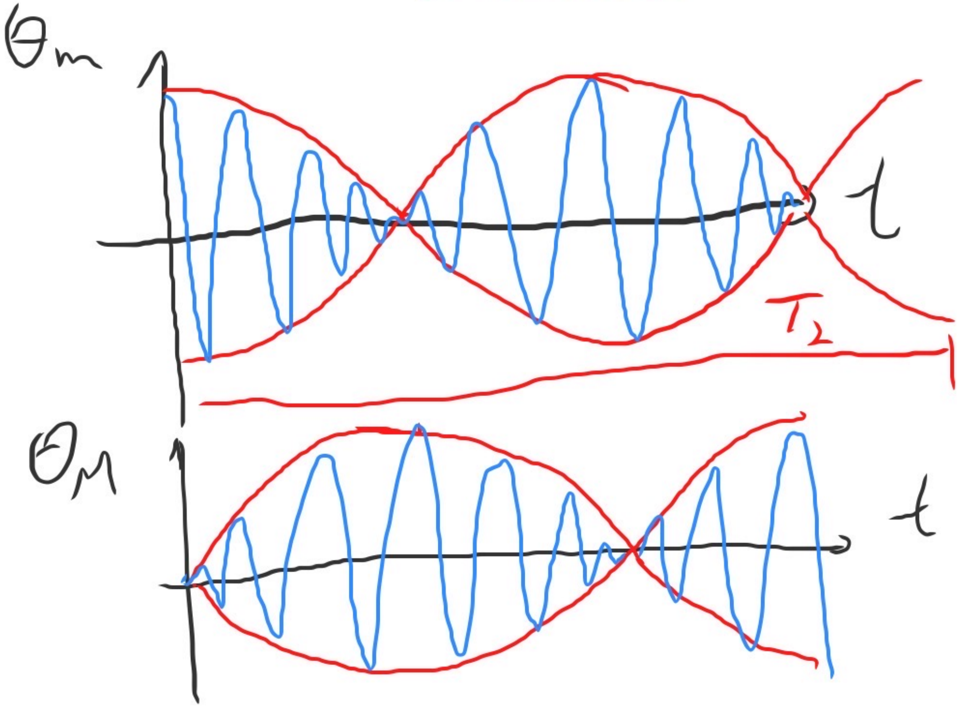
$-\omega_2 B\,\mathrm{sen}\,\varphi_2=0$ $\Rightarrow$ $\varphi_0=\varphi_2=0$; $\Rightarrow A=B \Rightarrow A+B=\alpha$

$\alpha=2A$ $\qquad A=\dfrac{\alpha}{2}$

$$\theta_m = \frac{a}{2}\left[\cos(\omega_0 t) + \cos(\omega_2 t)\right] \; ; \; \theta_M = \frac{a}{2}\left[\cos(\omega_0 t) - \cos(\omega_2 t)\right]$$

$$\bar{\omega} = \frac{\omega_0 + \omega_2}{2} \; ; \; \Delta\omega = \omega_2 - \omega_0$$

$$\theta_m = a \, \underline{\cos\left(\frac{\Delta\omega}{2} t\right)} \, \underline{\cos(\bar{\omega} t)} \; ; \; \theta_M = a \, \mathrm{sen}\left(\frac{\Delta\omega}{2} t\right) \mathrm{sen}(\bar{\omega} t)$$



Batimento = envoltória

$$T_2 = \frac{2\pi}{\Delta\omega} = \frac{2\pi}{\omega_2 - \omega_0}$$

$\omega_2 \to \omega_0 \to T_2 \to \infty$

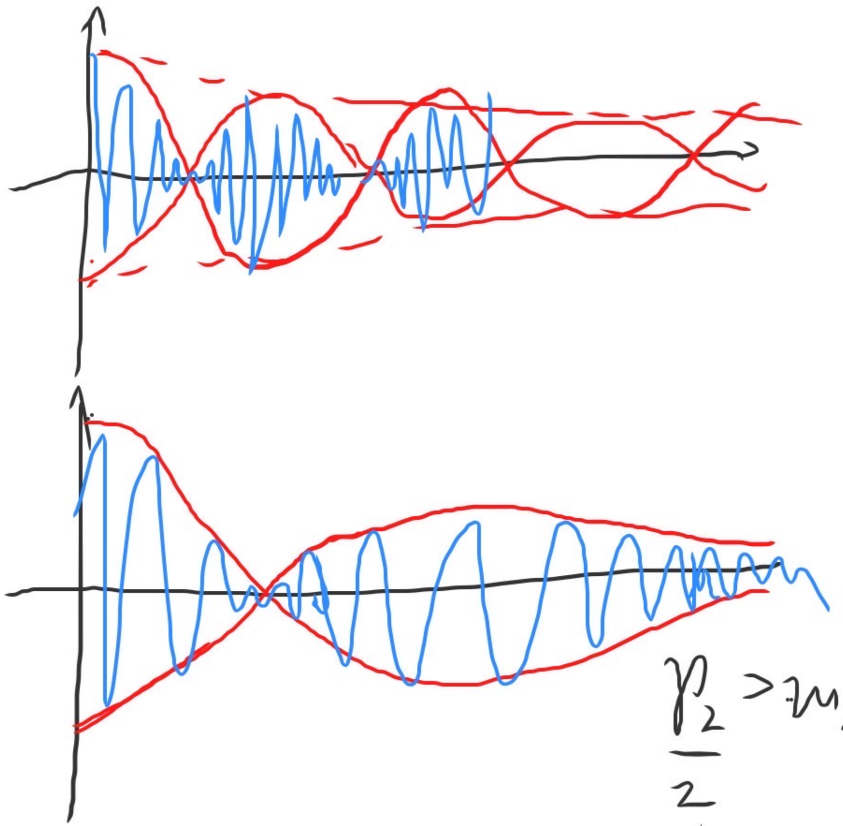
$\omega_2 = \sqrt{\frac{g}{\ell - h}}$ $\quad h \downarrow \; , \; T_2 \nearrow$

Amortecimento : $\frac{d}{dt}\bar{E} = -\gamma \bar{E}$

$$E = E_{\bar{\Theta}} + E_{\Delta\Theta}$$

$E_{\bar{\Theta}} \downarrow m_0 \downarrow$ amortecimento γ_0

$E_{\Delta\Theta} \downarrow m_2 \downarrow$ amortecimento $\gamma_2 \gg \gamma_0$



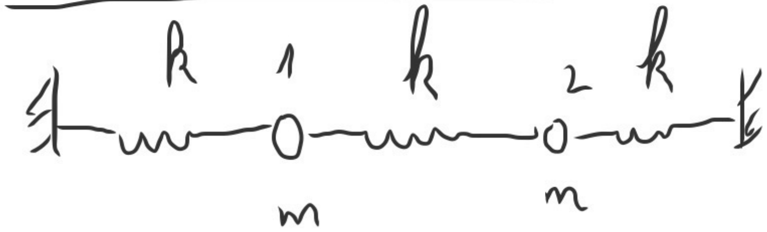
$\frac{\gamma_2}{2} > m_2$

amort. supercritico

$$E_{\bar{\Theta}} = \bar{E}_0 e^{-\gamma_0 t}$$

$$E_{\Delta\Theta} = E_{\Delta 0} \underbrace{e^{-\gamma_2 t}}$$ efeito de força dissipativa

Osciladores Acoplados



Modo simétrico

$\omega_0 = \sqrt{\frac{k}{m}}$

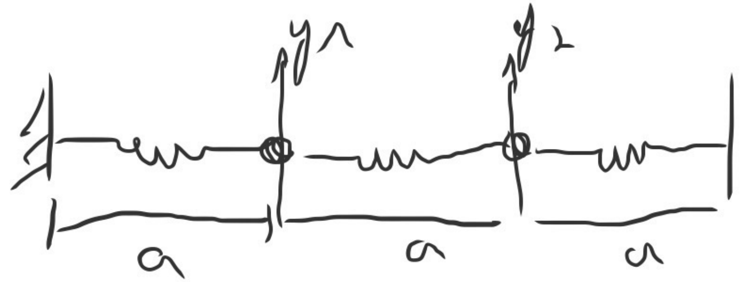
Modo Anti-simétrico

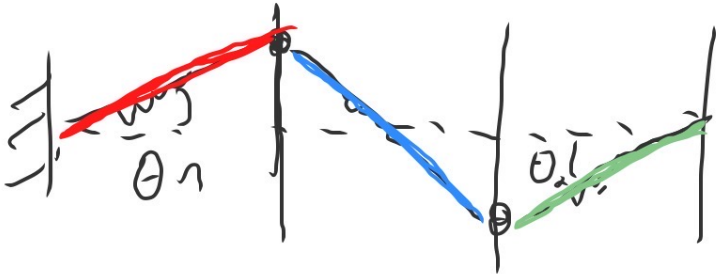
$$F_1 = -kx_1 - k(x_1 - x_2)$$

$$F_2 = -kx_2 - k(x_2 - x_1)$$

$$m\frac{d^2}{dt^2}(x_1 - x_2) = -3k(x_1 - x_2)$$

$\omega_2 = \sqrt{\frac{3k}{m}}$



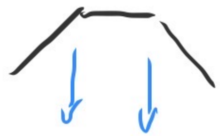


$$F_1 = -T_0 \,\mathrm{sen}\,\theta_1 \simeq -T_0 \frac{y_1}{a}$$

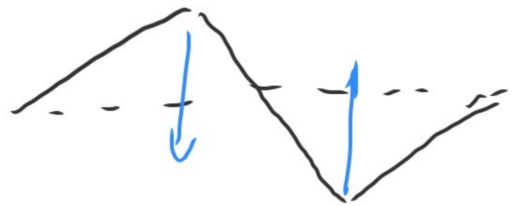
$$F_2 = -T_0 \,\mathrm{sen}\,\theta_2 \simeq -T_0 \frac{y_2}{a}$$

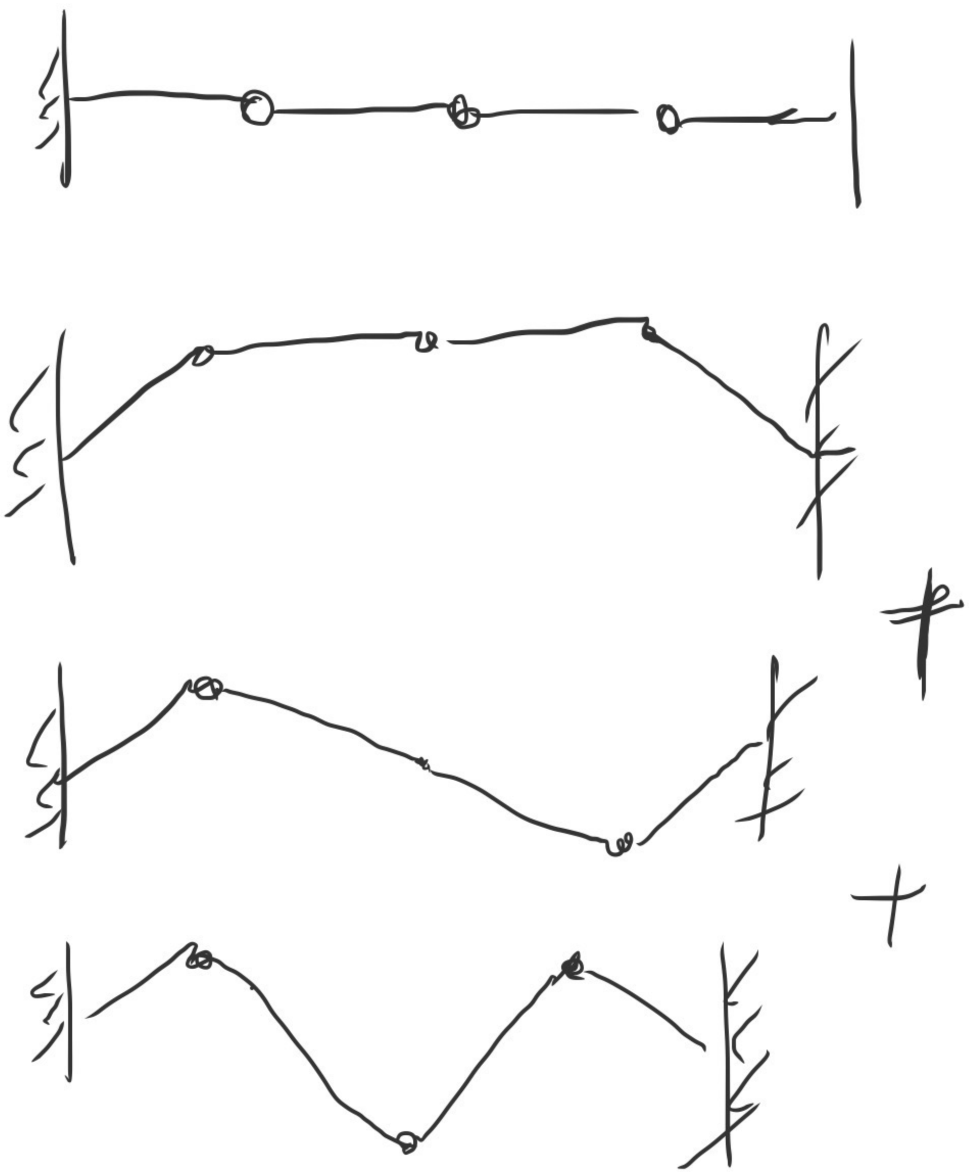
$$F_{12} = -T_0 \frac{(y_1 - y_2)}{a}$$

$$\omega_0 = \frac{T_0}{ma}$$



$$\omega_2 = \sqrt{3 \frac{T_0}{ma}}$$





3 graus de liberdade

⇒ 3 modos de oscilação!

N graus de liberdade

N modos de oscilação

$N \to \infty \Rightarrow$