PRIMEIRA PARTE.

Seja

$$A := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$ em $M_2(\mathbb{R})$. Note que

$$AJ = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -b & a \\ -d & c \end{pmatrix},$$

e que

$$JA = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ -a & -b \end{pmatrix}.$$

Logo,

$$AJ = JA \iff \begin{cases} c = -b \\ a = d \end{cases}.$$

Isto é, $AJ = JA \iff A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$.

SEGUNDA PARTE.

Sejam $A := \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ e $B := \begin{pmatrix} a' & b' \\ -b' & a' \end{pmatrix}$ matrizes que comutam com J. Note que

$$AB = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a' & b' \\ -b' & a' \end{pmatrix} = \begin{pmatrix} aa' - bb' & ab' + ba' \\ -(ba' + ab') & aa' - bb' \end{pmatrix},$$

e que

$$BA = \begin{pmatrix} a' & b' \\ -b' & a' \end{pmatrix}\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = \begin{pmatrix} a'a - b'b & a'b + b'a \\ -(b'a + a'b) & a'a - b'b \end{pmatrix}.$$

Logo, $AB = BA$.

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