Eletromagnetismo Avançado

1º de setembro

Leis de conservação

Conservação do momento

$$\overleftrightarrow{T} = \epsilon_0\left(\vec{\mathbf{E}}\otimes\vec{\mathbf{E}} - \frac{1}{2}E^2\mathbb{1}\right) + \frac{1}{\mu_0}\left(\vec{\mathbf{B}}\otimes\vec{\mathbf{B}} - \frac{1}{2}B^2\mathbb{1}\right)$$

$$\vec{\mathbf{f}} = \vec{\boldsymbol{\nabla}}\cdot\overleftrightarrow{T} - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

$$\underbrace{\vec{\mathbf{F}}}_{\frac{\mathrm{d}\vec{\mathbf{p}}_{mec}}{\mathrm{d}t}} = \int_S \hat{\mathbf{n}}\cdot\overleftrightarrow{T}\mathrm{d}a - \frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\epsilon_0\int_V\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)\mathrm{d}\tau}_{\vec{\mathbf{p}}_{EM}}$$

$$\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \int_S \hat{\mathbf{n}}\cdot\overleftrightarrow{T}\mathrm{d}a$$

$$\frac{\partial\vec{\mathcal{P}}}{\partial t} = \vec{\boldsymbol{\nabla}}\cdot\overleftrightarrow{T}$$

Conservação do momento

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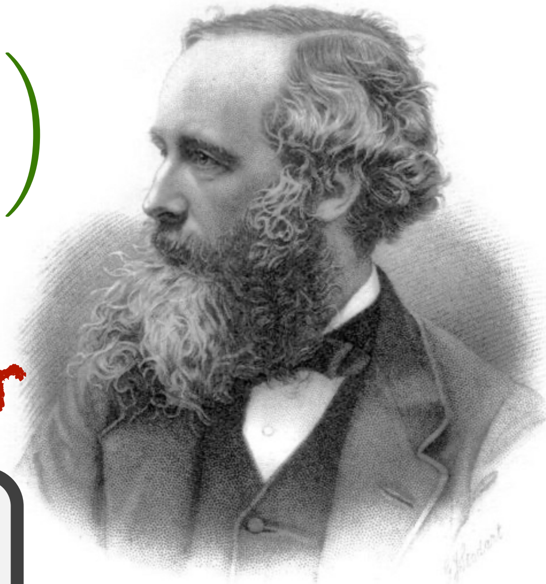

- $\overleftrightarrow{T}\cdot\hat{n}$ transforma $\hat{n}$ em outro vetor

$$\overleftrightarrow{T}_E\cdot\hat{\mathbf{n}} = \epsilon_0 E(\vec{\mathbf{E}}\cdot\hat{\mathbf{n}})\hat{\mathbf{E}} - \epsilon_0\frac{E^2}{2}\hat{\mathbf{n}}$$

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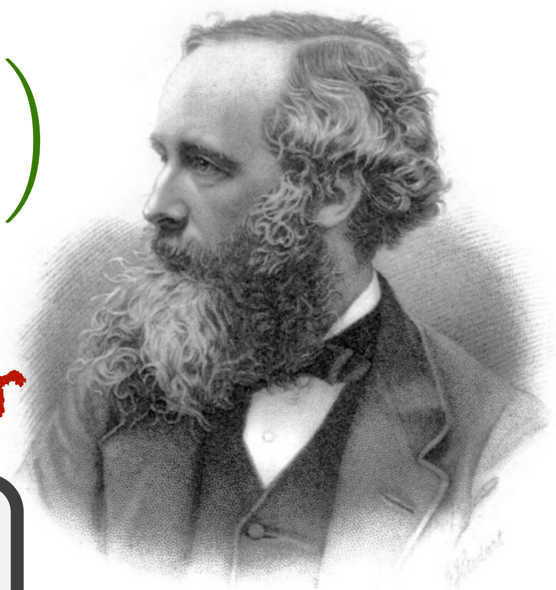


- $\overleftrightarrow{T}\cdot\hat{n}$ transforma $\hat{n}$ em outro vetor

$$\overleftrightarrow{T}_E\cdot\hat{\mathbf{n}} = \epsilon_0 E(\vec{\mathbf{E}}\cdot\hat{\mathbf{n}})\hat{\mathbf{E}} - \epsilon_0\frac{E^2}{2}\hat{\mathbf{n}}$$

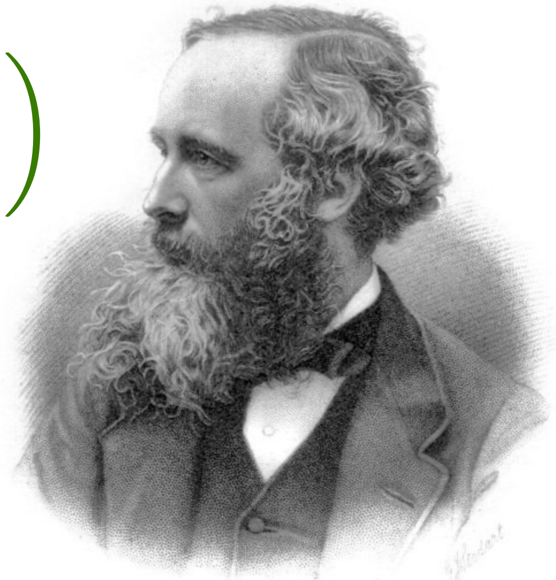
- Autovetores e autovalores de $\overleftrightarrow{T}_E$:

$$\hat{\mathbf{n}} = \hat{\mathbf{E}} \quad\Rightarrow\quad \lambda = \epsilon_0\frac{E^2}{2}$$

$$\hat{\mathbf{n}} \perp \hat{\mathbf{E}} \quad\Rightarrow\quad \lambda = -\epsilon_0\frac{E^2}{2}$$

Conservação do momento

$$\overleftrightarrow{T} = \epsilon_0\left(\vec{\mathbf{E}} \otimes \vec{\mathbf{E}} - \frac{1}{2}E^2\mathbb{1}\right) + \frac{1}{\mu_0}\left(\vec{\mathbf{B}} \otimes \vec{\mathbf{B}} - \frac{1}{2}B^2\mathbb{1}\right)$$



- $\overleftrightarrow{T}$ é tensor

- $\overleftrightarrow{T} \cdot \hat{n}$ transforma $\hat{n}$ em outro vetor

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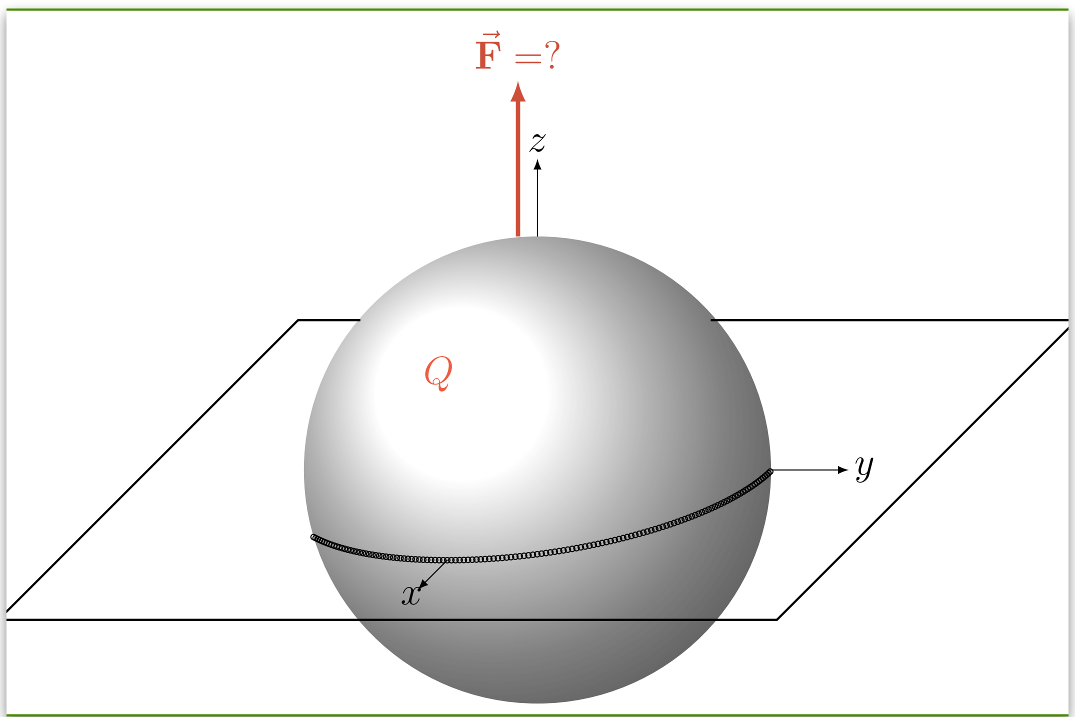
$$\hat{\mathbf{E}} \quad \Rightarrow \quad \lambda = \epsilon_0 \frac{E^2}{2}$$

$$\hat{\mathbf{v}} \quad (\hat{\mathbf{v}} \perp \hat{\mathbf{E}}) \quad \Rightarrow \quad \lambda = -\epsilon_0 \frac{E^2}{2}$$

Pratique o que aprendeu

$$\vec{\mathbf{F}} = \int_S \hat{\mathbf{n}} \cdot \overleftrightarrow{T} \mathrm{d}a - \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_V \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \right) \mathrm{d}\tau$$



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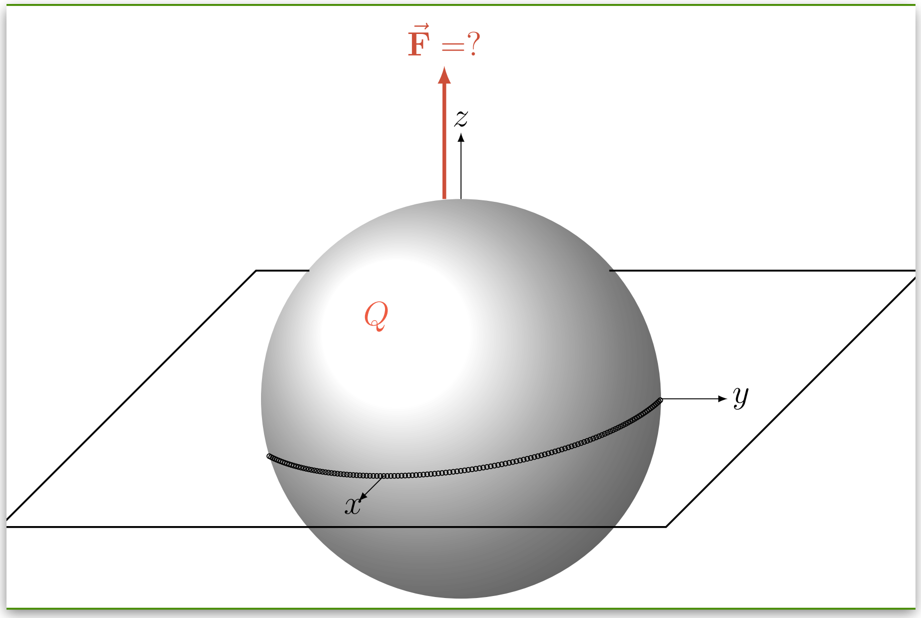
- Superfície de Gauss: hemisfério



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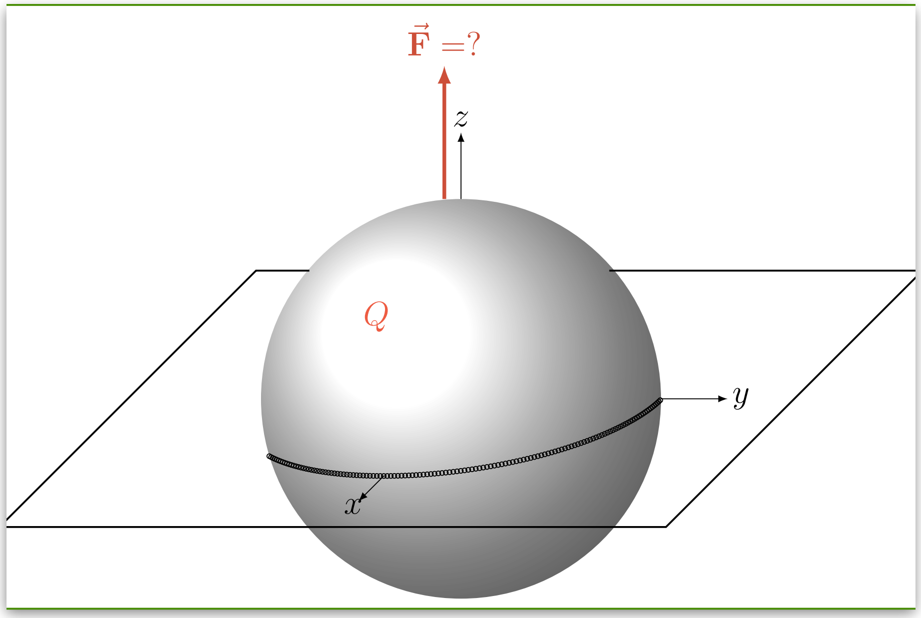
- Superfície de Gauss: hemisfério
  - Topo => hemisférico
  - Base => plana



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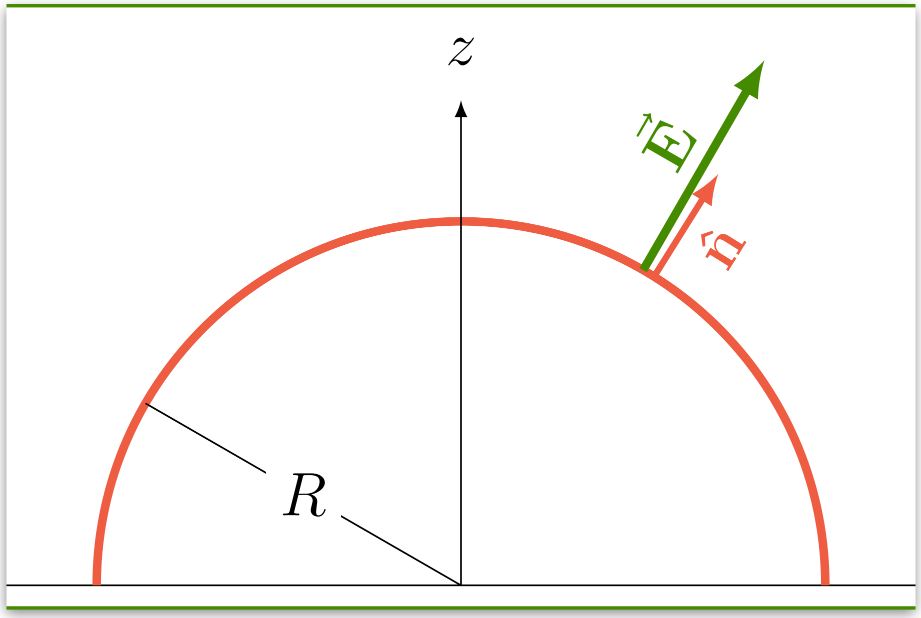
- Topo => hemisférico
- $\hat{n}$ paralelo ao campo



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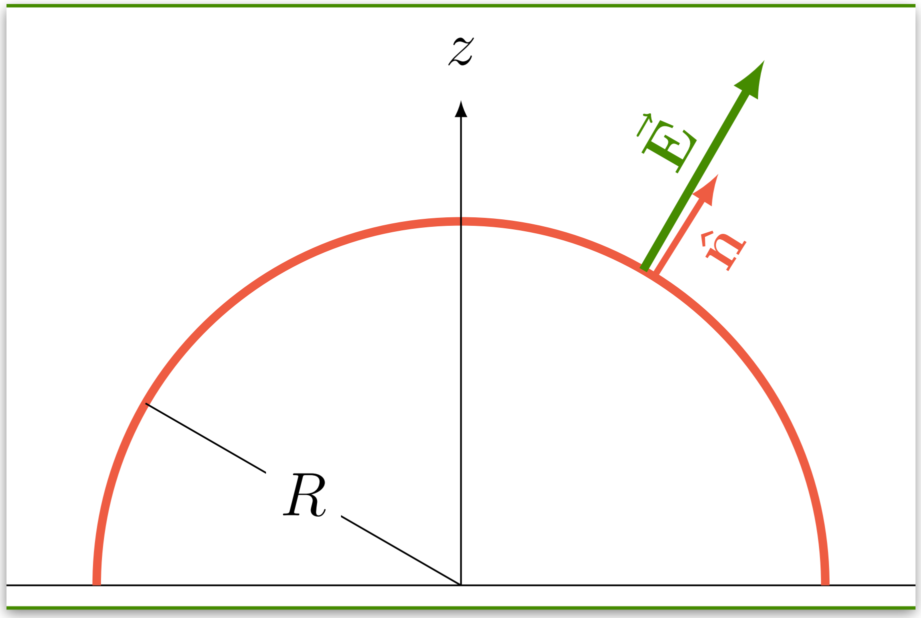
Topo => hemisférico

$$\overleftrightarrow{T} \cdot \hat{\mathbf{n}} = \epsilon_0 (\vec{\mathbf{E}} \cdot \hat{\mathbf{n}})\vec{\mathbf{E}} - \epsilon_0 \frac{E^2}{2} \hat{\mathbf{n}}$$

$$\hat{n} \parallel \overrightarrow{E}$$



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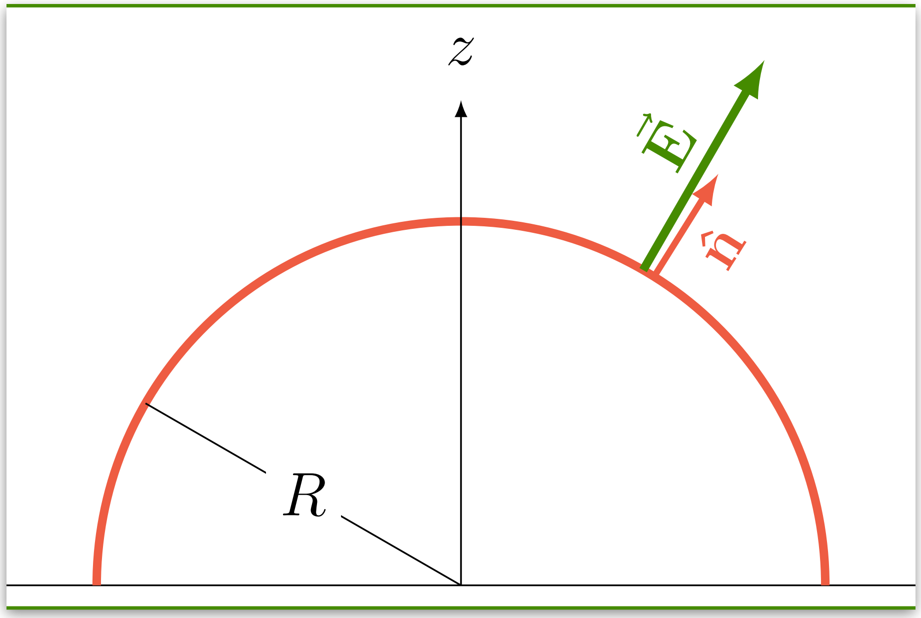

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Base => plana

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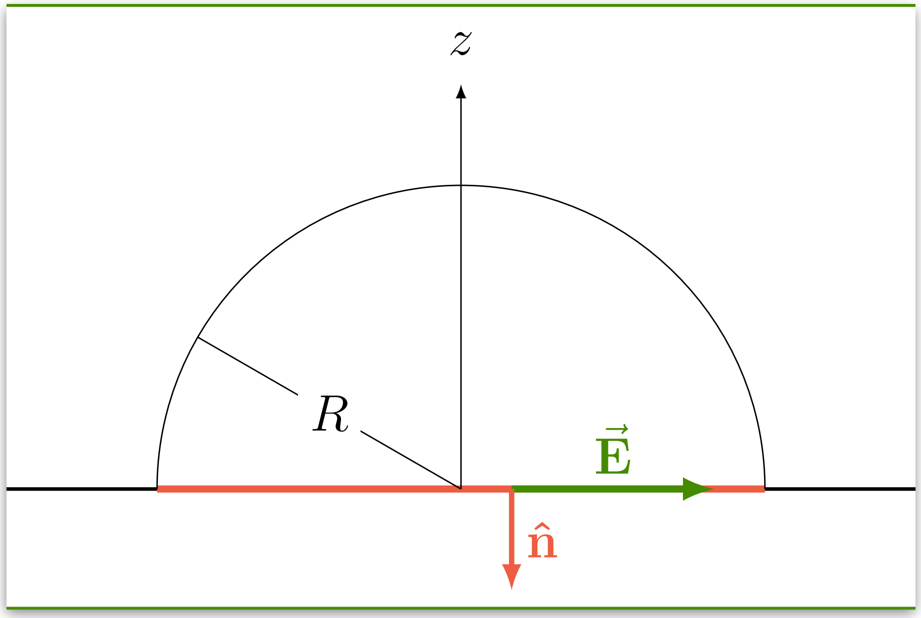

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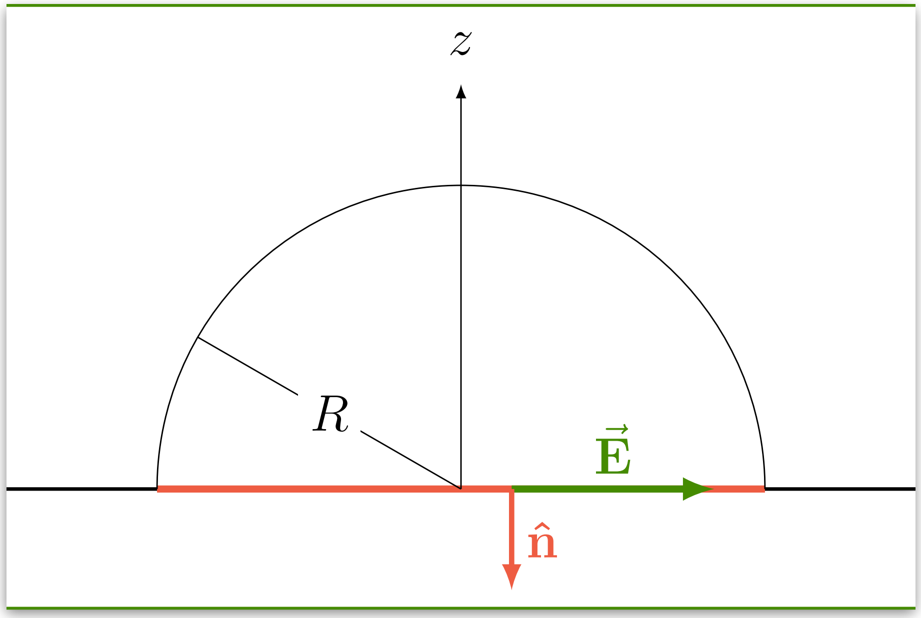

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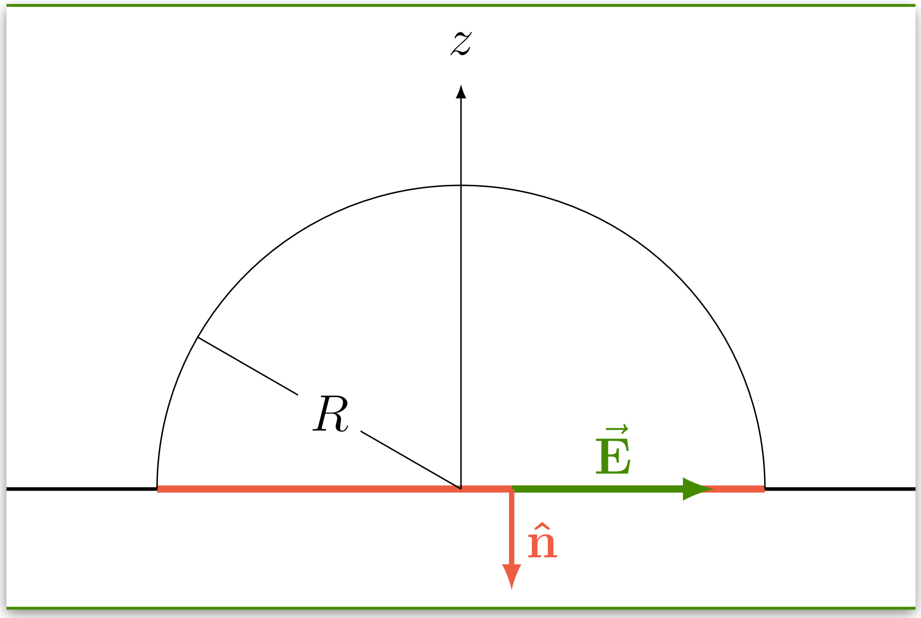

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Ondas eletromagnéticas

Equações de onda

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

+ Condições de contorno

Equações de onda

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

Condições livres

$$\nabla^2 \vec{\mathbf{B}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

Equações de onda

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$$\nabla^2 \vec{\mathbf{B}} = \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

Condições livres

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{E}}_0 \exp\Big(i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\Big)$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{B}}_0 \exp\Big(i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\Big)$$