5930300 – Química Quântica

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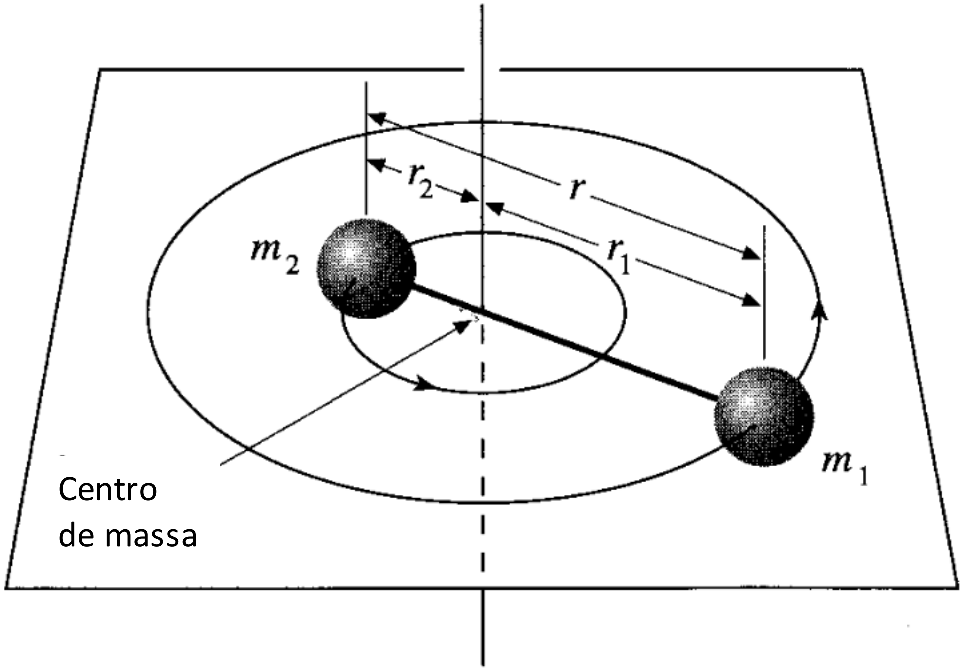
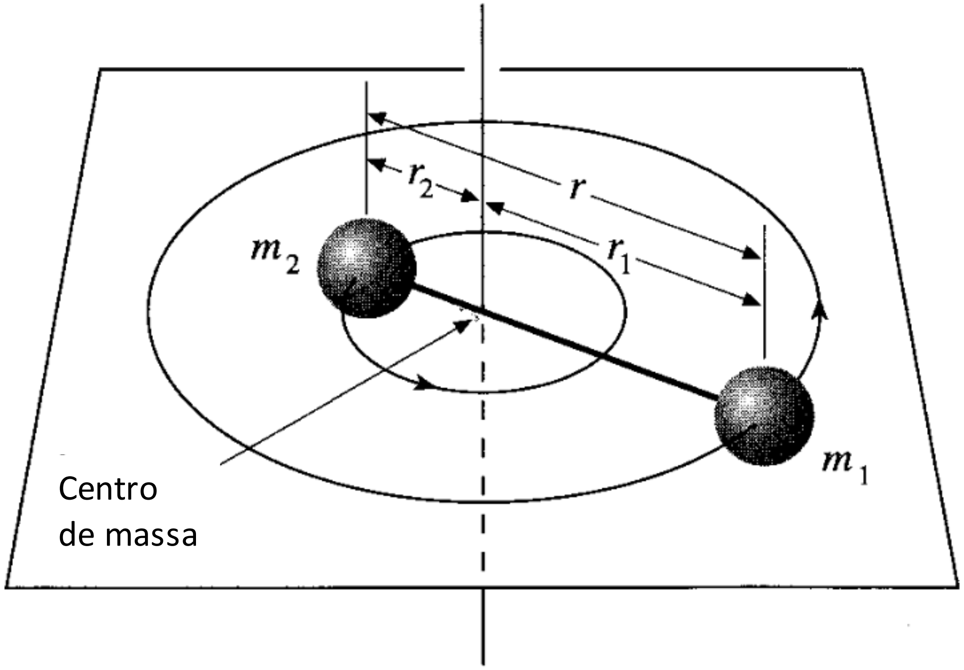
Rotor rígido
Rotações moleculares



- Massas m_1 e m_2 rodam ao redor do centro de massas.
- r_1 e r_2: distâncias fixas até o centro de massas.
- v_{rot}: frequência de rotação.
- Sem ação de forças externas.

Rotor rígido
Rotações moleculares



Velocidades

$$v_1 = 2\pi r_1 \nu_{\text{rot}} = r_1 \omega$$

$$v_2 = 2\pi r_2 \nu_{\text{rot}} = r_2 \omega$$

- $\omega = 2\pi\nu$: frequência angular

Rotor rígido

$$v_1 = r_1\omega \qquad\qquad v_2 = r_2\omega$$

Energia cinética (K)

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\left(m_1r_1^2\omega^2 + m_2r_2^2\omega^2\right) = \frac{1}{2}\left(m_1r_1^2 + m_2r_2^2\right)\omega^2$$

$$= \frac{1}{2}I\omega^2$$

$$I = m_1r_1^2 + m_2r_2^2 = \mu r^2$$

Momento de inércia

$$\mu = \frac{m_1m_2}{m_1 + m_2}$$

Massa reduzida

Rotor rígido

Energia cinética (K)

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

$$= \frac{1}{2} I \omega^2$$

Momento angular (L)

$$L = I\omega$$

$$K = \frac{L^2}{2I} \qquad V(x, y, z) = 0$$

Rotor rígido

$$E = K + V$$

Mecânica clássica

$$\hat{H} = \hat{K} + \hat{V}$$

Mecânica quântica

$$K = \frac{L^2}{2I}$$

$$\hat{K} = -\frac{\hbar^2}{2I}\nabla^2$$

(r constante)

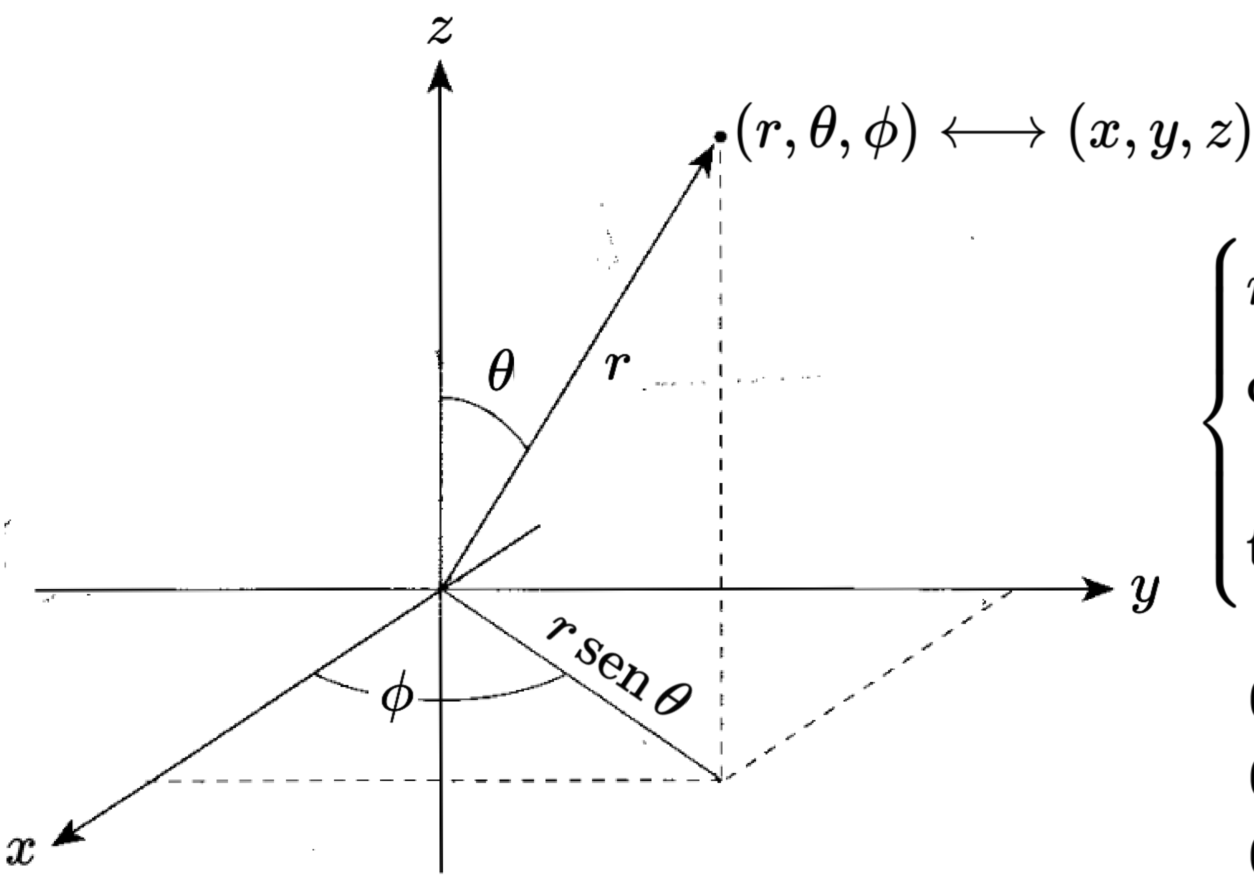
$$V(x, y, z) = 0$$

$$\hat{V} = \hat{O} = 0$$

$$\hat{L}^2 = -\hbar^2\nabla^2 = -\hbar^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\hat{H} = -\frac{\hbar^2}{2I}\nabla^2 = \frac{\hat{L}^2}{2I}$$

Coordenadas esféricas



$$\begin{cases} r = \left(x^2 + y^2 + z^2\right)^{1/2} \\ \cos\theta = \dfrac{z}{\left(x^2 + y^2 + z^2\right)^{1/2}} \\ \tan\phi = \dfrac{y}{x} \end{cases}$$

$$0 \leq r < \infty$$
$$0 \leq \theta \leq \pi$$
$$0 \leq \phi \leq 2\pi$$

$$\begin{cases} x = r \operatorname{sen}\theta \cos\phi & -\infty < x < \infty \\ y = r \operatorname{sen}\theta \operatorname{sen}\phi & -\infty < y < \infty \\ z = r\cos\theta & -\infty < z < \infty \end{cases}$$

Rotor rígido
Operador momento angular

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \begin{cases} x = r\,\mathrm{sen}\,\theta\cos\phi \\ y = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\phi \\ z = r\cos\theta \end{cases}$$

↓ Regra da cadeia

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)_{\theta,\phi} + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial}{\partial\theta}\right)_{r,\phi} + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta}$$

Para r constante:

$$\nabla^2 = \frac{1}{r^2}\frac{1}{\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2}\frac{1}{\mathrm{sen}^2\,\theta}\frac{\partial^2}{\partial\phi^2}$$

Rotor rígido
Operador momento angular

$$\nabla^2 = \frac{1}{r^2}\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2}\frac{1}{\operatorname{sen}^2\theta}\frac{\partial^2}{\partial\phi^2}$$

(para r constante)

$$\hat{L}^2 = -\hbar^2\nabla^2$$

$$\hat{H} = \frac{\hat{L}^2}{2I}$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]$$

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]$$

Rotor rígido
Equação de Schrödinger

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]$$

$$\hat{H}Y(\theta,\phi) = EY(\theta,\phi)$$

$$-\frac{\hbar^2}{2I}\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]Y(\theta,\phi) = EY(\theta,\phi)$$

Rotor rígido
Equação de Schrödinger

$$-\frac{\hbar^2}{2I}\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]Y(\theta,\phi)=EY(\theta,\phi)$$

$$\times \operatorname{sen}^2\theta$$

$$-\frac{\hbar^2}{2I}\left[\operatorname{sen}\theta\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right)+\left(\frac{\partial^2}{\partial\phi^2}\right)\right]Y(\theta,\phi)=E\operatorname{sen}^2\theta Y(\theta,\phi)$$

$$\left[\operatorname{sen}\theta\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right)+\left(\frac{\partial^2}{\partial\phi^2}\right)\right]Y(\theta,\phi)=-(2IE/\hbar^2)\operatorname{sen}^2\theta Y(\theta,\phi)$$

$$\beta=\frac{2IE}{\hbar^2}$$

$$\operatorname{sen}\theta\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial Y}{\partial\theta}\right)+\frac{\partial^2 Y}{\partial\phi^2}+\left(\beta\operatorname{sen}^2\theta\right)Y=0$$

Rotor rígido
Harmônicos esféricos

$$\operatorname{sen}\theta\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial Y}{\partial\theta}\right)+\frac{\partial^2 Y}{\partial\phi^2}+\left(\beta\operatorname{sen}^2\theta\right)Y=0$$

$$\beta=\frac{2IE}{\hbar^2}$$

Separação de variáveis

$$Y(\theta,\phi)=\Theta(\theta)\Phi(\phi)$$

$$\Phi(\phi)\operatorname{sen}\theta\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\operatorname{sen}\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+\Theta(\theta)\frac{\mathrm{d}^2 Y}{\mathrm{d}\phi^2}+\left(\beta\operatorname{sen}^2\theta\right)\Theta(\theta)\Phi(\phi)=0$$

$$\div\Theta(\theta)\Phi(\phi)$$

$$\frac{\operatorname{sen}\theta}{\Theta(\theta)}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\operatorname{sen}\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+\beta\operatorname{sen}^2\theta+\frac{1}{\Phi(\phi)}\frac{\mathrm{d}^2\Phi}{\mathrm{d}\phi^2}=0$$

Rotor rígido
Harmônicos esféricos

$$\underbrace{\frac{\operatorname{sen}\theta}{\Theta(\theta)}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\operatorname{sen}\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+\beta\operatorname{sen}^2\theta}_{\text{Só depende de }\theta}+\underbrace{\frac{1}{\Phi(\phi)}\frac{\mathrm{d}^2\Phi}{\mathrm{d}\phi^2}}_{\text{Só depende de }\phi}=0$$

$$\begin{cases}\dfrac{\operatorname{sen}\theta}{\Theta(\theta)}\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left(\operatorname{sen}\theta\dfrac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+\beta\operatorname{sen}^2\theta=m^2\\[2ex]\dfrac{1}{\Phi(\phi)}\dfrac{d^2\Phi}{\mathrm{d}\phi^2}=-m^2\end{cases}$$

m^2 é uma constante (de separação de variáveis).

Rotor rígido
Harmônicos esféricos

$$\frac{1}{\Phi(\phi)}\frac{d^2\Phi}{\mathrm{d}\phi^2} = -m^2$$

$$\frac{d^2\Phi}{\mathrm{d}\phi^2} = -m^2\Phi(\phi)$$

Qual função $\Phi(\phi)$ que derivada duas vezes resulta nela mesma vezes uma constante ao quadrado com o sinal trocado?

Fórmula de Euler

$$e^{\mathrm{i}x} = \cos x + \mathrm{i}\,\mathrm{sen}\, x$$

$$\Phi(\phi) = A_m e^{\mathrm{i}m\phi}$$

$$\Phi(\phi) = A_{-m} e^{-\mathrm{i}m\phi}$$

Rotor rígido
Harmônicos esféricos

$$\frac{d^2\Phi}{\mathrm{d}\phi^2} = -m^2\Phi(\phi)$$

$$\Phi(\phi) = A_m e^{im\phi}$$

$$\frac{\mathrm{d}(A_m e^{\mathrm{i}m\phi})}{\mathrm{d}\phi} = \mathrm{i}mA_m e^{im\phi} = \mathrm{i}m\Phi(\phi)$$

$$\frac{\mathrm{d}^2\left(A_m e^{\mathrm{i}m\phi}\right)}{\mathrm{d}\phi^2} = \frac{\mathrm{d}}{\mathrm{d}\phi}\frac{\mathrm{d}\left(A_m e^{\mathrm{i}m\phi}\right)}{\mathrm{d}\phi} = (\mathrm{i})^2 m^2\Phi(\phi)$$

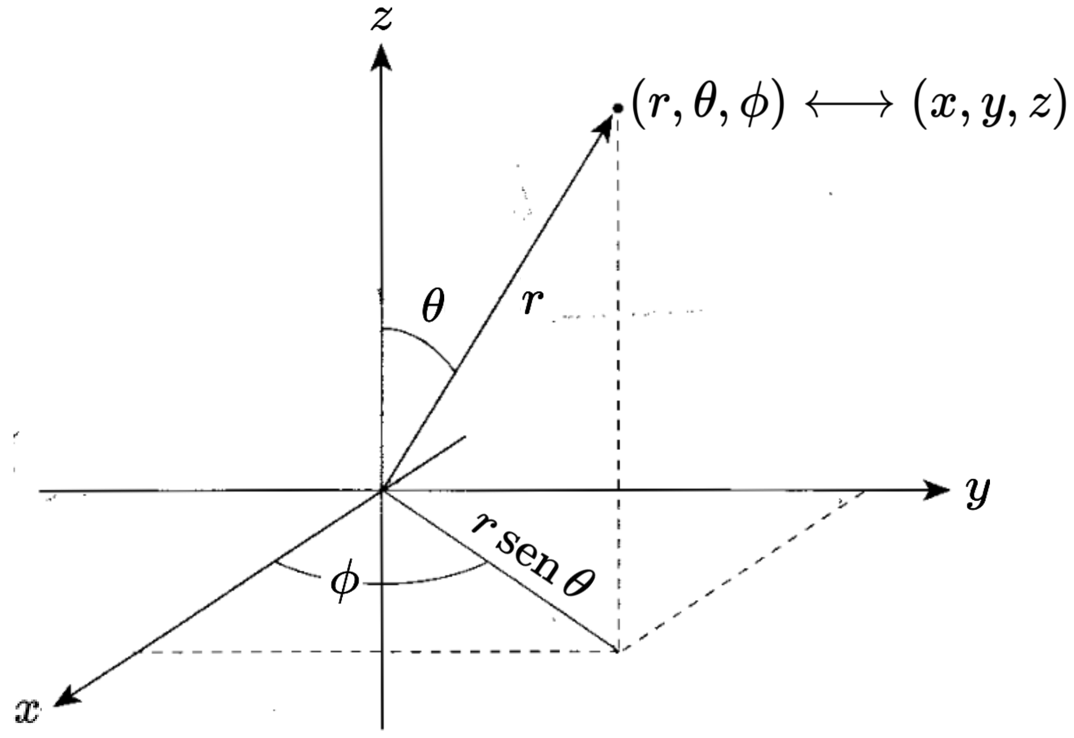
$$\frac{\mathrm{d}^2\left(A_m e^{\mathrm{i}m\phi}\right)}{\mathrm{d}\phi^2} = -m^2 A_m e^{\mathrm{i}m\phi}$$

O mesmo pode ser verificado para outra solução.

Rotor rígido
Harmônicos esféricos

$\Phi(\phi) = A_m e^{\mathrm{i}m\phi}$ deve ser função unívoca de ϕ.

$\Phi(\phi + 2\pi) = \Phi(\phi)$



Rotor rígido
Harmônicos esféricos

$$\Phi(\phi) = A_m e^{\mathrm{i}m\phi} \qquad \Phi(\phi) = A_{-m} e^{-\mathrm{i}m\phi} \qquad \Phi(\phi + 2\pi) = \Phi(\phi)$$

$$A_m e^{\mathrm{i}m(\phi+2\pi)} = A_m e^{\mathrm{i}m\phi}$$

$$e^{\mathrm{i}m(\phi+2\pi)} = e^{\mathrm{i}m\phi}$$

$$\frac{e^{\mathrm{i}m(\phi+2\pi)}}{e^{\mathrm{i}m\phi}} = 1$$

$$e^{\mathrm{i}2\pi m} = 1$$

$$A_{-m} e^{-\mathrm{i}m(\phi+2\pi)} = A_{-m} e^{-\mathrm{i}m\phi}$$

$$e^{-\mathrm{i}m(\phi+2\pi)} = e^{-\mathrm{i}m\phi}$$

$$\frac{e^{-\mathrm{i}m(\phi+2\pi)}}{e^{-\mathrm{i}m\phi}} = 1$$

$$e^{-\mathrm{i}2\pi m} = 1$$

$$e^{\pm\mathrm{i}2\pi m} = 1$$

Rotor rígido
Harmônicos esféricos

$$e^{\pm \mathrm{i}2\pi m} = 1$$

Fórmula de Euler

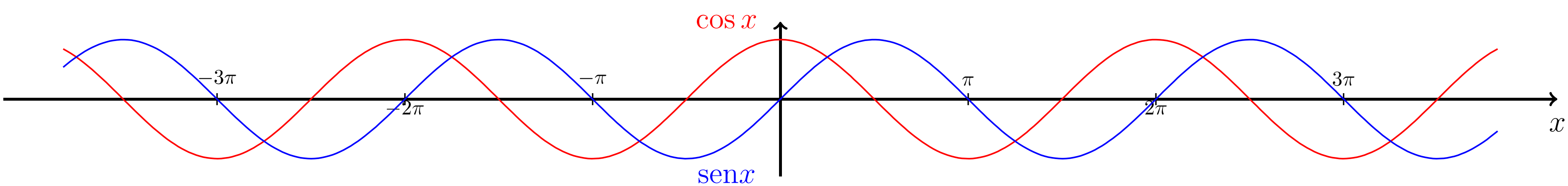
$$e^{\mathrm{i}x} = \cos x + \mathrm{i}\,\mathrm{sen}\,x$$

$$x = 2\pi m$$

$$\cos(2\pi m) \pm \mathrm{i}\,\mathrm{sen}(2\pi m) = 1$$

$$\begin{cases} \mathrm{sen}(2\pi m) = 0 \\ \cos(2\pi m) = 1 \end{cases}$$

$$m = 0, \pm 1, \pm 2, \ldots$$



Rotor rígido
Harmônicos esféricos

$$\Phi_m(\phi) = A_m e^{\mathrm{i}m\phi} \quad m = 0, \pm 1, \pm 2, \ldots$$

$\Phi_m(\phi)$ deve ser normalizada.

$$\int_0^{2\pi} \Phi_m^*(\phi)\Phi_m(\phi)d\phi = 1$$

$$|A_m|^2 2\pi = 1$$

$$\int_0^{2\pi} A_m^* e^{-\mathrm{i}m\phi} A_m e^{\mathrm{i}m\phi} \mathrm{d}\phi = 1$$

$$A_m = (2\pi)^{-1/2}$$

$$|A_m|^2 \int_0^{2\pi} d\phi = 1$$

$$\Phi_m(\phi) = \frac{1}{(2\pi)^{1/2}} e^{im\phi} \quad m = 0, \pm 1, \pm 2, \ldots$$

Rotor rígido
Harmônicos esféricos

$$\begin{cases} \dfrac{\operatorname{sen}\theta}{\Theta(\theta)}\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left(\operatorname{sen}\theta\dfrac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+\beta\operatorname{sen}^2\theta=m^2 \\ \dfrac{1}{\Phi(\phi)}\dfrac{d^2\Phi}{\mathrm{d}\phi^2}=-m^2 \end{cases}$$

$$\longrightarrow \quad \Phi_m(\phi)=\frac{1}{(2\pi)^{1/2}}e^{im\phi} \qquad m=0,\pm1,\pm2,\ldots$$

Equação para $\Theta(\theta)$ não tem coeficientes constantes: difícil de resolver.

$$\Theta(\theta)=\left[\frac{(2l+1)}{2}\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2}P_l^{|m|}(\cos\theta)$$

$P_l^{|m|}(x)$: funções associadas de Legendre

(polinômios em $\cos\theta$ e $\operatorname{sen}\theta$).

Condições da solução:

$$\beta=l(l+1)$$

$$l=0,1,2,\ldots$$

$$m=0,\pm1,\pm2,\ldots,\pm l$$

Rotor rígido
Harmônicos esféricos

$$Y_l^m(\theta,\phi) = \left[\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2} P_l^{|m|}(\cos\theta)e^{im\phi}$$

$$l = 0, 1, 2, \ldots$$

$$m = 0, \pm 1, \pm 2, \ldots, \pm l$$

Notação específica para rotor rígido:

$$l \to J = 0, 1, 2, \ldots$$

$$m \to M_J = 0, \pm 1, \pm 2, \ldots, \pm J$$

$$Y_l^m(\theta,\phi) \to Y_J^{M_J}(\theta,\phi)$$

Rotor rígido
Harmônicos esféricos

$$Y_0^0 = \frac{1}{(4\pi)^{1/2}}$$

$$Y_1^1 = \left(\frac{3}{8\pi}\right)^{1/2} \operatorname{sen}\theta e^{i\phi}$$

$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

$$Y_1^{-1} = \left(\frac{3}{8\pi}\right)^{1/2} \operatorname{sen}\theta e^{-i\phi}$$

$$Y_2^2 = \left(\frac{15}{32\pi}\right)^{1/2} \operatorname{sen}^2\theta e^{2i\phi}$$

$$Y_2^1 = \left(\frac{15}{8\pi}\right)^{1/2} \operatorname{sen}\theta \cos\theta e^{i\phi}$$

$$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$$

$$Y_2^{-1} = \left(\frac{15}{8\pi}\right)^{1/2} \operatorname{sen}\theta \cos\theta e^{-i\phi}$$

$$Y_2^{-2} = \left(\frac{15}{32\pi}\right)^{1/2} \operatorname{sen}^2\theta e^{-2i\phi}$$

Rotor rígido
Harmônicos esféricos
Autovalores e autofunções

$Y_J^{M_J}(\theta, \phi)$ são autofunções de $\hat{L}^2$.

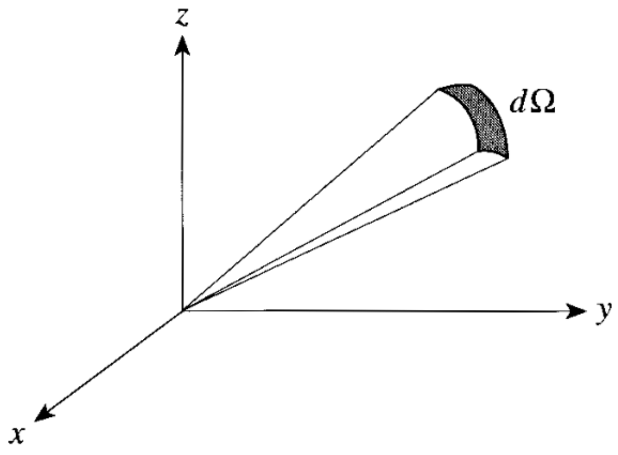
$$\underbrace{\hat{L}^2}_{\text{Operador}} \underbrace{Y_J^{M_J}(\theta, \phi)}_{\text{Autofunção}} = \underbrace{\hbar^2 J(J+1)}_{\text{Autovalor}} \underbrace{Y_J^{M_J}(\theta, \phi)}_{\text{Autofunção}}$$

Os valores possíveis de L^2 são $\hbar^2 J(J+1)$, com $J = 0, 1, 2, \ldots$

Rotor rígido
Harmônicos esféricos
Ortonormalidade

$$\int_0^{\pi} \mathrm{d}\theta \, \mathrm{sen}\, \theta \int_0^{2\pi} \mathrm{d}\phi Y_l^m(\theta,\phi)^* Y_n^k(\theta,\phi) = \delta_{ln}\delta_{mk}$$

Note que a integral é sobre uma superfície esférica, cujo elemento diferencial de área é $\mathrm{d}\Omega = \mathrm{sen}\,\theta \mathrm{d}\theta \mathrm{d}\phi$.



Rotor rígido
Energia

$$\hat{H}Y_J^{M_J}(\theta,\phi) = \frac{\hat{L}^2}{2I}Y_J^{M_J}(\theta,\phi) = EY_J^{M_J}(\theta,\phi)$$

$$-\frac{\hbar^2}{2I}\underbrace{\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\operatorname{sen}^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)\right]}Y_J^{M_J}(\theta,\phi) = EY_J^{M_J}(\theta,\phi)$$

$$\beta = \frac{2IE}{\hbar^2} = J(J+1) \qquad J = 0, 1, 2, \ldots$$

$$E_J = \frac{\hbar^2}{2I}J(J+1)$$

$$J = 0, 1, 2, \ldots$$

Rotor rígido
Energia

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

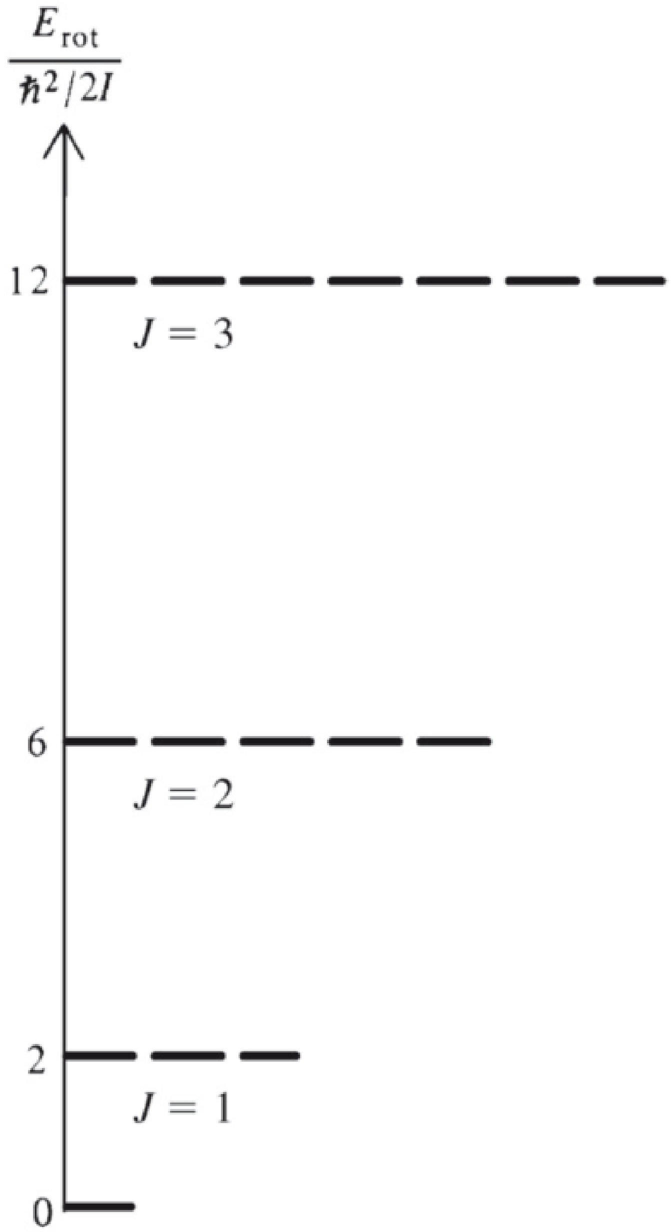
$$J = 0, 1, 2, \ldots$$

$$M_J = 0, \pm 1, \pm 2, \ldots, \pm J$$

O número quântico M_J não aparece na expressão da energia:

- Estados com mesmo J e diferentes M_J têm a mesma energia, são degenerados.

Degenerescência: $g_J = 2J + 1$



Espectroscopia rotacional

Regra de seleção: $\Delta J = \pm 1$ (Transição observada/permitida)

$$\Delta E = E_{J+1} - E_J$$

$$= \frac{\hbar^2}{2I}\{(J+1)(J+2) - J(J+1)\}$$

$$= \frac{\hbar^2}{2I}\{\cancel{J^2} + 3J + 2 - \cancel{J^2} - J\}$$

$$= \frac{h^2}{8\pi^2 I}\{2J+2\} = \frac{h^2}{4\pi^2 I}(J+1)$$

Espectroscopia rotacional

Regra de seleção: $\Delta J = \pm 1$ (Transição observada/permitida)

$$\Delta E = E_{J+1} - E_J \quad = \frac{h^2}{4\pi^2 I}(J+1)$$

$$\Delta E = h\nu$$

$$\nu = \frac{\Delta E}{h} = \frac{h}{4\pi^2 I}(J+1)$$

$$J = 0, 1, 2, \ldots$$

É comum reescrever assim:

$$\nu = 2B(J+1)$$

$$B = \frac{h}{8\pi^2 I}$$

Constante rotacional (frequência)

Espectroscopia rotacional

$$\nu = 2B(J+1) \qquad J = 0, 1, 2, \ldots$$

$$B = \frac{h}{8\pi^2 I}$$

Constante rotacional (frequência)

Em termos do número de onda ($\tilde{\nu}$)

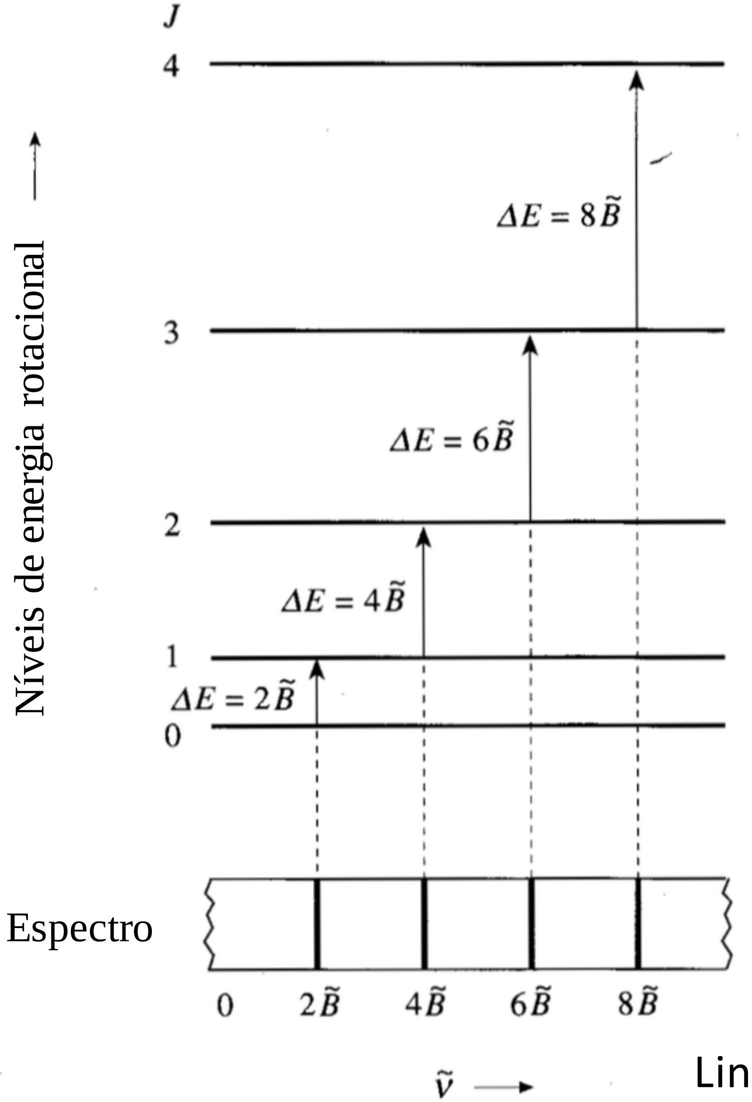
$$\tilde{\nu} = \frac{\nu}{c}$$

$$\tilde{\nu} = 2\tilde{B}(J+1) \qquad J = 0, 1, 2, \ldots$$

$$\tilde{B} = \frac{h}{8\pi^2 \tilde{c} I}$$

Constante rotacional (núm. de onda)

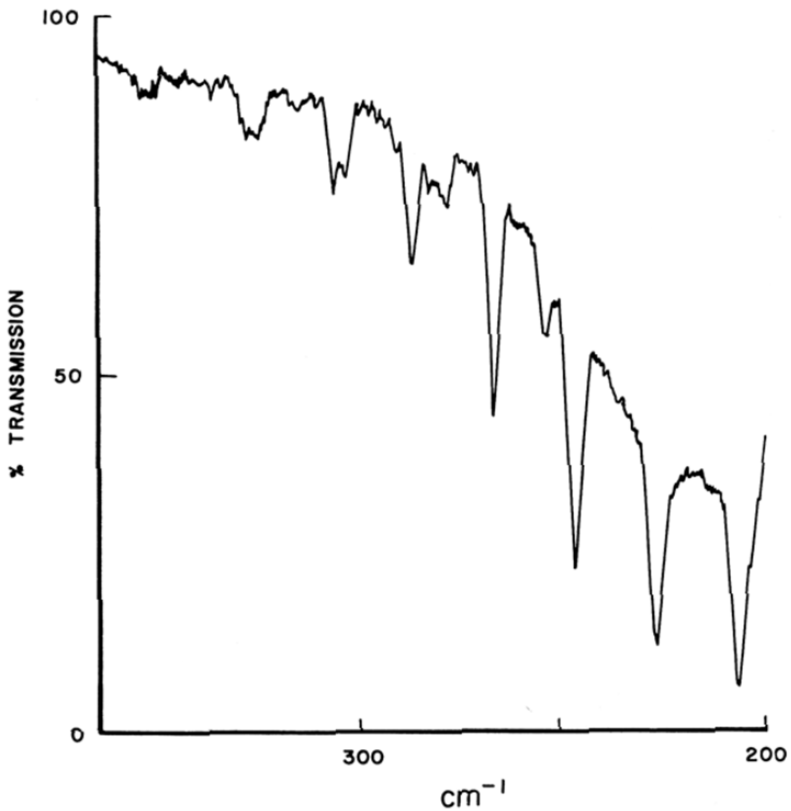
Espectroscopia rotacional



Linhas igualmente espaçadas de $2B$.

Espectroscopia rotacional
Exemplo

O espectro micro-ondas do $H^{35}Cl$ consiste numa série de de linhas igualmente espaçadas de 20,9 cm^{-1}. Determine o comprimento da ligação do $H^{35}Cl$.



$$2\tilde{B} = 20{,}9 \text{ cm}^{-1}$$

$$\tilde{B} = \frac{h}{8\pi^2 \tilde{c} I}$$

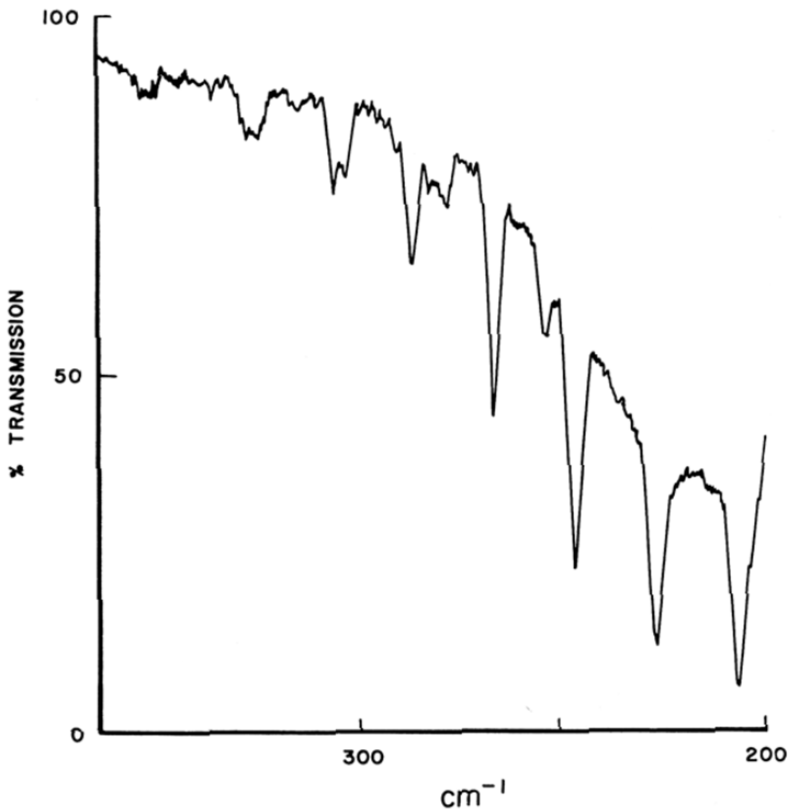
$$I = \mu R^2$$

$$2 \times \left(\frac{h}{8\pi^2 \tilde{c} \mu R^2} \right) = 20{,}9 \text{ cm}^{-1}$$

$$R = \left(\frac{h}{4\pi^2 \mu \tilde{c} (20{,}9 \text{ cm}^{-1})} \right)^{1/2}$$

Espectroscopia rotacional
Exemplo

O espectro micro-ondas do $H^{35}Cl$ consiste numa série de de linhas igualmente espaçadas de 20,9 cm^{-1}. Determine o comprimento da ligação do $H^{35}Cl$.



$$R = \left(\frac{h}{4\pi^2 \mu \tilde{c} \, (20{,}9 \text{ cm}^{-1})} \right)^{1/2}$$

$$\mu(^1\mathrm{H}^{35}\mathrm{Cl}) = 1{,}6266 \times 10^{-27} \text{ kg}$$

$$\tilde{c} = 2{,}99792458 \times 10^{10} \text{ cm s}^{-1}$$

$$h = 6{,}626\,070\,15 \times 10^{-34} \text{ J s}$$

$$R = 1{,}28 \times 10^{-10} \text{ m}$$

$$\boxed{R = 1{,}28 \text{ Å}}$$