Exercícios Capítulo 2

1. Considere o modelo de regressão linear passando pela origem

$$Y_i = \beta X_i + \varepsilon_i, \qquad (i = 1, \ldots, n).$$

Pede-se:

a) Mostre que a estimativa de quadrados mínimos de β é dada por:

$$\hat{\beta} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} {X_i}^2}.$$

b) Obtenha $\mathrm{Var}(\hat{\beta})$.

2. Seja

$$\begin{aligned} Y_1 &= \theta + \varepsilon_1 \\ Y_2 &= 2\theta - \phi + \varepsilon_2 \\ Y_3 &= \theta + 2\phi + \varepsilon_3 \end{aligned}$$

em que $\mathrm{E}(\varepsilon_i) = 0$ $(i = 1, 2, 3)$. Encontre as estimativas de quadrados mínimos de θ e ϕ.

3) Completar a tabela a seguir

i	Y_i	X_i	X_i^2	X_iY_i	x_i	x_i^2	y_i	x_iY_i	X_iy_i	x_iy_i
1	2,0	1,0								
2	5,0	3,0								
3	5,6	4,0								
4	8,5	6,0								
5	9,0	7,0								
6	13,0	10,0								
Soma										
Média			----	---	---	---	---	---	---	---

a. Obter as estimativas dos parâmetros $\hat{\beta}_1$ e $\hat{\beta}_0$ utilizando os seguintes estimadores

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{n}}{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} =$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i y_i}{\sum_{i=1}^{n} x_i^2} =$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i Y_i}{\sum_{i=1}^{n} x_i^2} =$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} =$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} =$$

Vimos também que, os parâmetros $\hat{\beta}_1$ e $\hat{\beta}_0$ são combinações lineares dos Y_i

$$\hat{\beta}_1 = \sum_{i=1}^{n} c_i Y_i \quad \text{em que: } c_i = \frac{x_i}{\sum_{i=1}^{n} x_i^2}$$

$$\hat{\beta}_0 = \sum_{i=1}^{n} d_i Y_i \quad \text{em que: } d_i = \frac{1}{n} - \bar{X} c_i$$

b. Complete a tabela a seguir e verifique as afirmações anteriores

i	Y_i	X_i	x_i	x_i^2	c_i	$c_i Y_i$	d_i	$d_i Y_i$
1	2,0	1,0						
2	5,0	3,0						
3	5,6	4,0						
4	8,5	6,0						
5	9,0	7,0						
6	13,0	10,0						
Soma					----		---	
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