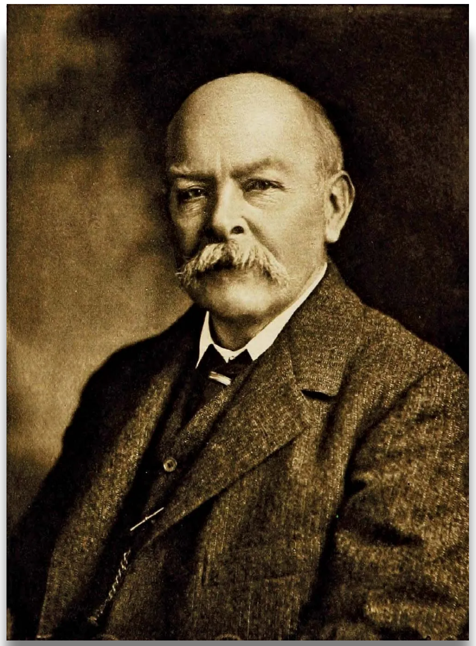
Eletromagnetismo Avançado

25 de agosto

Leis de conservação

Conservação do momento

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$



Conservação do momento

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\mathbf{f}} = \rho \vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}$$

$$\begin{aligned}\vec{\mathbf{f}} =& \epsilon_0 \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}} \left(\frac{E^2}{2} \right) \right) \\ &+ \frac{1}{\mu_0} \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \left(\frac{B^2}{2} \right) \right) \\ &- \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)\end{aligned}$$

Tensores

$$\overset{\leftrightarrow}{W} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} u_x v_x & u_x v_y & u_x v_z \\ u_y v_x & u_y v_y & u_y v_z \\ u_z v_x & u_z v_y & u_z v_z \end{bmatrix}$$

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$$\overleftrightarrow{W} = \vec{\mathbf{u}} \otimes \vec{\mathbf{v}}$$

$$W_{ij} = u_i v_j$$

$$t = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$t = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$$

$$t = \sum_i u_i v_i \equiv u_i v_i$$

Tensores

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$$\overleftrightarrow{W} = \vec{\mathbf{u}} \otimes \vec{\mathbf{v}} \qquad\qquad W_{ij} = u_i v_j$$

$$\vec{\mathbf{q}} \cdot \overleftrightarrow{W} = (\vec{\mathbf{q}} \cdot \vec{\mathbf{u}})\vec{\mathbf{v}}$$

Tensores

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$$\overleftrightarrow{W} = \vec{\mathbf{u}} \otimes \vec{\mathbf{v}} \qquad W_{ij} = u_i v_j$$

$$\vec{\mathbf{q}} \cdot \overleftrightarrow{W} = (\vec{\mathbf{q}} \cdot \vec{\mathbf{u}})\vec{\mathbf{v}} \qquad \left(\vec{\mathbf{q}} \cdot \overleftrightarrow{W}\right)_j = u_i v_i q_j$$

Tensores

$$\overleftrightarrow{W} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} u_x v_x & u_x v_y & u_x v_z \\ u_y v_x & u_y v_y & u_y v_z \\ u_z v_x & u_z v_y & u_z v_z \end{bmatrix}$$

$$\overleftrightarrow{W} = \vec{\mathbf{u}} \otimes \vec{\mathbf{v}}$$

$$W_{ij} = u_i v_j$$

$$\vec{\mathbf{q}} \cdot \overleftrightarrow{W} = (\vec{\mathbf{q}} \cdot \vec{\mathbf{u}})\vec{\mathbf{v}}$$

$$\left(\vec{\mathbf{q}} \cdot \overleftrightarrow{W}\right)_j = u_i v_i q_j$$

$$\overleftrightarrow{W} \cdot \vec{\mathbf{q}} = \vec{\mathbf{u}}(\vec{\mathbf{v}} \cdot \vec{\mathbf{q}}) = (\vec{\mathbf{v}} \cdot \vec{\mathbf{q}})\vec{\mathbf{u}}$$

Conservação do momento

$$\vec{\mathbf{f}} = \epsilon_0 \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}} \left(\frac{E^2}{2} \right) \right)$$

$$+ \frac{1}{\mu_0} \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \left(\frac{B^2}{2} \right) \right)$$

$$- \epsilon_0 \frac{\partial}{\partial t} \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \right)$$

Conservação do momento

$$\vec{\mathbf{f}} = \epsilon_0 \Big((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}} \left(\frac{E^2}{2} \right) \Big)$$

$$+ \frac{1}{\mu_0} \Big((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \left(\frac{B^2}{2} \right) \Big)$$

$$- \epsilon_0 \frac{\partial}{\partial t} \Big(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \Big)$$

$$\overleftrightarrow{T} = \epsilon_0 \Big(\vec{\mathbf{E}} \otimes \vec{\mathbf{E}} - \frac{1}{2} E^2 \mathbb{1} \Big) + \frac{1}{\mu_0} \Big(\vec{\mathbf{B}} \otimes \vec{\mathbf{B}} - \frac{1}{2} B^2 \mathbb{1} \Big)$$

Conservação do momento

$$\vec{\mathbf{f}} = \epsilon_0 \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}} \left(\frac{E^2}{2} \right) \right)$$

$$+ \frac{1}{\mu_0} \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \left(\frac{B^2}{2} \right) \right)$$

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$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} E^2 \delta_{ij} \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

Conservação do momento

$$T_{ij} = \epsilon_0\left(E_i\,E_j - \frac{1}{2}E^2\delta_{ij}\right) + \frac{1}{\mu_0}\left(B_i\,B_j - \frac{1}{2}B^2\delta_{ij}\right)$$

Conservação do momento

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} E^2 \delta_{ij} \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

$$\begin{aligned} \partial_i T_{ij} =& \epsilon_0 \left((\partial_i E_i) E_j + E_i \partial_i E_j - \frac{1}{2} \partial_i E^2 \right) \\ &+ \frac{1}{\mu_0} \left((\partial_i B_i) B_j + B_i \partial_i B_j - \frac{1}{2} \partial_i B^2 \right) \end{aligned}$$

Conservação do momento

$$T_{ij} = \epsilon_0\Big(E_i E_j - \frac{1}{2}E_i E_i \delta_{ij}\Big) + \frac{1}{\mu_0}\Big(B_i B_j - \frac{1}{2}B_i B_i \delta_{ij}\Big)$$

$$\partial_i T_{ij} = \epsilon_0\Big((\partial_i E_i)E_j + E_i \partial_i E_j - \frac{1}{2}\partial_i E^2\Big)$$
$$+\frac{1}{\mu_0}\Big((\partial_i B_i)B_j + B_i \partial_i B_j - \frac{1}{2}\partial_i B^2)$$

$$\vec{\boldsymbol{\nabla}}\cdot\overleftrightarrow{T} = \epsilon_0\Big((\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}}\left(\frac{E^2}{2}\right)\Big)$$
$$+\frac{1}{\mu_0}\Big((\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}}\left(\frac{B^2}{2}\right)\Big)$$

Conservação do momento

$$\vec{\mathbf{f}} = \epsilon_0 \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} - \vec{\boldsymbol{\nabla}} \left(\frac{E^2}{2} \right) \right)$$
$$+ \frac{1}{\mu_0} \left((\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \left(\frac{B^2}{2} \right) \right)$$
$$- \epsilon_0 \frac{\partial}{\partial t} \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \right)$$

$$\vec{\mathbf{f}} = \vec{\boldsymbol{\nabla}} \cdot \overleftrightarrow{T} - \epsilon_0 \frac{\partial}{\partial t} \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \right)$$