# Intervalo de confiança (IC) para $\mu$ de população $N(\mu, \sigma^2)$
## $\sigma^2$ - conhecido

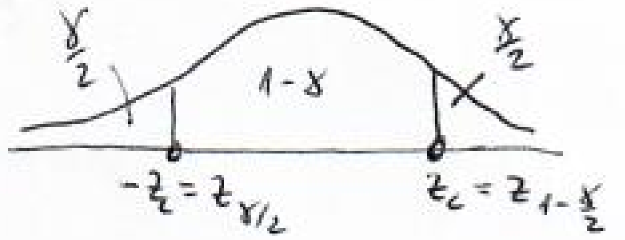
$$X \sim N(\mu, \sigma^2) \Rightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right) \Rightarrow \frac{\bar{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} = \sqrt{n}\,\frac{\bar{X}-\mu}{\sigma} \sim N(0,1)$$

**Def.** $P(|X-\mu| \le \varepsilon) = (1-\gamma)100\%$

$\varepsilon$ - erro, $\gamma$ - pequeno (0.05, 0.001, 0.1) e conhecido

$$P(|\bar{X}-\mu| \le \varepsilon) = P(-\varepsilon \le \bar{X}-\mu \le \varepsilon) = P(\bar{X}-\varepsilon \le \mu \le \bar{X}+\varepsilon)$$

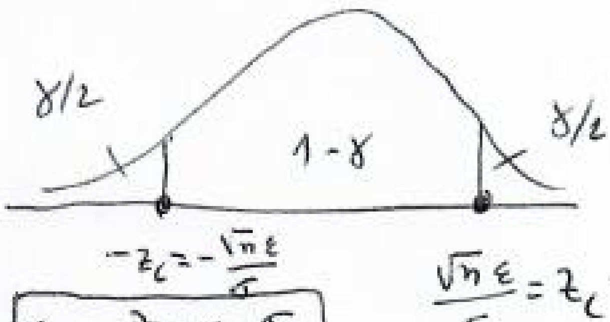
$\Rightarrow (1-\gamma)100\%$ IC $= [\bar{X}-\varepsilon, \bar{X}+\varepsilon]$ para $\mu$




Sabendo $\underline{\underline{\gamma}}$, determinamos $z_c$ via tabela $N(0,1)$

| $\gamma$ | $1-\gamma$ | $z_c = z_{1-\gamma/2} = -z_{\gamma/2}$ |
|---|---|---|
| 0.2 | 0.8 | $z_{0.9} = 1,28$ |
| 0.1 | 0.9 | $z_{0.95} = 1.64$ |
| 0.05 | 0.95 | $z_{0.975} = 1.96$ |
| 0.02 | 0.98 | $z_{0.99} = 2.33$ |
| 0.01 | 0.99 | $z_{0.995} = 2.58$ |

Verifiquem!




$\Rightarrow \boxed{\varepsilon = z_{1-\gamma/2}\frac{\sigma}{\sqrt{n}}}$

$$P(|\bar{X}-\mu| \le \varepsilon) = P(-\varepsilon \le \bar{X}-\mu \le \varepsilon) = P\left(-\frac{\varepsilon}{\sqrt{\frac{\sigma^2}{n}}} \le \frac{\bar{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{\sigma^2}{n}}}\right)$$

$$= P\left(-\frac{\sqrt{n}\,\varepsilon}{\sigma} \le Z \le \frac{\sqrt{n}\,\varepsilon}{\sigma}\right) = (1-\gamma)100\%$$

$\boxed{\text{Então } (1-\gamma)100\% \text{ IC para } \mu \text{ é } \left[\bar{X} - z_c\frac{\sigma}{\sqrt{n}}, \bar{X} + z_c\frac{\sigma}{\sqrt{n}}\right]}$

De $\underline{\varepsilon = z_{1-\frac{\gamma}{2}}\frac{\sigma}{\sqrt{n}}}$ $\Rightarrow$ determinamos tamanho de amostra $\boxed{n = \frac{z^2_{1-\frac{\gamma}{2}}\,\sigma^2}{\varepsilon^2}, \text{ se erro } \underline{\underline{\varepsilon}} \text{ é dado}}$

**Exemplo** $\bar{X} = 5$, $n = 25$. Determine 95% IC para $\mu$, se $\sigma^2 = 4$

95% IC $\Rightarrow \gamma = 0.95 \Rightarrow z_c = z_{0.975} = 1.96$

95% IC para $\mu$: $\left[5 - 1.96 \times \frac{2}{\sqrt{25}}, 5 - 1.96 \times \frac{2}{\sqrt{25}}\right] = [4.216; 5.784]$

Interpretação: Em 95% dos casos $\mu \in [4.216; 5.784]$

Se $\varepsilon = 0.5 \Rightarrow$ tamanho de amostra deve ser $n = \frac{(1.96)^2 2^2}{(0.5)^2} \simeq \underline{\underline{64}}$

Exemplo (casa): $X \sim N(\mu, \sigma^2 = 16)$. Amostra de tamanho $n = 36$ produziu $\bar{X} = 72$.

a) Determine 99% IC para $\mu$.
b) Qual seria o tamanho de amostra, se erro $\varepsilon = 2$?