A robust blade element momentum theory model for tidal stream turbines including tip and hub loss corrections

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A robust blade element momentum theory model for tidal stream turbines including tip and hub loss corrections

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Blade Element Momentum Theory (BEMT) performance models for wind turbines lead to a robust BEMT model of marine current or tidal stream turbines. It is shown that numerical convergence methods for models reported in the literature are problematic when away from the normal operating range and this paper reports a robust numerical scheme using combined Monte Carlo and sequential quadratic optimisation. The model is extended by Prandtl corrections for losses at the blade tip and hub. Results are validated against an industrial code, Garrad Hassan's Tidal Bladed Software (GH Tidal Bladed) evaluation version, and a lifting line theory model.

AUTHOR'S BIOGRAPHIES

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INTRODUCTION – THE MOTIVATION FOR A ROBUST AND CORRECTED BEMT MODEL

Horizontal axis tidal stream turbines (TST) are systems that convert the energy in flowing water into kinetic energy in the device itself. In order to differentiate these devices from barrage systems, TSTs operate in a free stream flow without the requirement for a significant dam structure. In most designs this is in the form of rotating hydrofoil blades which are used to power an electrical generator.

The aim of this paper is to develop a model to predict the performance and loadings on a TST rotor. It is intended that this would then be used in the design process to characterise the performance and loading requirements of a system, as well as testing specific features and control systems. Despite a lack of accuracy under certain conditions, the low computational demand has lead to a wide acceptance of blade element theory for modelling of turbine rotors. The work discussed here attempts to deal with some of the shortcomings of the method.

Several papers[1, 2] suggest that much can be learnt from the wind turbine industry when predicting the performance of tidal stream turbines. Initially the application of BEMT was marine and aviation propellers but was later applied to wind turbines. Griffiths and Woollard[3] present a clear approach to designing and modelling the performance of an optimally shaped wind turbine using BEMT. It is known[4] that BEMT can be imprecise as it is not able to accurately estimate wake effects or complex three-dimensional flows. In its standard form, BEMT does not take into account secondary effects from three-dimensional flows such as tip vortices and radial flow induced by the rotation of the blade system. By comparison, Navier-Stokes CFD-based methods give very good results as they are able to capture viscous and compressible flow effects but the high computational demand of such an approach limits its application.

BEMT CORRECTIONS

Kishinami *et al*[5] conducted a comparison of a small-scale wind turbine test with a modelling approach similar to Griffiths' approach. One significant difference was that rotor blade tip losses were taken into account. This was done by significantly reducing the lift coefficient of the aerofoil section from the standard empirical data for radius values larger than 97% of the total blade radius. The basis of deciding the reduction of the lift coefficients is not outlined but is possibly due to empirical data or prior knowledge. The experimental and theoretical results follow similar trends but the estimated error of the experimental results is approximately 18% and therefore firm conclusions cannot be drawn from the results.

In Mikkelsen's thesis,[6] a study of a lifting line technique was carried out. This theory and a Navier-Stokes approach were used to investigate some of the assumptions made in wind turbine BEMT. It is explained that there are some basic assumptions used in blade element theory that are not verified. These are that the flow can be divided into annular stream tubes, that the pressure in the wake far downstream is equal to the upstream pressure and that the induced velocity in the rotor plane is half that of the induced velocity in the far wake. It is also assumed that axial momentum theory can be applied in the differential form neglecting the resulting axial force of pressure acting on lateral boundaries of the stream tube and that conservation of circulation may be ignored. The effect of these assumptions was investigated and it was found that the maximum resulting error is 3% and therefore, despite the inherent assumptions of Blade Element Theory, the error is found to be acceptable for most operating conditions.

Shen *et al*[7] review tip loss corrections for wind turbines and quote Prandtl who developed an approach for tip correction using a factor 'F' to correct the aerodynamic force components. It is shown in Mikkelsen,[6] that this correction can be employed in the axial angular momentum equations of BEMT[8] to give updated axial and tangential flow interference factors. This approach is well accepted but does have some inherent inconsistencies. One such inconsistent characteristic is that as the radius of the blade element tends to the overall blade radius, the axial interference factor tends to unity, meaning that axial velocity becomes zero. This is inconsistent because the applied force at the tip is zero. Refinements[9, 10] were made but these are said to lack rigorous consistency near the tip. A mathematically consistent system was introduced[11] that overcomes the near tip inconsistency by considering a balance of momentum for a real rotor with a finite number of blades with real aerodynamic forces and ensures that the forces at the tip of the blade are zero. Although this approach is an improvement, it still does not completely model the real situation at the tip and includes a factor that must be calibrated using model testing.

Maalawi and Badawi[12] implement BEMT with Prandtl's tip loss factor and attempt to solve this equation system directly. Hence this lowers the computational demand compared to iterative techniques, but as the BEMT approach has a low computational demand compared to full CFD, analytical approaches such as this seem unnecessary. Madsen *et al*[13] compare BEMT to a Navier-Stokes CFD solution. Shortcomings in accuracy are shown to be limited to the blade root and blade tip. Correction approaches are presented to improve the accuracy of BEMT and with these implemented, it is shown that BEMT may be used as an accurate performance prediction approach.

As an alternative approach, Sharpe[14] published a lifting line theory with a prescribed wake. The theory is applied in order to create a wind turbine blade design code. The approach includes radial flow circulation and captures tip and hub loss effects.

As the wind industry is more mature than the marine turbine industry, it is unsurprising that there are commercial and academic codes available to model the performance of wind turbines. The systems discussed all take similar approaches to modelling turbine performance using BEMT with correction factors for tip loss, tower shadow, hub loss and stall delay to improve the accuracy of the systems. As has already been seen, much of wind turbine theory may be relatively simply adapted to apply to marine turbines, for example, Orme developed a tidal turbine model[1, 15] and validated it against a tow test.

BLADE ELEMENT MOMENTUM THEORY

BEMT has its origins in momentum theory and the development from this to BEMT is well explained in many texts.[16, 17] The mathematical basis presented here includes sufficient details of the implementation for the work to be repeated by the reader.

One dimensional momentum theory

This theory calculates the energy absorption of a wind or tidal turbine. The rotor in this case is modelled as a frictionless permeable 'actuator disc' which is assumed to impart no rotational velocity to the flow. A control volume surrounds the actuator disc. The control volume is bounded by a stream-tube, with two cross sections far upstream and far downstream of the disc. It is assumed that the stream-tube does not interact with the fluid outside of the stream-tube. The actuator disc removes energy from the stream-tube by providing a drag force that produces a pressure drop in the fluid just downstream of the disc. Both upstream and downstream surfaces of the stream-tube are assumed to be at ambient static pressure and so the flow speed must drop downstream to satisfy Bernoulli's equation.

If the free-stream flow velocity is equal to U, the downstream flow velocity is equal to U_1, the density ρ and the cross sectional areas at the two points are A_0 and A_1 respectively then the rate of change of momentum of the fluid due to the disk, may be written as equation (1):

$$F_A = U(\rho A_0 U) - U_1(\rho A_1 U_1) \tag{1}$$

where F_A is the axial force.

As mass flow rate $\dot{m}$ is conserved in the stream-tube this can be rewritten as equation (2):

$$F_A = \dot{m}(U - U_1) \tag{2}$$

F_A can also be defined in terms of the pressure differential immediately upstream and downstream of the actuator disc. The static pressures far upstream and far downstream of the actuator disc are equal to ambient, p_{amb}. Introducing the flow speed at the disc, u_{disc} and p_{ud}, p_{dd} as pressures immediately upstream and downstream of the disc and noting that Bernoulli's equation applies separately in the upstream and downstream regions leads to equations (3) and (4) respectively.

$$p_{amb} + \frac{1}{2}\rho U^2 = p_{ud} + \frac{1}{2}\rho u_{disc}^2 \tag{3}$$

$$p_{dd} + \frac{1}{2}\rho u_{disc}{}^2 = p_{amb} + \frac{1}{2}\rho U_1^2 \tag{4}$$

Assuming the cross sectional areas of the stream tubes just upstream and just downstream of the disc are effectively the area of the disc, A_{disc}, equations (3) and (4) can be used to derive equation (5) for F_A based on the pressure differential.

$$F_A = A_{disc}\frac{1}{2}\rho(U^2 - U_1^2) \tag{5}$$

Equating equation (5) to equation (2) yields equation (6):

$$u_{disc} = \frac{U + U_1}{2} \tag{6}$$

If the axial induction factor a, is defined as the fractional reduction in flow speed between free-stream and the actuator disc, then,

$$U_1 = U(1 - 2a) \tag{7}$$

$$u_{disc} = U(1 - a) \tag{8}$$

Using equation (5), the power generated, P, is the thrust multiplied by the flow velocity at the disc, either equation (9) or in terms of a as equation (10):

$$P = A_{disc}\frac{1}{2}\rho(U^2 - U_1^2)u_{disc} = A_{disc}\frac{1}{2}\rho u_{disc}(U - U_1)(U + U_1) \tag{9}$$

$$P = A_{disc}\frac{1}{2}\rho U^3 4a(1 - a)^2 \tag{10}$$

The axial force produced, equation (5), can be written in the same form as equation (11):

$$F_A = A_{disc}\frac{1}{2}\rho U^2 4a(1 - a) \tag{11}$$

Rotational momentum

Some of the energy lost from the axial flow is converted into rotational momentum of the stream-tube, as a reaction to the rotational torque imparted to the turbine rotor. It is generally assumed to be small in comparison to the rotational speed of the system; Manwell[17] states that this allows the assumption that the ambient pressure far upstream is equal to the pressure far downstream.

For the development of a model incorporating rotational effects, the stream-tube is divided into annular sections with local radius r and thickness dr, giving the area of the stream-tube annulus as $2\pi r dr$. A control volume rotating at the speed of the rotor, Ω, is employed to solve the problem, a derivation of the pressure differential just downstream and just upstream of the rotor based on Bernoulli's equation as is given by Glauert.[8] As the axial flow speed is effectively constant but the rotational flow increases by ω just downstream of the rotor, this pressure change may be written in terms of the imparted rotational momentum[18] as equation (12):

$$p_{ud} - p_{dd} = \rho(\Omega + \frac{1}{2}\omega)r^2\omega \tag{12}$$

The elemental thrust may then be calculated as the change in pressure multiplied by the annular area as in equation (13).

$$dF_A = \left(\rho(\Omega + \frac{1}{2}\omega)r^2\omega\right)2\pi r dr \tag{13}$$

The angular or tangential induction factor, b, is now introduced,

$$b = \frac{\omega}{2\Omega} \tag{14}$$

The annular thrust equation (13) can be re-written using equation (14) to give equation (15):

$$dF_A = \left(4b(1 + b)\tfrac{1}{2}\rho\Omega^2 r^2\right)2\pi r dr \tag{15}$$

As wake rotation is now included in the equations, it is possible to develop a formula for the torque produced on the rotor annulus, dT, as it must be equal to the change in angular momentum of the wake. For an annulus this may be written as equation (16):

$$dT = d\dot{m}(\omega r)r = \rho u_{disc} 2\pi r dr(\omega r)r \tag{16}$$

Using equation (8) and equation (14) in equation (16) an expression for elemental torque is obtained in terms of the upstream flow equation (17). The axial thrust in an annulus of area $2\pi r dr$ is derived from equation (11) and is shown in equation (18).

$$dT_1 = 4b(1 - a)\rho U\Omega r^2 \pi r dr \tag{17}$$

$$dF_{A1} = 2\pi r\frac{1}{2}\rho U^2 4a(1 - a)dr \tag{18}$$

These equations provide the torque and axial force of a rotor annulus if the free stream flow, axial induction factor and tangential induction factor are known.

Blade element theory

Unfortunately, a and b are not known *a priori* and so the equations derived from momentum theory are of little use in isolation. Blade element theory divides the rotor blades into discrete span-wise (along the blade length) elements. There is no fluid interaction between these two-dimensional aerofoil elements and, consequently, the loads on the blades can then be assumed to rely purely on the lift and drag characteristics of these foil shapes.

Fig 1 shows the lift and drag forces acting on an element. dL is the element lift force and dD is the element drag force. ϕ is the inclination of the resultant flow, V, to the horizontal axis, α is the angle of attack of the turbine blade from the resultant flow and θ is the combined pitch and twist of the blade.

The axial thrust of the blade element and the torque produced can be found by resolving the lift and drag forces dL and dD. Writing these in terms of lift and drag coefficients C_L, C_D and c the aerofoil chord length, expressions for dF_{A2} and dT_2 can now be obtained:

$$dF_{A2} = N\frac{1}{2}\rho V^2 c(C_L \cos\phi + C_D \sin\phi)dr \qquad (19)$$

$$dT_2 = N\frac{1}{2}\rho V^2 cr(C_L \sin\phi - C_D \cos\phi)dr \qquad (20)$$

where N is the number of blades. From Fig 1, it can be seen that:

$$\phi = \tan^{-1}\left(\frac{U(1-a)}{r\Omega(1+b)}\right) = \tan^{-1}\left(\frac{(1-a)}{\lambda(1+b)}\right) \qquad (21)$$

where $\lambda = \frac{r\Omega}{U}$, the dimensionless local speed ratio.

Those who are aware of previous BEMT work at Swansea should note that the definition of ϕ, the inclination angle of the incoming flow, given by Griffiths[3] is in fact different from the notation used here, which is consistent with the majority of other texts.[16–18]

V, the resultant fluid flow, can be calculated using Pythagoras' theory:

$$V = [(r\Omega(1+b))^2 + (U(1-a))^2]^{\frac{1}{2}} = U[\lambda^2(1+b)^2 + (1-a)^2]^{\frac{1}{2}} \qquad (22)$$

Numerical implementation

There are now two separate formulae for axial elemental force (18, 20) and two for elemental torque (17, 19), which can be set equal to each other and solved. Methods presented in the existing literature[16–18] solve the objective equations either for lift coefficient and angle of attack or to solve iteratively for a and b. These approaches work well as a simple iterative search loop but provide little control on search direction and they are both quite dependent on the selection of a good starting value. Experience shows they perform poorly at high angles of attack. An alternative approach is proposed here which can be directly used with a variety of pre-constructed solver routines that benefit from significant development. The particular routine used here is Matlab's built-in 'fmincon' function,[19] that employs sequential quadratic programming to solve the objective, giving a solution to an objective function in fewer iterations than a line search that is less likely to converge on a local minimum.

The torque equations (17, 19) and axial force equations (18, 20) equalities are combined into a single minimisation objective function in equation (23).

Minimise g:

$$g = (dF_{A1} - dF_{A2})^2 + (dT_1 - dT_2)^2$$

$$g = (4\pi r U^2(1-a)a - N\frac{1}{2}V^2 c(C_L \cos\phi + C_D \sin\phi))^2 + \ldots \qquad (23)$$

$$(4\pi\rho r^2 U\Omega(1-a)b - N\frac{1}{2}V^2 c(C_L \sin\phi - C_D \cos\phi))^2$$

In this objective function, a and b are the independent variables. The boundary constraints can be input to this routine; b may have any value but will generally be near zero, for conventional BEMT the maximum value of a is 0.5. A value for a over this point would imply that the flow was reversed downstream of the turbine, unless a high induction correction is employed.[20] In true operation, the flow incorporates fluid from the free stream and creates a high degree of turbulence over this limit. Lower bound constraints are also imposed on a and b to prevent excessively negative values of a and b being explored.

Where both a and b are negative, this implies that the rotor system is acting as a propeller. This can occur at high tip-speed ratios where the tip elements will reach high linear speeds. The outboard elements will produce drag under these conditions but elements of the blade further inboard may continue to produce a generative torque. There is a stage where

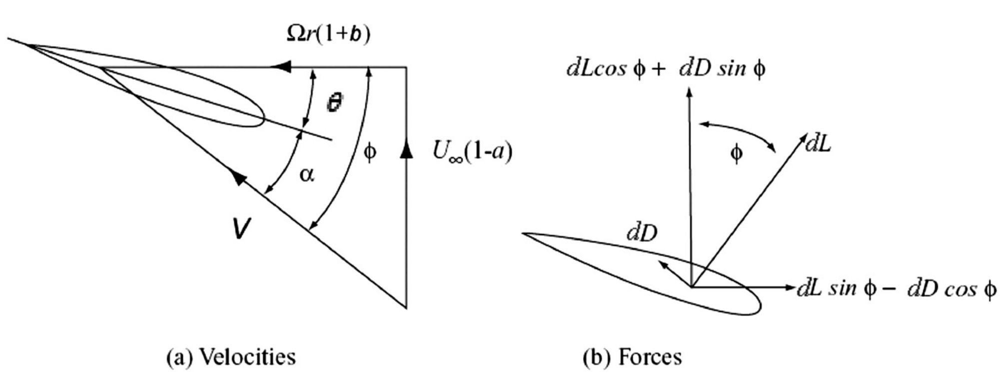


Fig 1: Lift and drag diagram for viscous flow[5]

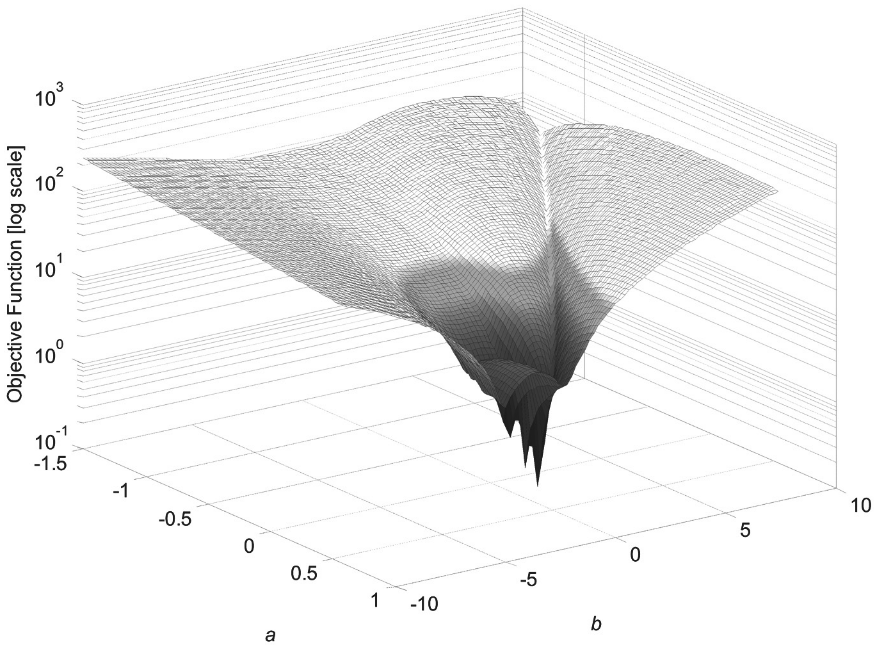


Fig 2: Surface of objective function for varying *a* and *b* for a case with multiple local minima

these two conditions balance out, giving a moving rotor with zero net torque (propeller brake state). It is therefore important that the solution system is capable of solving for all feasible cases.

The implementation, as previously described in Chapman,[21, 22] has a low computational demand but struggles to find properly converged solutions in some cases where tangential induction factors are high or where there are multiple local minima, as shown in Fig 2. A key motivation of this work is to develop a system that always converges to the best solution for every blade element and always avoids these local minima. Therefore, it is necessary to extend the robust optimiser to include a routine that provides an initial solution that is not within a local minima and close to the final solution. Hence, a Monte Carlo simulation is used to find initial *a* and *b* values before optimisation. This routine selects a number of random pairs of *a* and *b* values (this work routinely uses 1000) within the search space and returns the pair with the lowest value of *g()*. This is then used as the starting point for the optimiser.

Although the Monte Carlo routine adds complexity, it also reduces computational time spent in the optimiser so has little net increase in processor demand whilst increasing the solution robustness. Once values for *a* and *b* are found for each blade element, the torque, power and axial loads can be found for the complete rotor system. Post processing can yield axial loads, power coefficients and other information.

Lift and drag data for the foil being modelled must be selected to reflect the operating Reynolds number of the foil. It is common practice to select a fixed characteristic Reynolds number and use a single set of lift and drag data. This gives minimal errors if the Reynolds number operation range does not present massively different lift and drag data. The lift and drag data used in this study is that used by Orme in his thesis.[15] The data is for a NACA 4424 foil and data is obtained from foil tests given in Abbot and Von Doenhoff[23] and two-dimensional lifting surface theory from 'Profili,'[24] a graphical user interface for Drela's XFoil panel method code.[25] Post stall data was calculated using flat plate theory.[17] Implementation of the lift and drag coefficients in the objective function is undertaken by interpolation from a lookup table of lift data against angle of attack, α.

ACCOUNTING FOR LOSSES IN BEMT

Tip loss corrections

It is normally assumed that there is no flow along the span of the blade, but in reality this is not the case. Fluid will tend to flow around the tip from the pressure side to the suction side, and by implication, there is flow along the span. This flow reduces hydrodynamic efficiency near the tip, reducing lift and therefore torque force and ultimately power production near the blade tip.[17] The high density of water means that TST blades will tend to be relatively shorter than wind turbine blades and therefore, tip loss will be more significant for TSTs than for wind turbines.

With a finite blade, span-wise flow will shed a vortex from the tip in the same way as an aircraft wing. This shed vortex will follow the revolution of the rotor blade and produce a helical vortex[18] that is said to result in a high axial induction factor local to the blade tip relative to the induction factor in the rest of the annulus. This significant disagreement leads to the need for a correlation to be used in BEMT between local and average induction factor. There are a number of equations available to correct BEMT for this effect, all of these are variations on the Prandtl tip loss correction.

Hansen and Johansen[26] report on the effects of tip losses on wind turbine blades. In their paper a CFD model was compared to BEMT combined with Prandtl's tip loss model and an alternative model proposed by Shen *et al.*[7] Two different tip shapes are investigated, a swept tip and a squarer type of tip. The swept tip tends to shed the vortex over a larger span of the blade giving a more diffuse vortex than a square end tip, the more gradual change in chord forces the bound circulation to decrease in a more gradual manner. The main concern of the tip vortex in wind turbines is the acoustic noise it creates; this is less of a concern for tidal stream devices where downstream wake effects, cavitation and hydrodynamic efficiency are the primary concerns. CFD results for the swept wing show the entire tip operating in stalled conditions and a vortex emanating from the beginning of the sweep; in contrast the standard tip has attached flow until very close to the end of the blade. The power produced by the swept tip rotor was slightly larger than the standard tip but not by a significant amount.

The agreement between CFD and BEMT using the two different tip loss corrections is reported to be good with Shen's correction predicting a slightly lower performance than Prandtl's. This supports the argument for using BEMT with a tip loss correction for load estimation; it is reasonably accurate with a large computational saving compared to a CFD model.

Prandtl Tip loss correction

Although exact solutions of the tip loss behaviour exist, such as Goldstein[27] and Biot-Savart (clearly set out by Burton *et al*[18]), they do not lend themselves to inclusion in BEMT. Prandtl's approach approximates tip loss and can be employed with relative ease. The principle approximation is that in a fully developed wake, the shed vortex sheets, being impermeable, may be replaced by a series of disks moving at the speed of the wake and hence have no effect on the wake flow. At the wake edge, Glauert set the average fluid velocity to be $U(1-af(r))$ where $f(r)$ is a tip loss function. Near the rotor centre the average flow is unaffected by the free stream and so f is unity, near the tip, however, the interaction of the wake and free stream is significant and so $f(r)$ reduces with increasing r. The mathematical derivation of Prandtl's tip loss function is complex but results in a relatively simple closed solution given in equation (24).

$$f(r) = \frac{2}{\pi}\cos^{-1}[e^{-\pi(R_w/d - r/d)}] \tag{24}$$

Where R_w–r is the distance from the wake edge to the radial blade station and d is the normal distance between the vortex sheets equation (25).

$$d = \frac{2\pi R_w}{N}\sin\phi_s = \frac{2\pi R_w}{N}\frac{U(1-a)}{W_s} \tag{25}$$

Although this equation gives a useable formula for tip loss, calculation of the resultant wake velocity (W_s) and wake radius is complex. Glauert[8] proposed the approximate relationships

$$R_w \approx R \text{ and } \frac{r}{W} \approx \frac{R_w}{W_s}, \text{ where } W = \sqrt{(U(1-a))^2 + (\Omega r)^2}.$$

From this, we may obtain equation (26).

$$F_{Tip} = \frac{2}{\pi}\cos^{-1}[\exp-\left\{\frac{(N/2)[1-(r/R)]}{(r/R)\sin\phi}\right\}] \tag{26}$$

The blade element equations are derived from hydrofoil lift and drag forces and so remain unaffected by the tip loss correction, as they will automatically vary so long as the correct flow angle is found. Equating the updated momentum equations (27) and (28) and blade element equations as previously implemented without the correction factor equations (19) and (20), it is possible to solve the equations for axial and tangential induction factors with tip losses as before.

$$dT = 4Fb(1-a)\rho U\pi r^3\Omega dr \tag{27}$$

$$dF_A = F\rho U^2 4a(1-a)\pi r dr \tag{28}$$

Alternative tip loss models

Prandtl's model assumes that there is no wake expansion; this reduces the model's validity for high induction factor conditions where the wake will expand significantly. Glauert[8] showed that the accuracy of Prandtl's model decreased for blade numbers below three and for high Tip Speed Ratios. Hansen and Johansen[26] present some further correction factors. In Shen *et al*[7] the Prandtl and Glauert corrections are discussed. Glauert's tip loss equation is then investigated and it is noted that for a non-zero lift coefficient the flow angle at the tip will tend to zero and that relative axial velocity will always tend to zero at the tip. Although this causes no major mathematical problem it is physically unrealistic with axial flow that is non-zero at the tip and a vortex is shed from the tip into the wake.

Two further models are then discussed, the first being that of Wilson and Lissaman[9] which employs the concept of circulation to create alternative tip loss corrections. As circulation is primarily generated by lift, the tangential induction factor b is neglected. For axial induction, both mass flow and induced velocity are corrected. Shen *et al* note that the zero flow inconsistency at the tip is also present in this model. The second correction proposed by De Vries[10] attempts to address the issue that Wilson and Lissaman's model gives a non-orthogonal relationship between the velocity at the blade and the induced velocity. Shen *et al* state that the results given by De Vries' model are almost identical to Wilson and Lissaman's and the zero flow condition at the tip still exists. As a solution to this zero flow problem Shen *et al* propose a further correction, equation (29):

$$F_1 = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-g\frac{N(R-r)}{2R\sin\phi}\right)\right]$$

where (29)

$$g = \exp\left[-c_1(N\lambda - c_2)\right]$$

where c_1 and c_2 are experimentally determined coefficients. Using one rotor system for low TSR and one rotor system for higher TSR, Wilson and Lissaman arrive at values of $c_1 = 0.125$ and $c_2 = 21$. F_1 is then applied to the lift and drag forces in the objective function using the traditional Glauert correction.

Hub losses

In addition to tip loss, Moriarty and Hansen[16] suggest the use of a hub loss model. It corrects the induced velocity that is caused by a vortex being shed near the hub of the rotor. The loss factor equation (30) is applied to the momentum equations and only differs slightly from the tip loss model. The underlying theory is the same and is simply adjusted to work outwards from the blade root rather than inwards from the tip.

$$F_{Hub} = \frac{2}{\pi}\cos^{-1} e^{-f}$$

where (30)

$$f = \frac{N}{2}\frac{r - R_{hub}}{r\sin\phi}$$

Implementation of tip and hub loss

In practice, a blade element will be affected by both tip and hub losses. F, the total loss for the element, is the product of F_{tip} equation (26) and F_{hub} equation (30). If either loss factor is neglected then F_{tip} or F_{hub} is set to unity. Subsequently, equations (27) and (28) are equated to the lift and drag based equations for dT and dFa equations (19) and (20) to produce a modified objective function in equation (31).

Minimise g where:

$$g_1 = ((4\pi\rho rU^2(1-a)aFdr) - (N\frac{1}{2}\rho V^2 c(C_L\cos\phi + C_D\sin\phi)dr))^2$$
$$g_2 = ((4\pi\rho r^3U\Omega(1-a)bFdr) - (N\frac{1}{2}\rho V^2 cr(C_L\sin\phi - C_D\cos\phi)dr))^2 \quad (31)$$
$$g = g_1 + g_2$$

Variation of loss equations along the blade

A comparison of the magnitude of different hub and tip losses is given in Fig 3 and the combined tip and hub loss effect is shown in Fig 4. Fig 5 shows how the predicted tip and hub loss varies for differing angles of phi, it can be seen that the impact of the loss factors is far smaller for low angles of attack. It is worthwhile noting that the loss equations will both tend to infinity at $\phi = 0$, which can be avoided by setting ϕ to be a small value if it is calculated to be zero when coding.

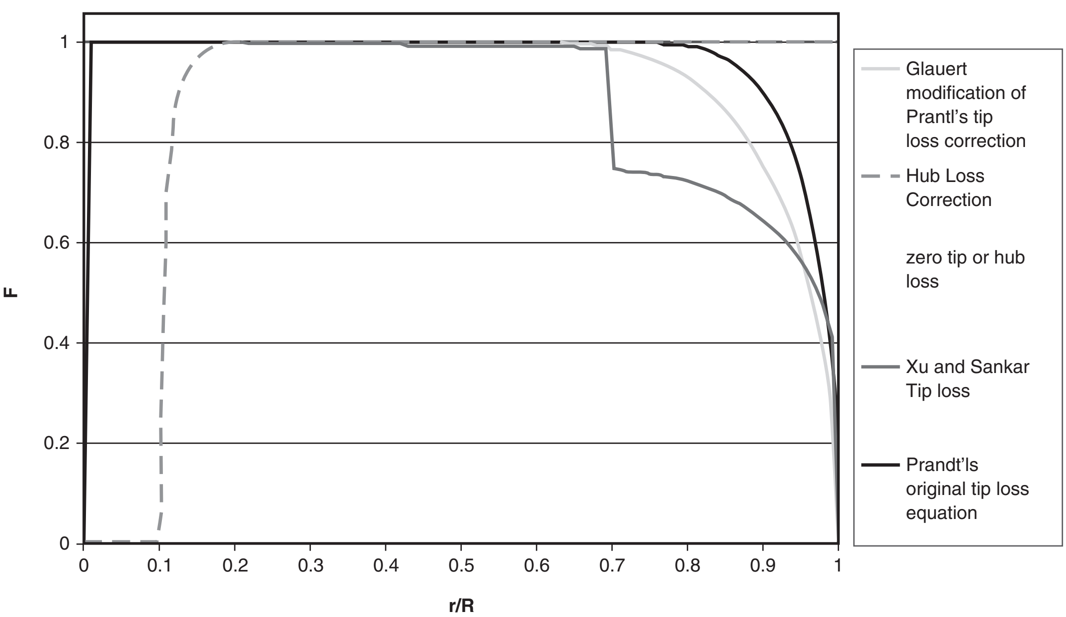
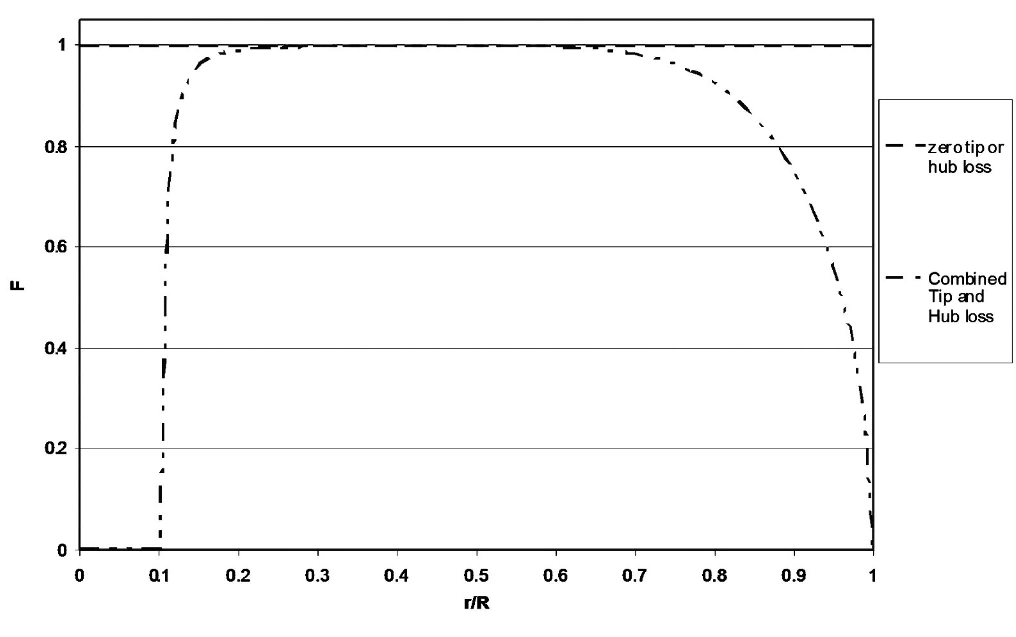


Fig 3: A comparison of tip and hub loss equations for a normalised blade radius with root at r/R = 0.1 and constant $\phi = 10°$

Fig 4: Combined tip and hub loss for a normalised blade with root at r/R = 0.1 and constant $\phi = 10°$ using Glauert's tip loss formula



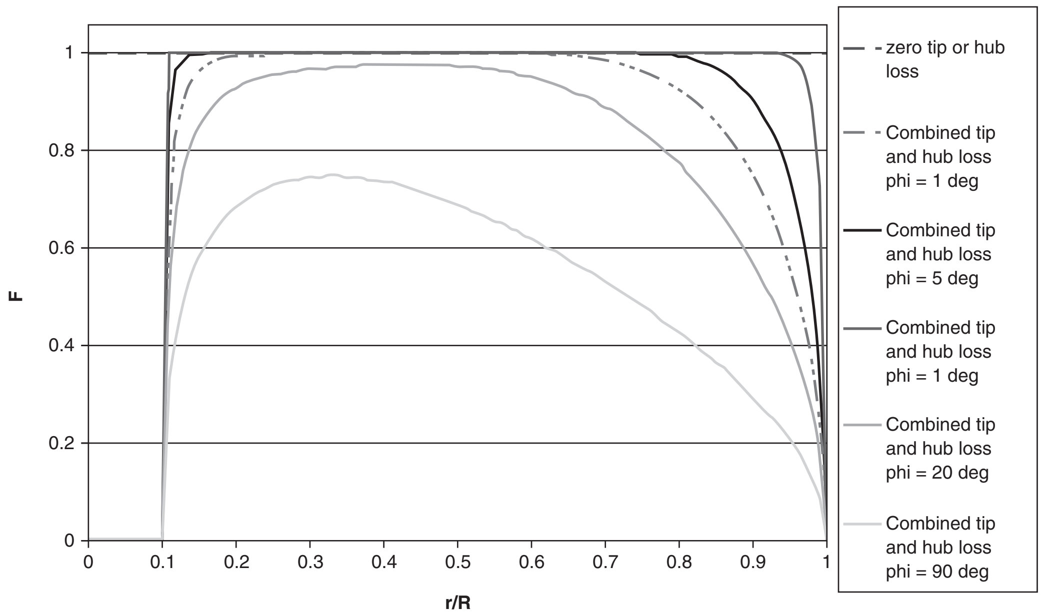
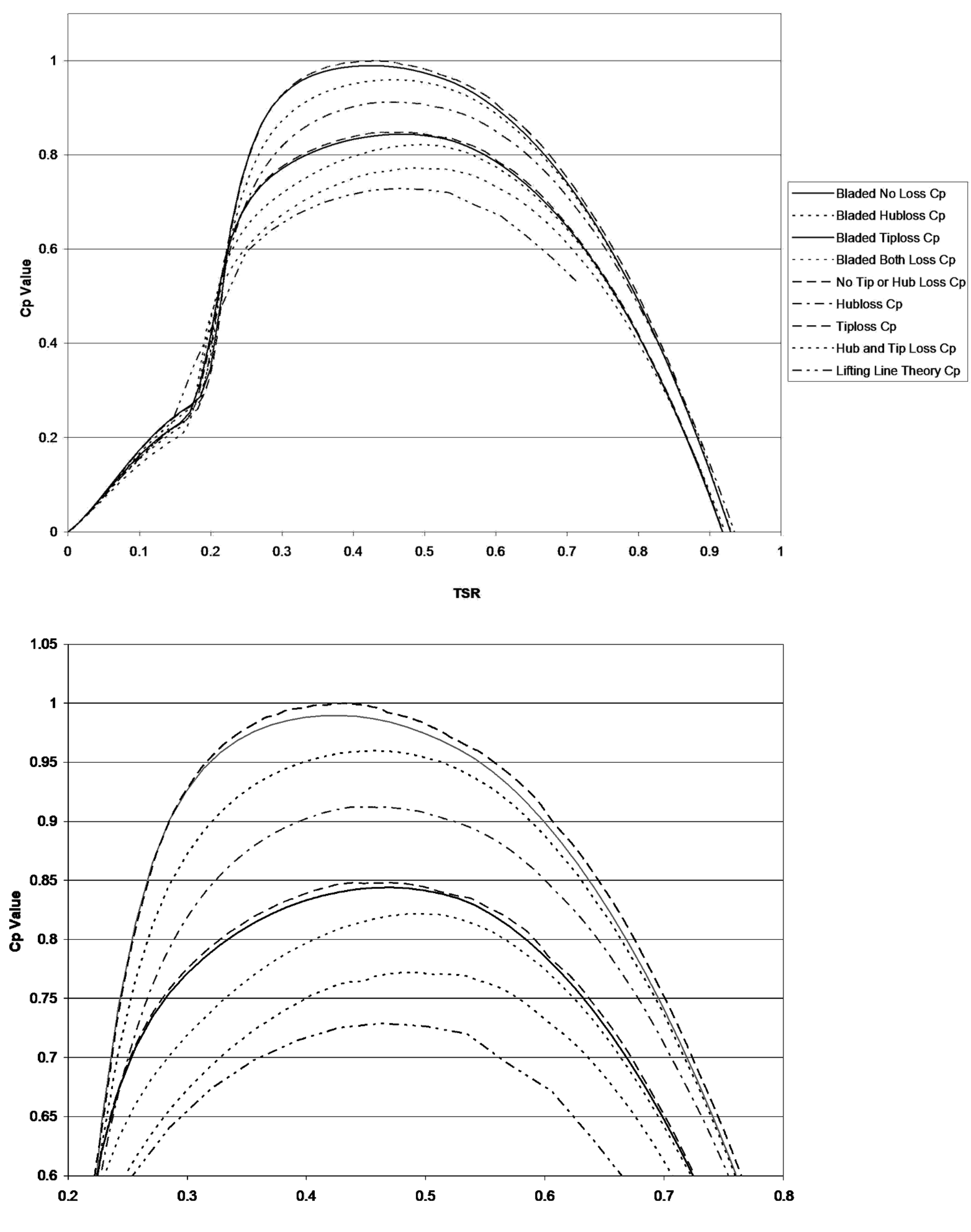


Fig 5: Combined tip and hub loss for a normalised blade with root at r/R = 0.1 for various values of ϕ using Glauert's tip loss formula

Fig 6: Comparison of normalised power coefficient against predictions from GH Tidal Bladed and Sharpe's lifting line theory code (detail view below Fig 6b)



RESULTS

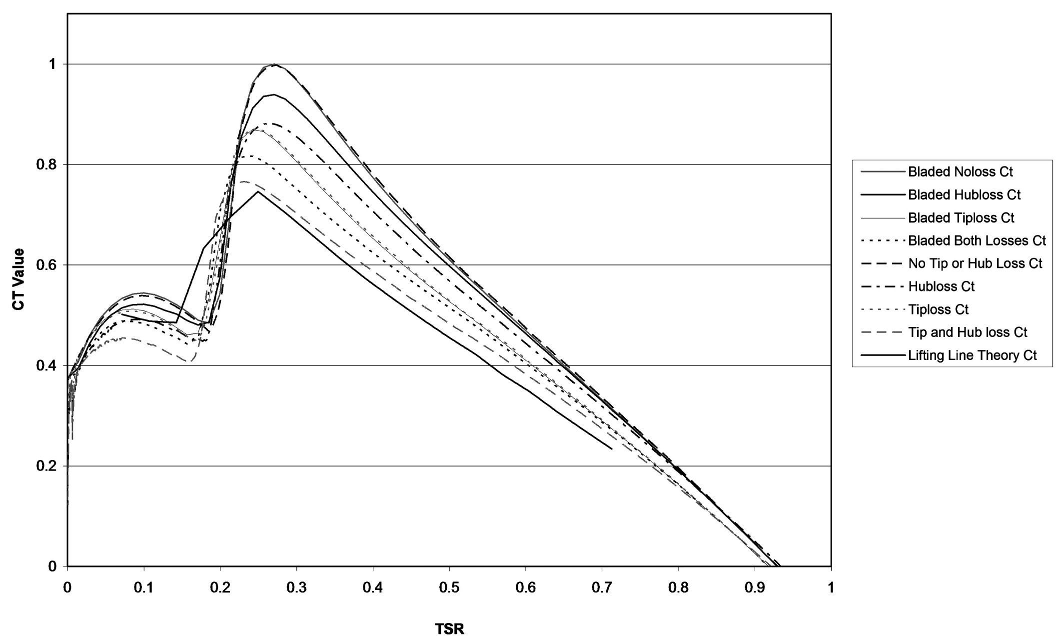
To assess the performance of the tip and hub loss correction factors, the predictions of blade performance made by the code with and without the corrections were compared to models from the GH Tidal Bladed program[28] and to a Lifting line theory model by Sharpe.[29] The results of this are shown in Figs 6 to 8.

It can be seen that the lossless solution compares very well between GH Tidal Bladed and the current work with disagreement between predicted values lower than one percent. The tip loss only curves also agree well, again with a disagreement of similar magnitude. The small disagreements in predicted values for these runs are expected to be primarily due to the different methods of blade geometry discretisation. In GH Tidal Bladed, the geometry is defined at element ends whereas with the current model the blade geometry at the element centre is defined; this leads to the slight disagreement in calculated performance even with a similar number of elements for each model. GH Tidal Bladed is based on the industry standard wind turbine design and certification software GH Bladed. This gives good confidence in the results from the developed code.

Disagreement between the two models for hub loss only is significant (up to around 7% for C_P). This disagreement is larger than can be attributed to differences in geometry definition and is most likely due to a different hub loss formula being employed by the two models. Furthermore, as the total loss factor is the product of tip and hub loss, a similar disagreement is seen for the combined loss model results.

Comparison to the lifting line model shows that both the developed model and GH Tidal Bladed predict higher torque, axial force and power coefficients than the lifting line theory.

Fig 7: Comparison of normalised torque coefficient against predictions from GH Tidal Bladed and Sharpe's lifting line theory code



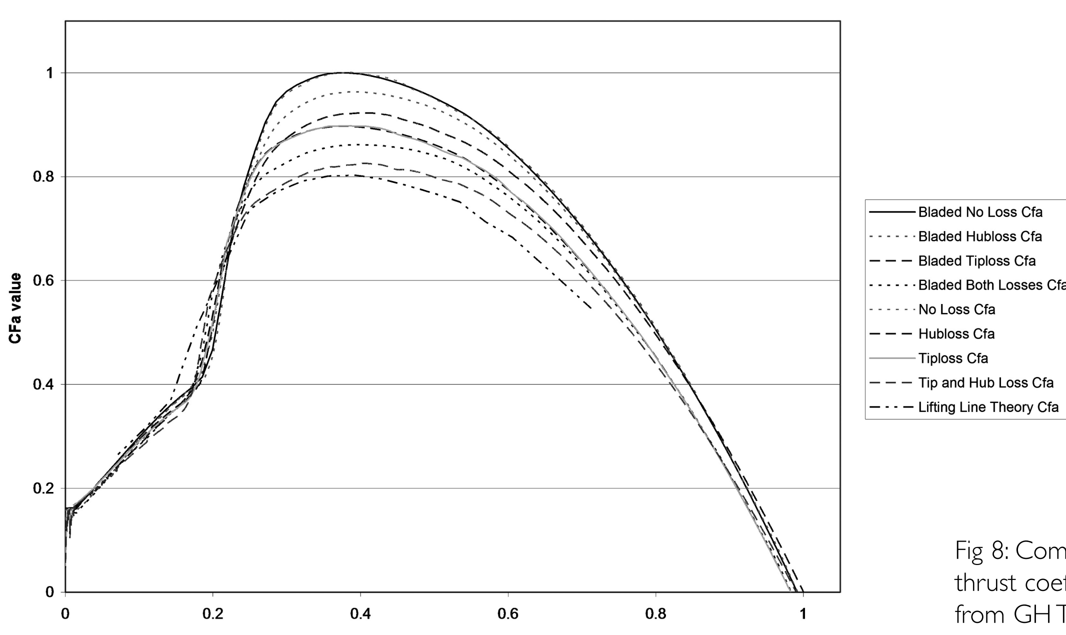


Fig 8: Comparison of normalised thrust coefficient with predictions from GH Tidal Bladed and Sharpe's lifting line model

This disagreement between the theories is discussed by Badreddinne, Ali and David[4] and is attributed to the fact that BEMT does not account for three dimensional flow effects induced on the rotor disc by the shed tip vortex or the induced radial flow created by rotation of the blades. When this work is compared to other work published on the modelling of marine rotors,[30] it shows excellent agreement between BEMT and experimental data in the operating range, but over predicts C_P at high TSR in over speed. Reference to Fig 2 gives one explanation; it is possible that simple search routines are converging to a local minima for some blade elements. Results at propeller brake state are not reported as there was no experimental data under these conditions.

CONCLUSION

In this paper a full description of a robust BEMT implementation is described and the code described in the paper has been improved to better predict the performance of a marine rotor under a range of operating conditions, from start up, through normal operation, to over speed and propeller brake state. Furthermore, the Prandtl tip and hub loss corrections were presented and were seen to improve accuracy when compared to a lifting line theory model. There is only a small computational cost of employing these corrections and so inclusion proves to be beneficial. The Prandtl corrections were selected as they do not rely on empirical data.

The output of the model using these corrections was compared with industrially certified code, GH Tidal Bladed and a lifting line theory model. The results showed a good correlation with the existing codes.

As more detailed site data becomes available,[31] the validation of a turbine design against a particular set of site sea state conditions will necessitate robust methodologies such as this computational model.

The authors are willing to make available the numerical implementation of this BEMT model for collaborative working on a non-commercial academic basis.

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