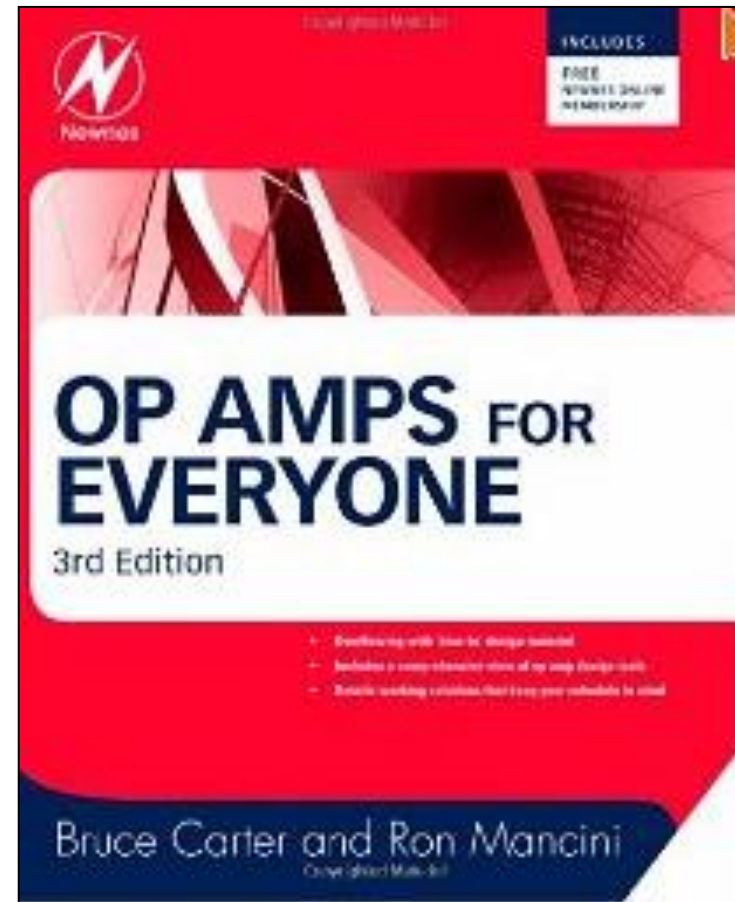


# **Laboratório 3**

## **Filtros Ativos Passa-Tudo**

## Referência

[OP AMPS for Everyone](#)  
Newnes, 2009



# Roteiro Experimental

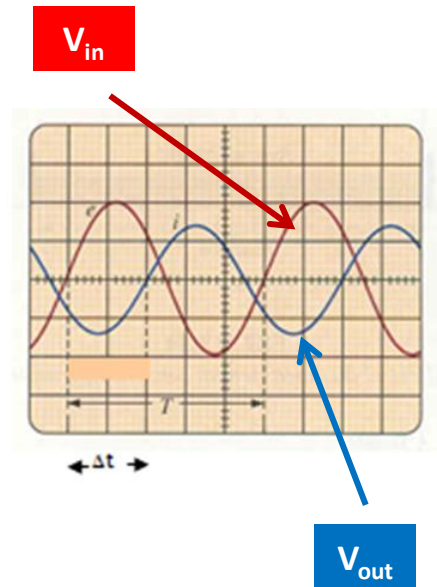
**SEL393 – Laboratório de Instrumentação Eletrônica I**  
**Escola de Engenharia de São Carlos - USP**  
**Departamento de Engenharia Elétrica**

**Laboratório 3 - Filtros Ativos Passa-Tudo**

Implemente em simulação um filtro passa-tudo para gerar um atraso de 2ms em um sinal com espectro de frequência  $0 < f < 1$  kHz. Para minimizar a distorção de fase a frequência de corte deve ser maior que 1 kHz.

- Plote em representação cartesiana a fase ( $\phi$ ) em função da frequência.
- Plote em representação Bode e meça o atraso de grupo não normalizado ( $T_{gr}$ ) de cada estágio do filtro.
- Compare o valor medido do atraso de grupo não normalizado ( $T_{gr}$ ) de cada estágio do filtro com o valor teórico.
- Plote em representação Bode e meça o atraso de grupo não normalizado ( $T_{gr}$ ) do filtro projetado.
- Compare o valor medido do atraso de grupo não normalizado ( $T_{gr}$ ) do filtro projetado com o valor teórico.
- Verifique que a defasagem muda linearmente com a frequência utilizando para este parâmetro uma escala linear.

## Phase Shift



$$\theta = \frac{\Delta t}{T} 360^\circ = \Delta t f 360^\circ$$

$\Delta t$  (delay) varies with the frequency.

To achieve **equal temporal delays** for all the frequencies, we need every frequency to have a different phase shift—namely, a phase shift that results in the same delay for every frequency. More specifically, we need a phase-shift response that *increases* linearly with frequency.

An **ideal linear-phase filter**, then, exhibits phase shift that increases linearly with frequency, and it thereby provides constant temporal delay (this applies primarily to the frequencies within the passband, i.e., the frequencies of interest).

**Group delay ( $t_{gr}$ )** is proportional to the derivative of the phase response with respect to frequency.

**The derivative of a linear function is a constant, which explains why a linear phase response is also referred to as constant group delay.**

$$t_{gr} = - \frac{d\phi}{d\omega}$$

Now consider a situation in which a filter will see signals composed of various different frequencies that work together. Problems could arise if these different frequencies experience different delays.

- 1 All-pass filter has a constant gain across the entire frequency range, and a phase response that changes linearly with frequency.
- 2 All-pass filters are used in circuits referred to as “**phase equalizers**” or “**delay equalizers**.” As discussed in [Understanding Linear-Phase Filters](#), it is sometimes important to ensure that all the frequency components in a signal experience **equal temporal delay**.

**Audio applications:** Frequencies representing different pitches must remain synchronized to ensure proper sound reproduction.

**Pitch** is an auditory sensation in which a listener assigns musical tones to relative positions on a musical scale based primarily on their perception of the frequency of vibration. **Pitch is closely related to frequency, but the two are not equivalent.** Frequency is an objective, scientific attribute that can be measured. **Pitch is each person's subjective perception of a sound wave, which cannot be directly measured.** However, this does not necessarily mean that most people won't agree on which notes are higher and lower.

**Digital communications:** The sinusoidal harmonic frequencies that constitute a square wave must experience constant delay to avoid distortion of the digital signal.

- 3 Similar to the low-pass filters, all-pass circuits of higher order consist of cascaded first-order and second-order all-pass stages.

$$A(s) = \frac{\prod_i (1 - a_i s + b_i s^2)}{\prod_i (1 + a_i s + b_i s^2)}$$

( $a_i$  and  $b_i$  being the coefficients of a partial filter)



4

**Group Delay ( $t_{gr}$ )**

It is the time by which the all pass filter delays each frequency within a band.

$$t_{gr} = - \frac{d\phi}{d\omega}$$

5

**Normalized Group Delay ( $T_{gr}$ )**

The frequency at which the normalized group delay drops to  $\frac{1}{\sqrt{2}}$  times its initial value is the corner frequency ( $f_c$ ) which corresponds to  $\Omega=1$ .

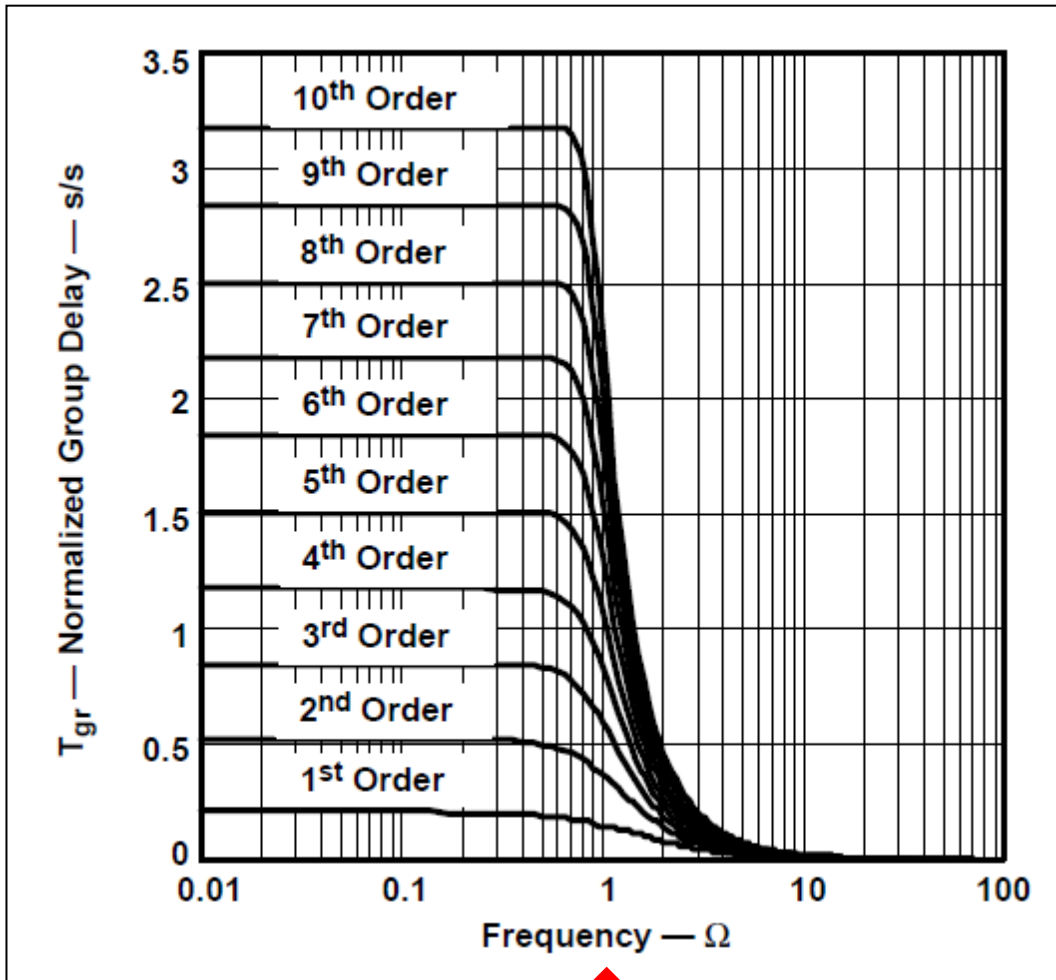
$$T_{gr} = \frac{t_{gr}}{T_c} = t_{gr} \cdot f_c = t_{gr} \cdot \frac{\omega_c}{2\pi}$$



$$T_{gr} = - \frac{1}{2\pi} \frac{d\phi}{d\Omega}$$

$$T_{gr} = - \frac{f_c}{2\pi} \frac{d\phi}{df}$$

$$T_{gr} = \frac{1}{\pi} \sum_i \frac{a_i(1 + b_i\Omega^2)}{1 + (a_1^2 - 2b_1)\cdot\Omega^2 + b_1^2\Omega^4}$$



At  $\Omega=1$  there is a -3dB decrease in  $T_{gr}$  !

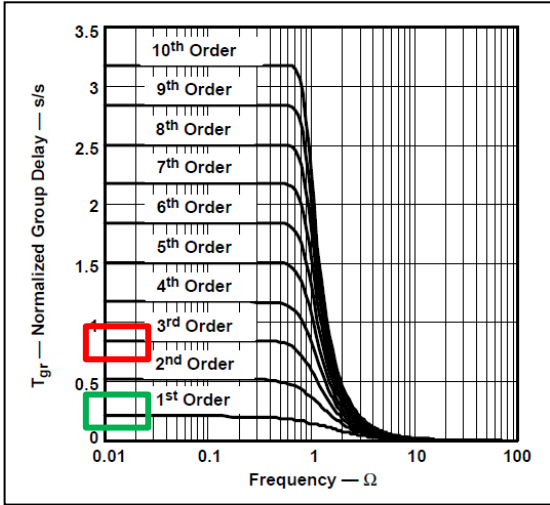
6

The  $T_{gr0}$  is the value of  $T_{gr}$  when  $\Omega < 0.1$



$$T_{gr0} = \frac{1}{\pi} \sum_i a_i$$

All Pass



n	l	a <sub>l</sub>	b <sub>l</sub>	f <sub>l</sub> /f <sub>c</sub>	Q <sub>l</sub>	T <sub>gr0</sub>	
1	1	0.6436	0.0000	1.554	—	0.2049	
	2	1	1.6278	0.8832	1.064	0.58	0.5181
		2	1.5092	1.0877	0.959	0.69	
		3	1.1415	0.0000	0.876	—	
3	1	2.3370	1.4878	0.820	0.52	1.1738	
	2	1	1.3506	1.1837	0.919		0.81
		2	1.2974	0.0000	0.771		—
		3	2.2224	1.5685	0.798		0.56
4	1	1.2116	1.2330	0.901	0.92	1.5060	
	2	1	2.6117	1.7763	0.750		0.51
		2	2.0706	1.6015	0.790		0.61
		3	1.0967	1.2596	0.891		1.02
5	1	1.3735	0.0000	0.728	—	2.1737	
	2	1	2.5320	1.8169	0.742		0.53
		2	1.9211	1.6116	0.788		0.66
		3	1.0023	1.2743	0.886		1.13
6	1	2.7541	1.9420	0.718	0.51	2.5084	
	2	1	2.4174	1.8300	0.739		0.56
		2	1.7850	1.6101	0.788		0.71
		3	0.9239	1.2822	0.883		1.23
7	1	1.4186	0.0000	0.705	—	2.8434	
	2	1	2.6979	1.9659	0.713		0.52
		2	2.2940	1.8282	0.740		0.59
		3	1.6644	1.6027	0.790		0.76
8	1	0.8579	1.2862	0.882	1.32	3.1786	
	2	1	2.8406	2.0490	0.699		0.50
		2	2.6120	1.9714	0.712		0.54
		3	2.1733	1.8184	0.742		0.62
9	1	1.5583	1.5923	0.792	0.81	3.1786	
	2	1	1.5583	1.5923	0.792		0.81
		2	1.5583	1.5923	0.792		0.81
		3	0.8018	1.2877	0.881		1.42

Examples:

n=1

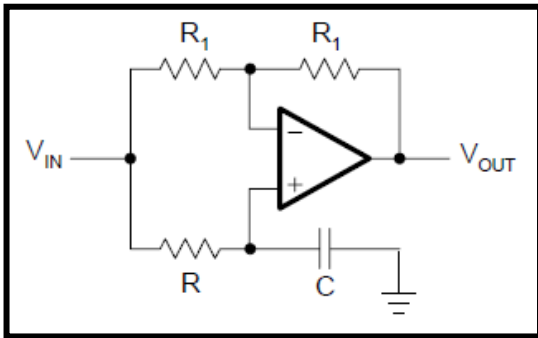
$$T_{gr0} = \frac{1}{\pi} (0,6436) = 0,2049$$

n=3

$$T_{gr0} = \frac{1}{\pi} (1,1415 + 1,5092)$$

$$T_{gr0} = 0,8437$$

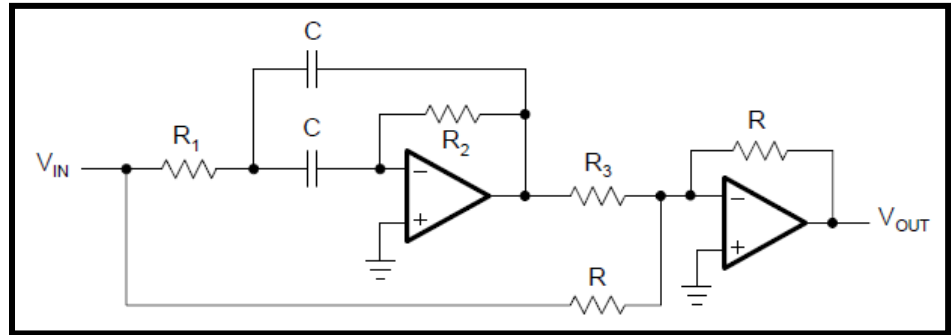
## First Order Topology



$$T_{gro} = 2RCf_C$$

$$R_1 = R$$

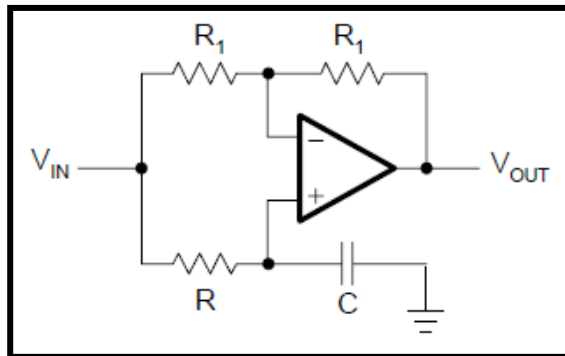
## Second Order Topology



$$T_{gro} = 4RCf_C$$

**Designing  
All Pass Filters  
(First Order Topology)**

## First Order Topology



$$A(s) = \frac{1 - RC\omega_c \cdot s}{1 + RC\omega_c \cdot s}$$

1 Specify  $f_c$  and  $C$

2 Calculate  $R$



$$R = \left| \frac{a_i}{2\pi f_c \cdot C} \right|$$

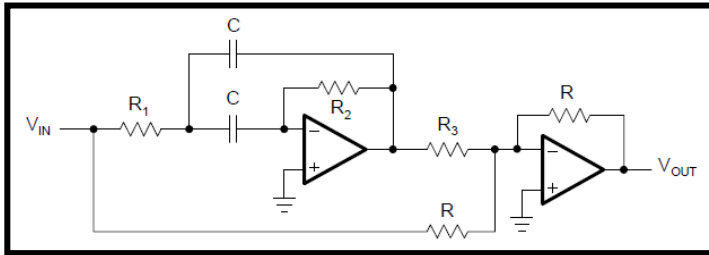
3 Delay group



$$T_{gro} = 2RCf_c$$

**Designing  
All Pass Filters  
(Second Order Topology)**

## Second Order Topology



$$A(s) = \frac{1 + (2R_1 - \alpha R_2)C\omega_c \cdot s + R_1 R_2 C^2 \omega_c^2 \cdot s^2}{1 + 2R_1 C \omega_c \cdot s + R_1 R_2 C^2 \omega_c^2 \cdot s^2}$$

**1** Specify  $f_c$  and  $C$

**2** Calculate  $R$

$$R = \left| \frac{a_1}{2\pi f_c \cdot C} \right|$$

**3** Calculate  $R_1, R_2, R_3$

$$R_1 = \frac{a_1}{4\pi f_c C}$$

$$R_2 = \frac{b_1}{a_1 \pi f_c C}$$

$$R_3 = \frac{R}{\alpha}$$

$$\alpha = \frac{a_1^2}{b_1}$$

**4** Maximum delay group

$$T_{gro} = 4RCf_c$$



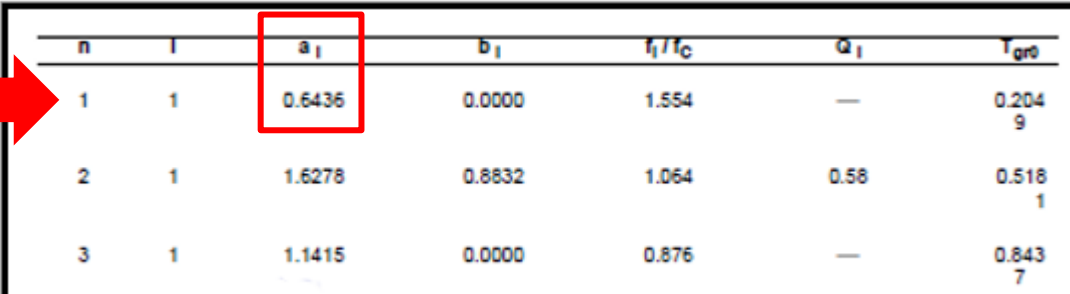
## Exemple 2:

Implemente no LTSPice um filtro passa tudo de ordem 1 com frequência de corte de 1KHz. Utilize  $C=10\text{nF}$ .

**1** Specify  $f_c$  and  $C$  and take  $a_1$

$$f_c = 1\text{KHz}, C=10\text{nF}, a_1 = 0,6436$$

$a_1$



n	l	$a_i$	$b_i$	$f_i/f_c$	$Q_i$	$T_{ord}$
1	1	0.6436	0.0000	1.554	—	0.2049
2	1	1.6278	0.8832	1.064	0.58	0.5181
3	1	1.1415	0.0000	0.876	—	0.8437

All Pass

**2** Calculate  $R$

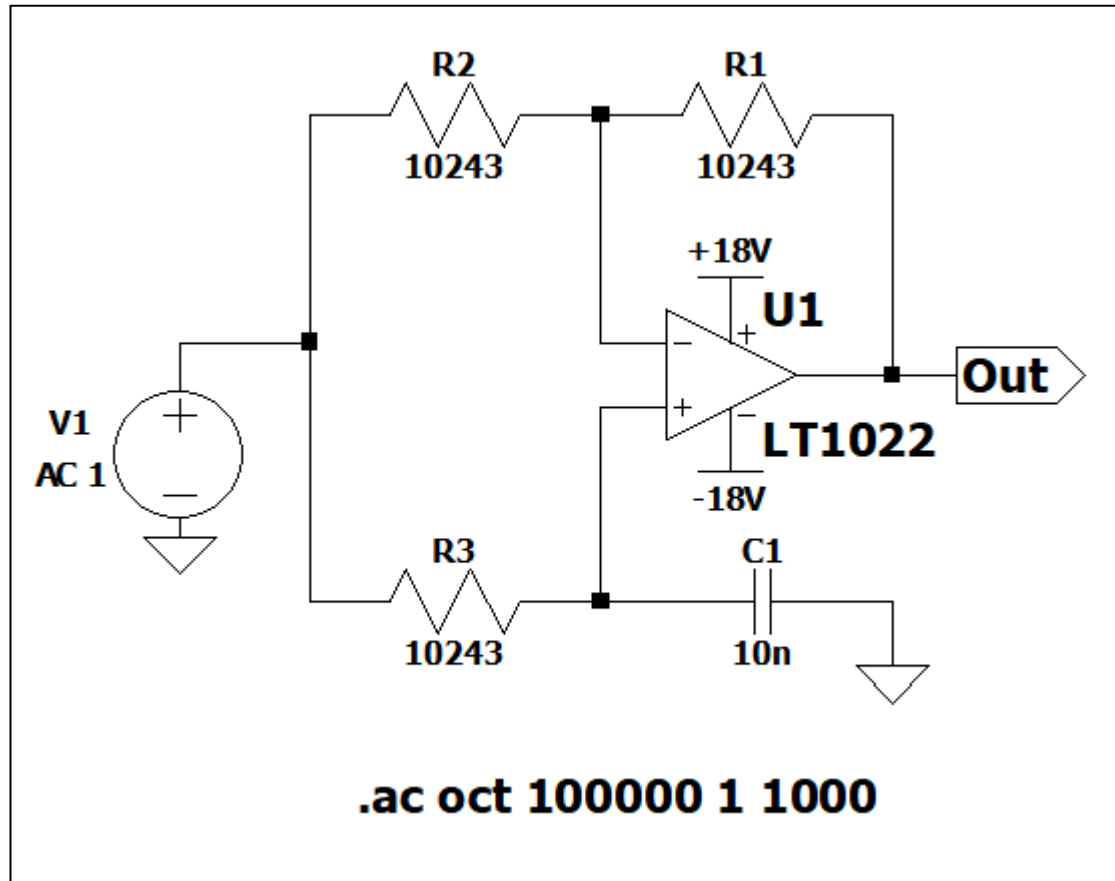
$$R = \left| \frac{a_i}{2\pi f_c \cdot C} \right|$$



$$R = \frac{0,6436}{2\pi \times 10^3 \times 10 \times 10^{-9}} = 10243$$

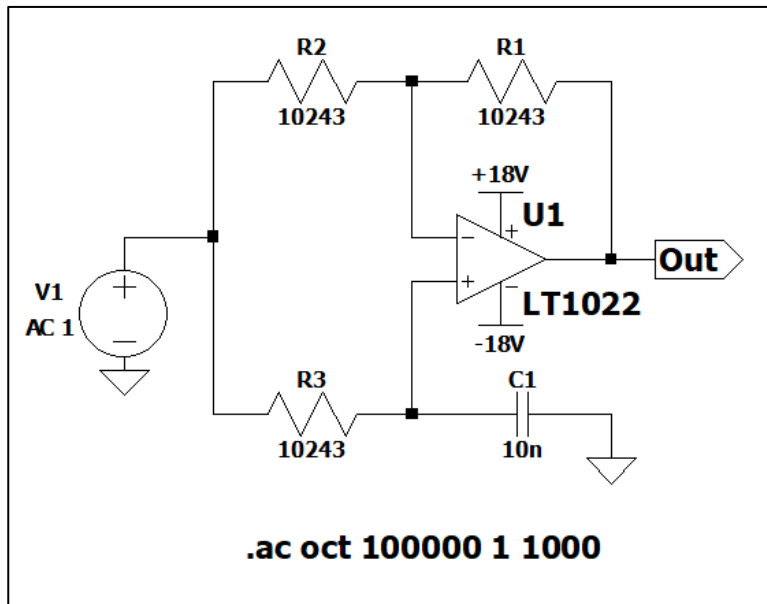
**3** Calculate  $T_{gro}$

$$T_{gro} = 2RCf_c \rightarrow T_{gro} = 2 \times 10243 \times 10 \times 10^{-9} \times 1000 = 0,20486$$



# LTSPice Simulation

# How to measure $\phi$ and $T_{gr}$ in the LTSPice ???



**Fase ( $\phi$ )**

$$\phi = \arctan(\text{Im}(V(\text{out}))/\text{Re}(V(\text{out})))$$

**Normalized Delay group ( $T_{gr}$ )**

$$T_{gr} = -\frac{f_c}{2\pi} \frac{d\phi}{df}$$

$$T_{gr} = (1/(2*\pi)) * \underbrace{d(\arctan(\text{Im}(V(\text{out}))/\text{Re}(V(\text{out}))))}_{\frac{d\phi}{df}} * 1000\text{Hz}$$

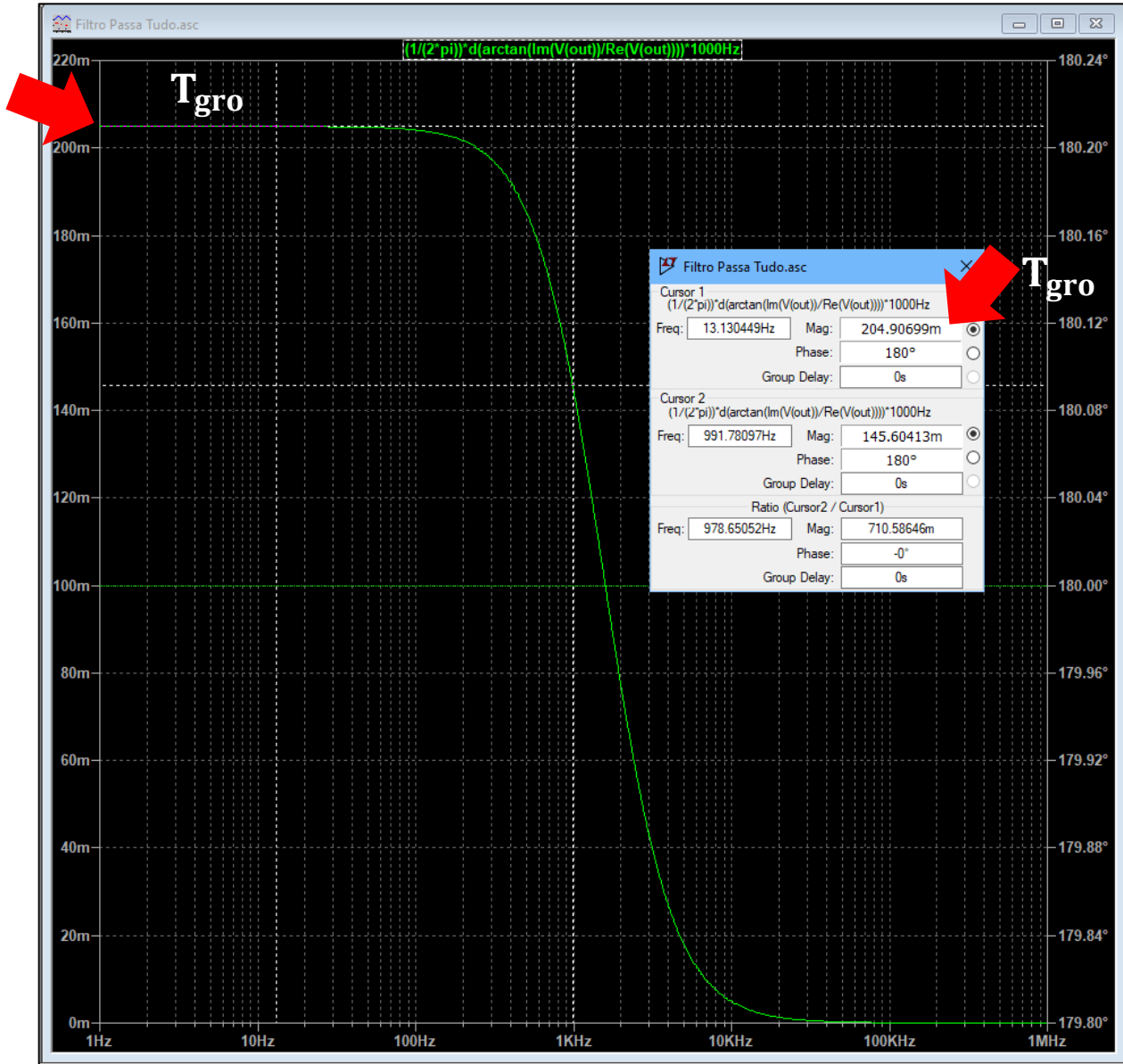
$$\frac{d\phi}{df}$$

Normalized delay group ( $T_{gr}$ )



$$T_{gr} = (1/(2*\pi)) * d(\arctan(\text{Im}(V(\text{out}))/\text{Re}(V(\text{out})))) * 1000\text{Hz}$$

Medida do  $T_{gro}$

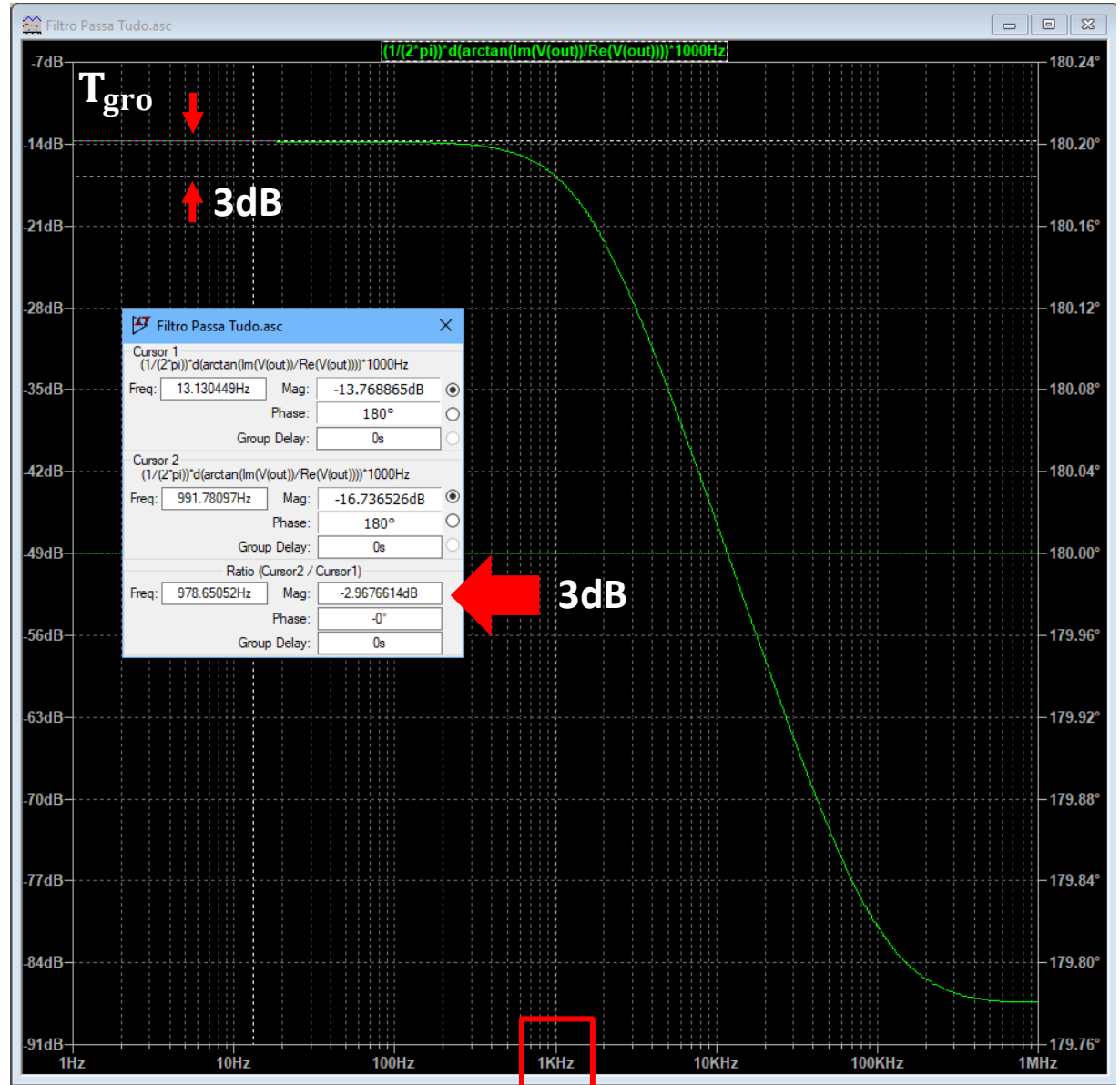


Normalized delay group ( $T_{gr}$ )



$$T_{gr} = (1/(2*\pi)) * d(\arctan(\text{Im}(V(\text{out}))/\text{Re}(V(\text{out})))) * 1000\text{Hz}$$

Verificação da queda de 3dB em  $f_c$  1KHz

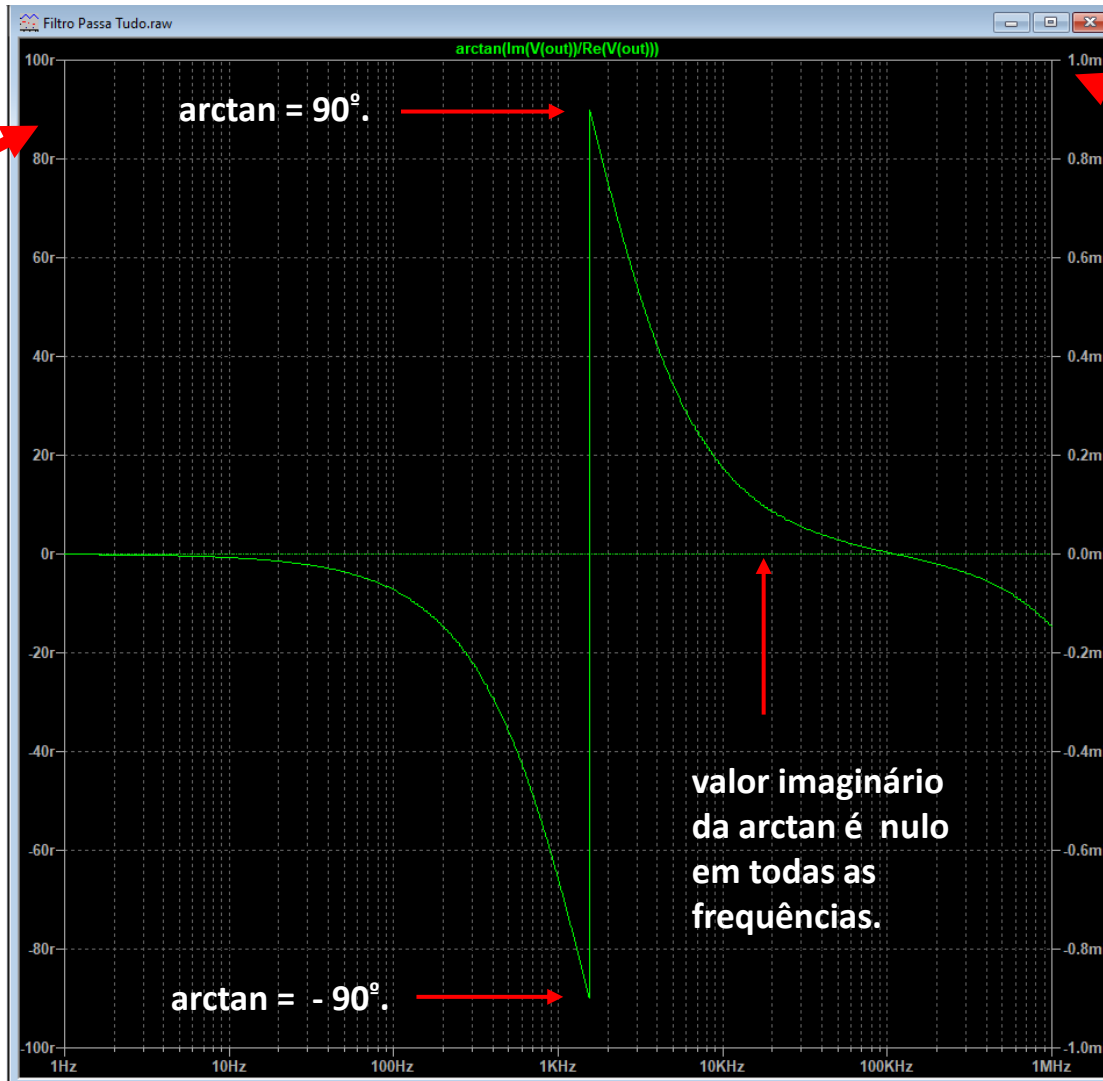


Fase ( $\phi$ )



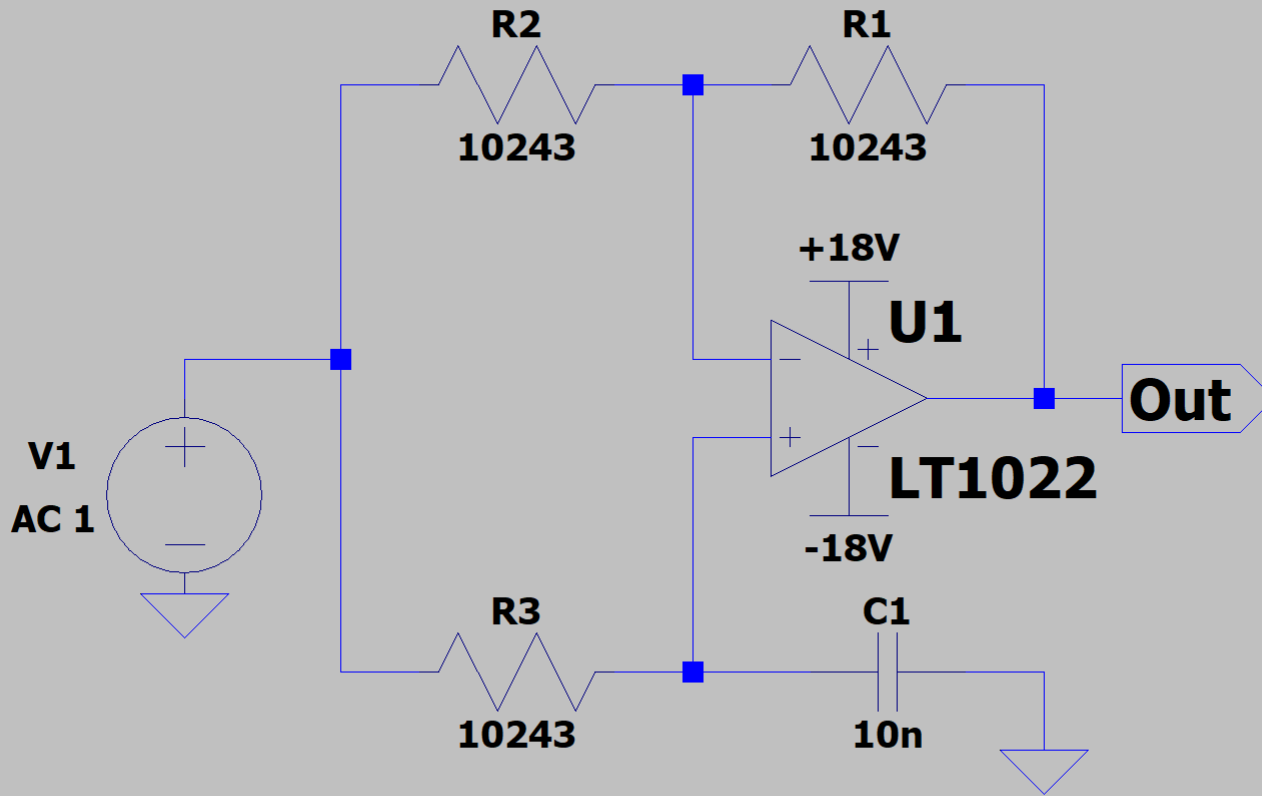
$$\phi = \arctan(\text{Im}(V(\text{out}))/\text{Re}(V(\text{out})))$$

$\phi \times f$  (utilizando representação cartesiana no LTSPice)



**Eixo vertical direito:** valor imaginário (i) do arctan que tem ser nulo em todas as frequências porque arctan é um número real.

**Eixo vertical esquerdo:** valor real (r) do arctan em função da frequência. O valor de arctan varia de  $90^\circ$  à  $-90^\circ$ .



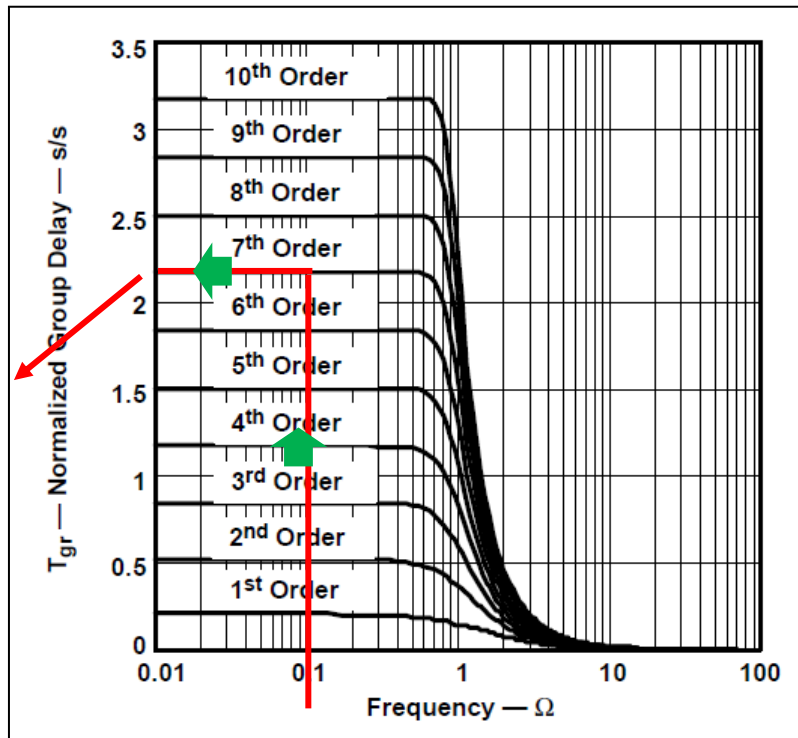
**.ac oct 100000 1 1Mega**



## Exemple 1:

A signal with the frequency spectrum,  $0 < f < 1$  kHz needs to be delayed by 2 ms. To keep the phase distortions at a minimum, the corner frequency of the all-pass filter must be  $f_c \geq 1$  kHz. Design a 2-ms delay all-pass filter.

**1** The figure below shows a seventh-order all-pass is needed to accomplish the desired delay.



$$T_{gr0} = 2.1737$$

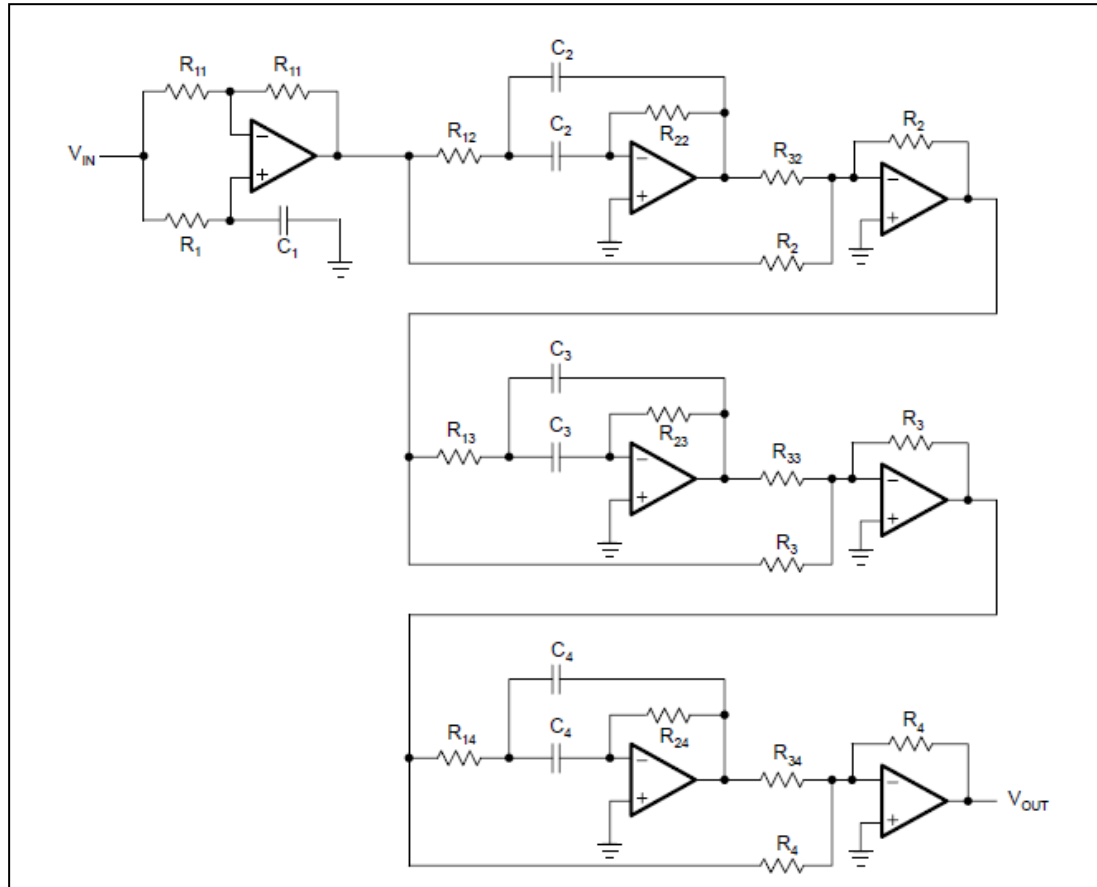
O filtro de ordem 6 tem  $T_{gr0}=2$ . Sendo  $t_{gr}=2$ ms resulta que a frequência de corte é  $f_c = 1$ kHz. No entanto  $f_c \geq 1$  kHz e, portanto, o filtro escolhido deverá ter ordem 7.

$$T_{gr0} = 2.1737$$

$$t_{gr} = 2 \text{ms}$$

$$f_c = \frac{T_{gr0}}{t_{gr0}} = 1.087 \text{ kHz}$$

# All Pass Filter – 7th Order



# All Pass Filter – 7th Order

