Laboratório 3
Filtros Ativos Passa-Baixa

Roteiro Experimental

ME3000 – Analog Electronic Courseware

ME3000 Analog Electronics Courseware

Out-of-Box Teaching Solution for Analog Electronics

Quick Start Guide

revision 1.31

Printed on 20 September 2012

Op-Amp Module (Module 2)

- 1st Order Active Filter
- 2nd Order Active Filter
- Buffer
- Inverting/Summing Amplifier
- Non-Inverting Amplifier
- Differential Amplifier

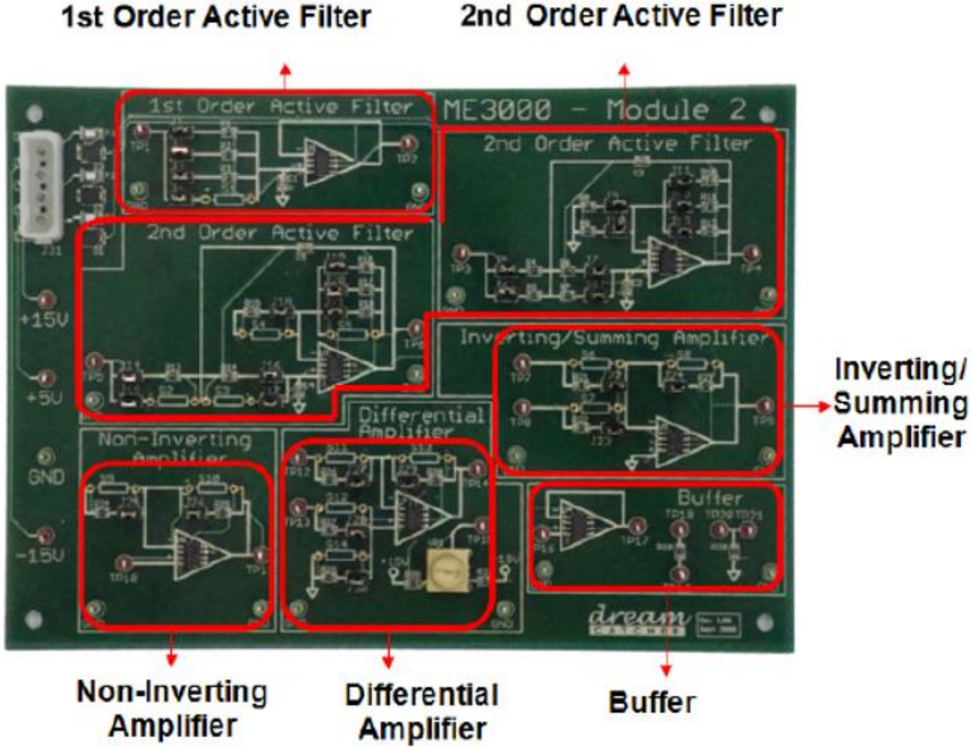

Figure A2 – ME3000-M2 Op-Amp Module

3.3 Lab Sheets

No	Lab Sheet	Objective	Duration
1	Diode Characteristics	To understand the characteristics of diodes	3 Hours
2	Rectifier Circuits	To understand the basic operations of rectifier circuits	3 Hours
3	BJT Characteristics	To understand the characteristics of a bipolar junction transistor (BJT)	3 Hours
4	DC Biasing	To demonstrate the effects of DC biasing on the AC operation of a common-emitter amplifier	3 Hours
5	Practical Op-Amp Circuits	To understand the typical configurations of operational amplifier circuits and their characteristics	3 Hours
6	RF Class A Tuned Amplifiers	To demonstrate the practical issues in designing an RF tuned amplifier and perform AC measurements on a Class A amplifier	3 Hours
7	555 Multivibrator Circuits	To understand the basic operations of a 555 Timer IC and design astable and monostable multivibrators using 555 timers	3 Hours
8	Active Filters	To understand the working principles of active filters and design an active low pass filter	3 Hours

SEL393 – Laboratório de Instrumentação Eletrônica I
Escola de Engenharia de São Carlos - USP
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Filtros Ativos Passa-Baixa

Introdução

As arquiteturas de filtros passa-baixa de 1a e 2a ordem disponíveis no kit educacional ME3000 / M2 para implementação de filtros de 1ª, 2ª, 3ª, 4ª e 5ª ordem são mostradas nas figuras 1a, 1b, 1cm, 1d e 1e.

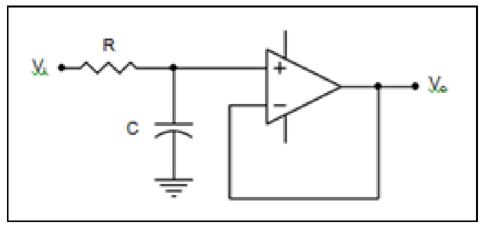


Fig. 1a – Esquemático Filtro Passa-Baixa de ordem 1 com ganho unitário

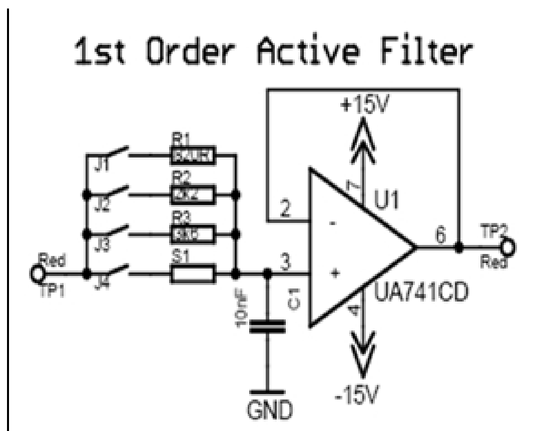


Fig. 1b – Filtro Passa-Baixa de ordem 1 disponível no Kit 3000 / M2

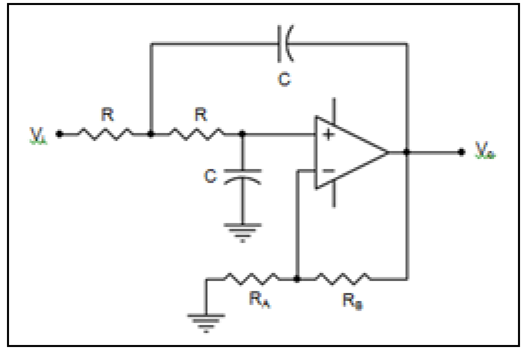


Fig. 1c – Esquemático Filtro Passa-Baixa de ordem 2 com ganho não unitário e arquitetura Sallen-Key

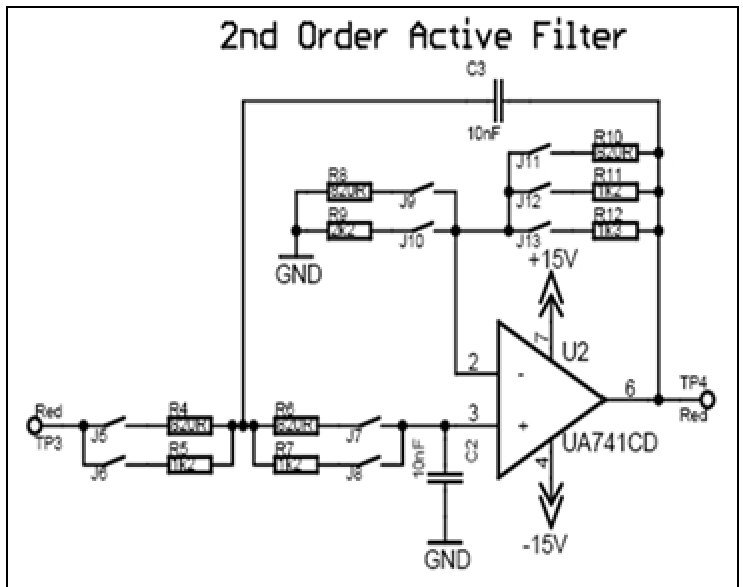


Fig. 1d – Filtro Passa-Baixa de ordem 2 com ganho não unitário e arquitetura Sallen-Key disponível no Kit 3000 / M2

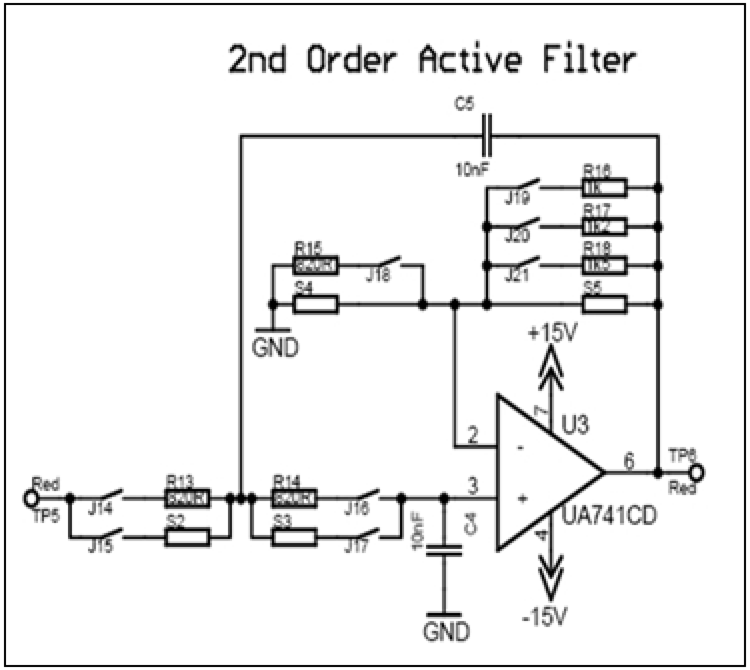


Fig. 1e – Filtro Passa-Baixa de ordem 2 com ganho não unitário e arquitetura Sallen-Key disponível no Kit 3000 / M2

No kit educacional ME3000 / M2 da Dream Catcher é possível implementar filtros passa-baixa com as especificações descritas na Tabela 1.1.

Tabela 1.1

Filtro	Passa-Baixa	Ordem Filtro	Frequência de corte (f_c - KHz)	Capacitor C (nF)
1	Butterworh	3ª	7.25	10
2	Butterworh	3ª	19.8	10
3	Butterworh	5ª	19.8	10
4	Chebyshev, 2dB ripple	3ª	14.5	10
5	Chebyshev, 2dB ripple	3ª	22	10
6	Chebyshev, 2dB ripple	5ª	20	10

Procedimento Experimental

Projete um dos filtros passa-baixa de 3ª ordem e um dos filtros passa-baixa de 5ª ordem com as especificações descritas na tabela 1.1 e com os componentes disponíveis nos circuitos das figuras 1a à 1e.

Implemente em protoboard os filtros projetados utilizando o kit educacional ME3000 / M2. Meça os seguintes parâmetros:

- Ganho total na faixa de passagem (A_0).
- Frequência de corte do filtro (f_c).
- Determine as frequências onde ocorrem os picos de ripple.

Simulação

1) Simule no LTSPice os filtros projetados. Meça os seguintes parâmetros:

- Ganho total na faixa de passagem (A_0).
- Frequência de corte do filtro (f_c).
- Determine as frequências onde ocorrem os picos de ripple.

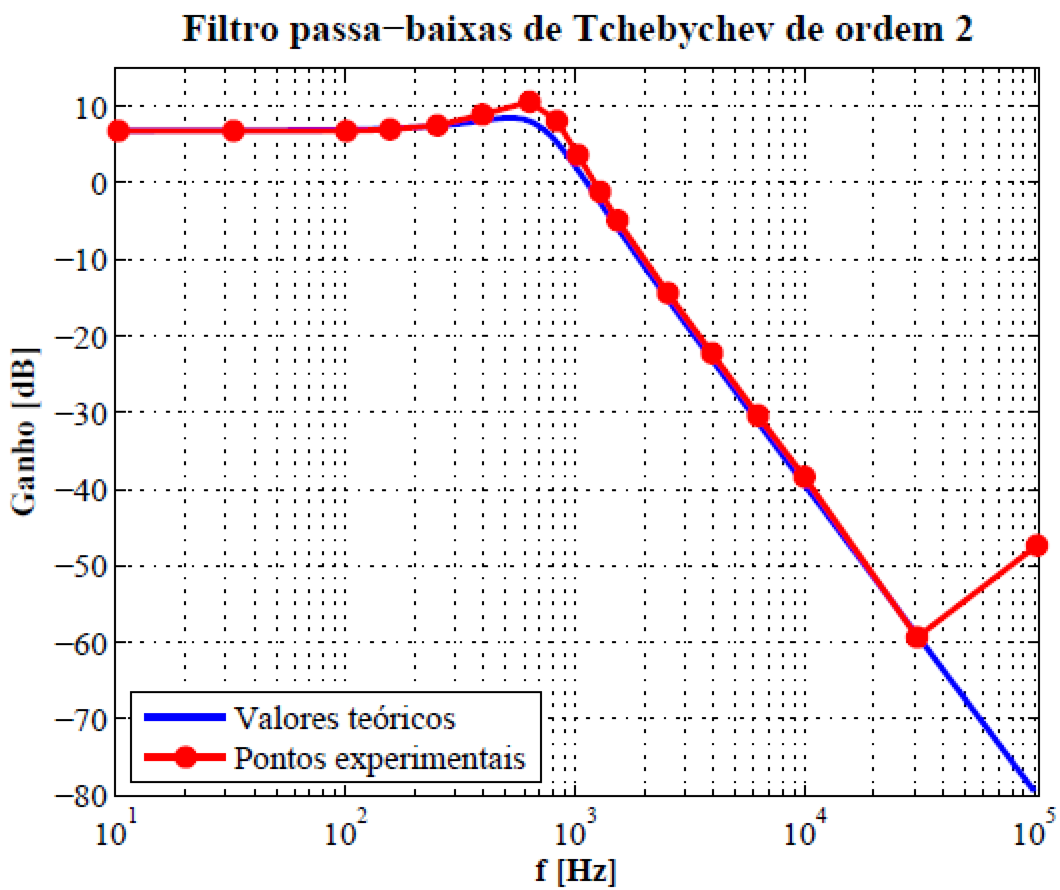
2) Compare os resultados experimentais e de simulação

3) Simule no LTSPice filtros passa-baixa de ordem 2 de Butterworth, Tchebychev (ripple de 3dB) e Bessel com topologia de Sallen-Key, ganho não unitário e frequência de corte de 20KHz. Utilize C=10nF.

Comente sobre a resposta de cada filtro na banda de passagem.

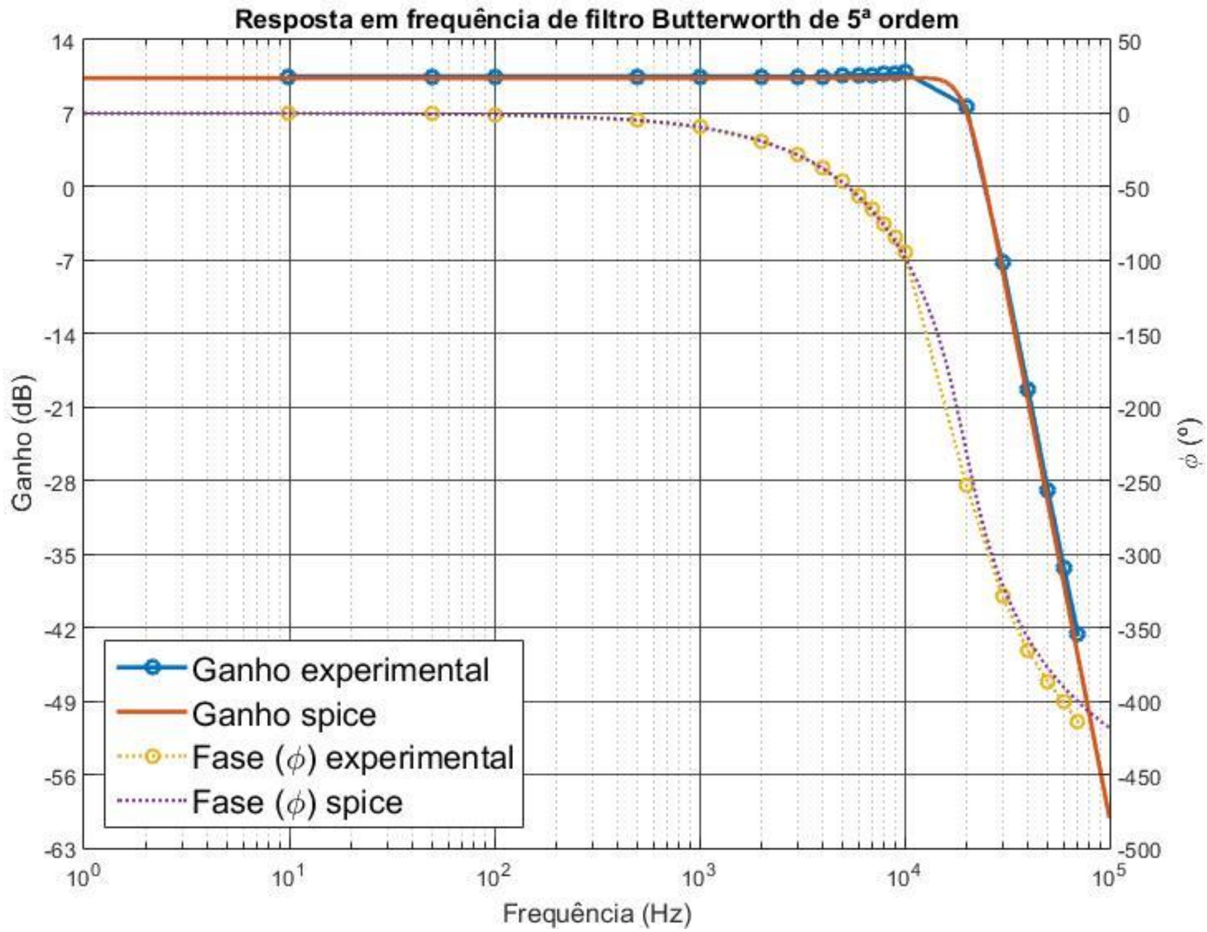
Comparação
Teoria x Experimento

Active Filters : Theory x Experimental



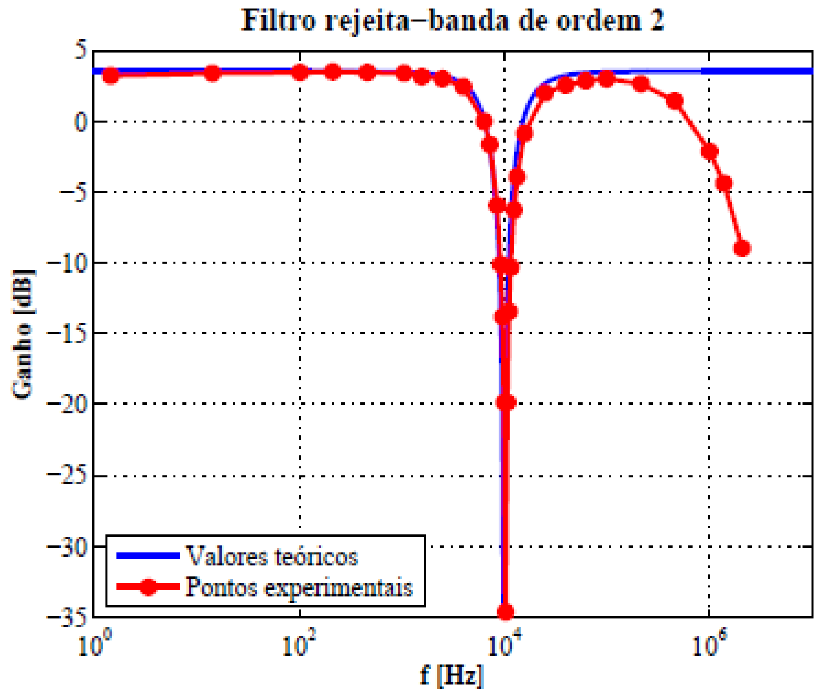
Resposta em frequência do filtro passa-baixas de Tchebychev de ordem 2

Active Filters : Theory x Experimental



Filtro Passa Baixa Butterwork, ordem n=5, f_c= 19.8KHz

Active Filters : Theory x Experimental



Resposta em frequência do filtro rejeita-banda de ordem 2 implementado.

Introduciton

1 What is a filter?

A filter is a device that passes electric signals at certain frequencies or frequency ranges while preventing the passage of others. — Webster.

2 **Applications (few examples):**

- In telecommunications **band pass filters** are used in the audio frequency range (0 – 20 KHz).

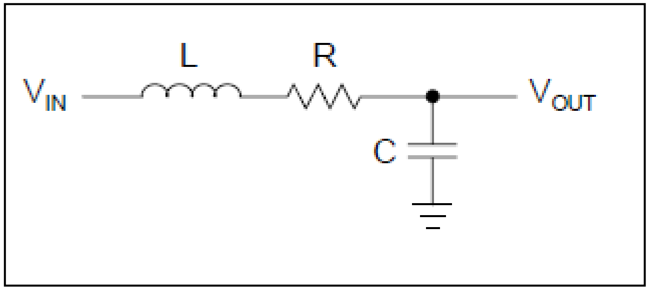
- In telephone centrals **high frequency band-pass filters** are used for channel selection.

3 There are filters that do not filter any frequencies of a complex input signal, but just add a linear phase shift to each frequency component, thus contributing to a constant time delay. These are called **all-pass filters**.

4 At high frequencies (> 1 MHz), all of these filters usually consist of passive components such as inductors (L), resistors (R), and capacitors (C). They are then called LRC filters.

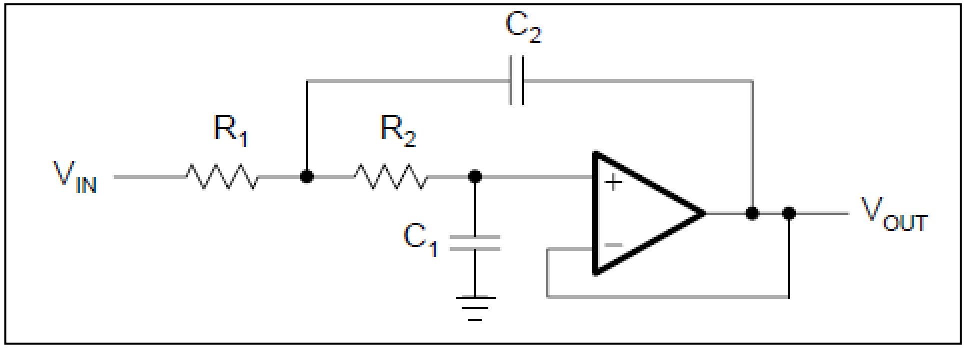
5 In the lower frequency range (1 Hz to 1 MHz), however, the inductor value becomes very large and the inductor itself gets quite bulky, making economical production difficult.

6 **Active filters are circuits that use an operational amplifier (op amp) as the active device in combination with some resistors and capacitors to provide an LRC-like filter performance at low frequencies.**



Second-Order Passive Low-Pass

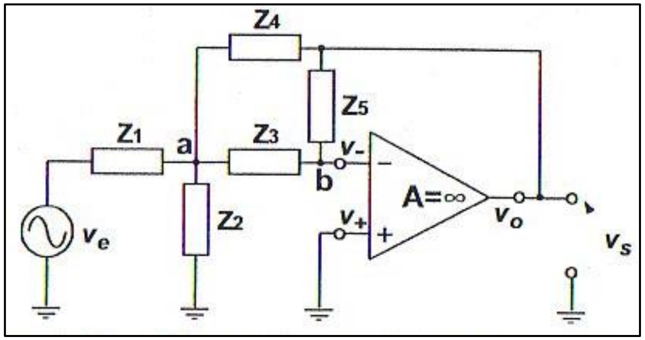
(passivo)



Second-Order Active Low-Pass

(ativo)

Determinação de Função de Transferência

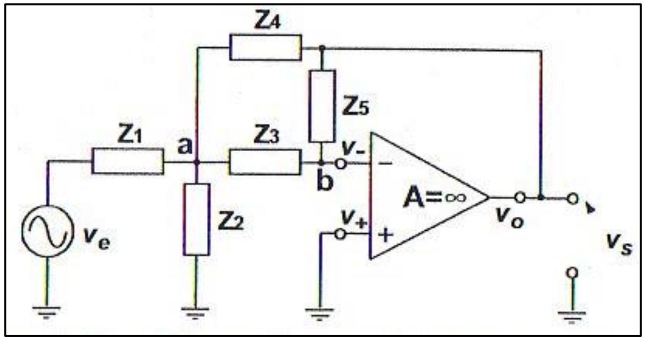


Filtro Ativo Genérico na Configuração Realimentação Múltipla

1 O ganho de tensão deste circuito pode ser facilmente obtido utilizando-se a LCK nos nós **a** e **b**, supondo que o amp op é ideal e aplicando-se a técnica de terra virtual.

2 **Nó a:**

$$\frac{v_a}{Z_2} - \frac{v_e - v_a}{Z_1} - \frac{v_s - v_a}{Z_4} + \frac{v_a}{Z_3} = 0 \longrightarrow v_a\left(\frac{1}{Z_1} - \frac{1}{Z_2} - \frac{1}{Z_3} + \frac{1}{Z_{34}}\right) - \frac{v_e}{Z_1} = \frac{v_s}{Z_4}$$



Filtro Ativo Genérico na Configuração Realimentação Múltipla

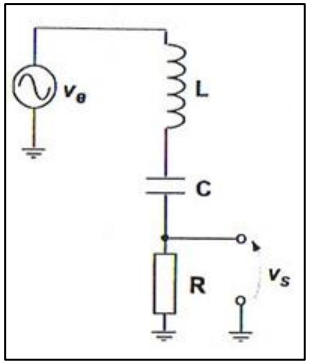
3 **Nó b:**

$$\frac{v_s}{Z_5} + \frac{v_a}{Z_3} = 0 \longrightarrow v_a = -\frac{Z_3}{Z_5} v_s \longrightarrow$$

$$\longrightarrow \boxed{G = \frac{v_s}{v_e} = \frac{\frac{1}{Z_1 Z_3}}{\frac{1}{Z_5}\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4}\right) + \frac{1}{Z_3 Z_4}}}$$

Filtro Passivo X Filtro Ativo

Mostrar que o filtro ativo passa-faixa com realimentação múltipla abaixo tem a mesma função de transferência do filtro passivo RLC.



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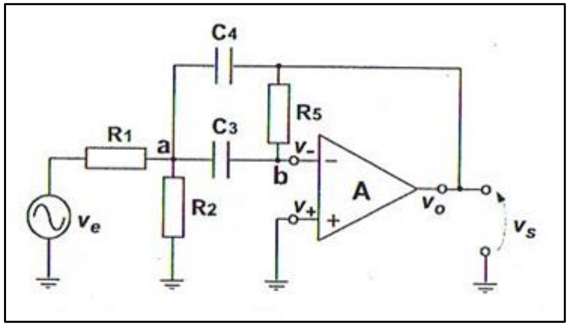
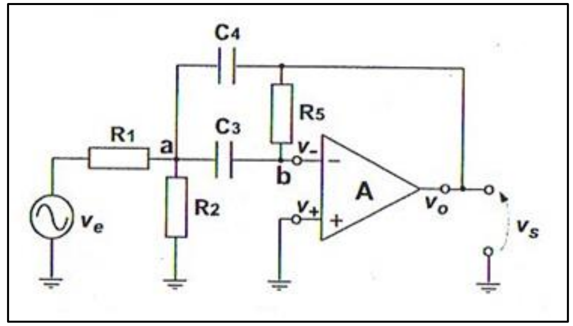


1 O ganho de tensão do **circuito passivo** é dado por:

$$G = \frac{v_s}{v_e} = \frac{R}{\left(jwL + R + \frac{1}{jwC}\right)} = \frac{\frac{jwR}{L}}{-w^2 + \frac{jwR}{L} + \frac{1}{LC}} = \frac{\frac{jww_o}{Q}}{-w^2 + \frac{jww_o}{Q} + w_o^2} \quad [1]$$

$$\boxed{w_o^2 = \frac{1}{LC}} \qquad \boxed{Q = \frac{w_o L}{R}}$$



2 O ganho de tensão do **circuito ativo**, como demonstrado, é dado por:

$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = 1/jwC_3$

$Z_4 = 1/jwC_4$

$Z_5 = R_5$

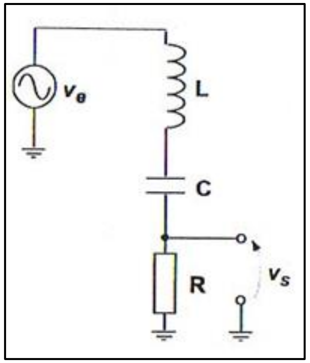
$$\rightarrow \quad G = \frac{v_s}{v_e} = \frac{\frac{1}{Z_1 Z_3}}{\frac{1}{Z_5}\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+\frac{1}{Z_4}\right)+\frac{1}{Z_3 Z_4}}$$

$$\rightarrow \quad G = \frac{v_s}{v_e} = -\frac{jwC_3}{-w^2 C_3 C_4 + jw\frac{C_3+C_4}{R_5}+\frac{1}{R_5}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

3 Se $C_3 = C_4 = C$ e $R_5 = 2R_1$ resulta:

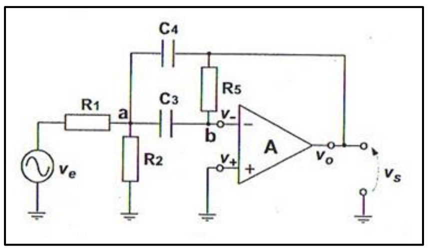
$$G = \frac{v_s}{v_e} = \frac{jw/(R_1 C)}{-w^2 + jw\frac{2}{R_5 C}+\frac{1}{R_5 C^2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{\frac{jww_0}{Q}}{-w^2+\frac{jww_0}{Q}+w_0^2}$$ [2]

$$\boxed{w_0^2 = \frac{1}{2R_5 C^2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} \quad \boxed{Q^2 = \frac{R_1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$



$$G = \frac{v_s}{v_e} = \frac{R}{\left(jwL + R + \frac{1}{jwC}\right)} = \frac{\frac{jwR}{L}}{-w^2 + \frac{jwR}{L} + \frac{1}{LC}} = \frac{\frac{jww_o}{Q}}{-w^2 + \frac{jww_o}{Q} + w_o^2}$$ [1]

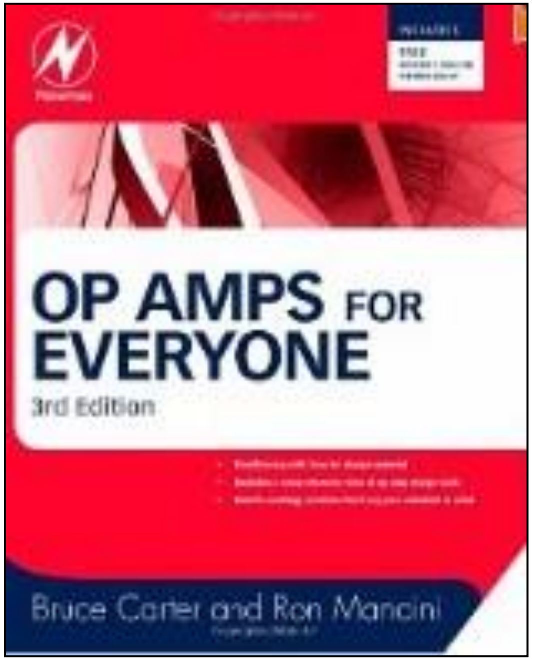
$$\boxed{w_o^2 = \frac{1}{LC}} \quad \boxed{Q = \frac{w_o L}{R}}$$



$$G = \frac{v_s}{v_e} = \frac{jw/(R_1C)}{-w^2 + jw\frac{2}{R_5C} + \frac{1}{R_5C^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{\frac{jww_0}{Q}}{-w^2 + \frac{jww_0}{Q} + w_0^2}$$ [2]

$$\boxed{w_0^2 = \frac{1}{2R_5C^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad \boxed{Q^2 = \frac{R_1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

➡ [1] = [2]



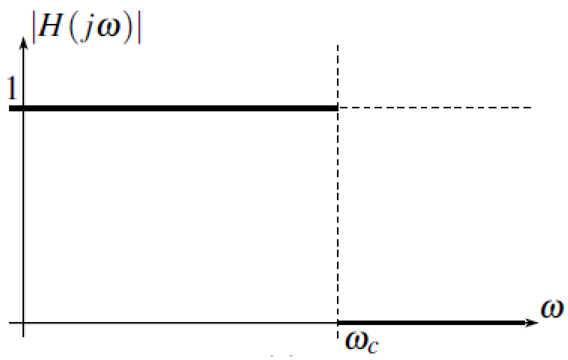
1 Rather than resembling just another filter book, the individual filter sections are written in a cookbook style, thus avoiding tedious mathematical derivations. Each section starts with the general transfer function of a filter, followed by the design equations to calculate the individual circuit components.

2 Introduction to three main filter optimizations (Butterworth, Tschebyscheff, and Bessel).

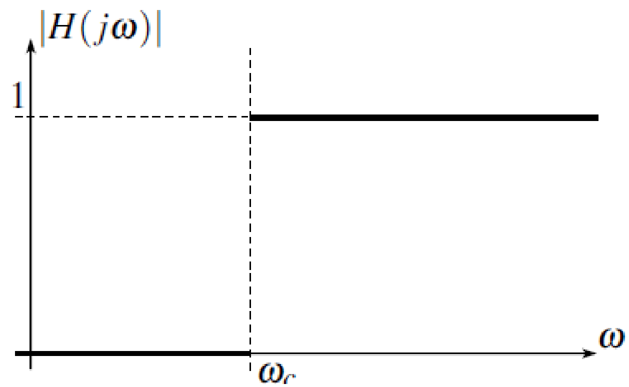
3 Description of the most common active filter applications:

low-pass
high-pass
band-pass
band-rejection
all-pass filters

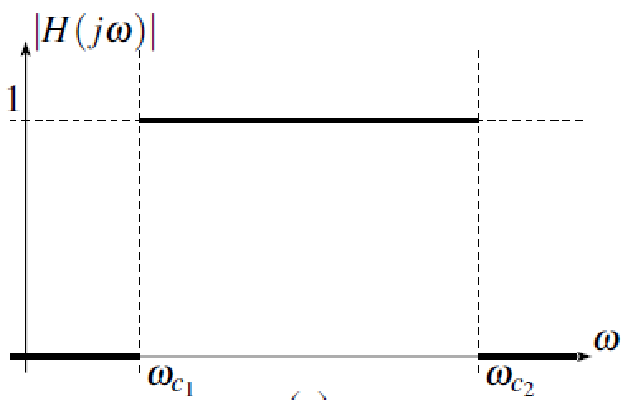
Active Filters



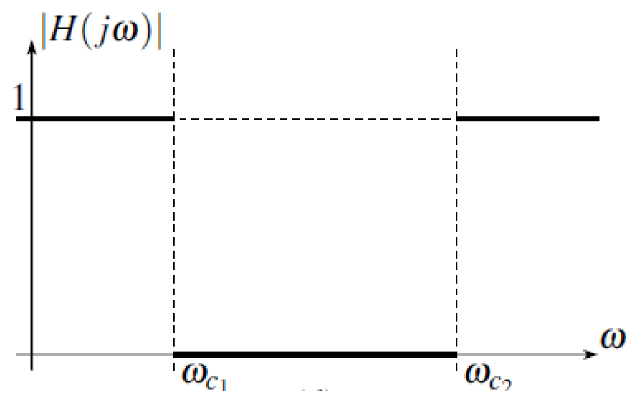
filtro passa-baixa



filtro passa-alta

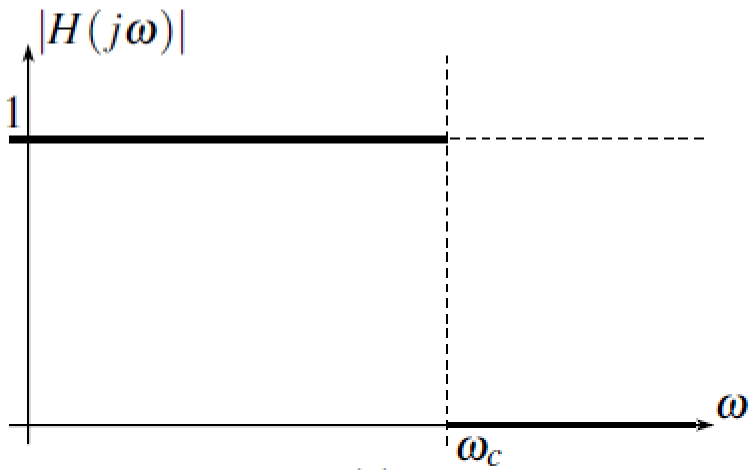


filtro passa-faixa

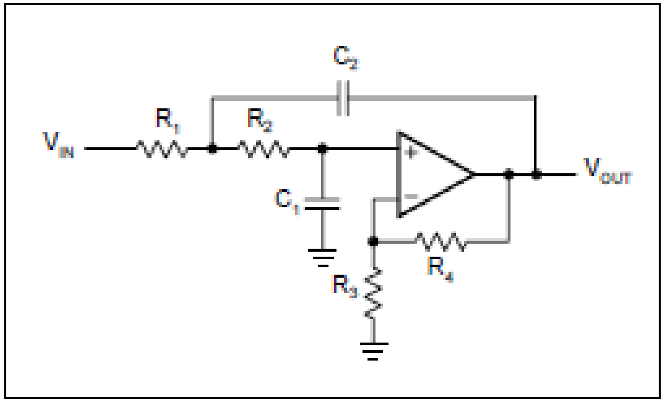


filtro rejeita-faixa

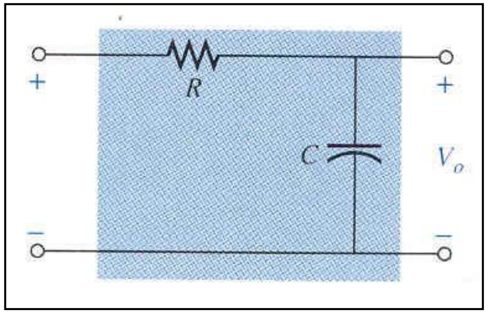
Active Filters



filtro passa-baixa

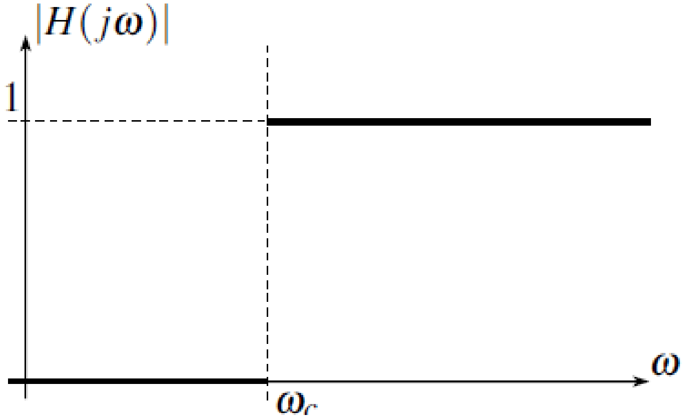


Active Filter
Sallen-Key Topology

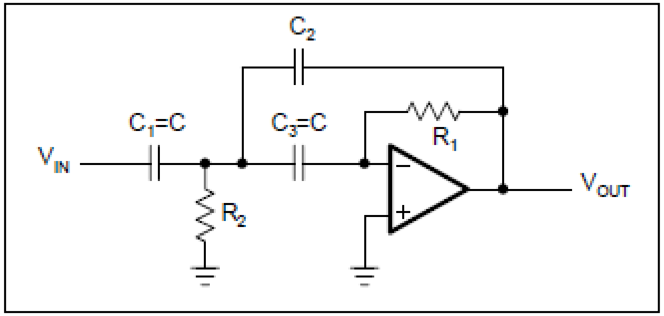


Passive Filter

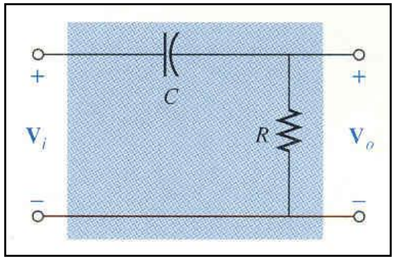
Active Filters



filtro passa-alta

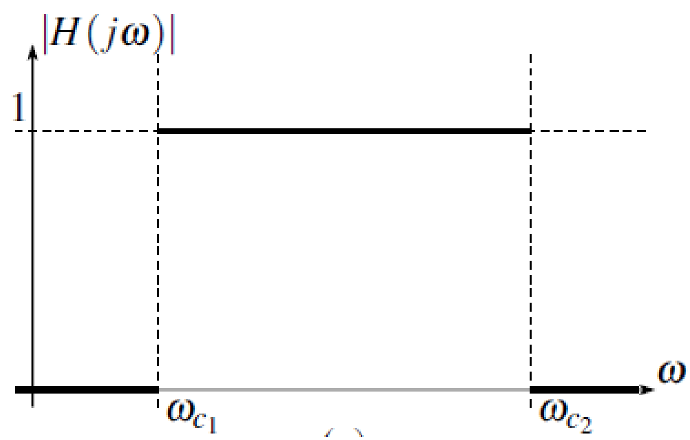


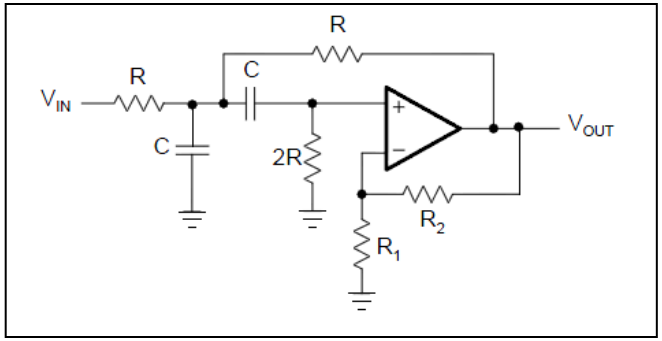
Active Filter
Multiple Feedback Topology



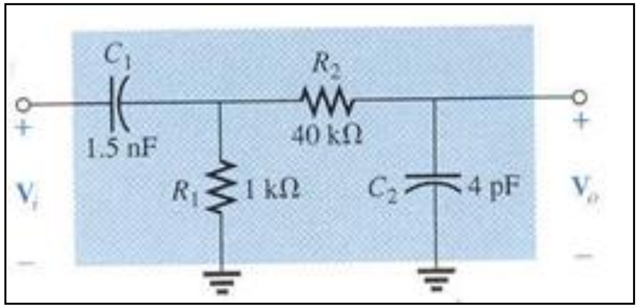
Passive Filter

Active Filters





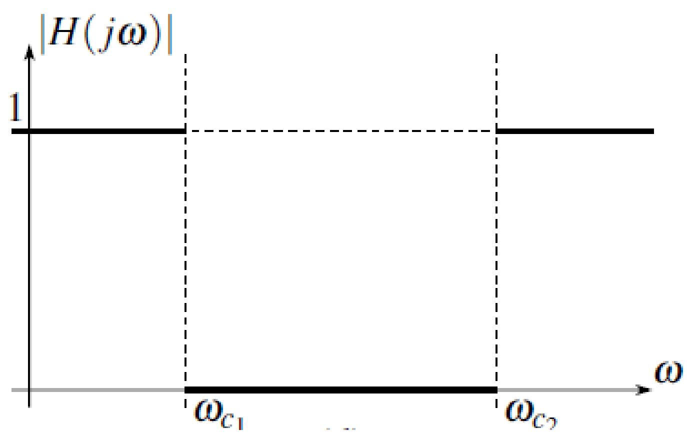
Active Filter
Sallen-Key Topology



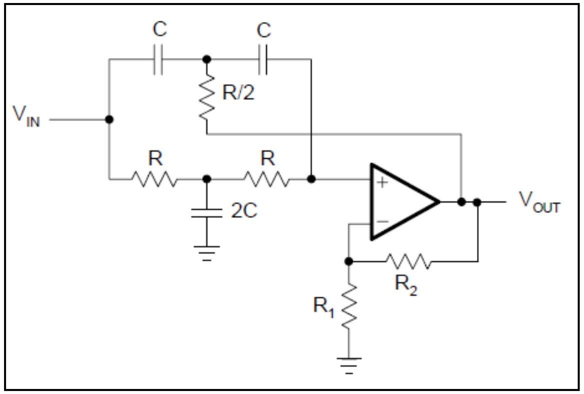
Passive Filter

filtro passa-faixa

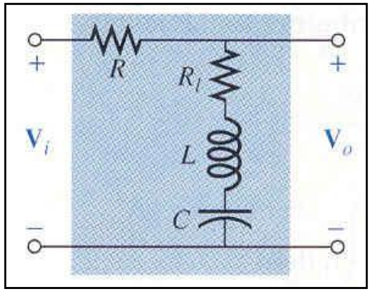
Active Filters



filtro rejeita-faixa

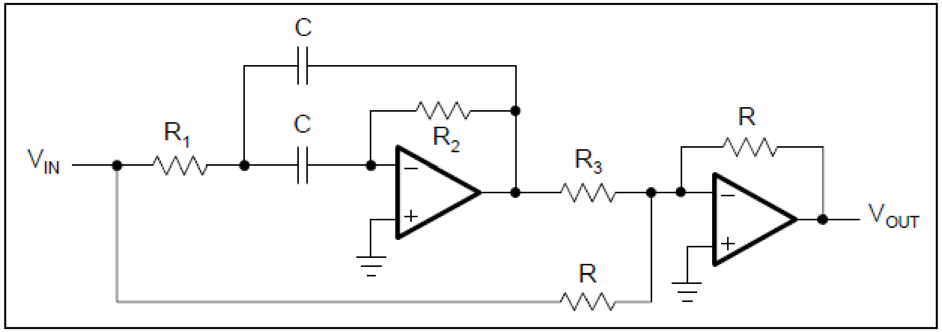


Active Filter
Twin-T Topology



Passive Filter

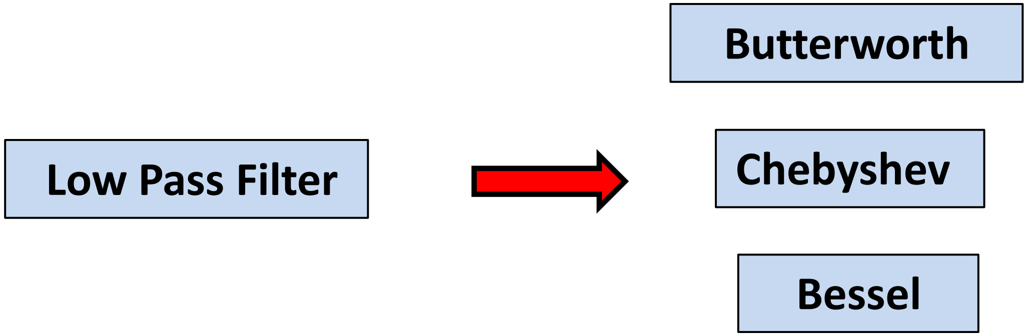
Active Filters



banda de passagem é todo o espectro de frequências mas com defasagens diferentes ao longo do espectro !

filtro passa-tudo

Low Pass Filters


Low Pass Filter
Butterworth
Chebyshev
Bessel

Low Pass Filter

General Transfer Function

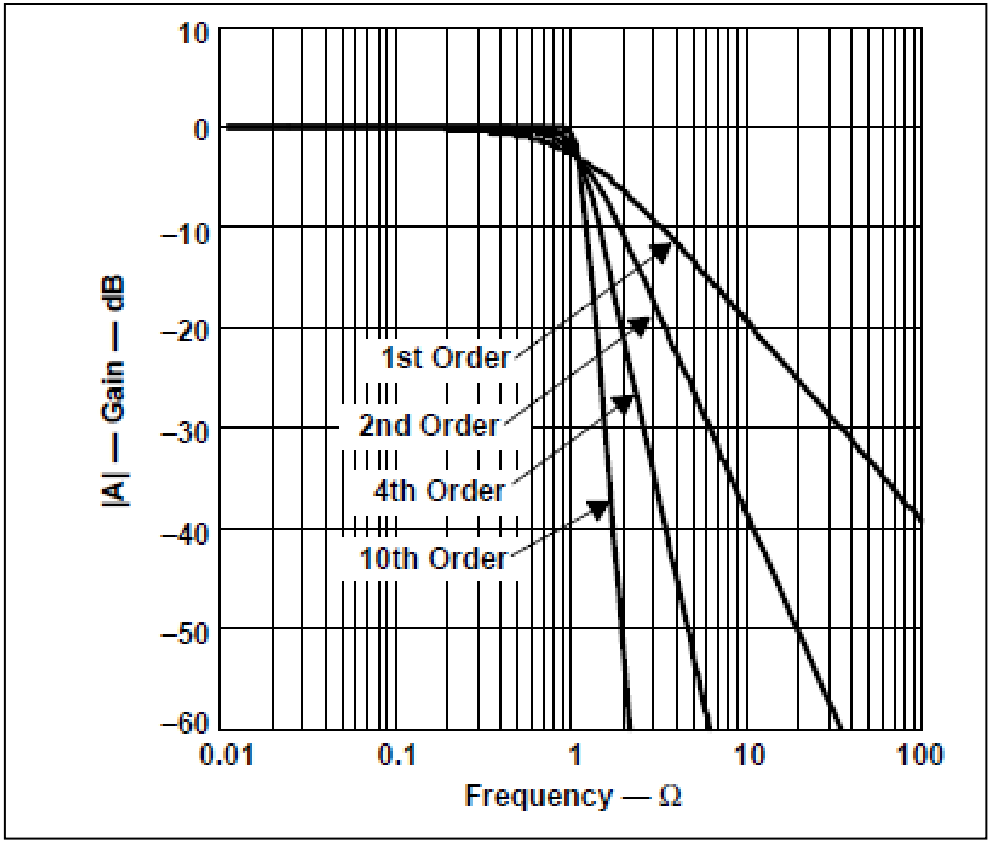
$$A(s) = \frac{A_0}{\prod_i \left(1 + a_i s + b_i s^2\right)}$$

1. The filter coefficients a_i and b_i distinguish between **Butterworth**, **Tschebyscheff**, and **Bessel** filters. The coefficients for all three types of filters are tabulated.

2. The order n determines the **gain rolloff above f_C ($n \cdot 20$ dB/decade)**

Butterworth Low Pass Filter

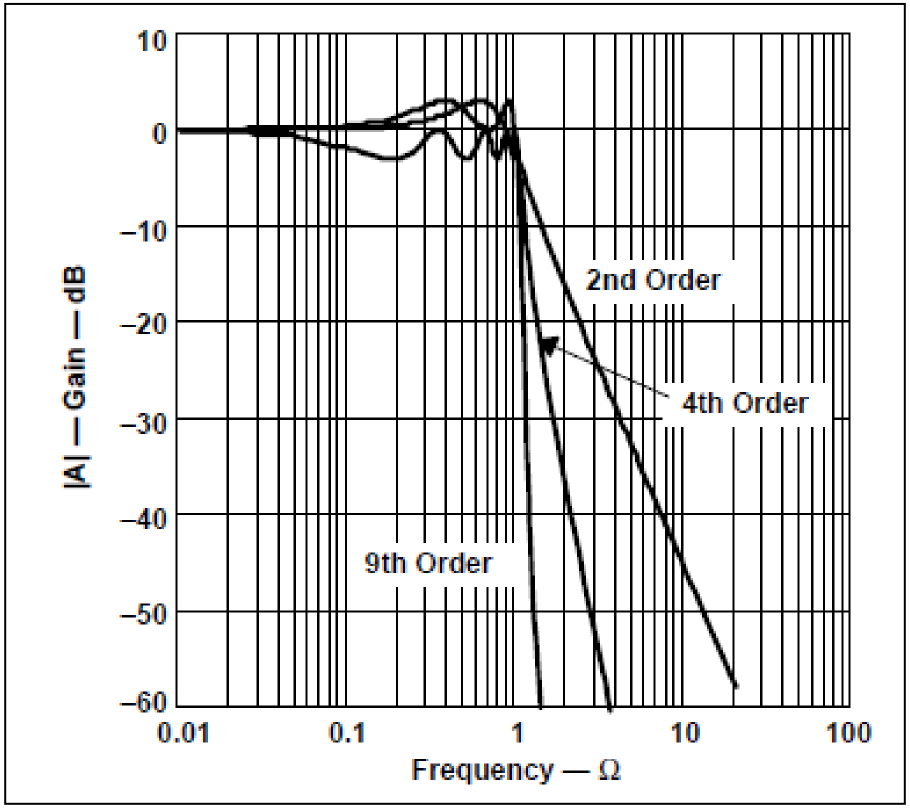
The Butterworth low-pass filter provides passband flatness.



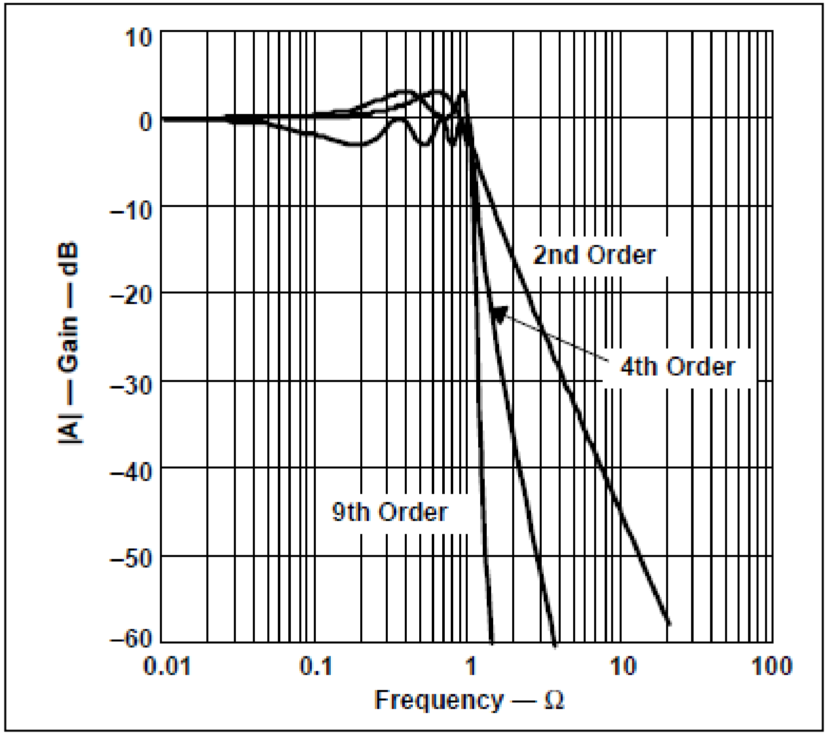
$\Omega = f / f_C$

Chebyshev Low Pass Filter

The Tschebyscheff low-pass filters provide an even higher gain rolloff above f_C. However, the passband gain is not monotone, but contains ripples of constant magnitude instead. For a given filter order, the higher the passband ripples, the higher the filter's rolloff.



$\Omega = f / f_C$



$\Omega = f / f_C$

1 For a given filter order, the higher the passband ripples, the higher the filter's rolloff.

2 Each ripple accounts for one second-order filter stage.

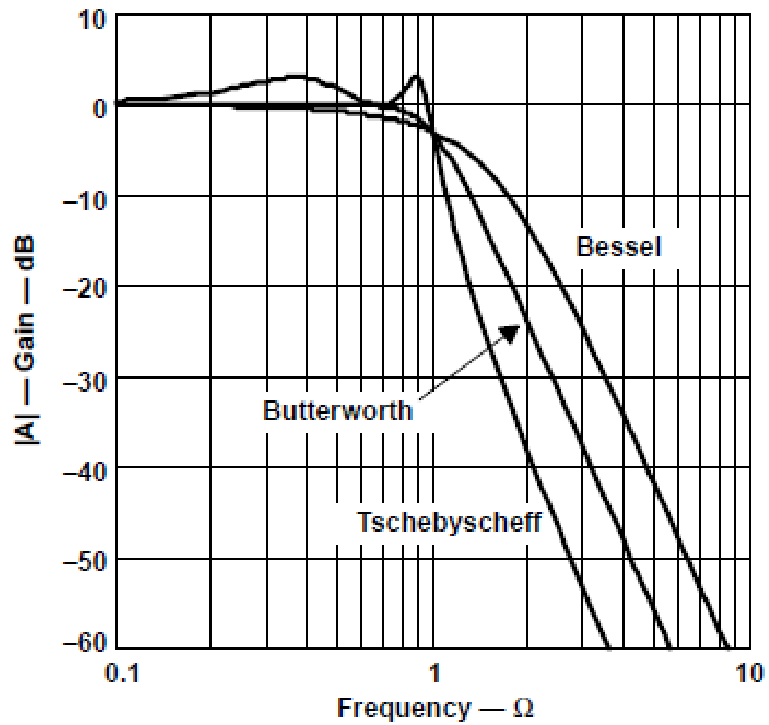
3 Filters with **even order numbers** generate **ripples above the 0-dB line.**

4 Filters with **odd order numbers** create **ripples below 0 dB**.

Bessel Low Pass Filter

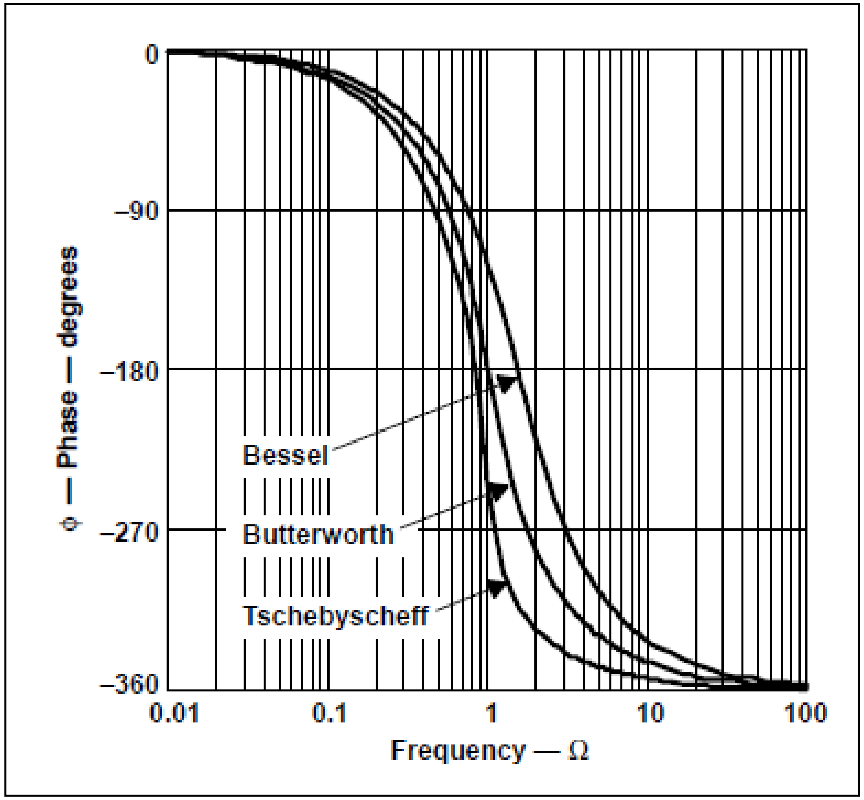
1. The passband gain of a Bessel low-pass filter is **not as flat as that of the Butterworth** low-pass

2. The transition from **passband to stopband is by far not as sharp as that of a Tschebyscheff** low-pass filter



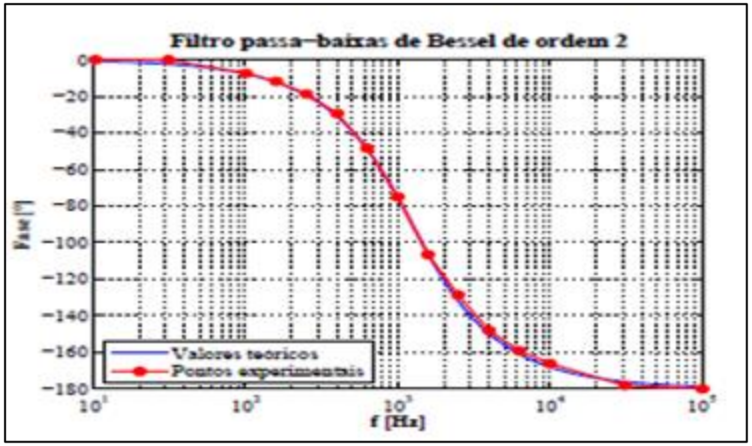
$\Omega = f / f_C$

3 The Bessel low-pass filters have a **linear phase response** over a wide frequency range

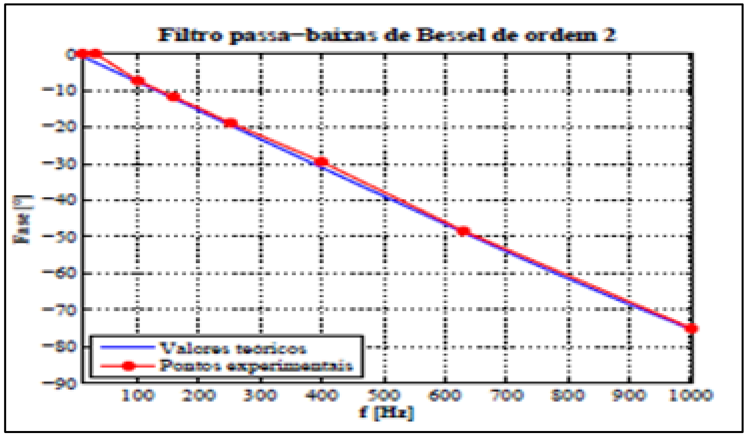


$\Omega = f / f_C$

4 The Bessel low-pass filters have a **linear phase response** over a wide frequency range



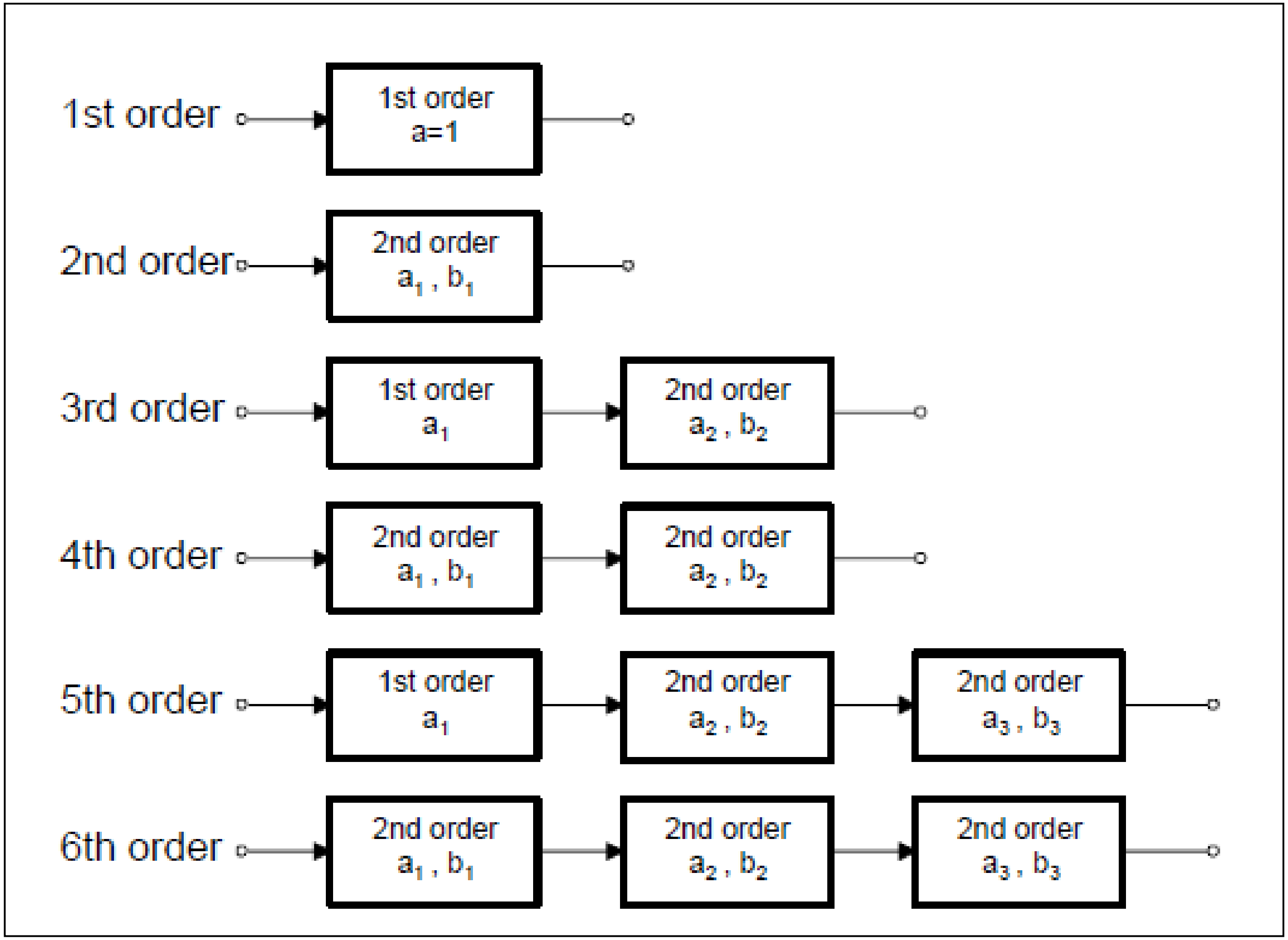
(monolog scale)



(linear scale)

Cascade Filters

The first-order and second-order filter stages are the building blocks for higher-order filters.



Designing
Low Pass Filters

Filters Coefficients

$$A(s) = \frac{A_0}{\prod_i \left(1 + a_i s + b_i s^2\right)}$$

Butterworth Coefficients

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62
6	1	1.9319	1.0000	0.676	0.52
	2	1.4142	1.0000	1.000	0.71
	3	0.5176	1.0000	1.479	1.93
7	1	1.0000	0.0000	1.000	—
	2	1.8019	1.0000	0.745	0.55
	3	1.2470	1.0000	1.117	0.80
	4	0.4450	1.0000	1.499	2.25
8	1	1.9616	1.0000	0.661	0.51
	2	1.6629	1.0000	0.829	0.60
	3	1.1111	1.0000	1.206	0.90
	4	0.3902	1.0000	1.512	2.56
9	1	1.0000	0.0000	1.000	—
	2	1.8794	1.0000	0.703	0.53
	3	1.5321	1.0000	0.917	0.65
	4	1.0000	1.0000	1.272	1.00
	5	0.3473	1.0000	1.521	2.88
10	1	1.9754	1.0000	0.655	0.51
	2	1.7820	1.0000	0.756	0.56
	3	1.4142	1.0000	1.000	0.71
	4	0.9080	1.0000	1.322	1.10
	5	0.3129	1.0000	1.527	3.20

Bessel Coefficients

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.3617	0.6180	1.000	0.58
3	1	0.7560	0.0000	1.323	—
	2	0.9996	0.4772	1.414	0.69
4	1	1.3397	0.4889	0.978	0.52
	2	0.7743	0.3890	1.797	0.81
5	1	0.6656	0.0000	1.502	—
	2	1.1402	0.4128	1.184	0.56
	3	0.6216	0.3245	2.138	0.92
6	1	1.2217	0.3887	1.063	0.51
	2	0.9686	0.3505	1.431	0.61
	3	0.5131	0.2756	2.447	1.02
7	1	0.5937	0.0000	1.648	—
	2	1.0944	0.3395	1.207	0.53
	3	0.8304	0.3011	1.695	0.66
	4	0.4332	0.2381	2.731	1.13
8	1	1.1112	0.3162	1.164	0.51
	2	0.9754	0.2979	1.381	0.56
	3	0.7202	0.2621	1.963	0.71
	4	0.3728	0.2087	2.992	1.23
9	1	0.5386	0.0000	1.857	—
	2	1.0244	0.2834	1.277	0.52
	3	0.8710	0.2636	1.574	0.59
	4	0.6320	0.2311	2.226	0.76
	5	0.3257	0.1854	3.237	1.32
10	1	1.0215	0.2650	1.264	0.50
	2	0.9393	0.2549	1.412	0.54
	3	0.7815	0.2351	1.780	0.62
	4	0.5604	0.2059	2.479	0.81
	5	0.2883	0.1665	3.466	1.42

Tschebyscheff Coefficients (0.5 dB Passband ripple)

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.3614	1.3827	1.000	0.86
3	1	1.8636	0.0000	0.537	—
	2	0.0640	1.1931	1.335	1.71
4	1	2.6282	3.4341	0.538	0.71
	2	0.3648	1.1509	1.419	2.94
5	1	2.9235	0.0000	0.342	—
	2	1.3025	2.3534	0.881	1.18
	3	0.2290	1.0833	1.480	4.54
6	1	3.8645	6.9797	0.366	0.68
	2	0.7528	1.8573	1.078	1.81
	3	0.1589	1.0711	1.495	6.51
7	1	4.0211	0.0000	0.249	—
	2	1.8729	4.1795	0.645	1.09
	3	0.4861	1.5676	1.208	2.58
	4	0.1156	1.0443	1.517	8.84
8	1	5.1117	11.9607	0.276	0.68
	2	1.0639	2.9365	0.844	1.61
	3	0.3439	1.4206	1.284	3.47
	4	0.0885	1.0407	1.521	11.53
9	1	5.1318	0.0000	0.195	—
	2	2.4283	6.6307	0.506	1.06
	3	0.6839	2.2908	0.989	2.21
	4	0.2559	1.3133	1.344	4.48
	5	0.0695	1.0272	1.532	14.58
10	1	6.3648	18.3695	0.222	0.67
	2	1.3582	4.3453	0.689	1.53
	3	0.4822	1.9440	1.091	2.89
	4	0.1994	1.2520	1.381	5.61
	5	0.0563	1.0263	1.533	17.99

Tschebyscheff Coefficients (1 dB Passband ripple)

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.3022	1.5515	1.000	0.96
3	1	2.2156	0.0000	0.451	—
	2	0.5442	1.2057	1.353	2.02
4	1	2.5904	4.1301	0.540	0.78
	2	0.3039	1.1697	1.417	3.56
5	1	3.5711	0.0000	0.280	—
	2	1.1280	2.4896	0.894	1.40
	3	0.1872	1.0814	1.486	5.56
6	1	3.8437	8.5529	0.366	0.76
	2	0.6292	1.9124	1.082	2.20
	3	0.1296	1.0766	1.493	8.00
7	1	4.9520	0.0000	0.202	—
	2	1.6338	4.4899	0.655	1.30
	3	0.3987	1.5834	1.213	3.16
	4	0.0937	1.0432	1.520	10.90
8	1	5.1019	14.7608	0.276	0.75
	2	0.8916	3.0426	0.849	1.96
	3	0.2806	1.4334	1.285	4.27
	4	0.0717	1.0432	1.520	14.24
9	1	6.3415	0.0000	0.158	—
	2	2.1252	7.1711	0.514	1.26
	3	0.5624	2.3278	0.994	2.71
	4	0.2076	1.3166	1.346	5.53
	5	0.0562	1.0258	1.533	18.03
10	1	6.3634	22.7468	0.221	0.75
	2	1.1399	4.5167	0.694	1.86
	3	0.3939	1.9665	1.093	3.56
	4	0.1616	1.2569	1.381	6.94
	5	0.0455	1.0277	1.532	22.26

Tschebyscheff Coefficients (2 dB Passband ripple)

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.1813	1.7775	1.000	1.13
3	1	2.7994	0.0000	0.357	—
	2	0.4300	1.2036	1.378	2.55
4	1	2.4025	4.9862	0.550	0.93
	2	0.2374	1.1896	1.413	4.59
5	1	4.6345	0.0000	0.216	—
	2	0.9090	2.6036	0.908	1.78
	3	0.1434	1.0750	1.493	7.23
6	1	3.5880	10.4648	0.373	0.90
	2	0.4925	1.9622	1.085	2.84
	3	0.0995	1.0826	1.491	10.46
7	1	6.4760	0.0000	0.154	—
	2	1.3258	4.7649	0.665	1.65
	3	0.3067	1.5927	1.218	4.12
	4	0.0714	1.0384	1.523	14.28
8	1	4.7743	18.1510	0.282	0.89
	2	0.6991	3.1353	0.853	2.53
	3	0.2153	1.4449	1.285	5.58
	4	0.0547	1.0461	1.518	18.39
9	1	8.3198	0.0000	0.120	—
	2	1.7299	7.6580	0.522	1.60
	3	0.4337	2.3549	0.998	3.54
	4	0.1583	1.3174	1.349	7.25
	5	0.0427	1.0232	1.536	23.68
10	1	5.9618	28.0376	0.226	0.89
	2	0.8947	4.6644	0.697	2.41
	3	0.3023	1.9858	1.094	4.66
	4	0.1233	1.2614	1.380	9.11
	5	0.0347	1.0294	1.531	29.27

Tschebyscheff Coefficients (3 dB Passband ripple)

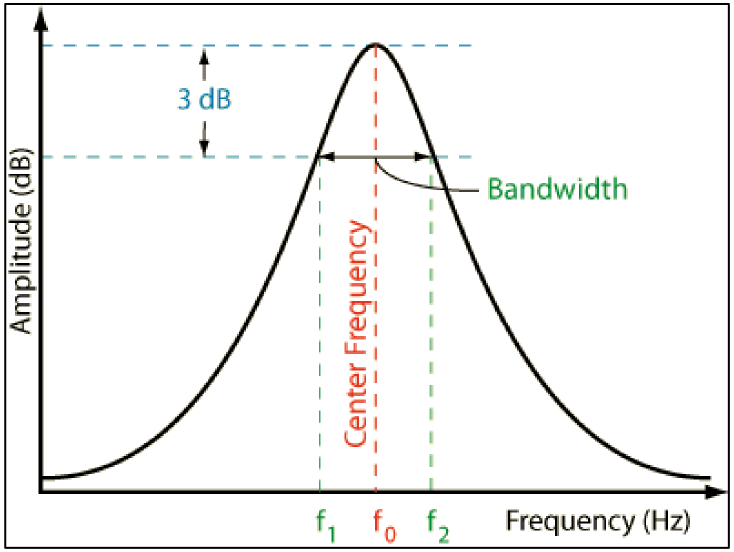
n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.0650	1.9305	1.000	1.30
3	1	3.3496	0.0000	0.299	—
	2	0.3559	1.1923	1.396	3.07
4	1	2.1853	5.5339	0.557	1.08
	2	0.1964	1.2009	1.410	5.58
5	1	5.6334	0.0000	0.178	—
	2	0.7620	2.6530	0.917	2.14
	3	0.1172	1.0686	1.500	8.82
6	1	3.2721	11.6773	0.379	1.04
	2	0.4077	1.9873	1.086	3.46
	3	0.0815	1.0861	1.489	12.78
7	1	7.9064	0.0000	0.126	—
	2	1.1159	4.8963	0.670	1.98
	3	0.2515	1.5944	1.222	5.02
	4	0.0582	1.0348	1.527	17.46
8	1	4.3583	20.2948	0.286	1.03
	2	0.5791	3.1808	0.855	3.08
	3	0.1765	1.4507	1.285	6.83
	4	0.0448	1.0478	1.517	22.87
9	1	10.1759	0.0000	0.098	—
	2	1.4585	7.8971	0.526	1.93
	3	0.3561	2.3651	1.001	4.32
	4	0.1294	1.3165	1.351	8.87
	5	0.0348	1.0210	1.537	29.00
10	1	5.4449	31.3788	0.230	1.03
	2	0.7414	4.7363	0.699	2.94
	3	0.2479	1.9952	1.094	5.70
	4	0.1008	1.2638	1.380	11.15
	5	0.0283	1.0304	1.530	35.85

All Pass

n	i	a_i	b_i	f_i / f_C	Q_i	T_{gr0}
1	1	0.6436	0.0000	1.554	—	0.2049
2	1	1.6278	0.8832	1.064	0.58	0.5181
3	1	1.1415	0.0000	0.876	—	0.8437
	2	1.5092	1.0877	0.959	0.69	
4	1	2.3370	1.4878	0.820	0.52	1.1738
	2	1.3506	1.1837	0.919	0.81	
5	1	1.2974	0.0000	0.771	—	1.5060
	2	2.2224	1.5685	0.798	0.56	
	3	1.2116	1.2330	0.901	0.92	
6	1	2.6117	1.7763	0.750	0.51	1.8395
	2	2.0706	1.6015	0.790	0.61	
	3	1.0967	1.2596	0.891	1.02	
7	1	1.3735	0.0000	0.728	—	2.1737
	2	2.5320	1.8169	0.742	0.53	
	3	1.9211	1.6116	0.788	0.66	
	4	1.0023	1.2743	0.886	1.13	
8	1	2.7541	1.9420	0.718	0.51	2.5084
	2	2.4174	1.8300	0.739	0.56	
	3	1.7850	1.6101	0.788	0.71	
	4	0.9239	1.2822	0.883	1.23	
9	1	1.4186	0.0000	0.705	—	2.8434
	2	2.6979	1.9659	0.713	0.52	
	3	2.2940	1.8282	0.740	0.59	
	4	1.6644	1.6027	0.790	0.76	
	5	0.8579	1.2862	0.882	1.32	
10	1	2.8406	2.0490	0.699	0.50	3.1786
	2	2.6120	1.9714	0.712	0.54	
	3	2.1733	1.8184	0.742	0.62	
	4	1.5583	1.5923	0.792	0.81	
	5	0.8018	1.2877	0.881	1.42	

Quality Factor

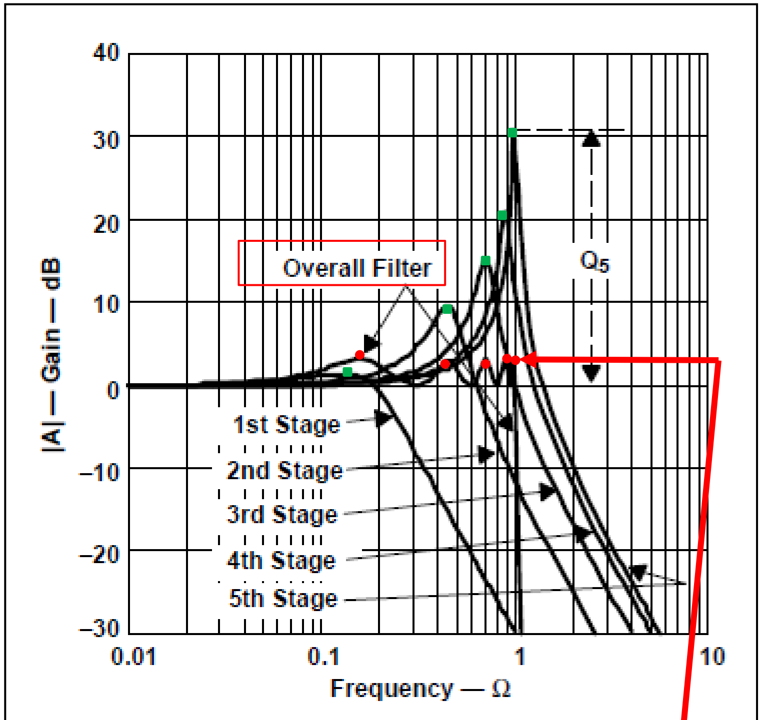
Band Pass Filters → $$Q = \frac{f_o}{f_2 - f_1}$$



Low and High Pass Filters → $$Q = \frac{\sqrt{b_i}}{a_i}$$

Example: low pass filter with **5 stages** **(Tschebyscheff – 2dB)**

Tschebyscheff Coefficients (2 dB Passband ripple)



n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.1813	1.7775	1.000	1.13
3	1	2.7994	0.0000	0.357	—
	2	0.4300	1.2036	1.378	2.55
4	1	2.4025	4.9862	0.550	0.93
	2	0.2374	1.1896	1.413	4.59
5	1	4.6345	0.0000	0.216	—
	2	0.9090	2.6036	0.908	1.78
	3	0.1434	1.0750	1.493	7.23
6	1	3.5880	10.4648	0.373	0.90
	2	0.4925	1.9622	1.085	2.84
	3	0.0995	1.0826	1.491	10.46
7	1	6.4760	0.0000	0.154	—
	2	1.3258	4.7649	0.665	1.65
	3	0.3067	1.5927	1.218	4.12
	4	0.0714	1.0384	1.523	14.28
8	1	4.7743	18.1510	0.282	0.89
	2	0.6991	3.1353	0.853	2.53
	3	0.2153	1.4449	1.285	5.58
	4	0.0547	1.0461	1.518	18.39
9	1	8.3198	0.0000	0.120	—
	2	1.7299	7.6580	0.522	1.60
	3	0.4337	2.3549	0.998	3.54
	4	0.1583	1.3174	1.349	7.25
	5	0.0427	1.0232	1.536	23.68
10	1	5.9618	28.0376	0.226	0.89
	2	0.8947	4.6644	0.697	2.41
	3	0.3023	1.9858	1.094	4.66
	4	0.1233	1.2614	1.380	9.11
	5	0.0347	1.0294	1.531	29.27

$$Q = \frac{\sqrt{b_i}}{a_i}$$

$$\boxed{Q_5 = \frac{\sqrt{b_5}}{a_5}}$$

$$\boxed{Q_5(\mathrm{dB}) = 20 \log Q_5}$$

ripple with 5 peaks

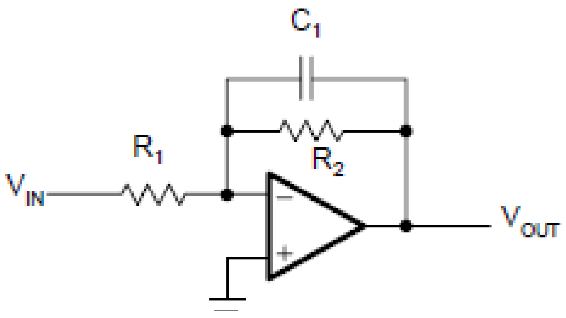
Transfer Function

First order filter → $A(s) = \dfrac{A_0}{1 + a_1 s}$

Second order filter → $A(s) = \dfrac{A_0}{1 + a_i s + b_i s^2}$

Low Pass Filters (First Order Topology)

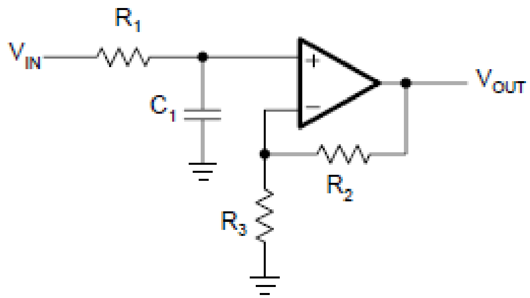
Inverting



$$A(s) = \frac{-\frac{R_2}{R_1}}{1 + \underbrace{\omega_c R_2 C_1}_{a_1} s}$$

$$A_0 = -\frac{R_2}{R_1}$$ **(DC gain)**

Noninverting

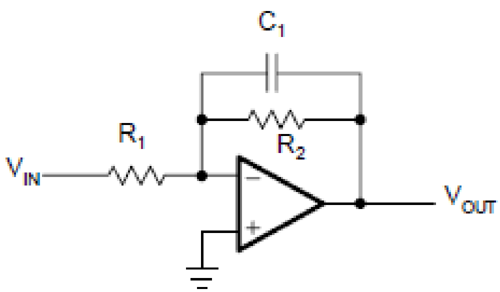


$$A(s) = \frac{1 + \frac{R_2}{R_3}}{1 + \underbrace{\omega_c R_1 C_1}_{a_1} s}$$

$$A_0 = 1 + \frac{R_2}{R_3}$$ **(DC gain)**

Designing Low Pass Filters (First Order Topology)

Inverting

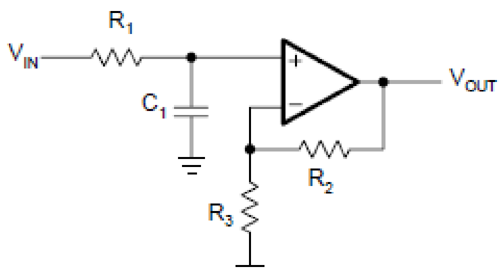


1 Specify f_C, A_0 , C_1

2 $R_2 = \dfrac{a_1}{2\pi f_c C_1}$

3 $R_1 = -\dfrac{R_2}{A_0}$

Noninverting

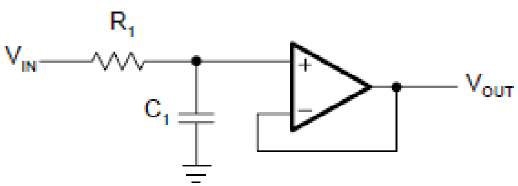


(Ganho ≠ 1)

1 Specify f_C, A_0, C_1

2 $R_1 = \dfrac{a_1}{2\pi f_c C_1}$

3 $R_2 = R_3(A_0 - 1)$



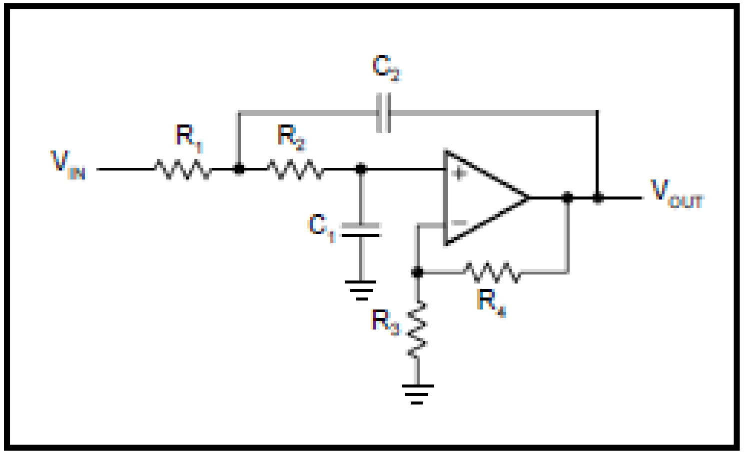
(Ganho = 1)

1 Specify f_C, C_1

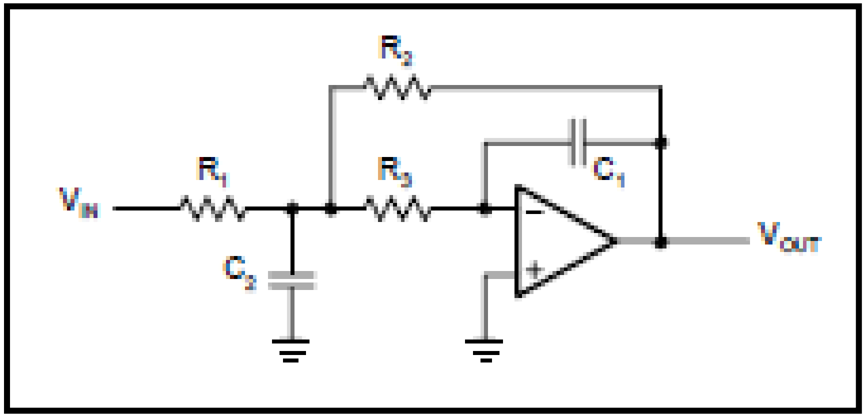
2 $R_1 = \dfrac{a_1}{2\pi f_c C_1}$

Low Pass Filters
(Second Order Topology)

Sallen-Key Topology

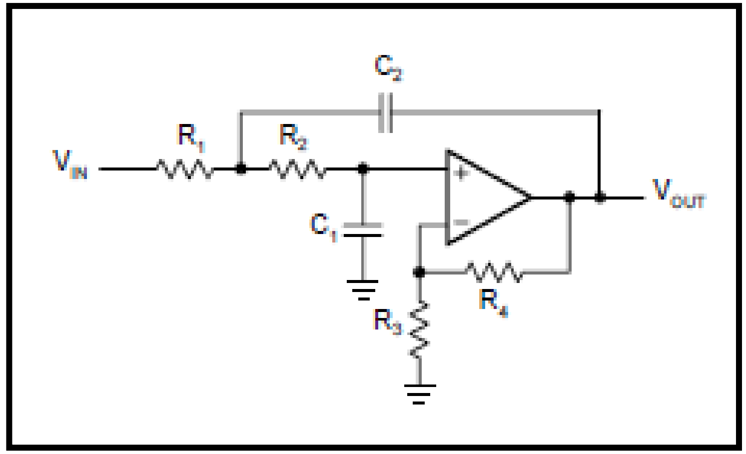


Multiple Feedbak Topology



Sallen-Key Topology ($A_o \neq 1$)

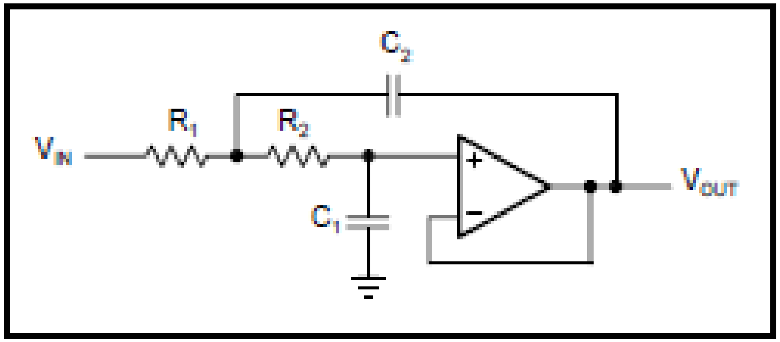
- The SK topology is commonly used in filters.



$$A(s) = \frac{A_0}{1 + \underbrace{\omega_c\left[C_1(R_1 + R_2) + (1 - A_0)\,R_1C_2\right]}_{a_1} s + \underbrace{\omega_c^2\,R_1R_2C_1C_2 s^2}_{b_1}}$$

Sallen-Key Topology (A_o=1)

- The SK topology is commonly used in filters.



$$A(s) = \frac{1}{1 + \underbrace{\omega_c C_1 (R_1 + R_2)}_{a_1} s + \underbrace{\omega_c^2 R_1 R_2 C_1 C_2}_{b_1} s^2}$$

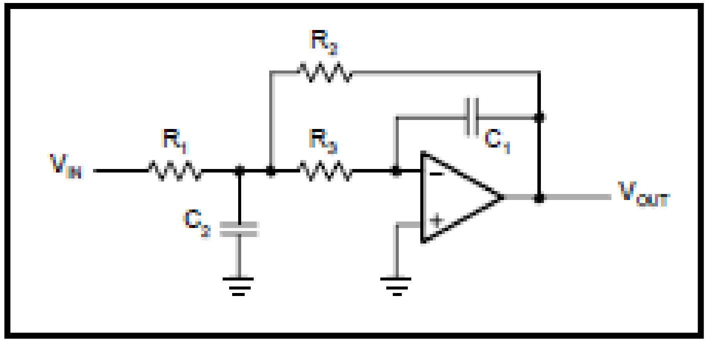
$$\boxed{A_0 = 1}$$

$$\boxed{a_1 = \omega_c C_1 (R_1 + R_2)}$$

$$\boxed{b_1 = \omega_c^2 R_1 R_2 C_1 C_2}$$

Multiple Feedbak Topology

- The MFB topology is commonly used in filters that have high Qs and require a high gain.



$$A(s) = -\frac{\frac{R_2}{R_1}}{1 + \underbrace{\omega_c C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)}_{a_1} s + \underbrace{\omega_c^2\, C_1 C_2 R_2 R_3}_{b_1} s^2}$$

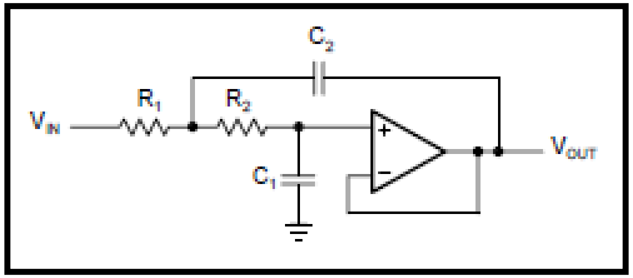
$$A_0 = -\frac{R_2}{R_1}$$

$$a_1 = \omega_c C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)$$

$$b_1 = \omega_c^2\, C_1 C_2 R_2 R_3$$

Designing Low Pass Filters (Second Order) Sallen-Key Topology

Sallen-Key Topology ($A_o = 1$)



1 Pick C_1

2 Determine filter coefficients a_1 e b_1

3 In order to obtain real values under the square root, C2 must satisfy the following condition:

$$C_2 \geq C_1 \frac{4b_1}{a_1^2}$$

4 Calculate R_1 and R_2

$$R_1 = \frac{a_1 C_2 - \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2}$$

$$R_2 = \frac{a_1 C_2 + \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2}$$

Exemple 1:

Design a second order unity gain Tschebyscheff low pass filter with a corner frequency of 3KHz and a 3dB passband ripple

1 Pick C_1 =22nF

2 Get the Tschebyscheff filters coefficient tables: a_1 = 1.0650 and b_1 = 1,9305

Table 16–9. Tschebyscheff Coefficients for 3-dB Passband Ripple

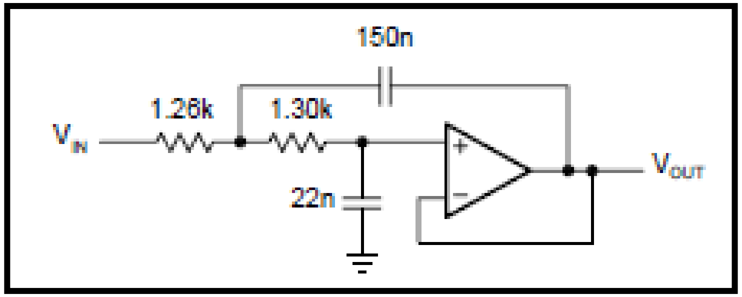
n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.0650	1.9305	1.000	1.30
3	1	3.3496	0.0000	0.299	—
	2	0.3559	1.1923	1.396	3.07

3 $C_2 \ge C_1 \frac{4b_1}{a_1{}^2}$ $\Rightarrow$ $C_2 \ge C_1 \frac{4b_1}{a_1{}^2} = 22 \cdot 10^{-9} nF \cdot \frac{4 \cdot 1.9305}{1.065^2} \cong 150\ nF$

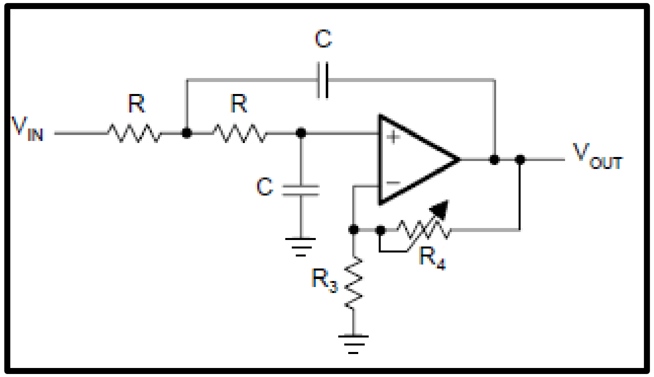
4 Calculate R_1 and R_2

$$R_1 = \frac{1.065 \cdot 150 \cdot 10^{-9} - \sqrt{(1.065 \cdot 150 \cdot 10^{-9})^2 - 4 \cdot 1.9305 \cdot 22 \cdot 10^{-9} \cdot 150 \cdot 10^{-9}}}{4\pi \cdot 3 \cdot 10^3 \cdot 22 \cdot 10^{-9} \cdot 150 \cdot 10^{-9}} = 1.26\ k\Omega$$

$$R_2 = \frac{1.065 \cdot 150 \cdot 10^{-9} + \sqrt{(1.065 \cdot 150 \cdot 10^{-9})^2 - 4 \cdot 1.9305 \cdot 22 \cdot 10^{-9} \cdot 150 \cdot 10^{-9}}}{4\pi \cdot 3 \cdot 10^3 \cdot 22 \cdot 10^{-9} \cdot 150 \cdot 10^{-9}} = 1.30\ k\Omega$$



Sallen-Key Topology
($A_o \neq 1$)



(R_1=R_2= R and C_1=C_2=C)

$$A(s) = \frac{A_0}{1 + \omega_c \underbrace{\left[C_1(R_1 + R_2) + (1 - A_0)\, R_1 C_2\right]}_{a_1} s + \underbrace{\omega_c^2\, R_1 R_2 C_1 C_2}_{b_1} s^2}$$

$$A(s) = \frac{A_0}{1 + \omega_c RC(3 - A_0)s + (\omega_c RC)^2 s^2}$$

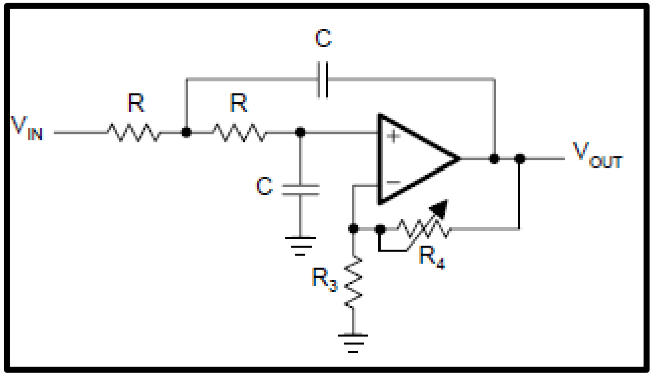
$$A_o = 3 - \frac{a_n}{\sqrt{b_n}}$$

$$A_0 = 1 + \frac{R_4}{R_3}$$

$$a_1 = \omega_c RC(3 - A_0)$$

$$b_1 = (\omega_c RC)^2$$

Sallen-Key Topology ($A_o \neq 1$)



1. Get a_n and b_n

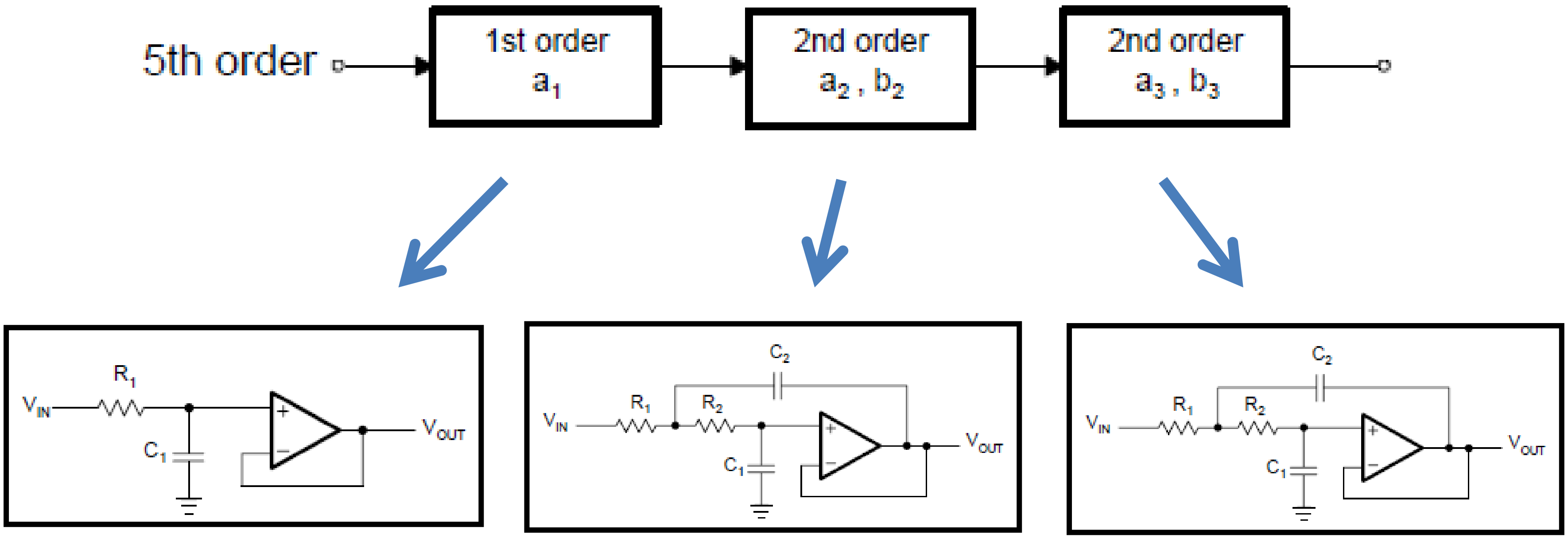
2. Pick C and get R → $$R = \frac{\sqrt{b_1}}{2\pi f_c C}$$

3. Calculate A_0 and R_4/R_3 → $$A_0 = 3 - \frac{a_1}{\sqrt{b_1}}$$ $$A_0 = 1 + \frac{R_4}{R_3}$$

4. Calculate Q → $$Q = \frac{1}{3 - A_0}$$

Designing Low Pass Filters (Higher Order) Sallen-Key Topology

Exemple 1:
Design a fifth order unity gain Butterworth low pass filter with a corner frequency of 50KHz.



1

Get Butterworth Coefficients

Table 16–5. Butterworth Coefficients

n	i	a_i	b_i	$k_i = f_{Ci} / f_C$	Q_i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62

First Filter: first order

2 Pick $C_1 = 1nF$

3 Get R_1

$$R_1 = \frac{a_1}{2\pi f_c C_1} = \frac{1}{2\pi \cdot 50 \cdot 10^3 Hz \cdot 1 \cdot 10^{-9} F} = 3.18\ k\Omega$$

Second Filter: Sallen-Key second order

4 Pick C_1 = 820pF

5 $C_2 \geq C_1 \frac{4b_1}{{a_1}^2}$ $\Rightarrow$ $C_2 \geq C_1 \frac{4b_2}{{a_2}^2} = 820 \cdot 10^{-12} F \cdot \frac{4 \cdot 1}{1.618^2} = 1.26\ nF$

6 With C_1 = 820pF and C_2 = 1.5nF calculate R_1 and R_2 $\Rightarrow$

$$\begin{cases} R_1 = \dfrac{a_1 c_2 - \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2} = 1.87\text{K}\ \Omega \\[2ex] R_2 = \dfrac{a_1 c_2 + \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2} = 4.42\ \text{K}\Omega \end{cases}$$

Third Filter: Sallen-Key second order

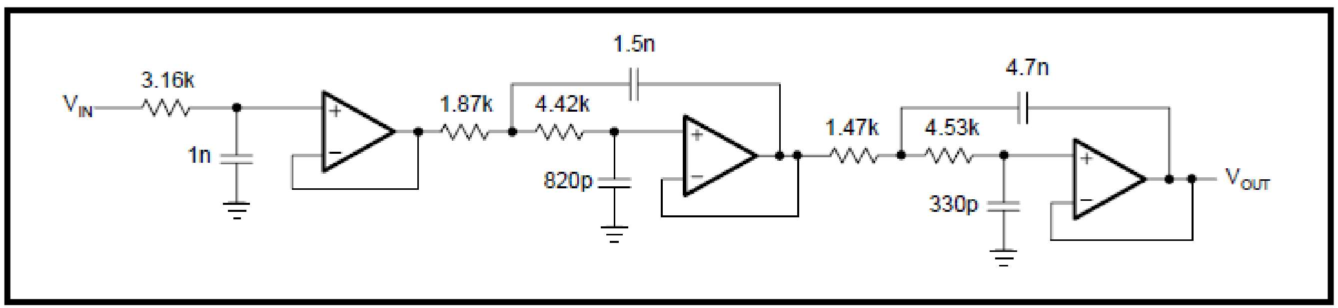
7 Pick C_1 = 330pF

8 $C_2 \geq C_1 \frac{4b_1}{a_1{}^2}$ → $C_2 \geq C_1 \frac{4b_3}{a_3{}^2} = 330 \cdot 10^{-12} F \cdot \frac{4 \cdot 1}{0.618^2} = 3.46\ nF$

9 With C_1 = 330pF and C_2 = 4.7nF calculate R_1 and R_2 →

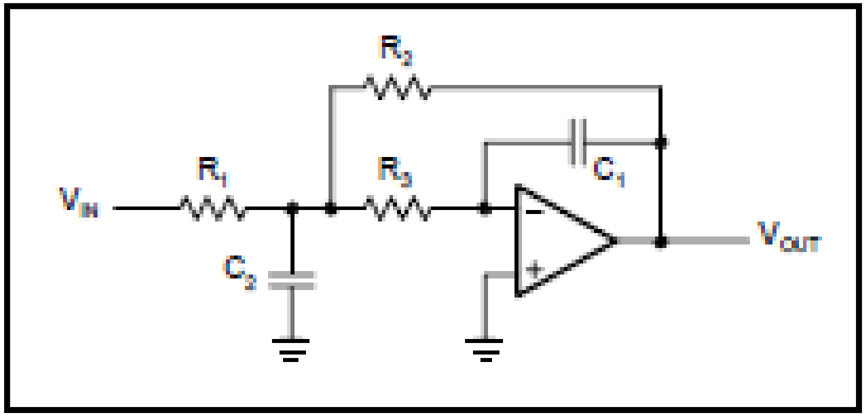
$$R_1 = \frac{a_1 C_2 - \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2} = 1.47K\ \Omega$$

$$R_2 = \frac{a_1 C_2 + \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4\pi f_c C_1 C_2} = 4.53\ K\Omega$$



Designing Low Pass Filters (Second Order) Multiple Feedback Topology

Multiple Feedbak Topology



$$A(s) = -\frac{\frac{R_2}{R_1}}{1 + \underbrace{\omega_c C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)}_{\boxed{a_1}} s + \underbrace{\omega_c^2 C_1 C_2 R_2 R_3}_{\boxed{b_1}} s^2}$$

$$A_0 = -\frac{R_2}{R_1}$$

$$a_1 = \omega_c C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)$$

$$b_1 = \omega_c^2 C_1 C_2 R_2 R_3$$

1 Choose C_1 , C_2

2 Calculate R_1, R_2, R_3

$$\boxed{R_2 = \frac{a_1 C_2 - \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2 (1 - A_0)}}{4\pi f_c C_1 C_2}}$$

$$\boxed{R_1 = \frac{R_2}{-A_0}}$$

$$\boxed{R_3 = \frac{b_1}{4\pi^2 f_c^2 C_1 C_2 R_2}}$$