

Laboratório 5

Oscilador Pierce

Referências

Veronese R, Osciladores, In: Notas de Aula da SEL318 – Circuitos Eletrônicos III.

Crystal Oscillators – Digital Eletronics Autumn 2014

Sedra AS, Smith KC, Microelectronics Circuits, Sixth Edition, Oxiford University Press, 2011

Roteiro Experimental

SEL0318 - Laboratório de Circuitos Eletrônicos III

Laboratório 6 – Oscilador Pierce

Osciladores baseados em filtros de ressonância são a melhor forma de se gerar oscilações em alta frequência com grande qualidade harmônica e com a possibilidade de se controlar a sua frequência por intermédio de uma tensão. As principais deficiências desses osciladores são as imprecisões devido ao espalhamento (tolerância) de fabricação dos componentes e a baixa qualidade presente nos indutores práticos.

Componentes especiais, **os cristais de quartzo (construídos de óxido de silício em cristal) são feitos para possuírem ressonâncias internas em frequências bem específicas, com pouquíssima variação e espalhamento de fabricação.** Essas ressonâncias internas, em duas frequências bem definidas e próximas, garantem que um oscilador que emprega um cristal de quartzo irá oscilar na frequência do cristal, o que é essencial para sincronização de microprocessadores, transmissão serial de dados, sistemas de comunicação.

1. Implementação em Protobord

1.1) Implemente o circuito da figura 1 em bancada com alimentação simples de 5V e transistor bipolar.

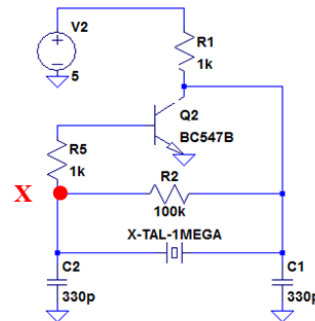


Fig. 1 - Oscilador Pierce

- 1.2) Capture as formas de onda no coletor do transistor (X) e na entrada do amplificador (Y).
- 1.3) Faça uma análise da distorção harmônica das duas formas de onda medindo a frequência fundamental das duas ondas, a amplitude pico a pico e a fase de uma com relação à outra.
- 1.4) Substitua o cristal de 1MHz por outros de 1.8432MHz, 4MHz e 10MHz. Ao utilizar o cristal de 10 MHz substitua os capacitores C_1 e C_2 por 56pF. Repita os procedimentos dos itens 1.2 e 1.3.

2. Simulação no LTSpice

2.1) Implemente no simulador o cristal de quartzo utilizando o componente “xtal”. Esse componente é tratado como um capacitor pelo simulador onde devem ser incluídos os quatro parâmetros que determinam as ressonâncias do dispositivo. Conforme apostila “Osciladores” do Prof. Paulo Roberto Veronese, um cristal de 1MHz apresenta as seguintes características:

$C_s = 9,95358 \text{ fF}$, $R = 640 \Omega$, $L = 2,54648 \text{ H}$ e $C_p = 2,4884 \text{ pF}$.

2.2) Implemente o circuito da Fig. 2 com uma fonte de varredura AC sendo aplicada ao cristal. Faça a análise .AC e plote a impedância do cristal com uma faixa de frequência adequada e resolução suficiente para a visualização das duas ressonâncias (série e paralelo). Determine o valor da frequência dessas ressonâncias.

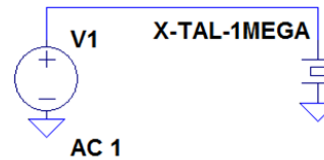


Fig. 2 - Simulação da resposta em frequência do cristal

OBS: Um circuito semelhante ao da Fig. 2 pode ser simulado para a visualização das duas ressonâncias (série e paralelo) utilizando-se as seguintes características de um cristal de 8 MHz:

$C_s = 27 \text{ fF}$, $R = 8\Omega$, $L = 14\text{mH}$ e $C_p = 5,6 \text{ pF}$.

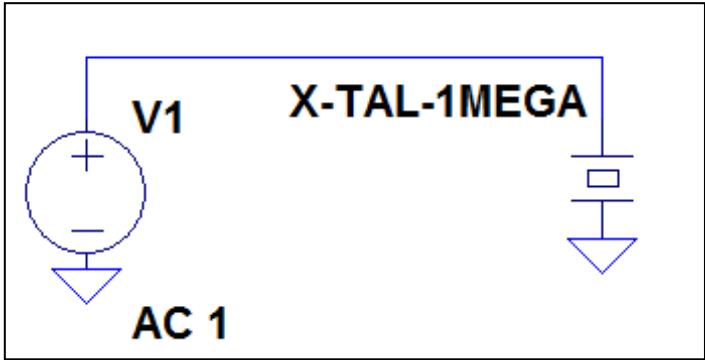
2.3) Qual a característica da impedância do cristal antes e depois das frequências de ressonância e entre as frequências de ressonância ?

2.4) Implemente o circuito da Fig. 1. 1 e faça a análise transiente em um tempo adequado utilizando o **comando startup** para garantir a oscilação do circuito.

2.5) Plote a forma de onda no nó X.

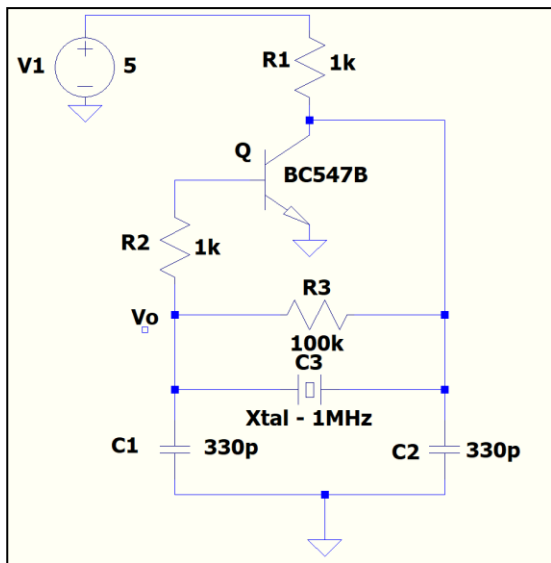
2.6) Faça a análise de distorção harmônica do sinal de entrada e de saída através das respectivas FFTs destacando a frequência da fundamental.

Simulação

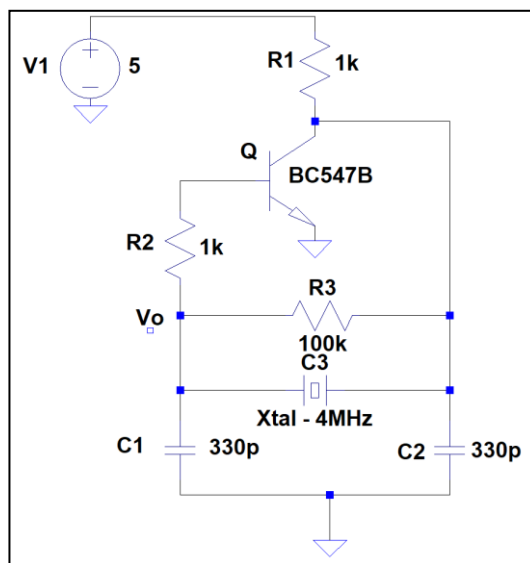


Implementação em Protoboard

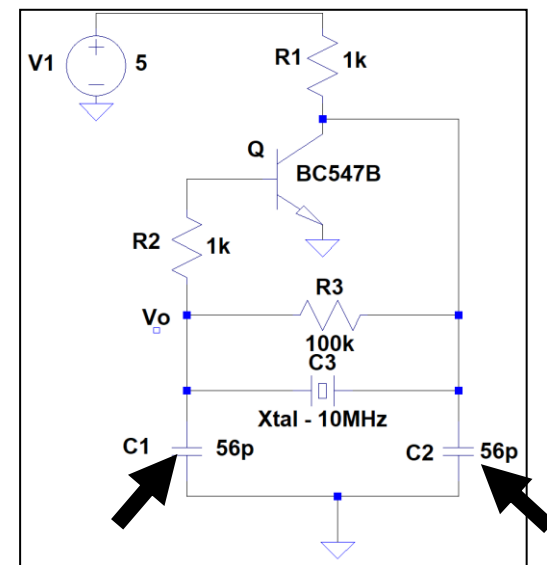
Inversor com BJT



1MHz



4MHz



10MHz

1

Os osciladores Colpitt e Hartley são osciladores senoidais construídos com células ressonantes *LC* e projetados para altas frequências.

2

O **Colpitt**, pela ausência de acoplamentos magnéticos entre bobinas, **apresenta estabilidade superior ao Hartley**, além de mais simples para ser projetado.

Por possuir mais ramos capacitivos de baixa reatância em altas frequências e nenhum acoplamento magnético indutivo, **o oscilador Colpitts apresenta, em geral, uma senóide de saída muito mais pura do que o oscilador Hartley e pode alcançar oscilações em frequências muito mais elevadas, como, por exemplo, na faixa de microondas.**

Se precisar ser sintonizado, no entanto, a sintonia deve ser feita através da variação da indutância da bobina, que nem sempre é viável ou simples.

3

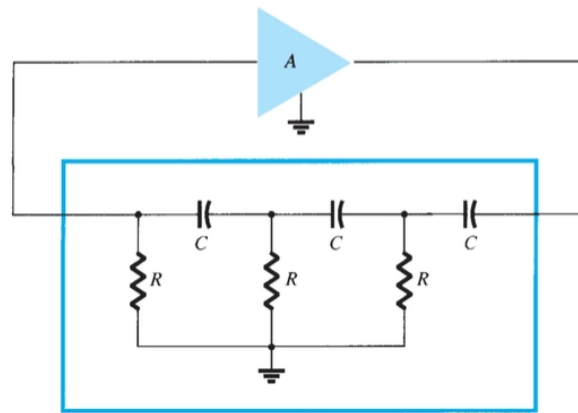
O **Hartley, por ser baseado em indutores, é preferido para trabalhar em faixas de frequência mais baixas**, como ondas médias e curtas de *RF*. Se precisar ser sintonizado **a sintonia é feita através de apenas um capacitor, o que facilita o projeto.**

Type of Oscillators

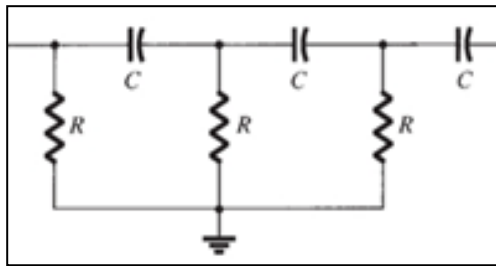
- 1 RC Oscillators
- 2 LC Oscillators
- 3 Relaxation Oscillators
- 4 Crystal Oscillators

RC Oscillators

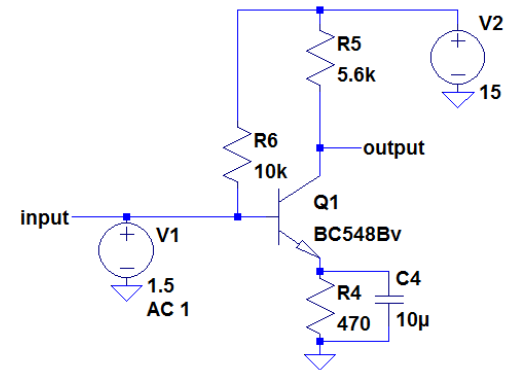
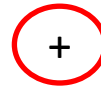
- In an *RC oscillator* circuit, the filter is a network of resistors and capacitors. *RC* oscillators are mostly used to generate lower frequencies, for example in the audio range. Common types of *RC* oscillator circuits are the phase shift oscillator and the Wien bridge oscillator.



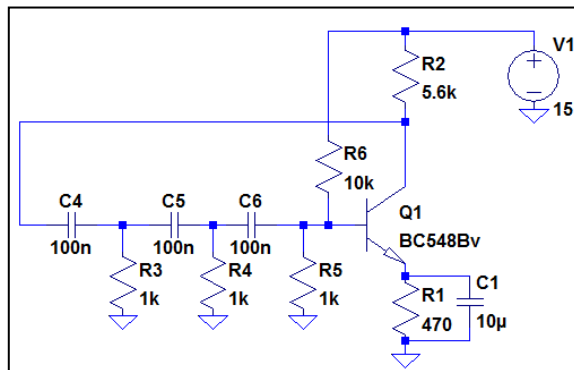
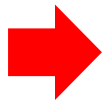
Malha de realimentação



Filtro Passivo RC



amplificador emissor comum
com ganho inversor



Oscilador com deslocador
de fase com malha de
realimentação RC e
amplificador emissor
comum com ganho inversor

$$f = \frac{1}{2\pi RC\sqrt{6}}$$

LC Oscillators

- In an *LC oscillator* circuit, the filter is a tuned circuit (often called a tank circuit) consisting of an inductor and capacitor connected together.

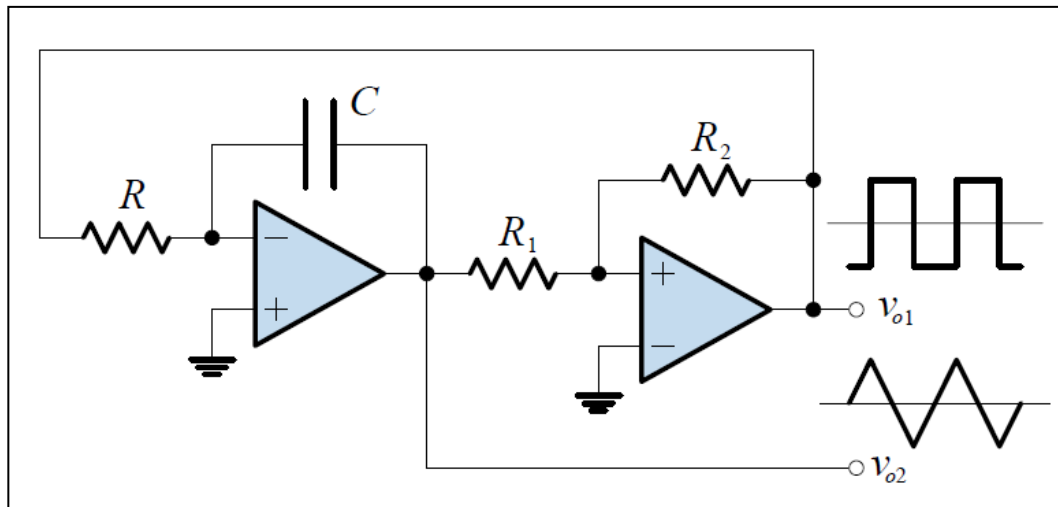
Charge flows back and forth between the capacitor's plates through the inductor, so the tuned circuit can store electrical energy oscillating at its resonance frequency. There are small losses in the tank circuit, but the amplifier compensates for those losses and supplies the power for the output signal. *LC* oscillators are often used at radio frequencies, when a tunable frequency source is necessary, such as in signal generators, tunable radio transmitters and the local oscillators in radio receivers.

Typical *LC* oscillator circuits are the Hartley, Colpitts and Clapp circuits.

Relaxation Oscillators

Relaxations Oscillators

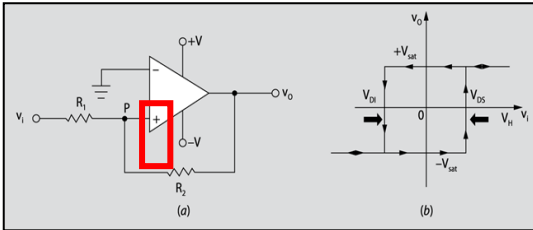
A *relaxation* oscillator produces a non-sinusoidal output, such as a square, sawtooth or triangle wave. It contains an energy-storing element (a capacitor or, more rarely, an inductor) and a nonlinear trigger circuit (a latch, Schmitt trigger, or negative resistance element) that periodically charges and discharges the energy stored in the storage element thus causing abrupt changes in the output waveform.



Oscilador de Relaxação

Geração de Onda Quadrada e Triangular com Comparador Não Inversor Regenerativo

Determinação da Frequência (f) de Oscilação



Comparador Não Inversor Regenerativo

$$V_{DS} = \frac{R_1}{R_2} \cdot (+V_{sat})$$

$$V_{DI} = \frac{R_1}{R_2} \cdot (-V_{sat})$$

- Os dois diodos Zener na saída limitam a tensão, de forma que a amplitude máxima do sinal de saída é:

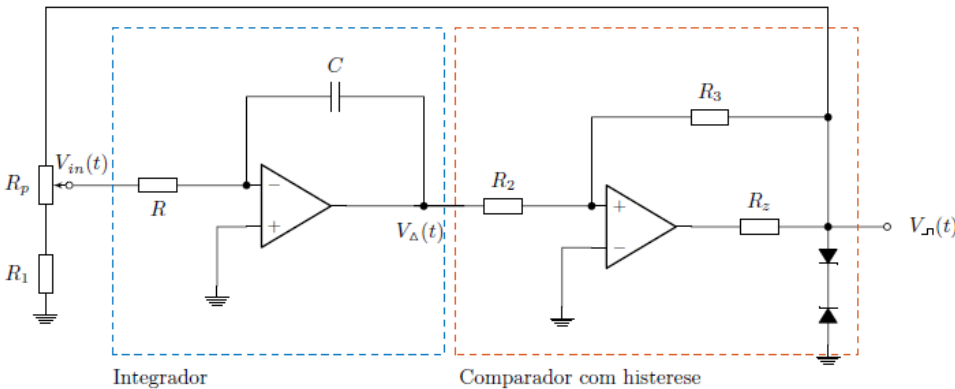
$$V_{\neg}^{\max}(t) = V_z + 0.7$$

- Devido ao potenciômetro R_p , a entrada do integrador ($V_{in}(t)$) é proporcional à saída do comparador, onde k representa o ajuste do potenciômetro:

$$V_{in}(t) = kV_{\neg}(t)$$

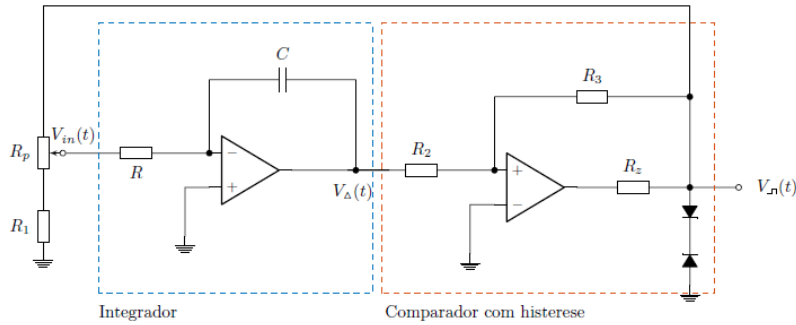
- No circuito integrador obtém-se:

$$V_{\Delta}(t) = -\frac{k}{RC} \int_0^t V_{\neg}(t) dt + V_{\Delta}(0)$$

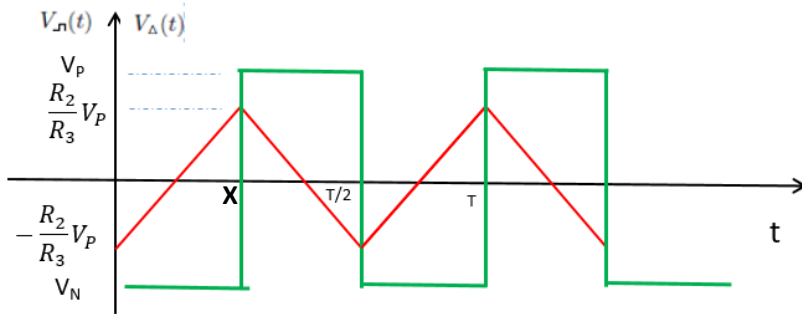


$$V_{DS} = \frac{R_2}{R_3} (+V_{sat})$$

$$V_{DI} = \frac{R_2}{R_3} (-V_{sat})$$



A figura abaixo representa as duas saídas do circuito.



4 $V_n(t)$ é uma onda quadrada de $-V_p$ a $+V_p$ com período T:

$$V_n(t) = \begin{cases} +V_p & T/2 > t > 0 \\ -V_p & T > t > T/2 \end{cases}$$

5 No instante $t=T/2$

$$V_\Delta(t) = -\frac{k}{RC} \int_0^t V_n(t) dt + V_\Delta(0)$$

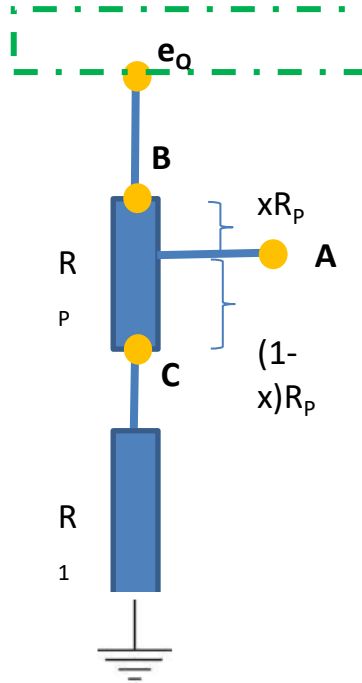
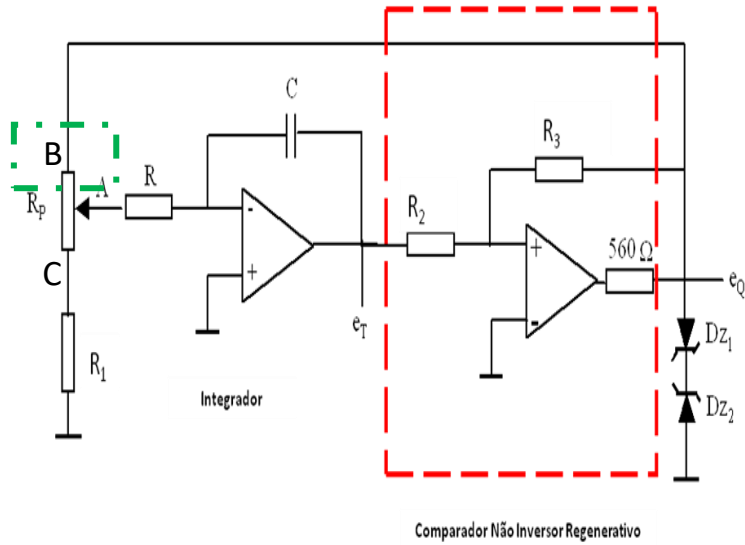
$$-\frac{R_2}{R_3} V_p = -\frac{k}{RC} \int_0^{T/2} V_p dt + \frac{R_2}{R_3} V_p$$

$$\frac{2R_2 V_p}{R_3} = \frac{kTV_p}{2RC} \Rightarrow T = \frac{4R_2 RC}{kR_3}$$

$$\rightarrow f = \frac{kR_3}{4R_2 RC} \text{ (Hz)}$$

$$\rightarrow \text{Se } R_2 = R_3 \rightarrow f = \frac{k}{4RC}$$

6 Cálculo de K



$$V_A = \frac{e_Q}{R_1 + R_p} (R_1 + (1-x)R_p)$$

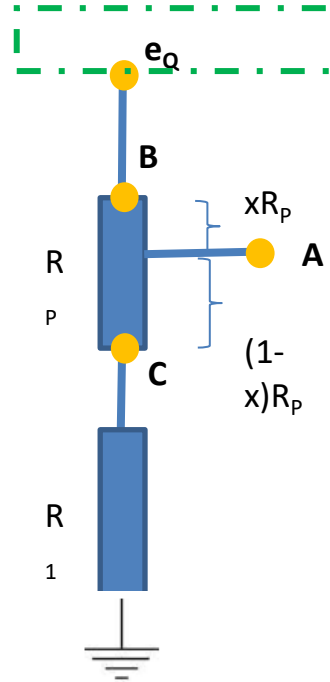
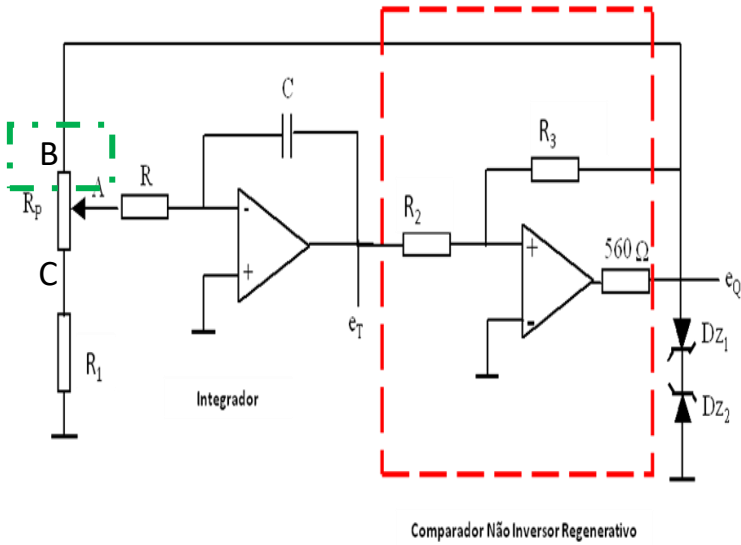
$$V_A = K e_Q$$

$$K = \frac{R_1 + (1-x)R_p}{R_1 + R_p}$$

Se $x=0$ (ponto B) $\rightarrow K = K_{\max} = 1$

Se $x=1$ (ponto C) $\rightarrow K_{\min} = \frac{R_1}{R_1 + R_p}$

7 Cálculo de K



$$V_A = \frac{e_Q}{R_1 + R_p} (R_1 + (1-x)R_p)$$

$$V_A = Ke_Q$$

$$K = \frac{R_1 + (1-x)R_p}{R_1 + R_p}$$

Se $x=0$ (ponto B) \rightarrow

$$K = K_{\max} = 1$$

Se $x=1$ (ponto C) \rightarrow

$$K_{\min} = \frac{R_1}{R_1 + R_p}$$

Resultados de Simulação (Onda Quadrada e Triangular)

; PARAMETROS DE ENTRADA

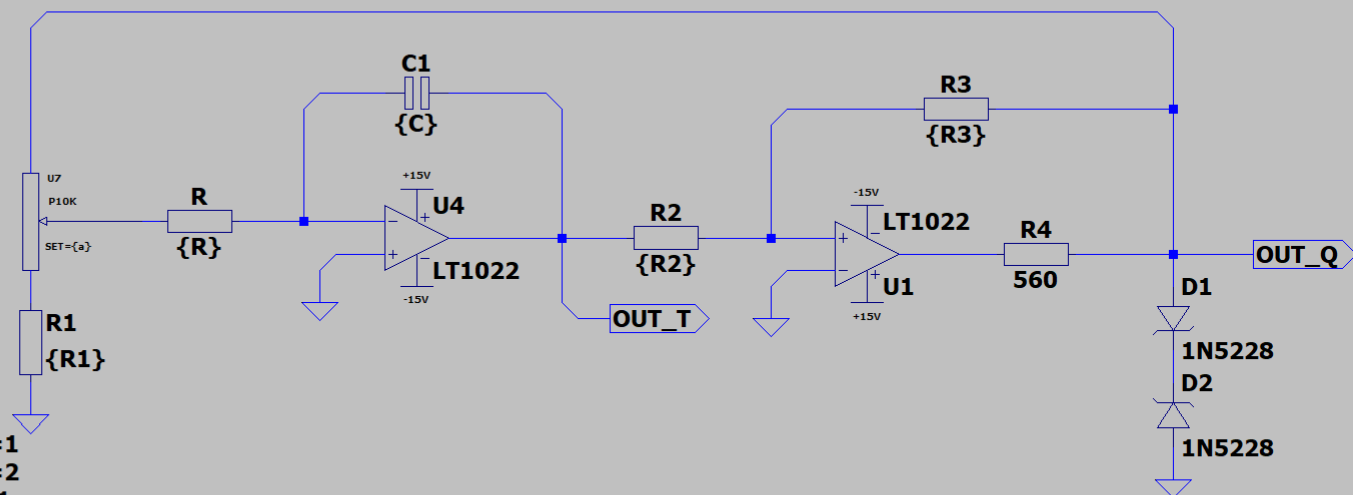
```
.param C = 22n
.param R = 12k
.param R1 = 1k
.param R2 = 10k
.param R3 = 10k
.param a = 1
```

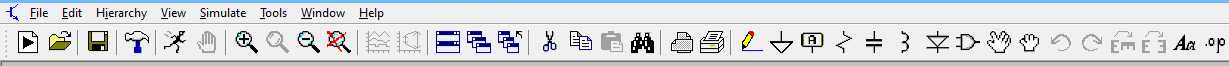
; SIMULACOES REALIZADAS

```
.tran 0 60m 10m 1u startup
.op
```

; MEDICOES REALIZADAS

```
.meas tran Aq max V(out_q)
.meas tran At max V(out_t)
.meas tran T1 when V(out_q)=Aq*0.9 rise=1
.meas tran T2 when V(out_q)=Aq*0.9 rise=2
.meas tran T3 when V(out_t)=At*0.9 rise=1
.meas tran T4 when V(out_t)=At*0.9 rise=2
.meas Freq_q param 1/(T2-T1)
.meas Freq_t param 1/(T4-T3)
.param realim = (R1+10k*a)/(10k+R1)
.meas Freq_teo param realim*R3/(R2*4*C*R)
```

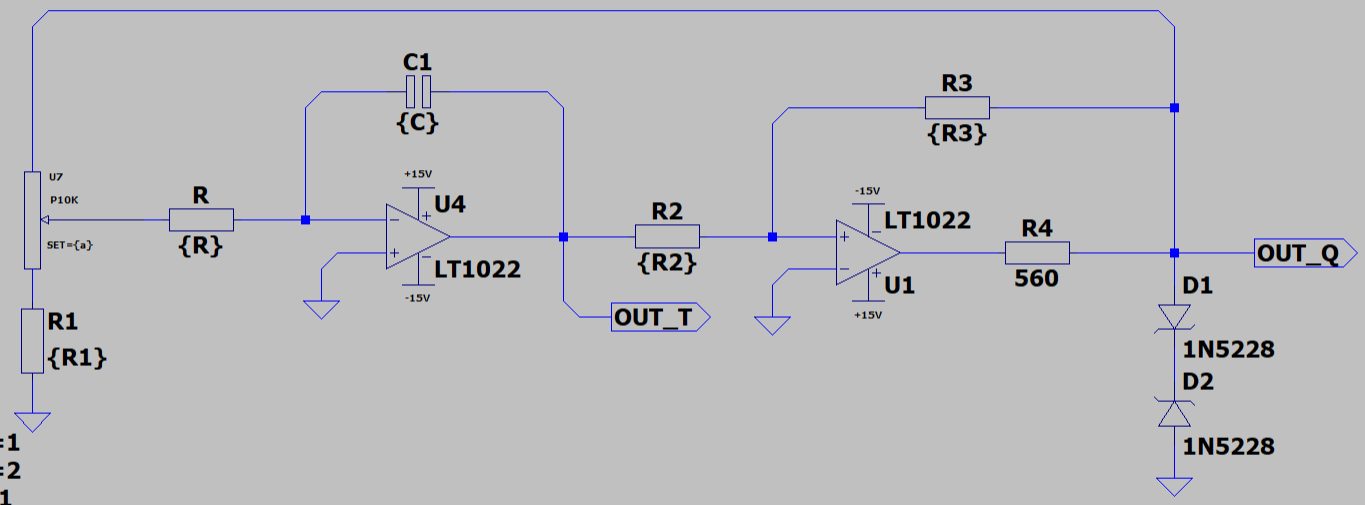


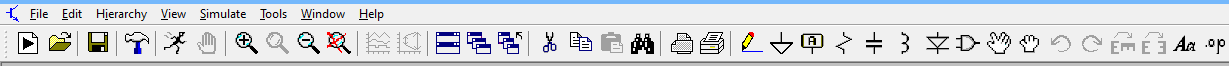


```
; PARAMETROS DE ENTRADA  
.param C = 22n  
.param R = 12k  
.param R1 = 1k  
.param R2 = 10k  
.param R3 = 22k  
.param a = 1
```

```
; SIMULACOES REALIZADAS  
.tran 0 60m 10m 1u startup  
.op
```

```
; MEDICOES REALIZADAS  
.meas tran Aq max V(out_q)  
.meas tran At max V(out_t)  
.meas tran T1 when V(out_q)=Aq*0.9 rise=1  
.meas tran T2 when V(out_q)=Aq*0.9 rise=2  
.meas tran T3 when V(out_t)=At*0.9 rise=1  
.meas tran T4 when V(out_t)=At*0.9 rise=2  
.meas Freq_q param 1/(T2-T1)  
.meas Freq_t param 1/(T4-T3)  
.param realim = (R1+10k*a)/(10k+R1)  
.meas Freq_teo param realim*R3/(R2*4*C*R)
```

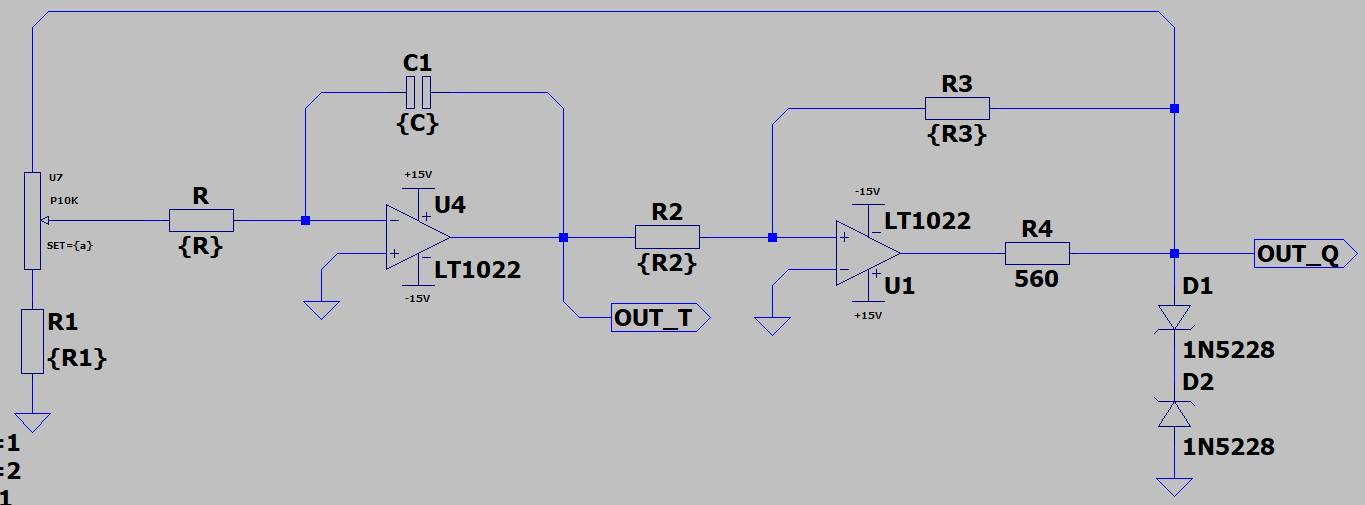




```
; PARAMETROS DE ENTRADA  
.param C = 22n  
.param R = 12k  
.param R1 = 1k  
.param R2 = 22k  
.param R3 = 10k  
.param a = 1
```

```
; SIMULACOES REALIZADAS  
.tran 0 60m 10m 1u startup  
.op
```

```
; MEDICOES REALIZADAS  
.meas tran Aq max V(out_q)  
.meas tran At max V(out_t)  
.meas tran T1 when V(out_q)=Aq*0.9 rise=1  
.meas tran T2 when V(out_q)=Aq*0.9 rise=2  
.meas tran T3 when V(out_t)=At*0.9 rise=1  
.meas tran T4 when V(out_t)=At*0.9 rise=2  
.meas Freq_q param 1/(T2-T1)  
.meas Freq_t param 1/(T4-T3)  
.param realim = (R1+10k*a)/(10k+R1)  
.meas Freq_teo param realim*R3/(R2*4*C*R)
```



Crystal Oscillators

- A *crystal oscillator* is a circuit that uses a piezoelectric crystal (commonly a quartz crystal) as a frequency selective element. The crystal mechanically vibrates as a resonator, and its frequency of vibration determines the oscillation frequency. Crystals have a very high Q -factor and also better temperature stability than tuned circuits, so crystal oscillators have much better frequency stability than LC or RC oscillators. They are used to stabilize the frequency of most radio transmitters, and to generate the clock signal in computers and quartz clocks. Crystal oscillators often use the same circuits as LC oscillators, with the crystal replacing the tuned circuit. Surface acoustic wave (SAW) devices are another kind of piezoelectric resonator used in crystal oscillators, which can achieve much higher frequencies. They are used in specialized applications which require a high frequency reference, for example, in mobile phones.

Abbreviation	Meaning
O	oscillator
X	crystal
LC	inductor capacitor
VC	voltage controlled
OC	oven controlled
ALC	automatic level controlled

Table 4.1 – Oscillator abbreviations

These basic abbreviations can be combined to form a new abbreviation. For example, an OCXO would be an oven-controlled crystal oscillator.

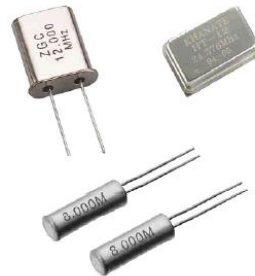
Crystal Oscillators

Crystal Oscillators

1

A “crystal” is a carefully oriented and dimensioned piece of quartz or other suitable piezoelectric material to which adherent electrodes have been applied.

The crystals are held within sealed enclosures by mounting supports that also serve as connections between the electrodes and the external leads. Crystal enclosures are designated by HC- numbers (Holder, Crystal).



2

Crystals are unique not only because of the achievable combinations of circuit parameter values (i.e. high frequencies of resonance, small capacitance, etc.) but also because of other important features such as cost, size, and stability with time, temperature and other environmental changes.

Frequency Range

Crystal resonators are available to cover frequencies from below 1 kHz to over 200 MHz. At the low-frequency end wristwatch and real-time clock applications operate at 32.768 kHz and powers of two times this frequency. More conventional resonators span the range 80 kHz to 200 MHz; these utilise what is known as *bulk acoustic waves* (BAWs) that propagate within the crystal. Surface acoustic waves (SAWs) travel along the surface. Devices based on SAWs are available for the range above 50 MHz into the low GHz region.

4

Frequency Stability

Precision quartz oscillators, held at constant temperature and protected from environmental disturbances, have fractional stabilities from 10^{-10} to 10^{-12} .

Measurement Terminology

Many hams think of frequency error, if they think of it at all, in terms like "parts per million." Time-nuts think in terms of **fractional frequency offset**, which is calculated by dividing the frequency error (*i.e.*, the measured frequency minus the nominal frequency) by the nominal frequency. In other words:

$$\frac{\text{Frequency}_{(\text{measured})} - \text{Frequency}_{(\text{nominal})}}{\text{Frequency}_{(\text{nominal})}}$$

So, an error of +1.54 Hertz at 100 MHz is:

$$\frac{100\ 000\ 001.54 - 100\ 000\ 000}{100\ 000\ 000}$$

The result is 1.54×10^{-8} , often written in ASCII as $1.54 \times 10\text{e-}8$. You'll often hear rough performance described as "parts in the eleventh" or something similar. Using exponential notation like this gets you to very tiny numbers, very quickly. Here are some convenient reference points:

Offset:	Error of:
1×10^{-3}	1 Hz at 1 kHz
1×10^{-6}	1 Hz at 1 MHz
1×10^{-7}	1 Hz at 10 MHz
1×10^{-8}	1 Hz at 100 MHz
1×10^{-9}	1 Hz at 1 GHz
1×10^{-10}	1 Hz at 10 GHz
1×10^{-11}	1 Hz at 100 GHz

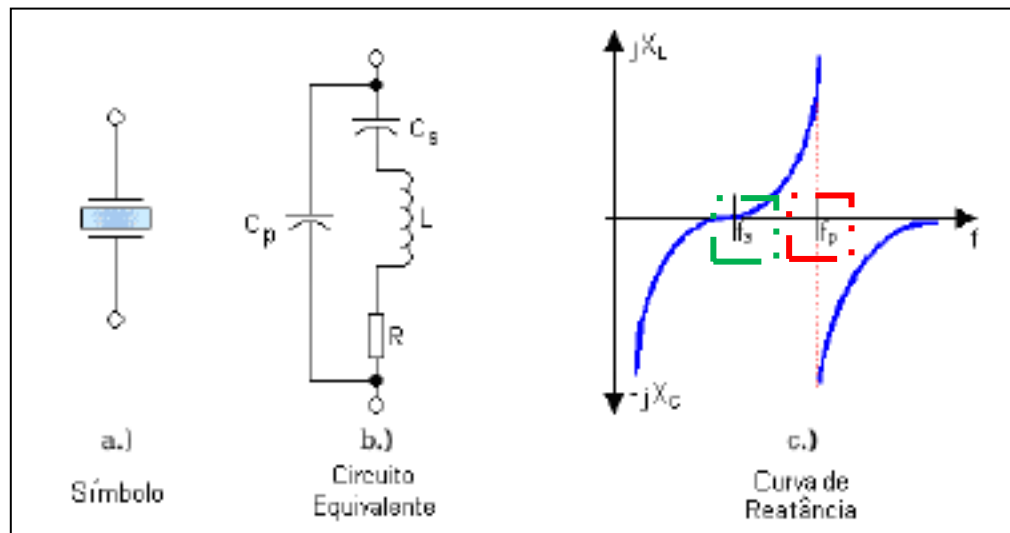
Aging Effects

Slow changes in frequency with time are referred to as aging. The principal causes of aging are contamination within the enclosure that is redistributed with time, slow leaks in the enclosure, mounting and electrode stresses that are relieved with time, and changes in atmospheric pressure. Changes in the quartz are usually negligible for most applications.

Cut	Frequency Range	Description
AT	0.5 – 300 MHz	<p>The most common cut, developed in 1934. The frequency-temperature curve is a sine-shaped curve with an inflection point at around 25–35°C. The <i>optimum</i> AT cut has a frequency variation with temperature of only ± 12 ppm from -50°C to +100°C. Most (estimated over 90%) of all crystals are this variant. Sensitive to mechanical stresses, whether caused by external forces or by temperature gradients. The upper limit for the fundamental mode of vibration is around 30-40 MHz.</p>
SC	0.5 – 200 MHz	<p>A special cut (Stress Compensated), developed in 1974, is for oven-stabilized oscillators with low phase noise and good aging characteristics. Compared to the AT cut, it is less sensitive to mechanical stresses, has a faster warm-up speed, higher Q, better close-in phase noise, less sensitivity to spatial orientation against gravity, and less sensitivity to vibrations. The frequency-temperature curve has an inflection point at 96°C and the optimum SC cut has a much lower temperature sensitivity than the optimum AT cut. It is suitable for OCXOs, e.g. space and GPS systems. Aging characteristics are 2 to 3 times better than of the AT cuts.</p>
BT	0.5 – 200 MHz	<p>A special cut, similar to the AT cut. It has well known and repeatable characteristics. It has poorer temperature characteristics than the AT cut. It is used for oscillators vibrating at a fundamental mode which is higher than the AT cut, up to over 50 MHz.</p>

7

Osciladores Pierce são, geralmente, osciladores Colpitts nos quais o indutor foi substituído por um cristal de quartzo. A figura abaixo apresenta o símbolo elétrico de um cristal de quartzo e o seu circuito equivalente.



Cristal de Quartzo e Curva de Reatância

$$f_s = \frac{1}{2\pi\sqrt{LC_S}} \quad [\text{Hz}]$$

$$f_p = \frac{1}{2\pi\sqrt{L\frac{C_S C_P}{C_S + C_P}}}$$

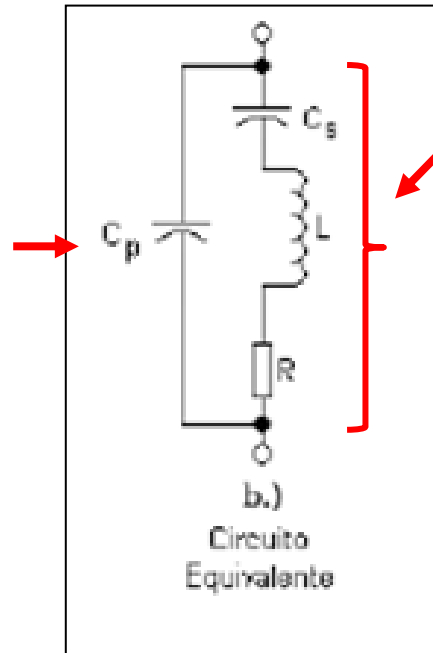
8

O cristal de quartzo é um isolante para todas as frequências de sinais elétricos exceto para uma determinada frequência fundamental à qual ele se apresenta essencialmente como um curto-circuito.

9 Uma propriedade muito importante do quartzo é a **piezoeletricidade** que é a capacidade que alguns cristais apresentam de **gerarem uma diferença de potencial elétrico entre pares de faces opostas, em resposta a uma pressão mecânica**. O efeito é bilateral, isto é, quando sujeitos a diferenças de potencial elétrico esses cristais se deformam proporcionalmente.

10

Esse dispositivo é um capacitor físico C_p , composto por duas placas condutoras isoladas entre si por uma fina lâmina de quartzo clivado.



O circuito equivalente RLC_s aparece devido ao efeito piezoeletrico do cristal.

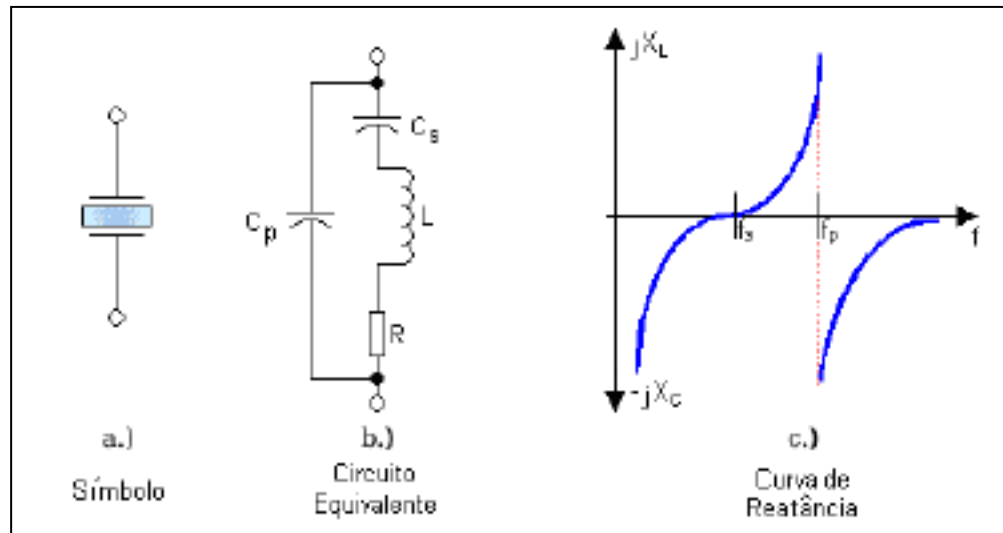
A bobina L é originária da inércia da vibração das moléculas do cristal.

A capacitância C_s tem origem na rigidez do cristal.

A resistência R equivale ao atrito entre moléculas na vibração.

11

Quartzo é um mineral muito abundante na natureza e é composto por sílica (SiO_2) no estado cristalino.



12

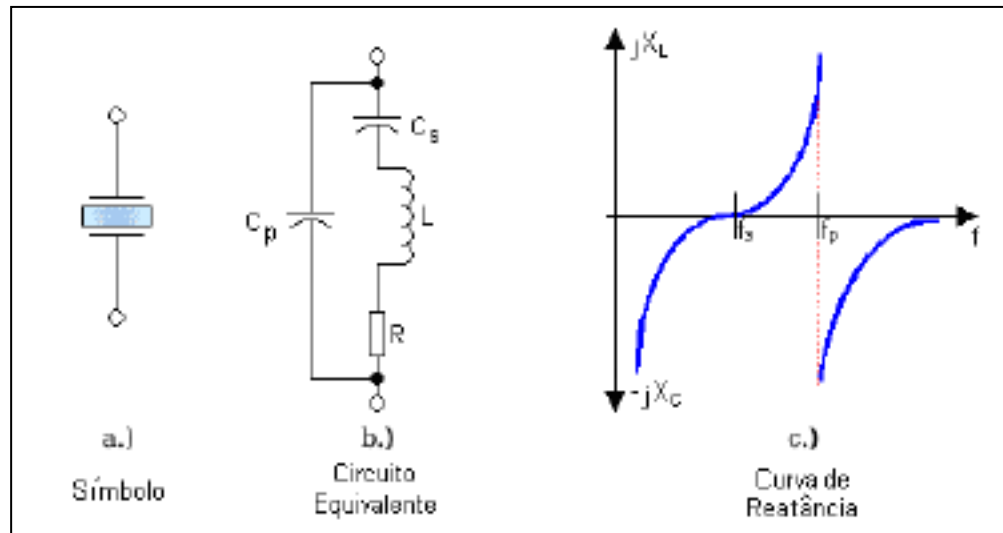
$C_p \gg C_s$, embora $f_p > f_s$, essas frequências são muito próximas.

Na análise do circuito equivalente observa-se que a **parte reativa do circuito é capacitiva em praticamente todas as frequências, exceto para a faixa $f_s < f < f_p$, na qual a reatância do circuito é indutiva.**

As frequências de ressonância dependem inversamente da espessura da lâmina e da orientação cristalina da clivagem do cristal.

Se o indutor do oscilador Colpitts for substituído por um cristal de quartzo obrigatoriamente o oscilador deverá oscilar em uma frequência $f_s < f_o < f_p$, na qual o circuito é indutivo. Como essas frequências são muito próximas, o oscilador oscilará com grande estabilidade.

Exemplo



Um cristal de quartzo de 1 MHz típico apresenta as seguintes características:

$L = 2,54648\text{H}$, $C_s = 9,95358\text{ fF}$, $C_p = 2,4884\text{ pF}$ e $R = 640\Omega$

As frequências de ressonância são $f_s = 0,99968\text{ MHz}$ e $f_p = 1,00168\text{ MHz}$.

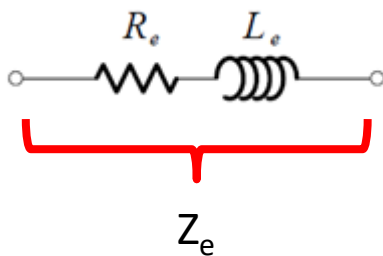
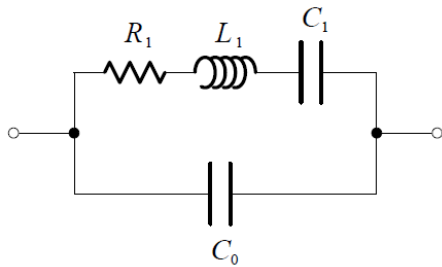
O fator de qualidade (Q) é igual a 25000 .

$$Q = \frac{\omega L}{R} = \frac{X_L}{R}$$

Os capacitores C_1 e C_2 , que fazem parte do divisor capacitivo do oscilador, têm seus valores recomendados pelo fabricante do cristal.

Equivalent Circuit Near Resonance

The crystal is inductive in the parallel-resonance region between f_{rs} and f_{rp} , and so in this “region of operation” it seems appropriate to represent the crystal by its equivalent series resistance and inductance:



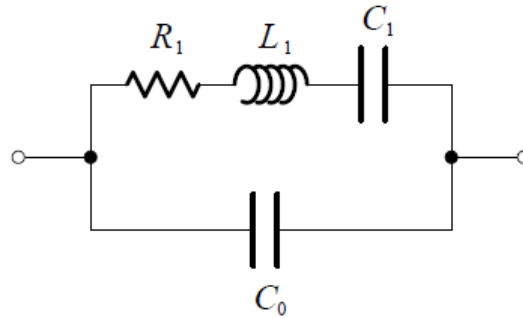
Seja
$$jX_1 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$$

$$Z_e = \frac{jX_{C_0}(R_1 + jX_1)}{R_1 + j(X_1 + X_{C_0})}$$

$$= \underbrace{\frac{R_1 X_{C_0}^2}{R_1^2 + (X_1 + X_{C_0})^2}}_{R_e} + j \underbrace{\frac{X_{C_0} [R_1^2 + X_1(X_1 + X_{C_0})]}{R_1^2 + (X_1 + X_{C_0})^2}}_{X_e}$$

$$= R_e + jX_e$$

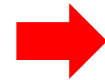
Determinação de f_p e f_s



1

$$Z_0 = \frac{1}{j\omega C_0}$$

$$Z_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1}$$

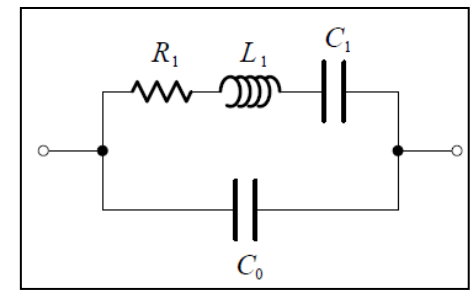


$$Z_e = \frac{Z_1 Z_0}{Z_1 + Z_0}$$



$$Z_e = \frac{\left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) \frac{1}{j\omega C_0}}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_0}} = \frac{\frac{\omega L_1 - 1/\omega C_1}{\omega C_0} - j \frac{R_1}{\omega C_0}}{R_1 + j \left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_0} \right)}$$

A expressão anterior de Z_e pode ser expressa por:



$$Z_e = \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd} = \frac{ac + bd + j(bc - ad)}{c^2 + d^2} = R_e + jX_e$$

2

Para que ocorra uma ressonância Z_e deve ser resistiva, então $X_e = 0$:

$$X_e = \frac{bc - ad}{c^2 + d^2} = 0 \quad \rightarrow \quad \frac{-R_1^2}{\omega_r C_0} = \left(\frac{\omega_r L_1 - 1/\omega_r C_1}{\omega_r C_0} \right) \left(\omega_r L_1 - \frac{1}{\omega_r C_1} - \frac{1}{\omega_r C_0} \right)$$

$$-R_1^2 = \left(\omega_r L_1 - \frac{1}{\omega_r C_1} \right) \left(\omega_r L_1 - \frac{1}{\omega_r C_1} - \frac{1}{\omega_r C_0} \right)$$

$$\omega_r^2 L_1^2 - \frac{L_1}{C_1} - \frac{L_1}{C_0} - \frac{L_1}{C_1} + \frac{1}{\omega_r^2 C_1^2} + \frac{1}{\omega_r^2 C_1 C_0} + R_1^2 = 0$$

$$\omega_r^4 + \left(\frac{R_1^2}{L_1^2} - \frac{2}{L_1 C_1} - \frac{1}{L_1 C_0} \right) \omega_r^2 + \frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} = 0$$

$$\omega_r^4 + \left(\frac{R_1^2}{L_1^2} - \frac{2}{L_1 C_1} - \frac{1}{L_1 C_0} \right) \omega_r^2 + \frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} = 0$$

3

Usando a fórmula quadrática obtem-se para w_r :

$$\omega_r = \left\{ \left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right) \right.$$



$$\pm \left[\left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} \right) \right]^{1/2} \Bigg\}^{1/2}$$

4

Na expressão a raiz quadrada mais interna pode ser reescrita:

$$\begin{aligned} & \left[\left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} \right) \right]^{1/2} \\ &= \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} - \frac{R_1^2}{L_1^3 C_1} + \frac{1}{4L_1^2 C_0^2} - \frac{R_1^2}{2L_1^3 C_1 C_0} + \frac{R_1^4}{4L_1^4} - \frac{1}{L_1^2 C_1^2} - \frac{1}{L_1^2 C_1 C_0} \right)^{1/2} \\ &= \left[\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \frac{R_1^2}{L_1^3 C_1} \right]^{1/2} \end{aligned}$$




$$\omega_r = \left\{ \left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right) \pm \left[\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \frac{R_1^2}{L_1^3 C_1} \right]^{1/2} \right\}^{1/2}$$

$$\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 \gg \frac{R_1^2}{L_1^3 C_1}$$

For example, typical values for an 8 MHz crystal are $L_1 = 14 \text{ mH}$, $C_1 = 27 \text{ fF}$, $R_1 = 8 \Omega$ and $C_0 = 5.6 \text{ pF}$. Thus the inequality above is $7.972 \times 10^{21} \gg 1.693 \times 10^{17}$, which is certainly true.

Então:
$$\left[\left(\frac{1}{2L_1C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \frac{R_1^2}{L_1^3C_1} \right]^{1/2} \approx \left(\frac{1}{2L_1C_0} - \frac{R_1^2}{2L_1^2} \right)$$


$$\omega_r \approx \left\{ \left(\frac{1}{L_1C_1} + \frac{1}{2L_1C_0} - \frac{R_1^2}{2L_1^2} \right) \pm \left(\frac{1}{2L_1C_0} - \frac{R_1^2}{2L_1^2} \right) \right\}^{1/2}$$

5

Esta equação tem duas frequências de ressonância.

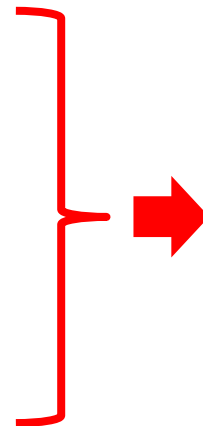
A primeira, utilizando o sinal negativo, é a **ressonância série** (ω_{rs}):

$$\omega_{rs} \approx \frac{1}{\sqrt{L_1C_1}}$$

A segunda, utilizando o sinal positivo, é a **ressonância paralela** (ω_{rp}):

$$\omega_{rp} \approx \sqrt{\frac{1}{L_1C_1} + \frac{1}{L_1C_0} - \frac{R_1^2}{L_1^2}}$$

$$\frac{1}{L_1C_0} \gg \frac{R_1^2}{L_1^2}$$



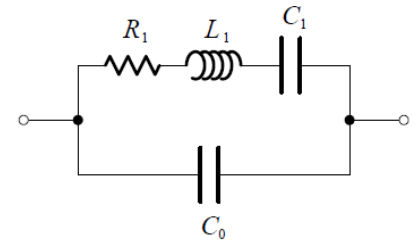
$$\omega_{rp} \approx \sqrt{\frac{1}{L_1C_1} + \frac{1}{L_1C_0}}$$

(For example, for the crystal considered earlier, $1.276 \times 10^{13} \gg 3.265 \times 10^5$)

6

Furthermore, the parallel resonance frequency can be written as:

$$\omega_{rp} \approx \sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_0}} = \underbrace{\frac{1}{\sqrt{L_1 C_1}}}_{\omega_{rs}} \left(1 + \frac{C_1}{C_0}\right)^{1/2}$$



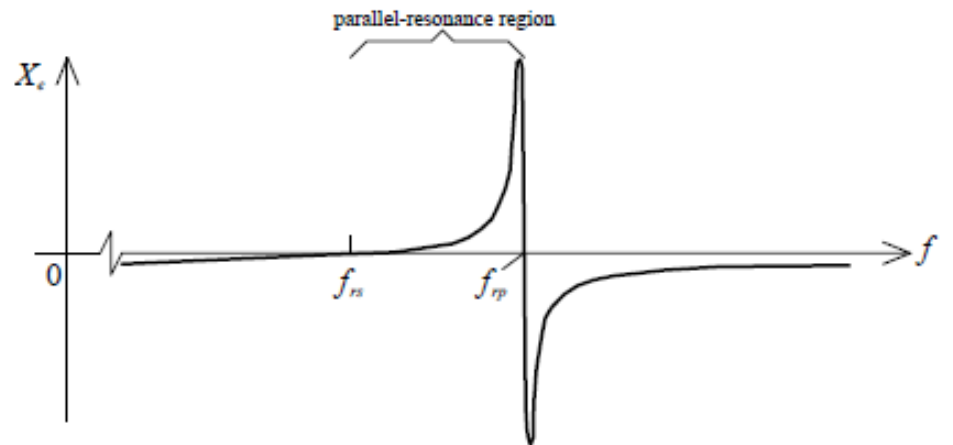
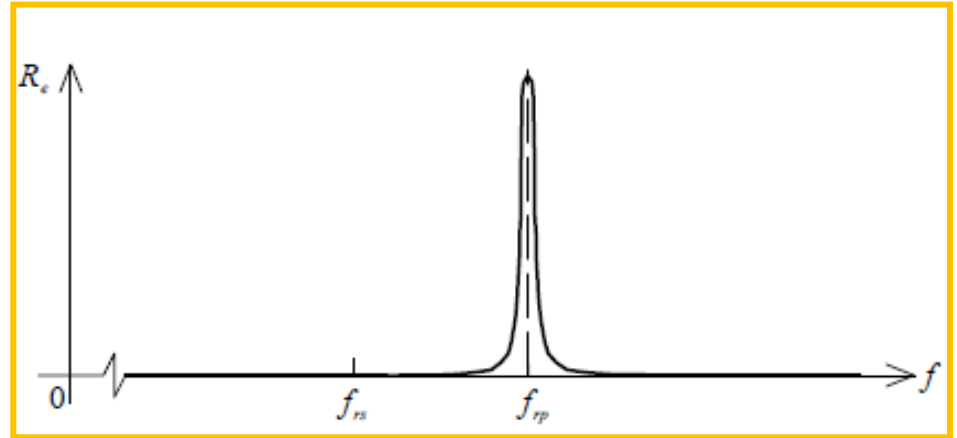
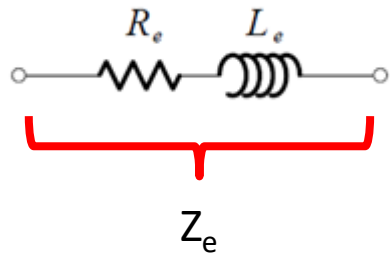
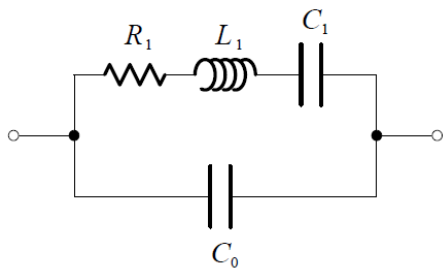
But $C_1/C_0 \ll 1$. Therefore, the binomial approximation:

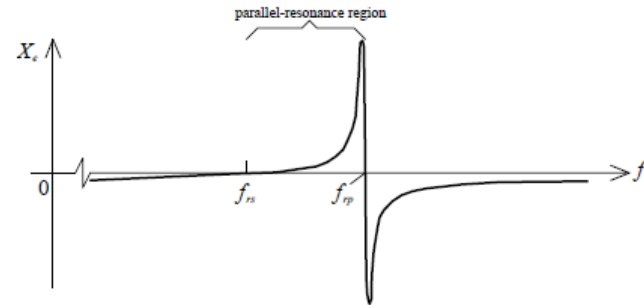
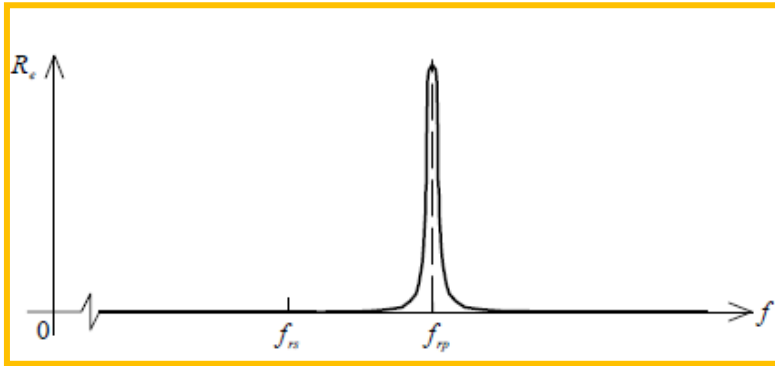
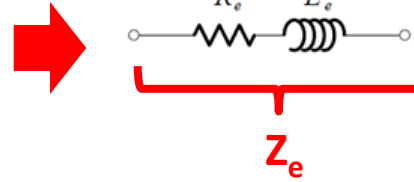
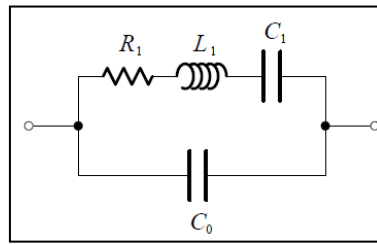
$$(1+x)^n \approx 1+nx \quad \text{if} \quad x \ll 1$$

may be used, and:

$$\omega_{rp} \approx \omega_{rs} \left(1 + \frac{C_1}{2C_0}\right) \quad \rightarrow \quad \Delta f = f_{rp} - f_{rs} \approx f_{rs} \left(\frac{C_1}{2C_0}\right)$$

The frequency separation is very small, typically $<0.3\%$ of the series resonance frequency. The frequency range Δf is known as the *pulling range*.





7

At the **series resonant frequency (f_{rs})**, $X_{L1} = -X_{C1}$ and the crystal branch impedance is simply R_1 . The resistor R_1 is also called the equivalent series resistance at the series resonance. The resistor R_1 appears parallel with the reactance C_0 , however $|X_{C0}| \gg R_1$ at this frequency, so the crystal essentially appears resistive.

8

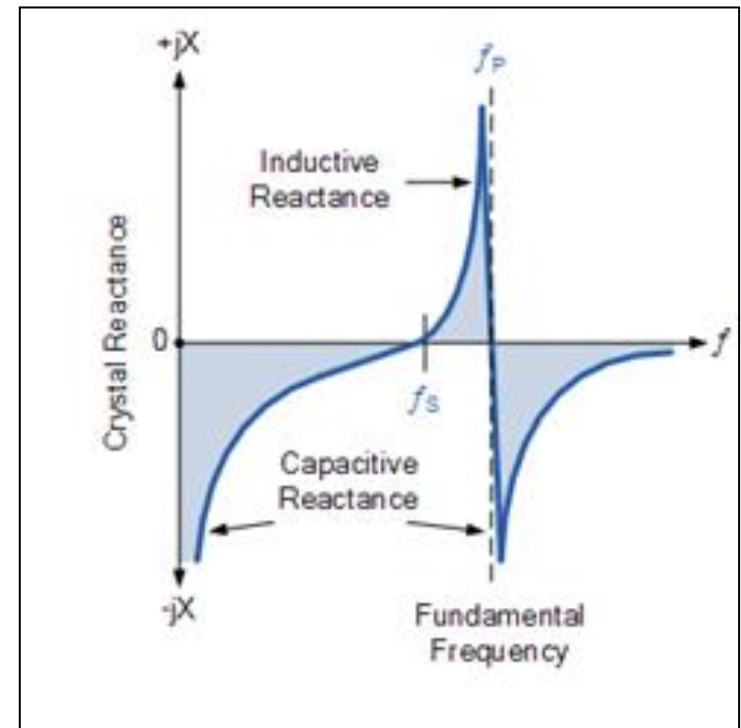
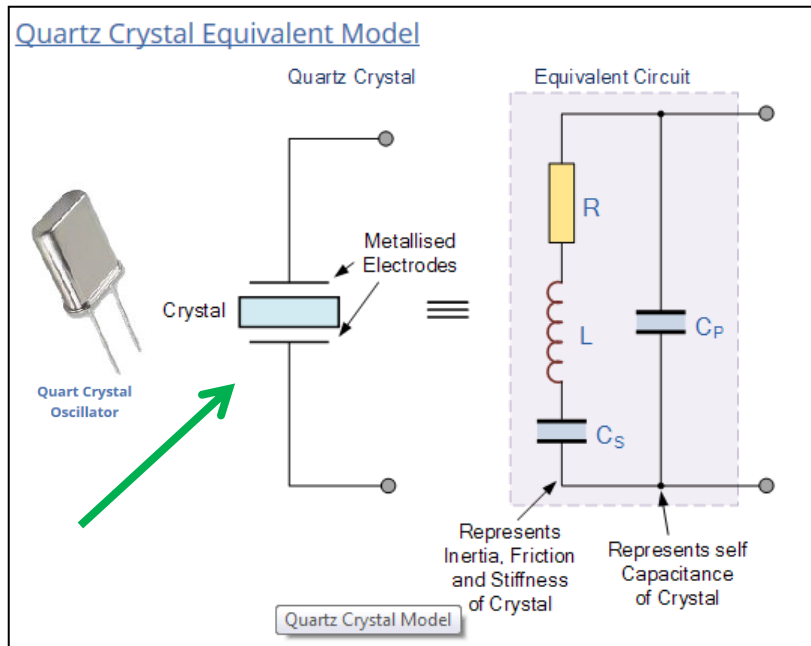
Between f_{rs} and f_{rp} the impedance is inductive with the phase being close to 90° . This is an important region of operation called **parallel-resonance region**.

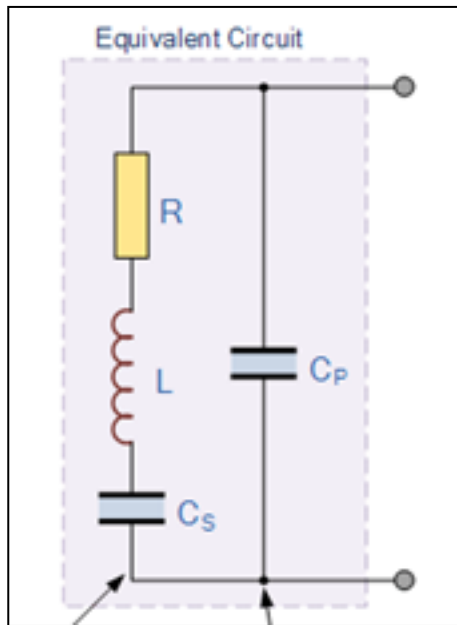
Cristal de Quartzo

As frequências de ressonância dependem da espessura do cristal e da sua orientação cristalina.

O cristal de quartzo comporta-se como um isolante (capacitor) para todas as frequências com exceção da frequência de ressonância.

Quando analisado eletricamente, o cristal de quartzo apresenta duas frequências de ressonância (série e paralela).





$$f_s = \frac{1}{2\pi\sqrt{LC_S}}$$

$$f_p = \frac{1}{2\pi\sqrt{L\frac{C_S C_P}{C_S + C_P}}}$$

$C_p \gg C_s$ resulta que f_p e f_s são muito próximas.

R não é desprezível

L é grande

C_s e C_p são pequenos

Q é alto

Exemplo:

$$f_o = 1\text{MHz}$$

$$L = 2,54468\text{ H}$$

$$R = 640\ \Omega$$

$$C_s = 9,95348\text{ fF}$$

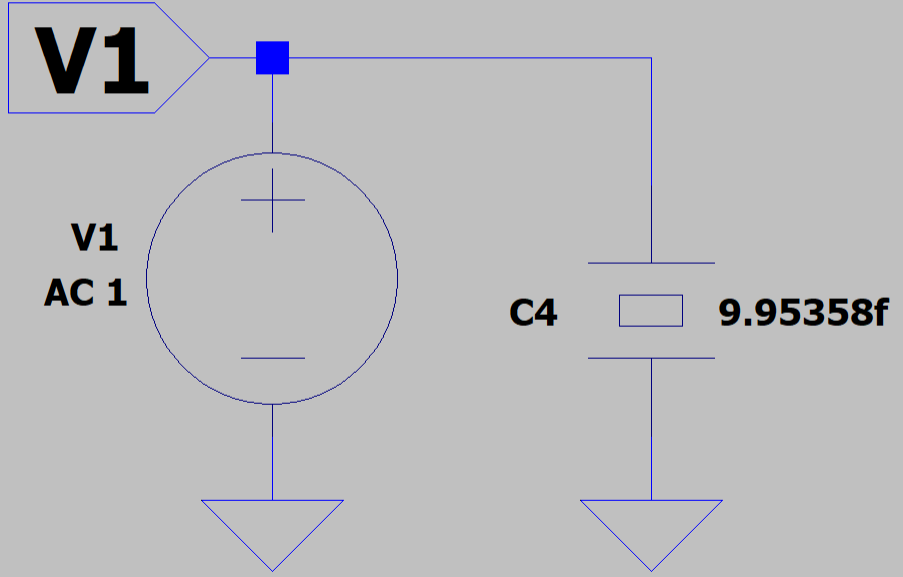
$$C_p = 2,4884\text{ pF}$$

$$f_s = 0,99968\text{ MHz}$$

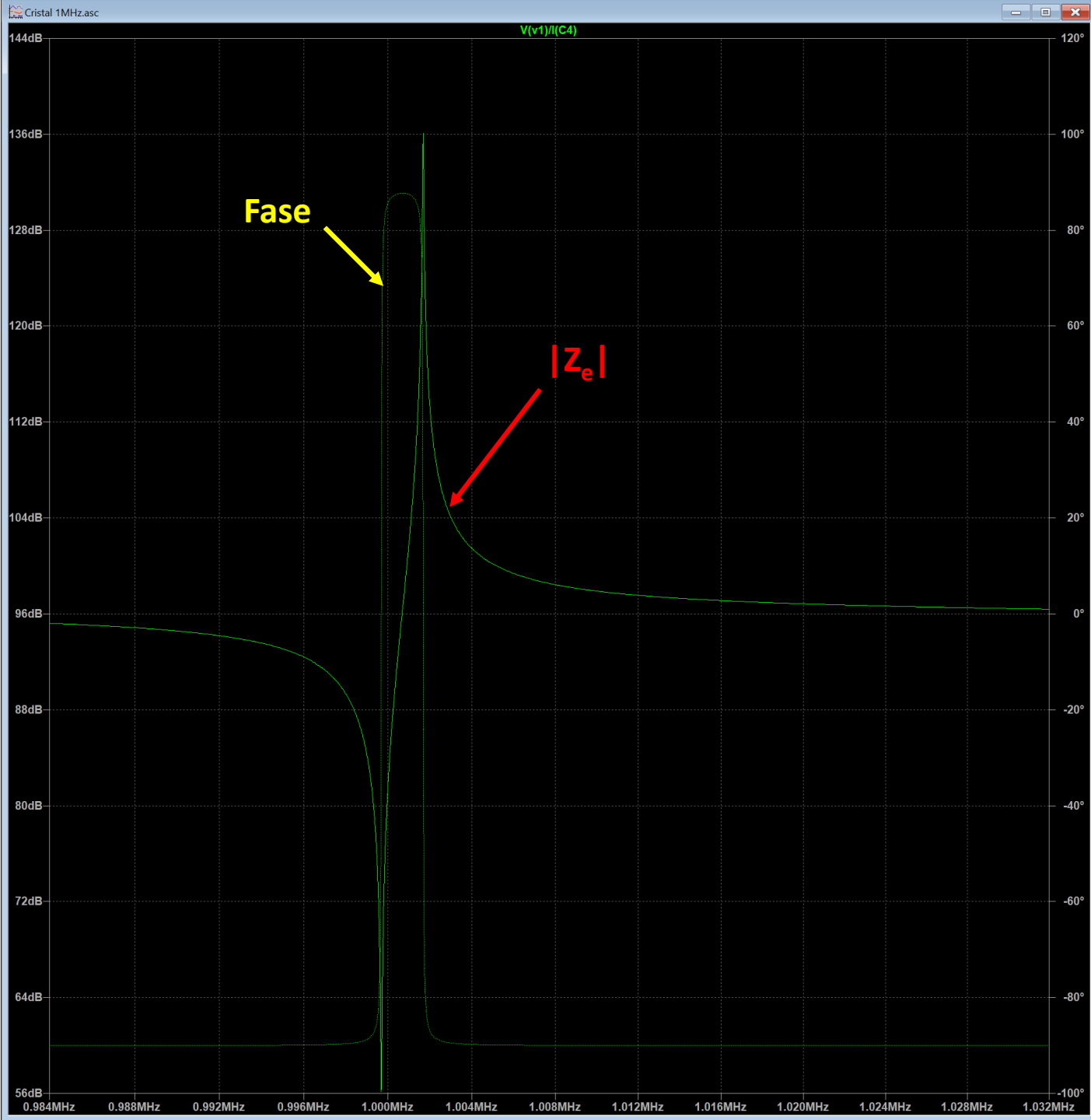
$$f_p = 1,00168\text{ MHz}$$

Crystal 1MHz

Resultados (Simulação LTSpice)



.ac dec 10Mega 0.9Mega 1.1Mega

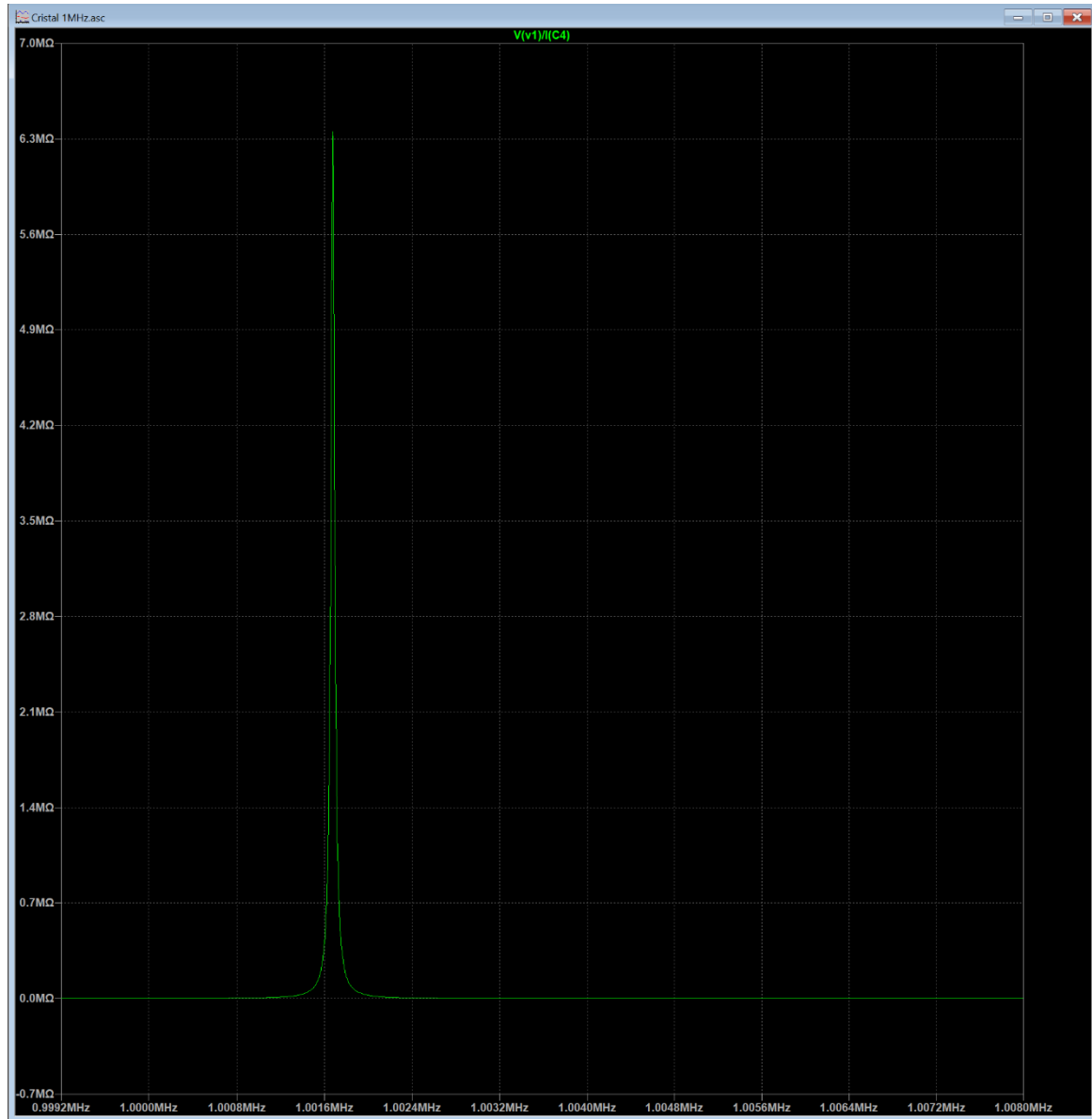
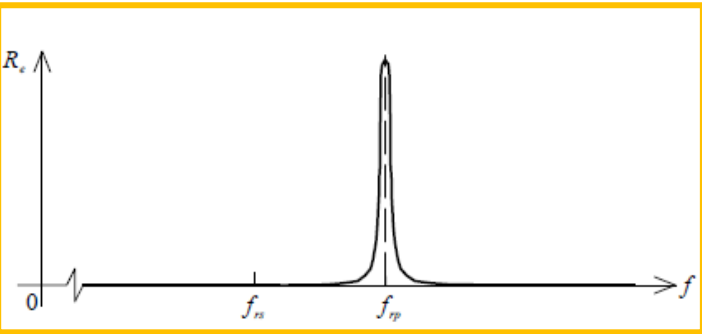


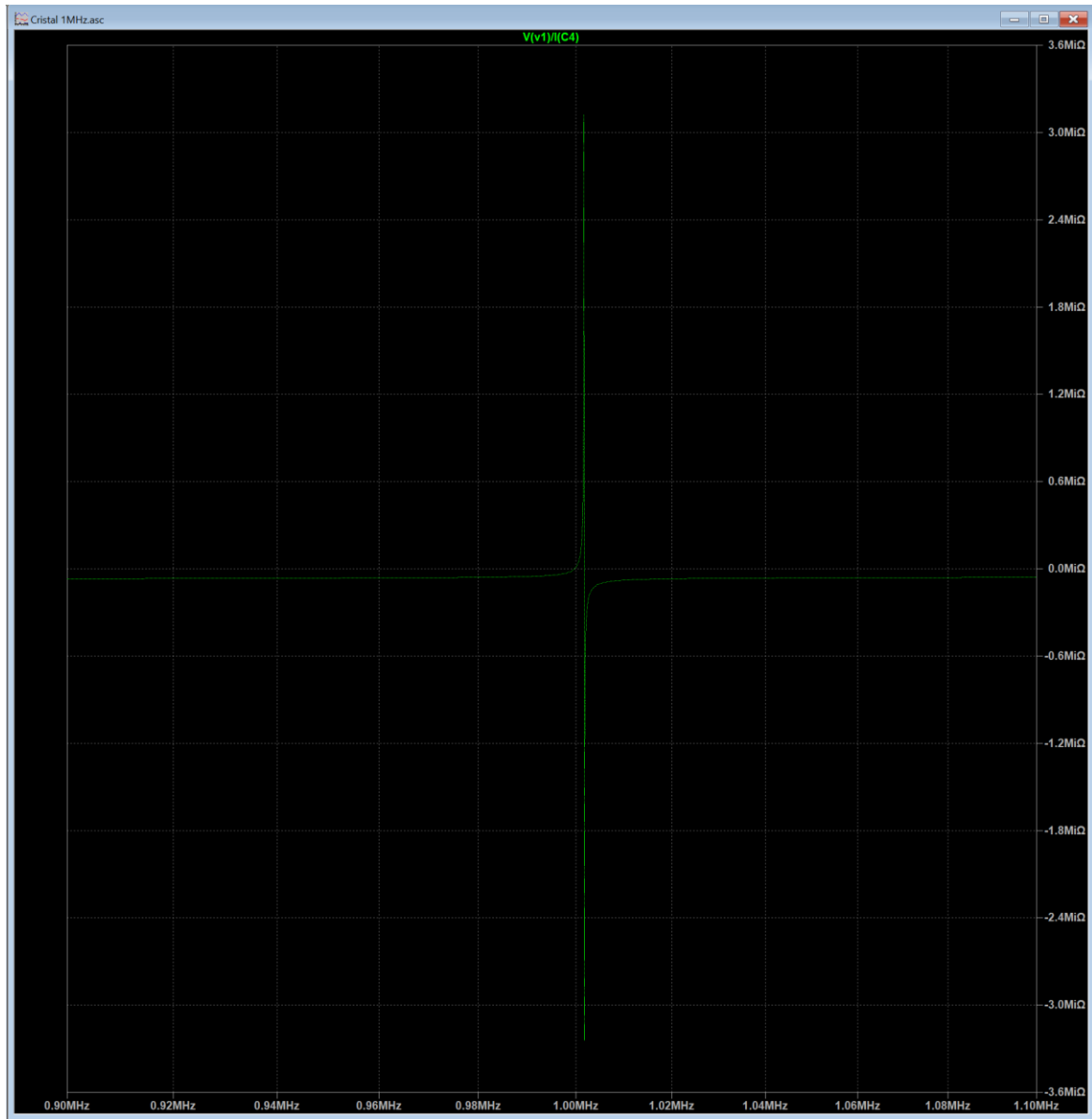
Bode Diagram

Z_e

Cartesian Diagram
(without imaginary component)

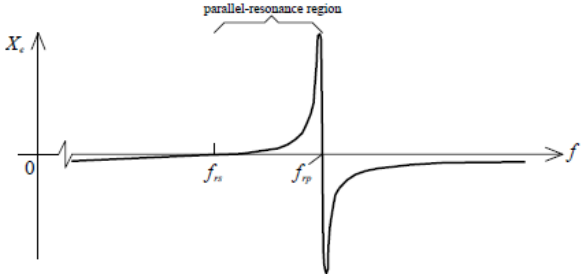
R_e





**Cartesian Diagram
(without real component)**

X_e

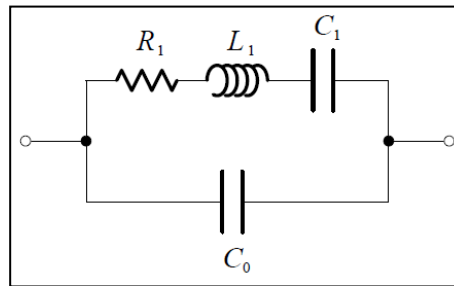


Oscilador Pierce

São osciladores Colpitts nos quais o indutor foi substituído por um cristal de quartzo.

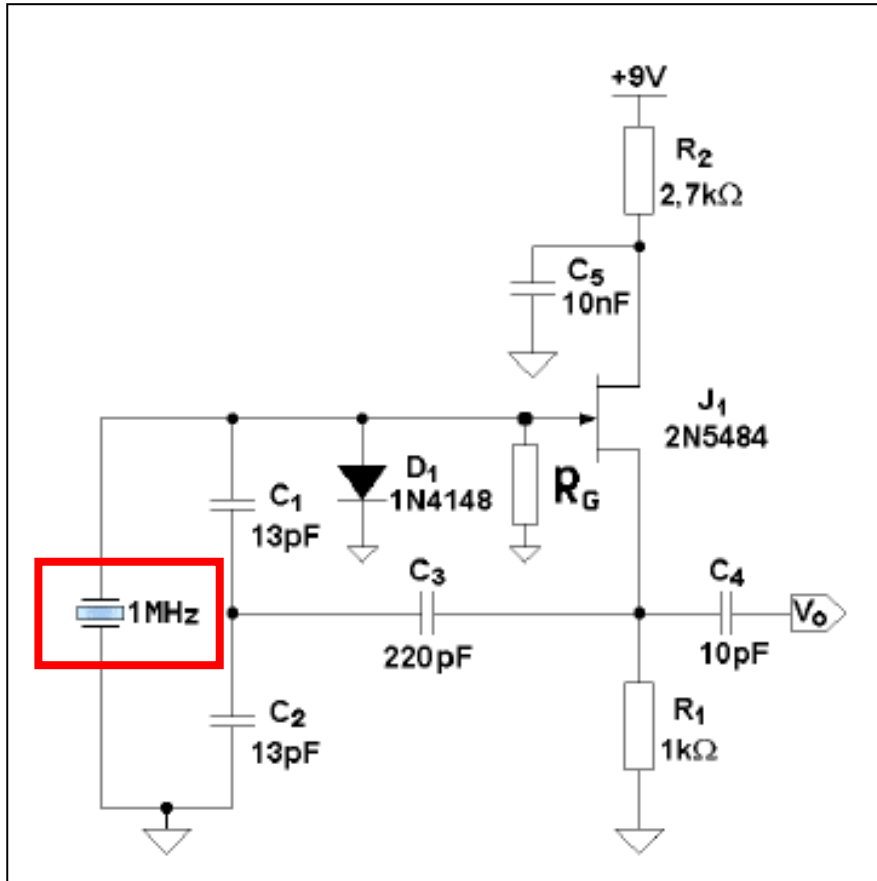
Se o indutor do oscilador Colpitts (L_R) for substituído por um cristal de quartzo a oscilação ocorrerá em $f_s < f < f_p$ onde o circuito é indutivo . Como essas frequências são próximas a oscilação terá grande estabilidade.

Os capacitores C_i e C_o do oscilador não influenciam a frequência de oscilação.

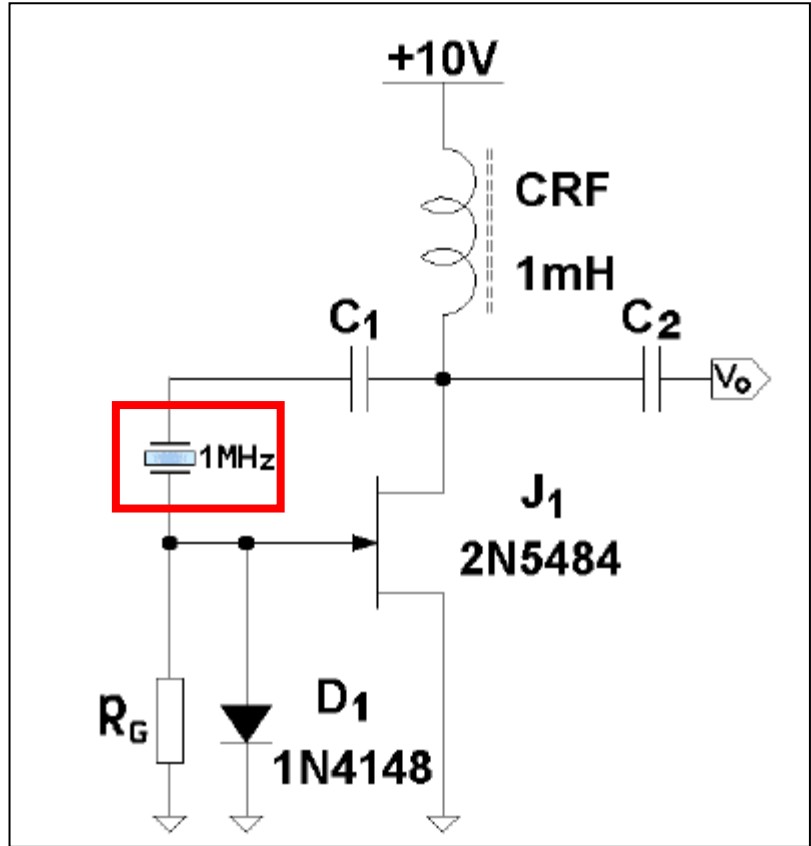


Oscilador Pierce com FET

Não Inversor com FET

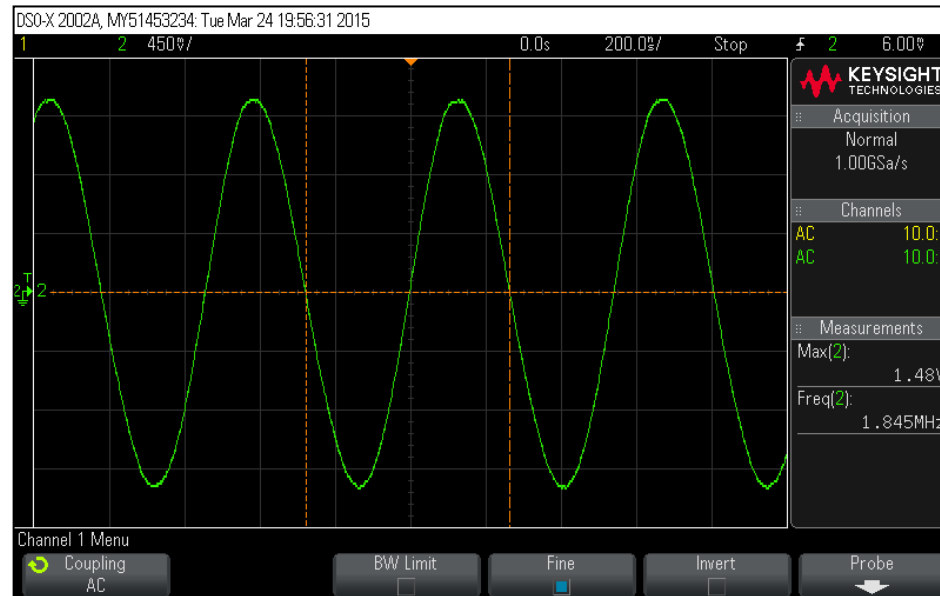


Inversor com FET



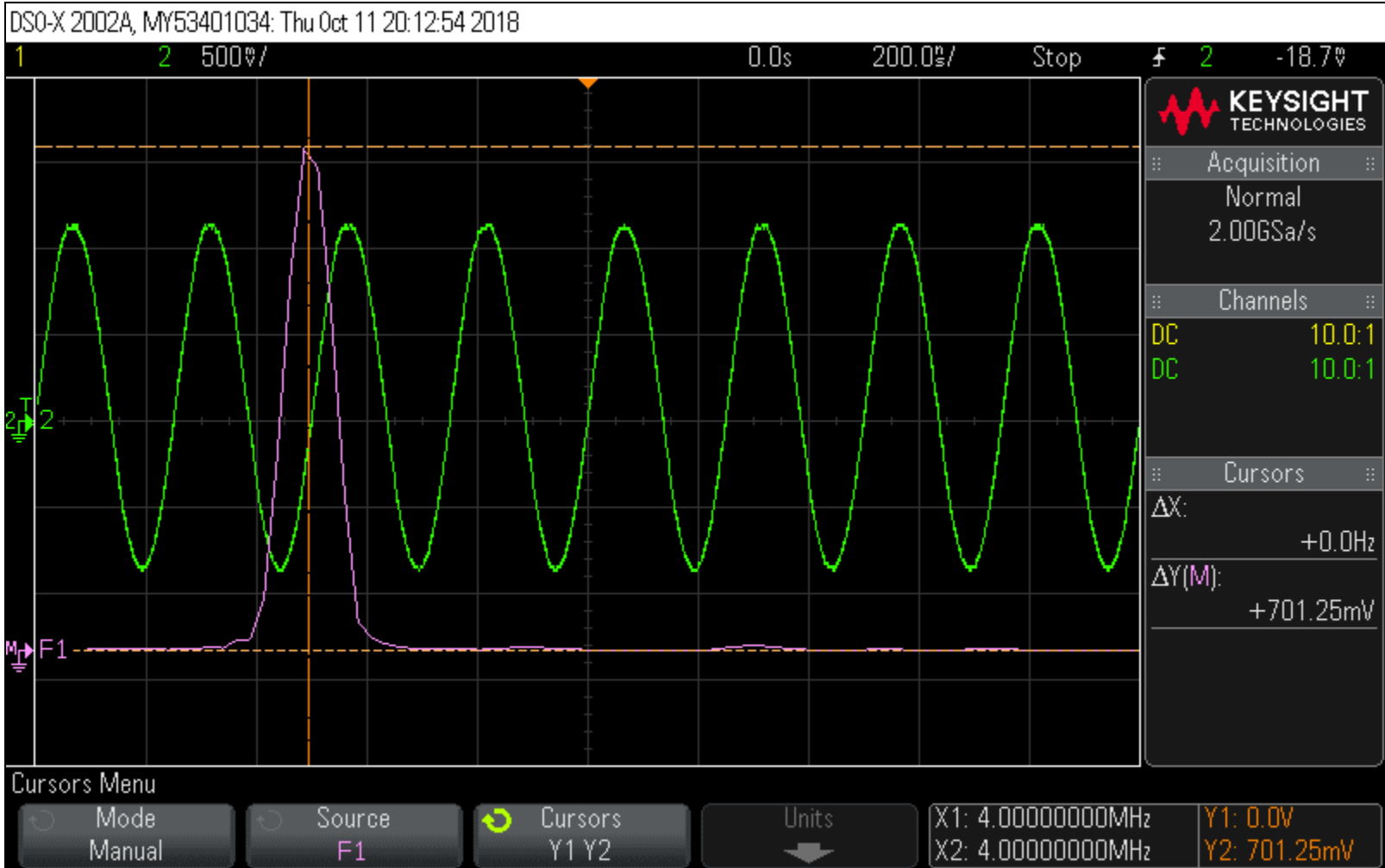
Resultados (Protoboard)

A frequência de oscilação é determinada pela frequência do cristal !



saida do filtro ressonante

$$f_{\text{cristal}} = 1.845\text{MHz}$$

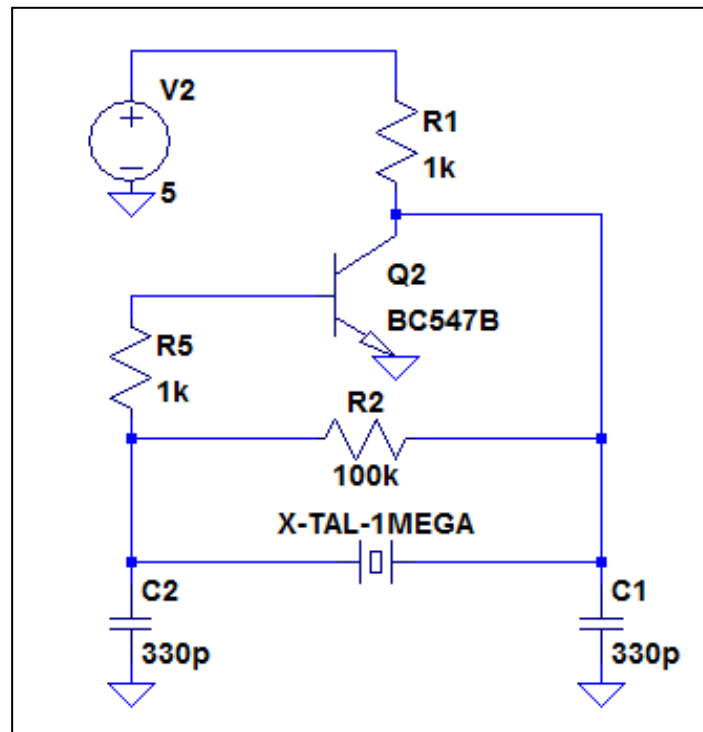


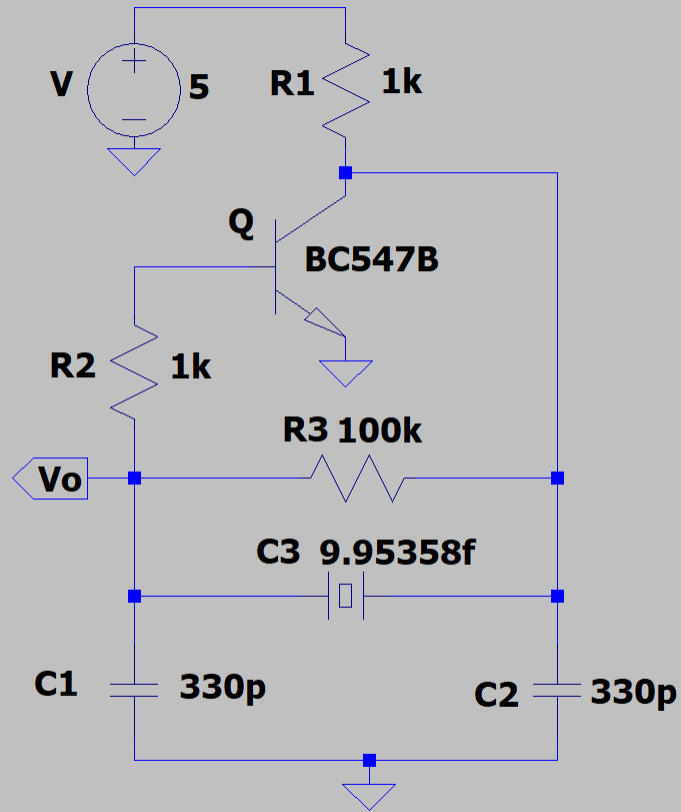
Saída do filtro ressonante
($f_{\text{cristal}} = 4\text{MHz}$)



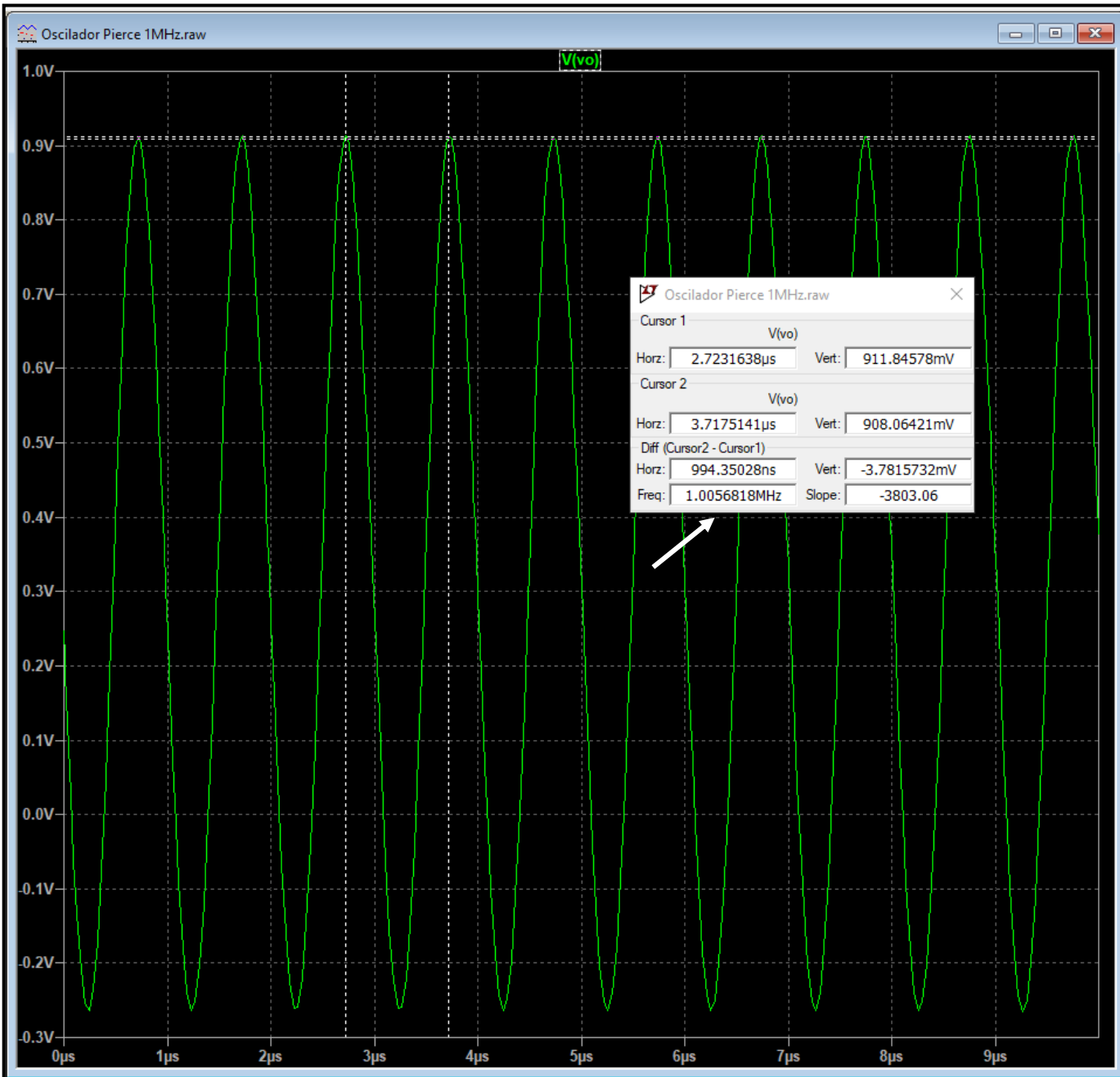
Resultados (Simulação LTSpice)

Oscilador Pierce com BJT





```
.four 1MEG V(Vo)
.tran 0 50.01m 50m
```



FFT

