



PEF-3202 INTRODUÇÃO À MECÂNICA DOS SÓLIDOS

Diagramas de esforços solicitantes: vigas poligonais

Turma 2: Valério Almeida

Agosto/2023

Equações Diferenciais de Equilíbrio

2

$$\frac{dN(x)}{dx} = -p_x(x) \quad \frac{dV(x)}{dx} = -p(x) \quad \frac{d^2M(x)}{dx^2} = -p(x) \quad \frac{dM(x)}{dx} = V(x)$$

a) Caso $p(x) = 0$

Sem carga distribuída no trecho $x_1 < x < x_2$

$V(x) = C_1 = cte \rightarrow$ Função (diagrama) de esforço cortante constante

$M(x) = C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor linear

b) Caso $p(x) = p = cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

$V(x) = -p x + C_1 \rightarrow$ Função (diagrama) de esforço cortante linear

$M(x) = -p \frac{x^2}{2} + C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor é parábola

c) Generalização para $\forall p(x)$ é imediata

d) Caso $p_x(x) = 0$

Sem carga distribuída no trecho $x_1 < x < x_2$

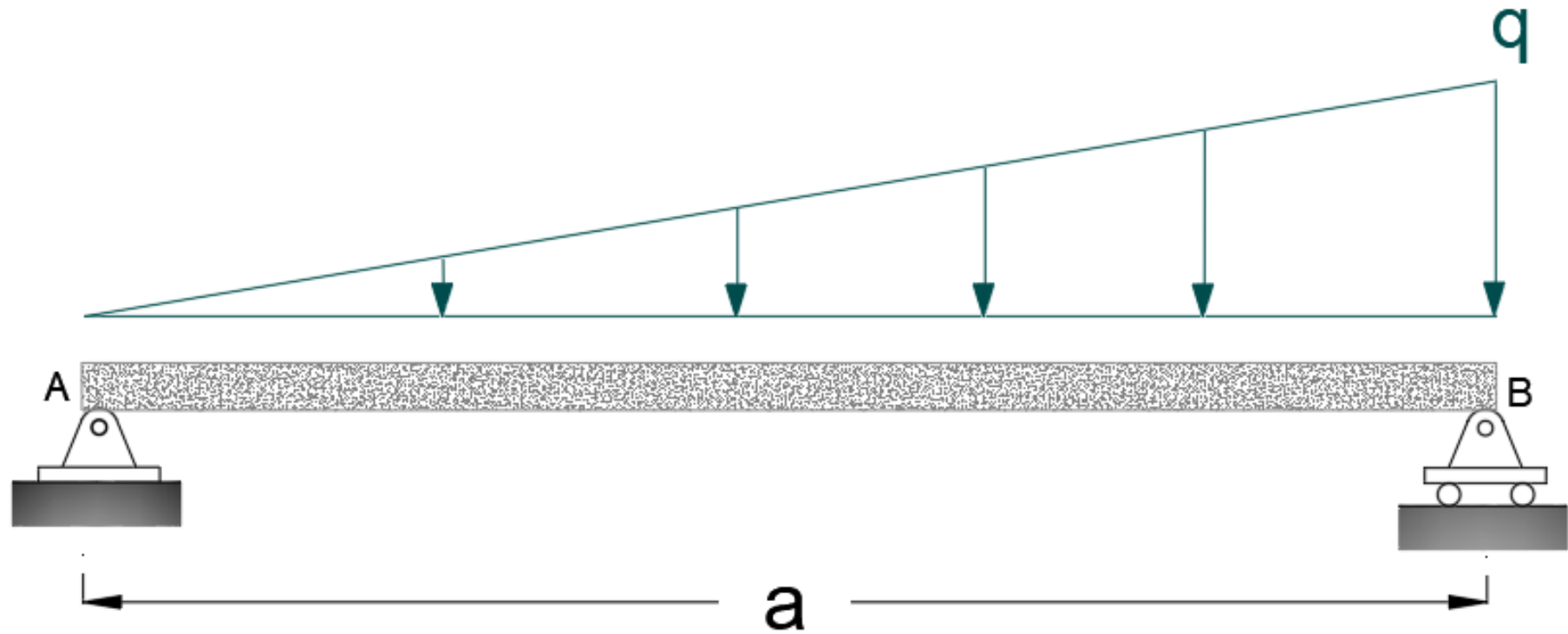
$N(x) = \text{constante}$

e) Caso $p_x(x) = cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

$N(x) = \text{linear (reta)}$

Para a estrutura a seguir, sabendo que $q = 12 \text{ kN/m}$ e $a = 6 \text{ m}$, obtenha os diagramas de esforços solicitantes



$$q(x) = 2x$$

$$A_y = 12 \text{ kN}$$

$$B_y = 24 \text{ kN}$$

$$dV/dx = -q(x)$$

$$V(x) = -x^2 + 12$$

$$V(0) = 12 \text{ kN}; V(6) = -24 \text{ kN}$$

$$V(x) = -x^2 + 12 = 0; x = 2\sqrt{3} = 3,46 \text{ m}$$

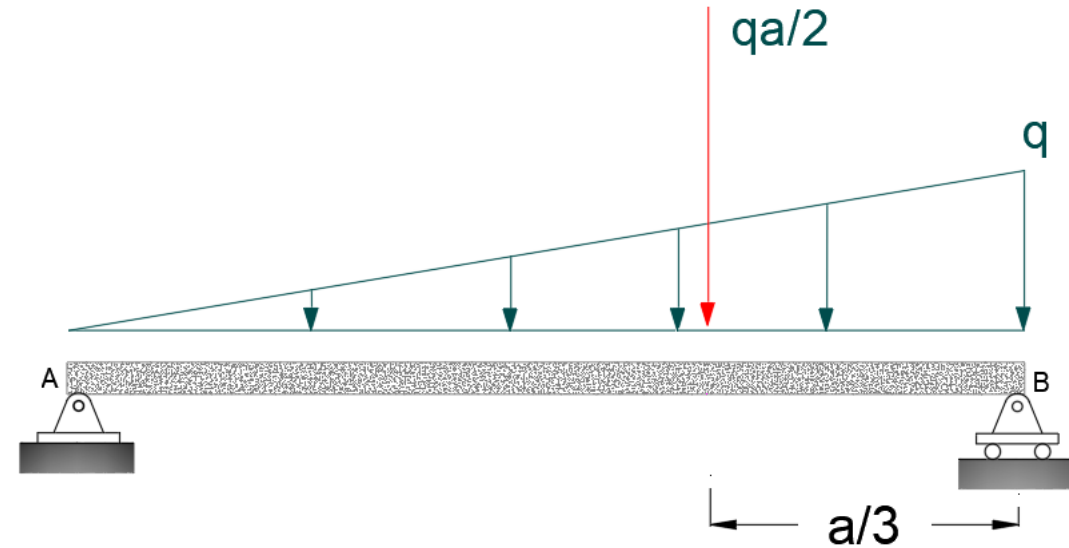
$$M(x) = -x^3/3 + 12x$$

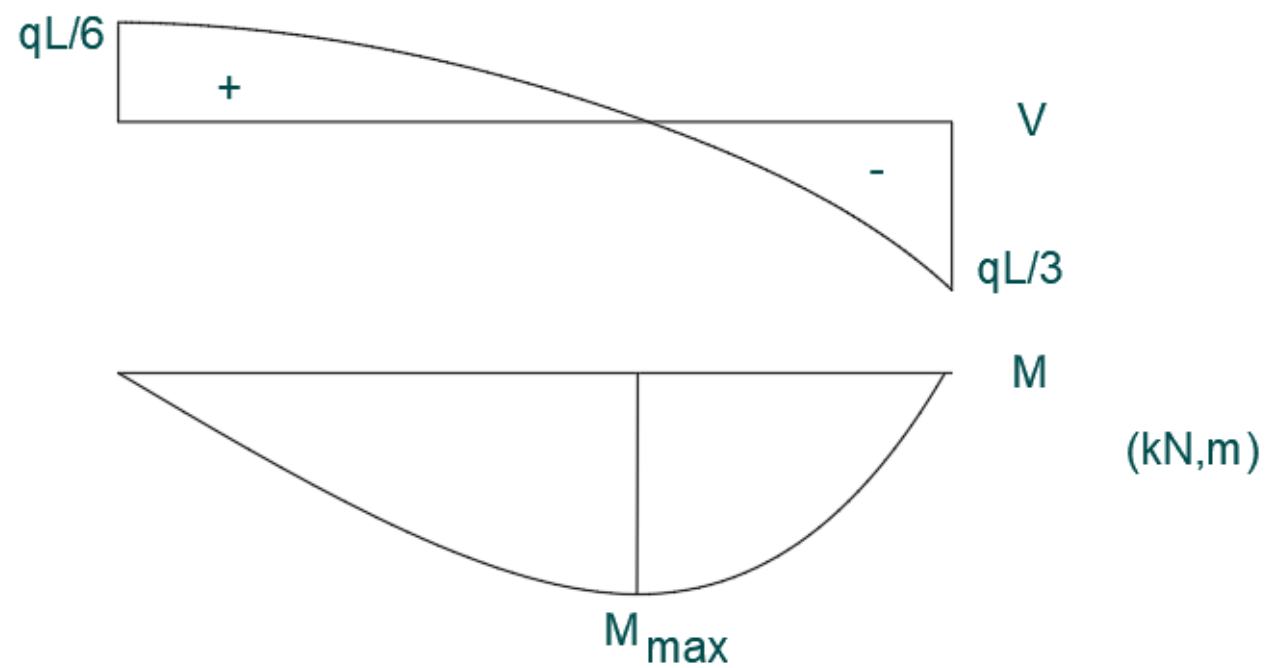
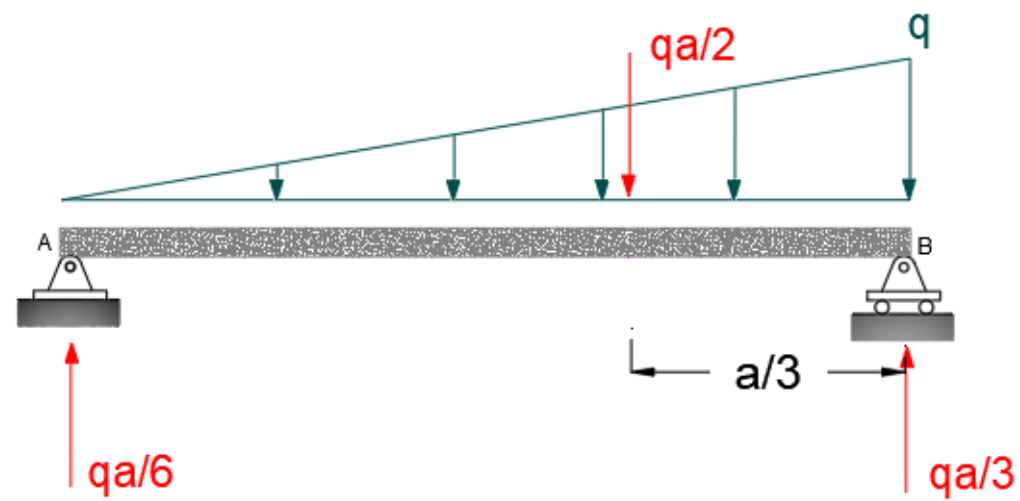
$$M(0) = 0; M(6) = -x^3/3 + 12x = 0$$

$$M'(x) = -3x^2/3 + 12 = 0; x = \pm 2\sqrt{3} = \pm 3,46 \text{ m}$$

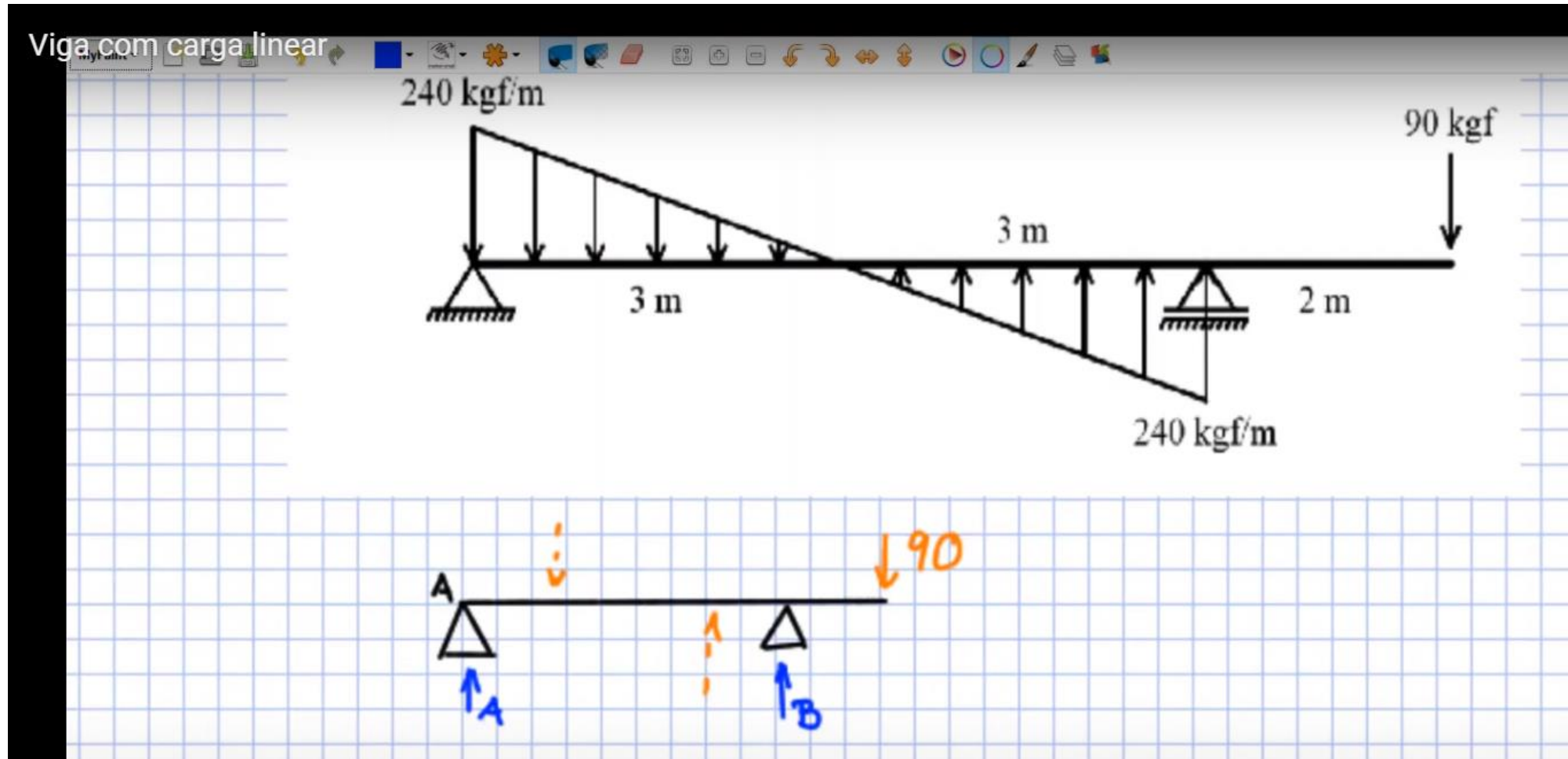
$$M(3,46) = -x^3/3 + 12x = 27,71 \text{ kNm}$$

$$M''(x) = -6x^2/3 = -2x \text{ (concavidade para cima entre } x=0 \text{ e } x=6)$$



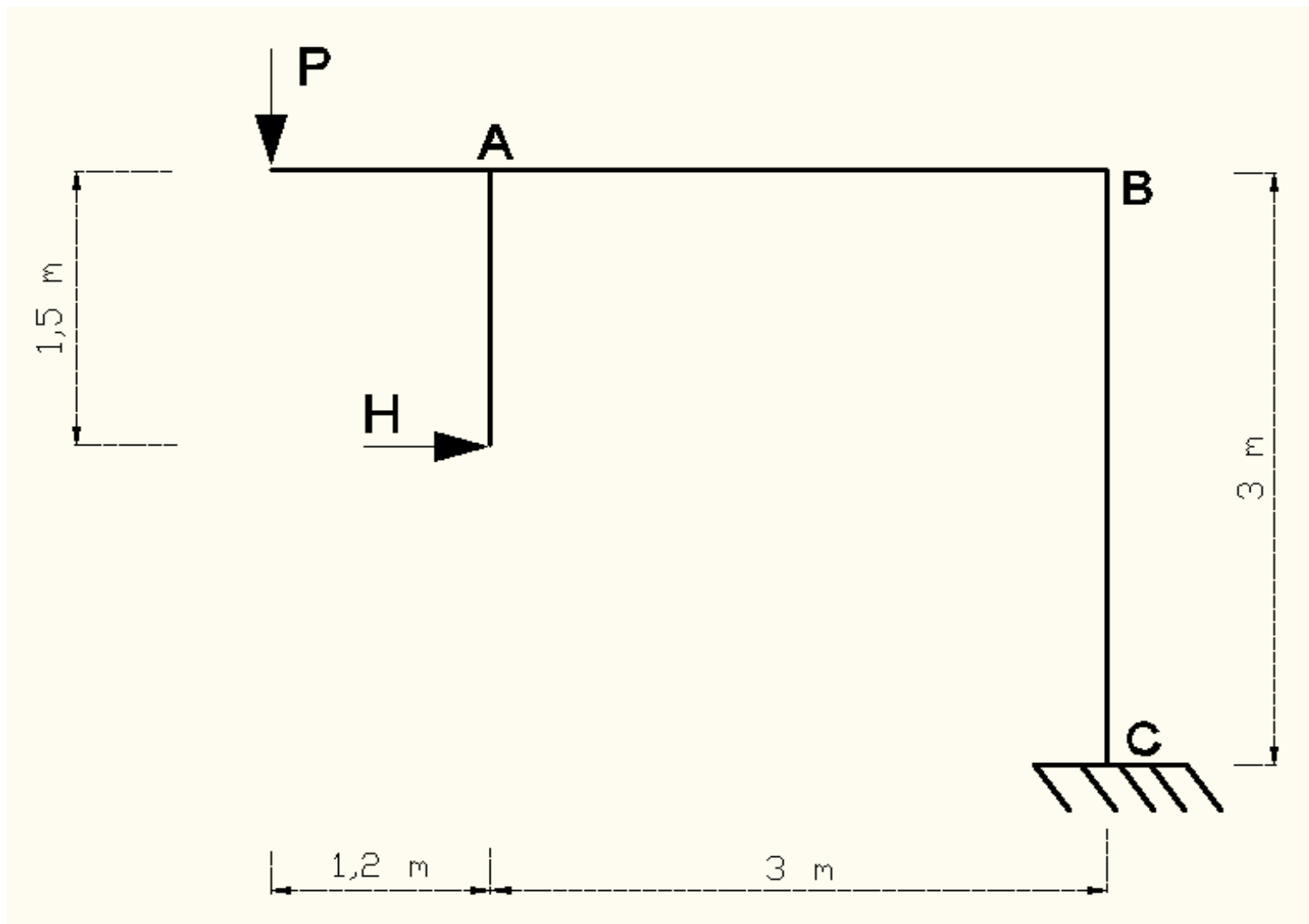


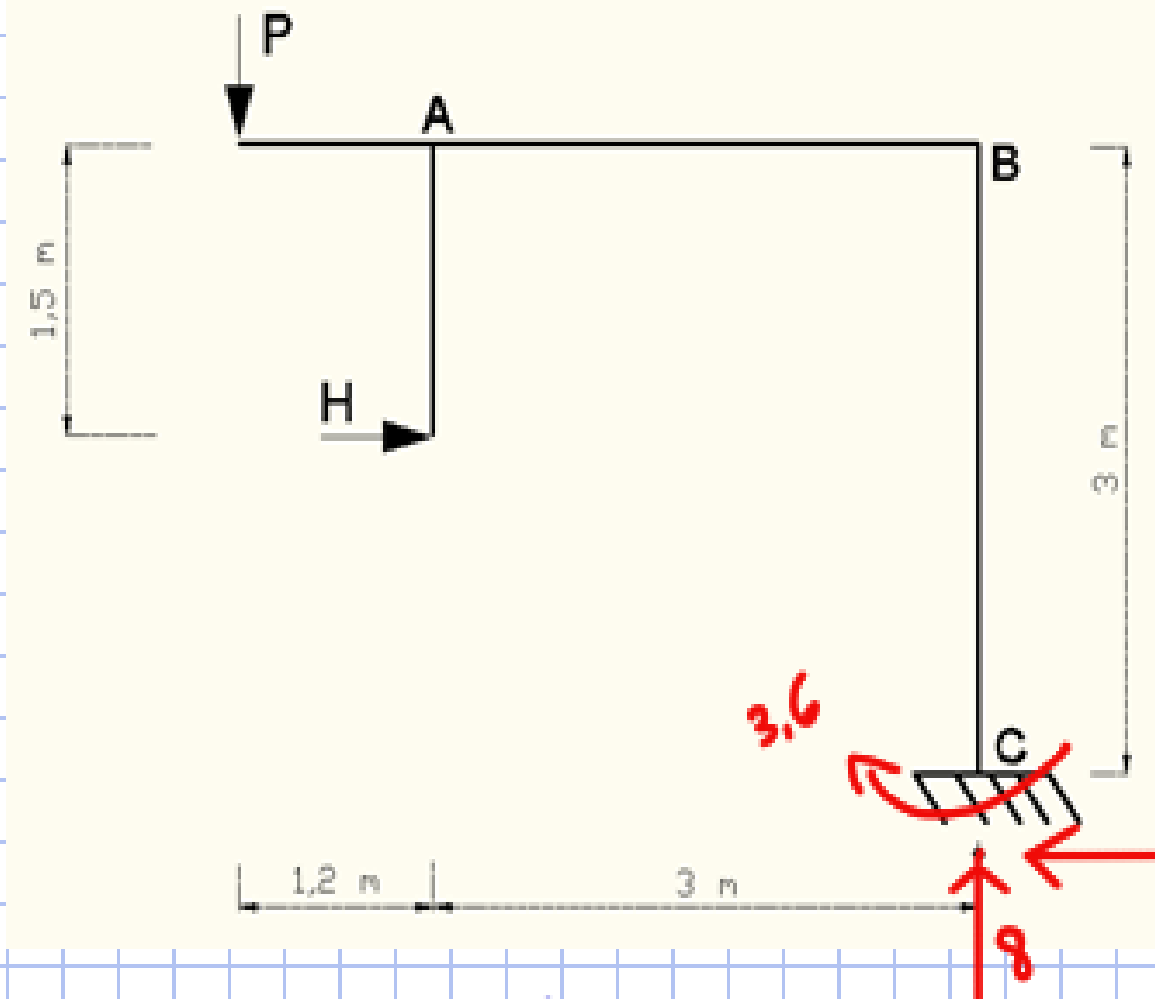
Exemplo: Viga com carga linear – Ver resolução no **YouTube!** Está no E-disciplinas o link



<https://www.youtube.com/watch?v=ZKmarvUU5pl>

Exemplo 21: Determinar os esforços solicitantes (N , V e M) na estrutura esquematizada a seguir, sob a ações das cargas indicadas. Indique explicitamente os valores e os pontos de momentos extremos nos desenhos em destaque. Dados: $P = 8 \text{ kN}$; $H = 20 \text{ kN}$.



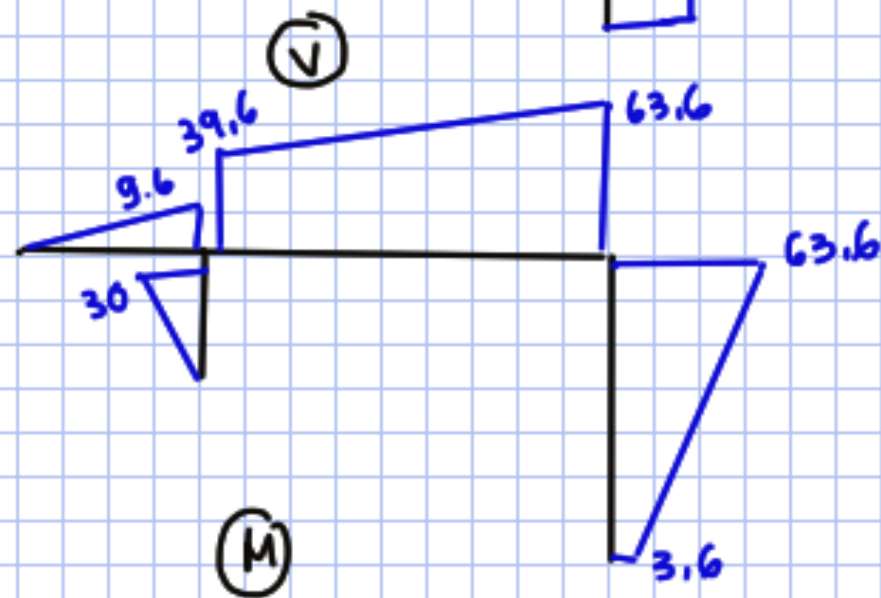
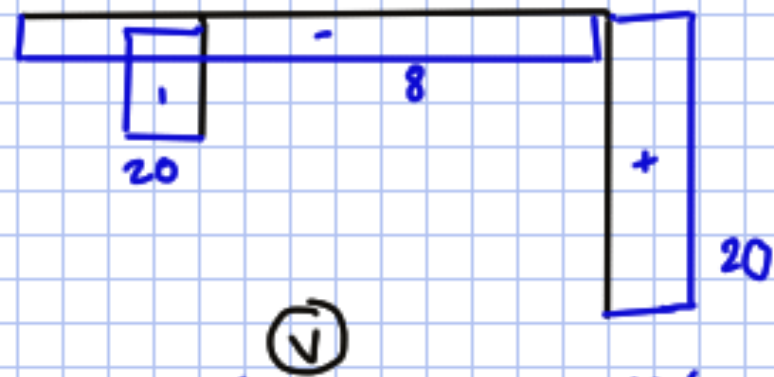
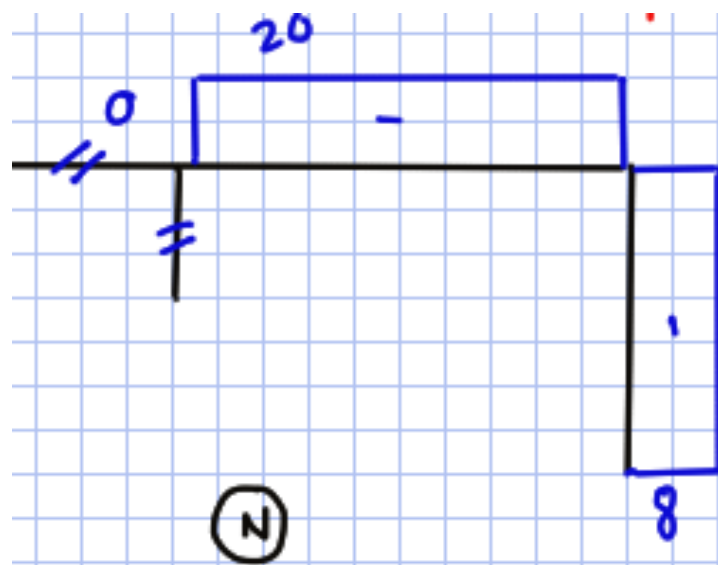


$$\sum M_C = 0:$$

$$M_C + H \cdot 1.5 = P \cdot 4.2$$

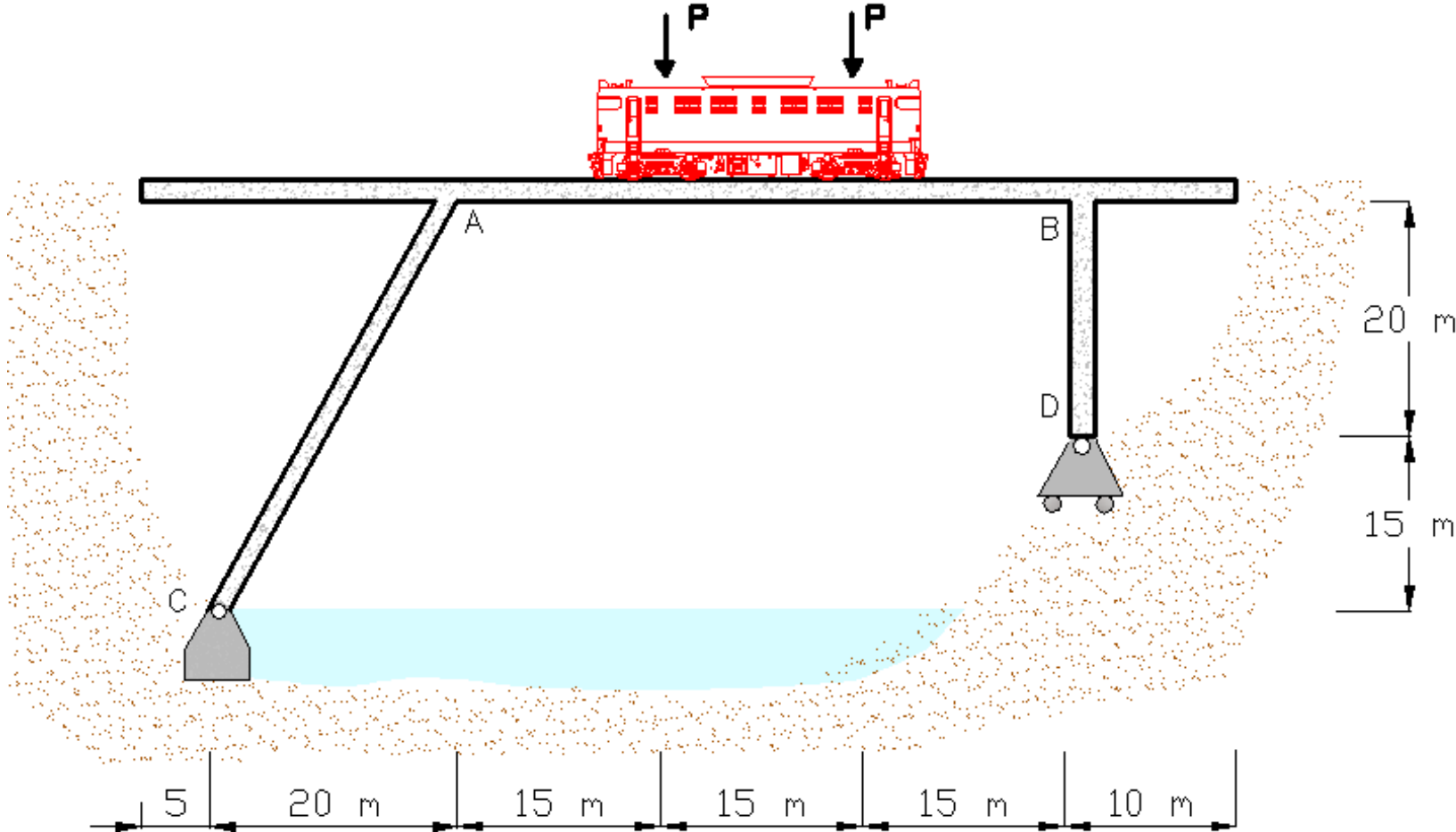
$$M_C = 8 \cdot 4.2 - 20 \cdot 1.5$$

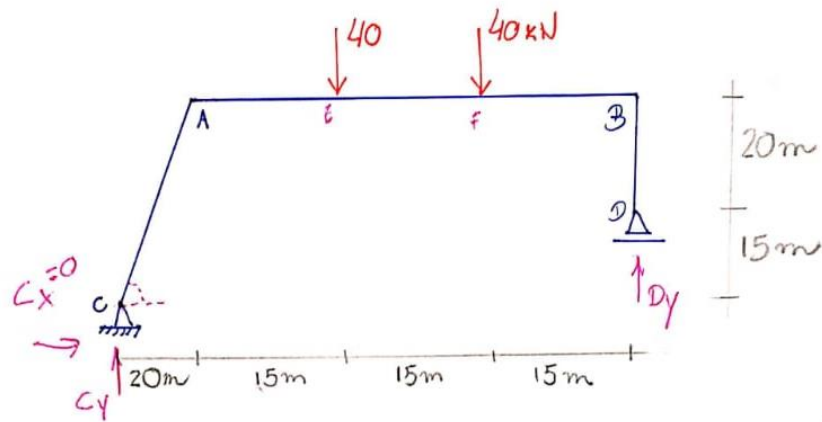
$$M_C = 3.6 \text{ kNm}$$



(kN, m)

Exemplo 15: A ponte está sujeita ao peso da locomotiva considerado como duas forças concentradas de valor $P = 40 \text{ kN}$. Obtenha os diagramas de esforços normal, cortante e momento fletor para o trecho CABD.



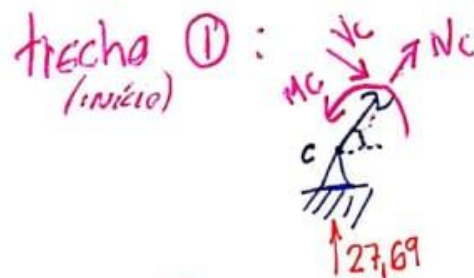


$$+\circlearrowleft \sum M_C = 0 : 40 \cdot 35 + 40 \cdot 50 = D_y \cdot 68 \rightarrow D_y = 52,31 \text{ kN}$$

$$\sum F_y = 0 : C_y + D_y = 80 \rightarrow C_y = 27,69 \text{ kN}$$

DIVIDIR EM TRECHOS :

CA	①
AE	②
EF	③
FB	④
BD	⑤

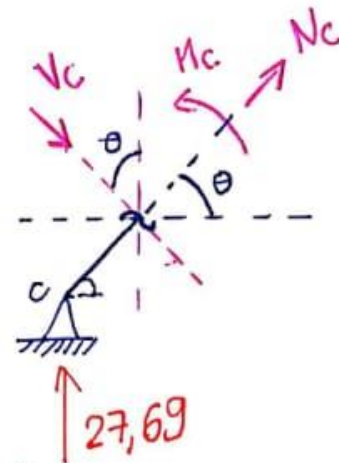


$$\sum F_x = 0$$

$$N_c \cdot \cos \theta + V_c \cdot \sin \theta = 0 \quad (1)$$

$$\sum F_y = 0$$

$$N_c \cdot \sin \theta - V_c \cdot \cos \theta + 27,69 = 0 \quad (2)$$



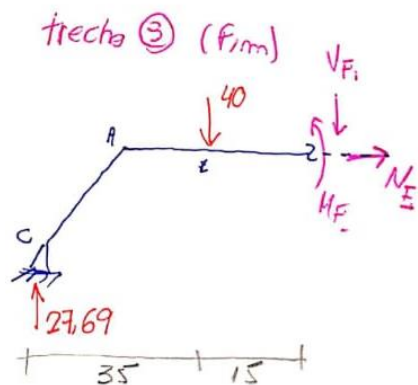
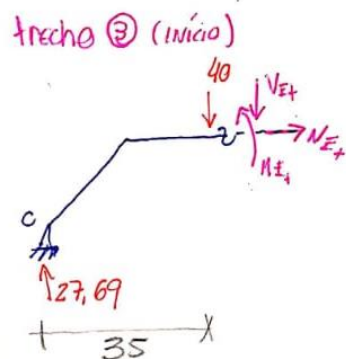
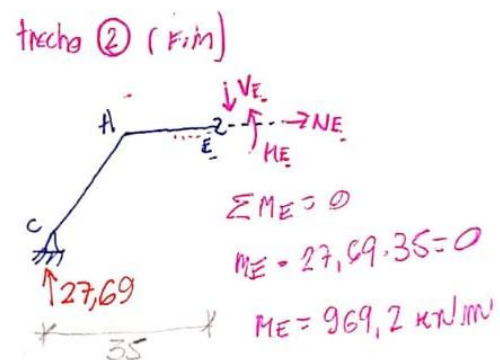
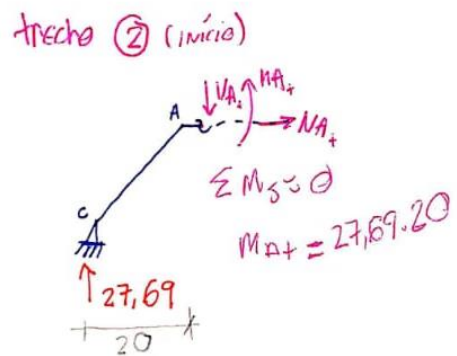
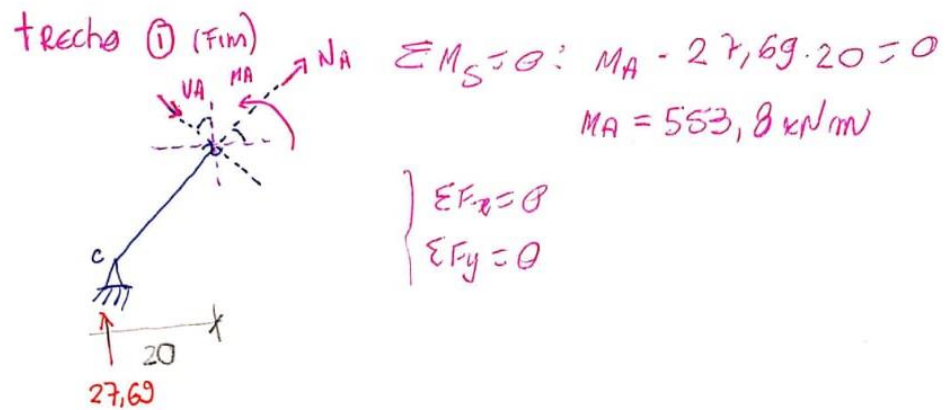
$$\cos \theta = 0,4961$$

$$\sin \theta = 0,8682$$

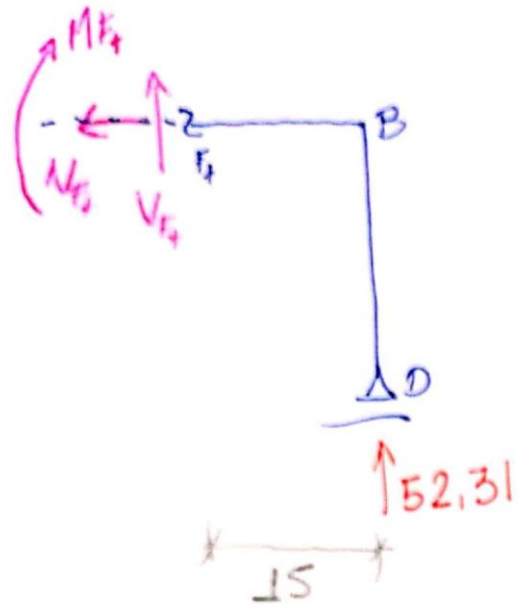
$$N_c = -24 \text{ kN}$$

$$V_c = 13,74 \text{ kN}$$

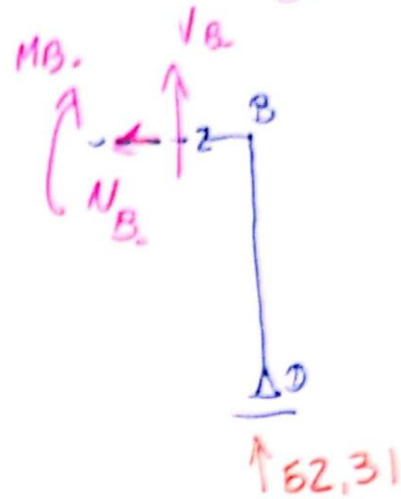
$$\sum M_A = 0 : M_C = 0$$



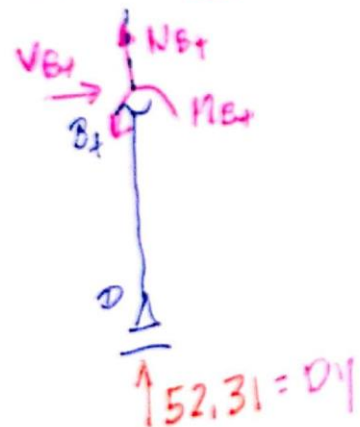
trecho (4) (início)



trecho (4) (Fim)



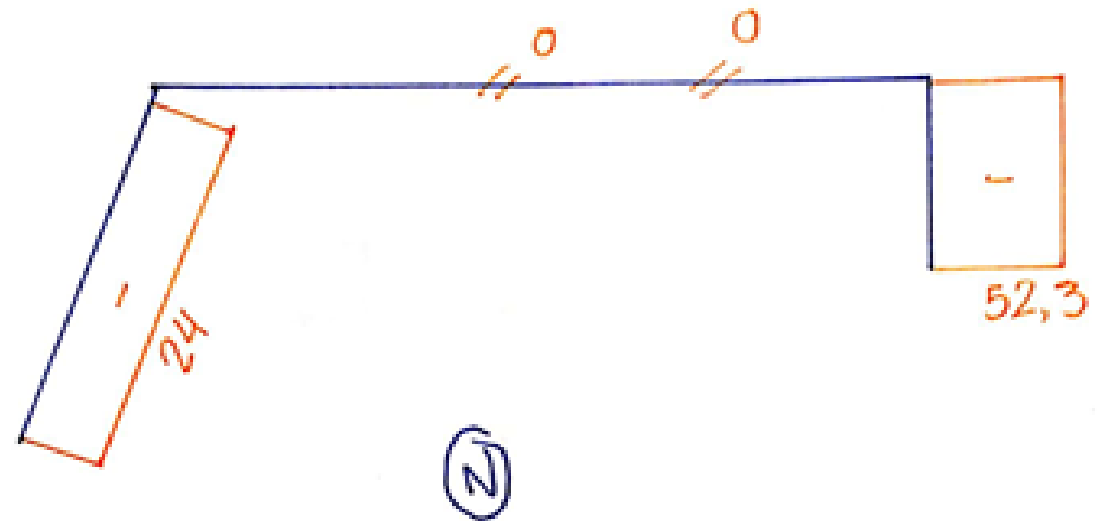
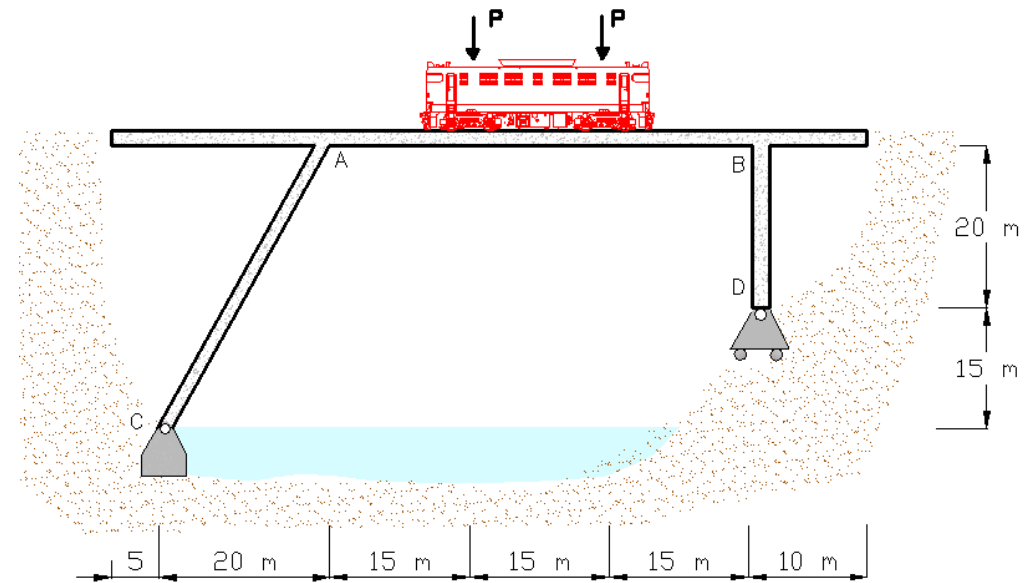
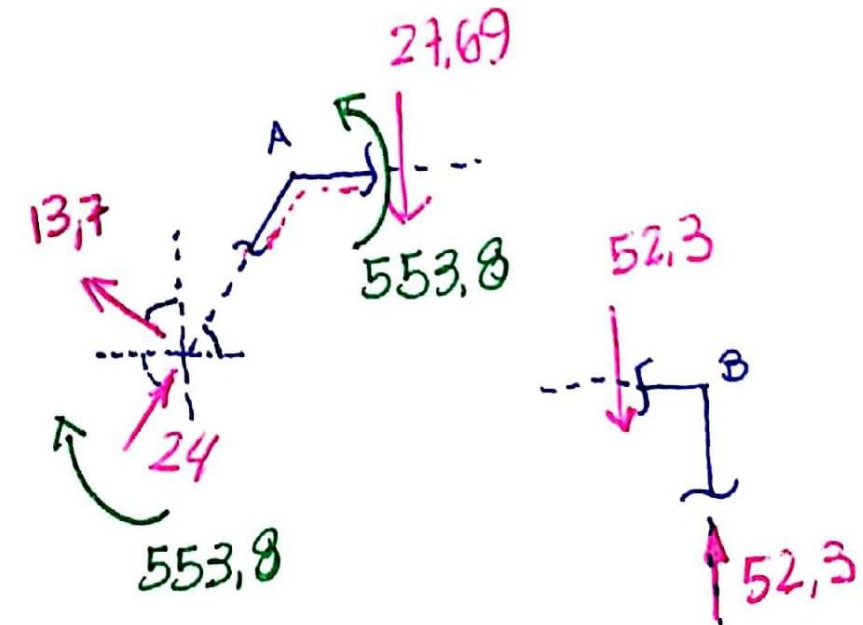
trecho (5) (início)

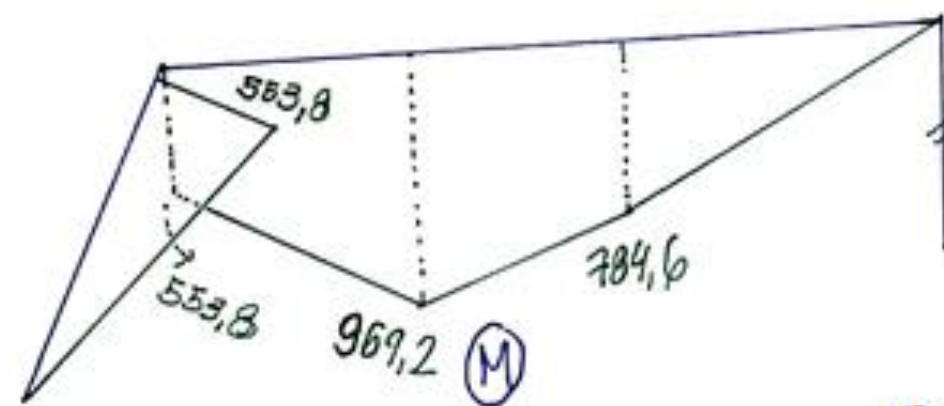
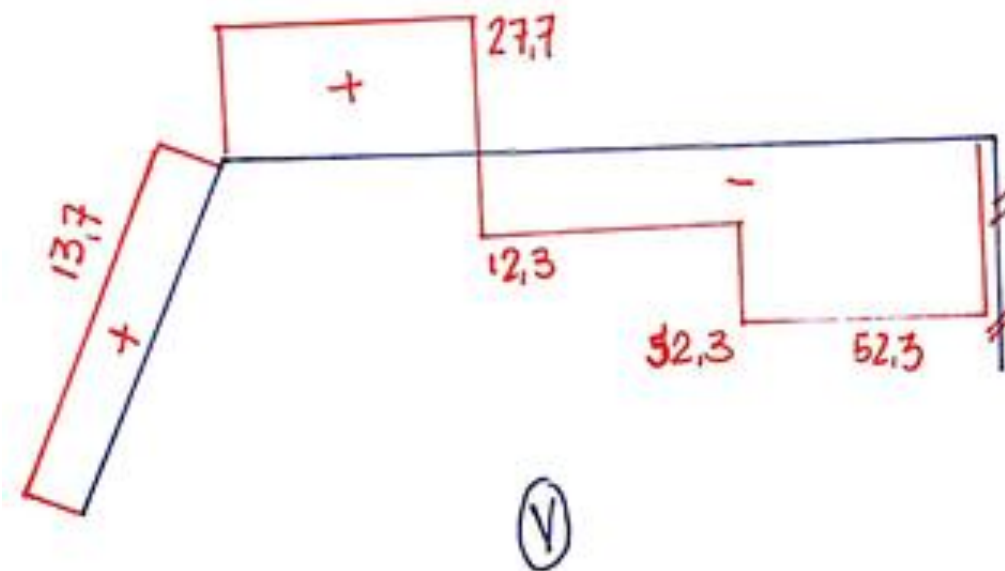
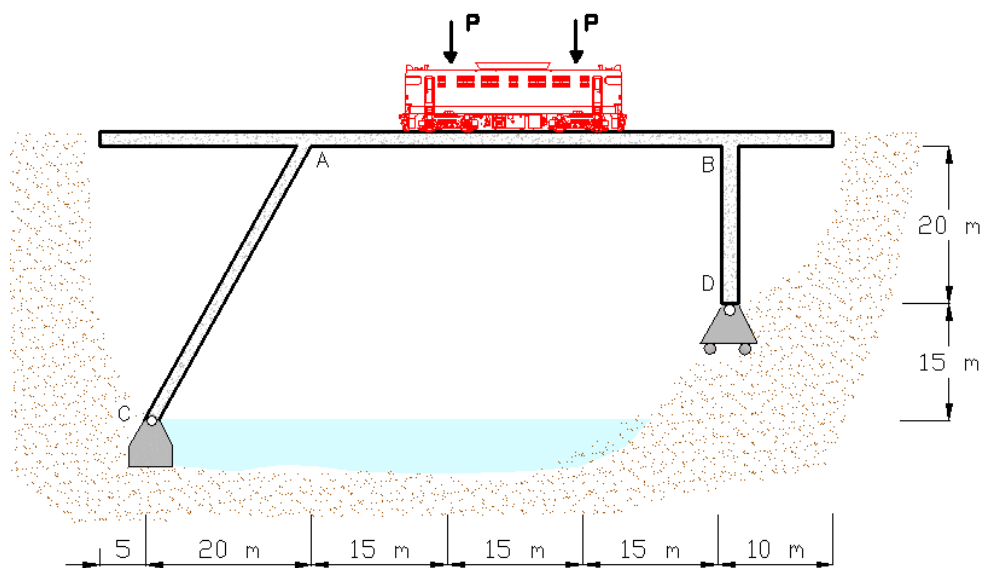


trecho (5) (Fim)



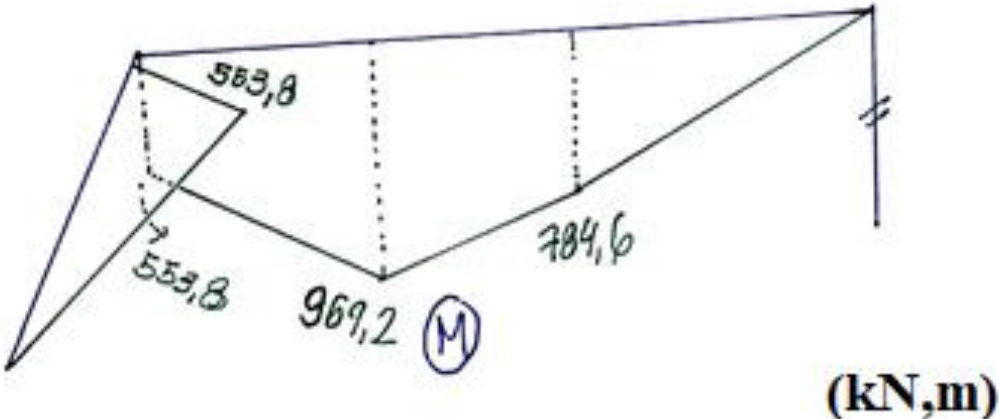
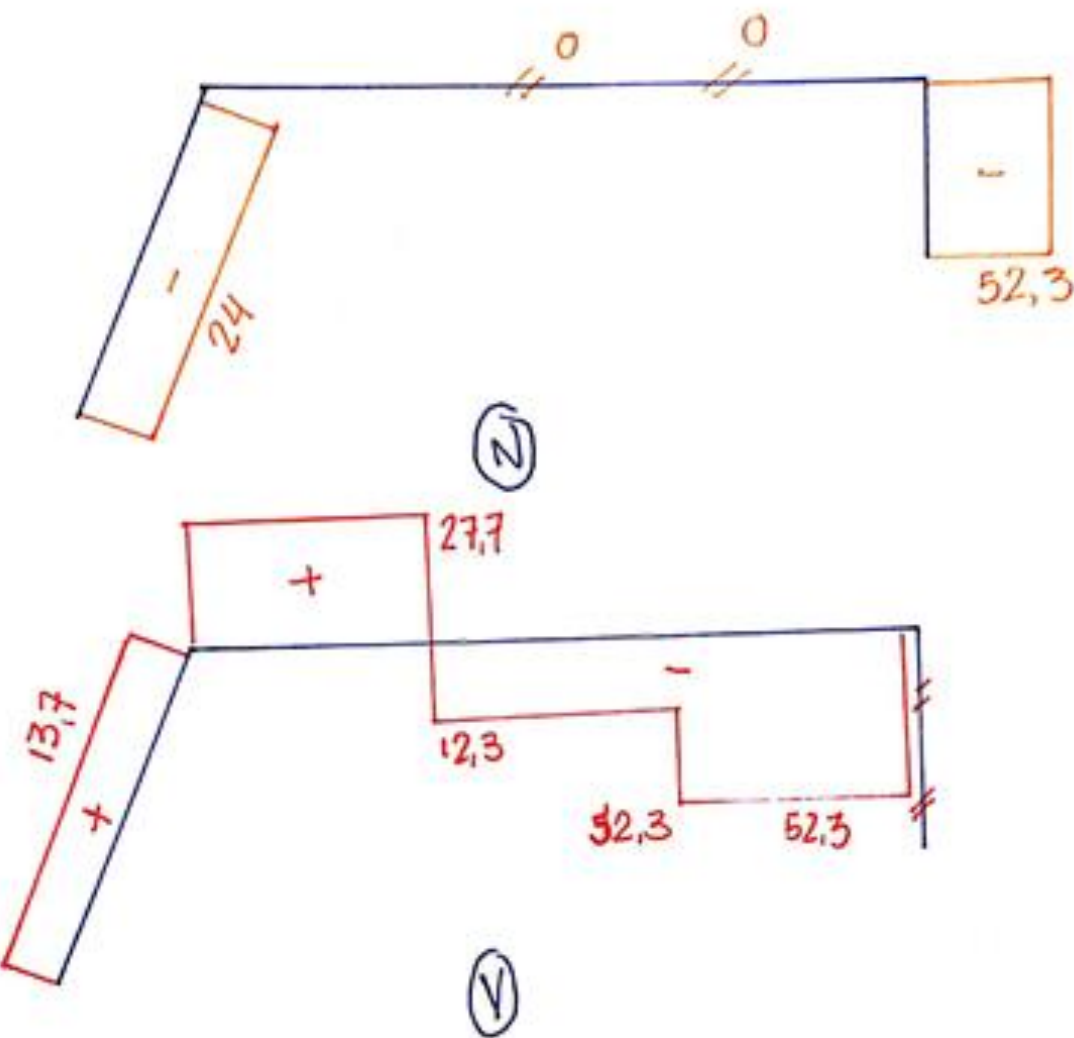
Verifique equilíbrio junto a A e B:





(kN,m)

Diagramas



Exemplo 16

1.7.21. O pórtico plano ABCD serve como contenção de terra. O solo exerce uma carga no muro CD conforme indicado no desenho e seu valor máximo é dado pela relação $q_e = \gamma_{\text{solo}} \cdot H \cdot b \cdot k_0$, com γ_{solo} sendo seu peso específico, k_0 é o coeficiente de empuxo do solo, H a altura do muro e b sua largura. Sobre a viga BC age uma carga constantemente distribuída devido a uma ação permanente de 40 kN/m. Determinar os diagramas de esforço normal, cortante e momento fletor apenas no trecho BC. Considere: $\gamma_{\text{solo}} = 22 \text{ kN/m}^3$, $H = 6 \text{ m}$, $b = 1 \text{ m}$, $k_0 = 0,33$.

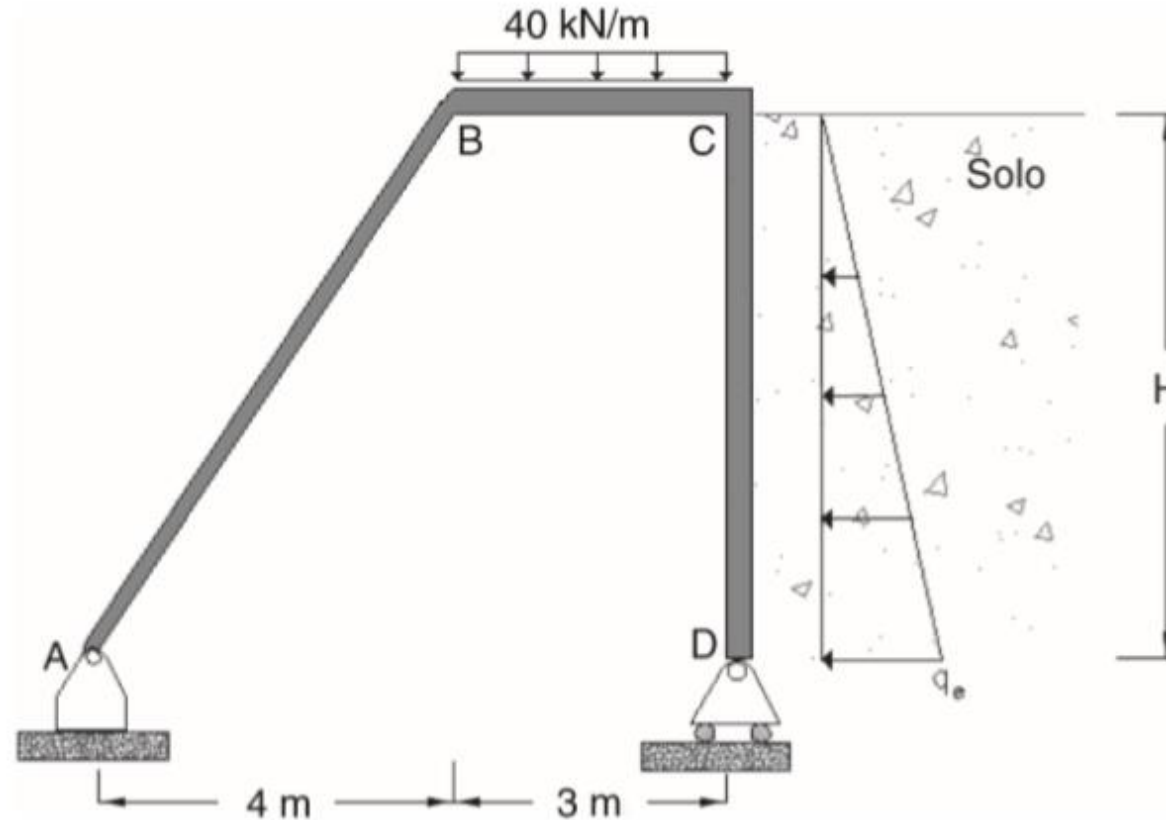


FIGURA 1.60A Pórtico biapoiado submetido às forças distribuídas constante e linear.

Resolução

A carga distribuída é dada por: $q_e = 22 \cdot 6 \cdot 1 \cdot 0,33 = 43,56 \text{ kN/m}$, a qual possui resultante de $F_e = 130,68 \text{ kN}$, aplicada a 2 m acima do ponto D. Usando as três equações de equilíbrio, determinam-se as reações:

$$\sum M_A = 0 : \rightarrow 7 \cdot D_y + 2 \cdot 130,68 = 40 \cdot 3 \cdot 5,5 \rightarrow D_y = 56,95 \text{ kN } (\uparrow)$$

$$\sum F_y = 0 : \rightarrow A_y + D_y = 40 \cdot 3 \rightarrow A_y = 63,05 \text{ kN } (\uparrow)$$

$$\sum F_x = 0 : \rightarrow A_x = 130,68 \text{ kN } (\rightarrow)$$

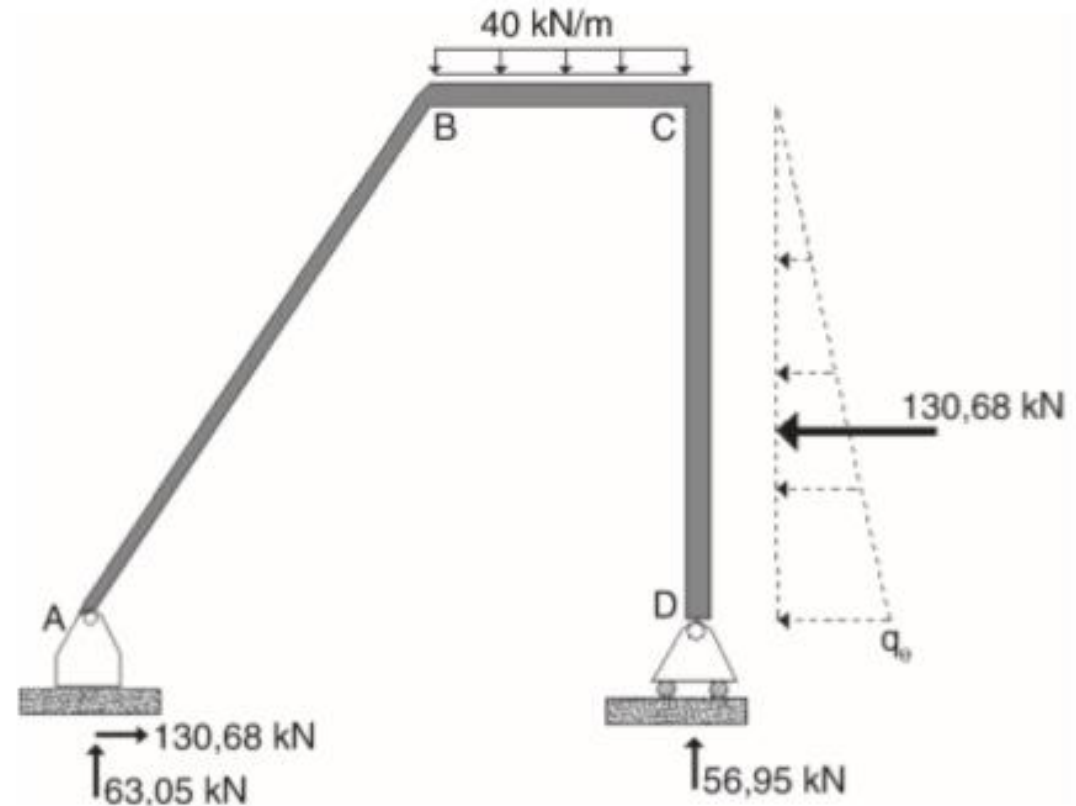
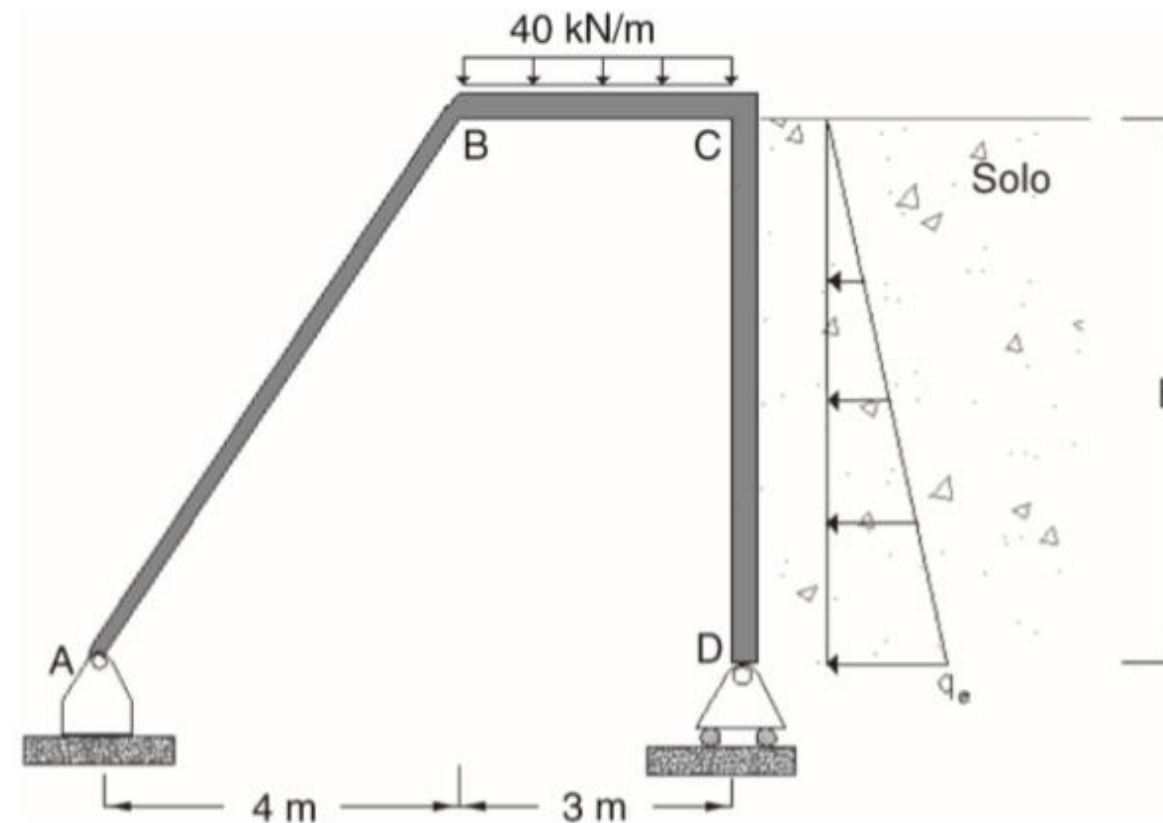
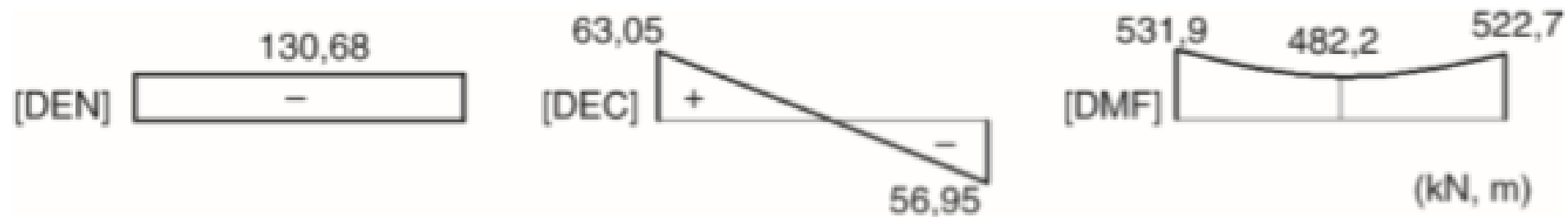


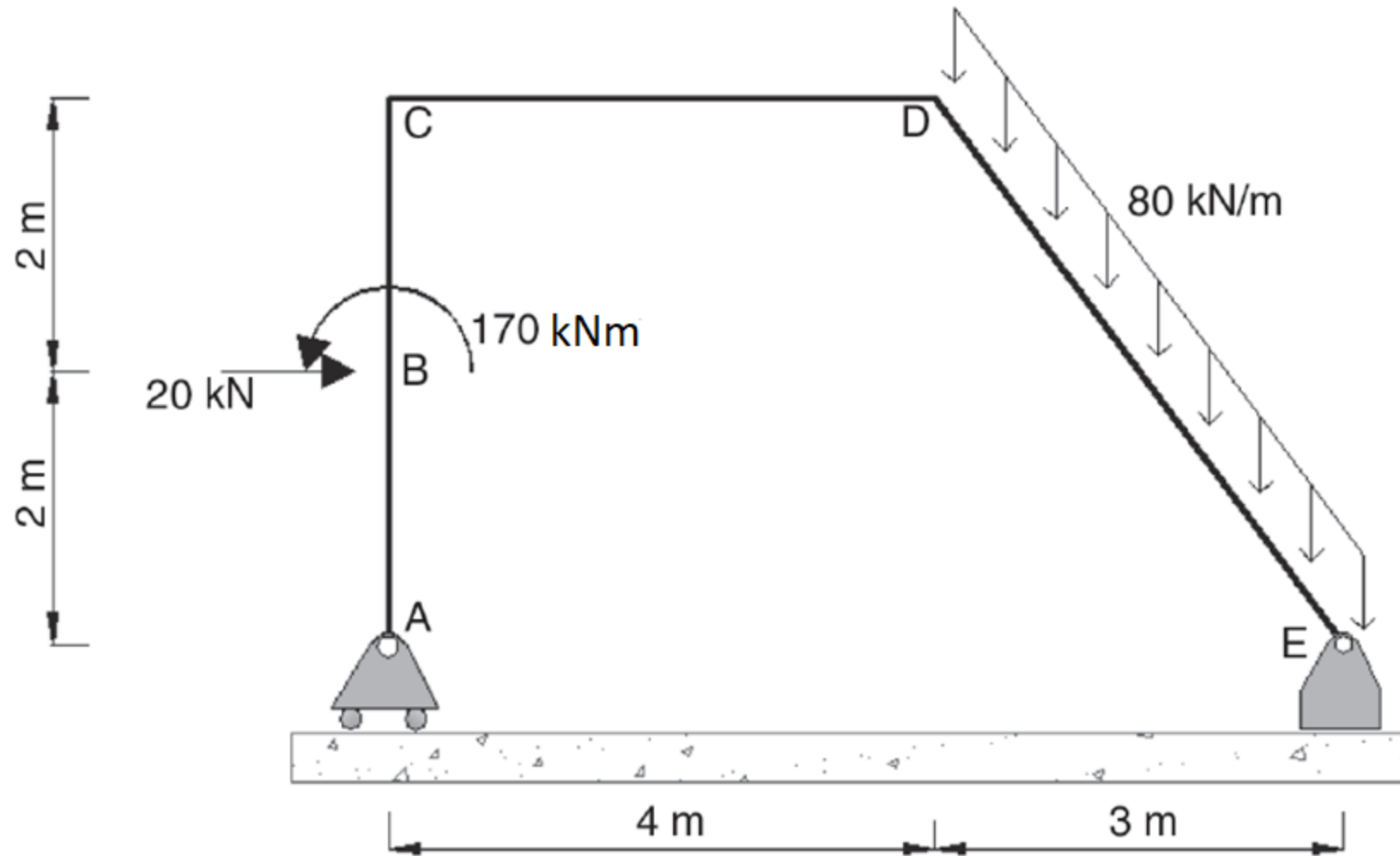
FIGURA 1.60B Indicação das ações resultantes e reações do pórtico biapoado.



Diagramas



Exemplo 19: Determinar os diagramas de esforços solicitantes de toda a estrutura plana*

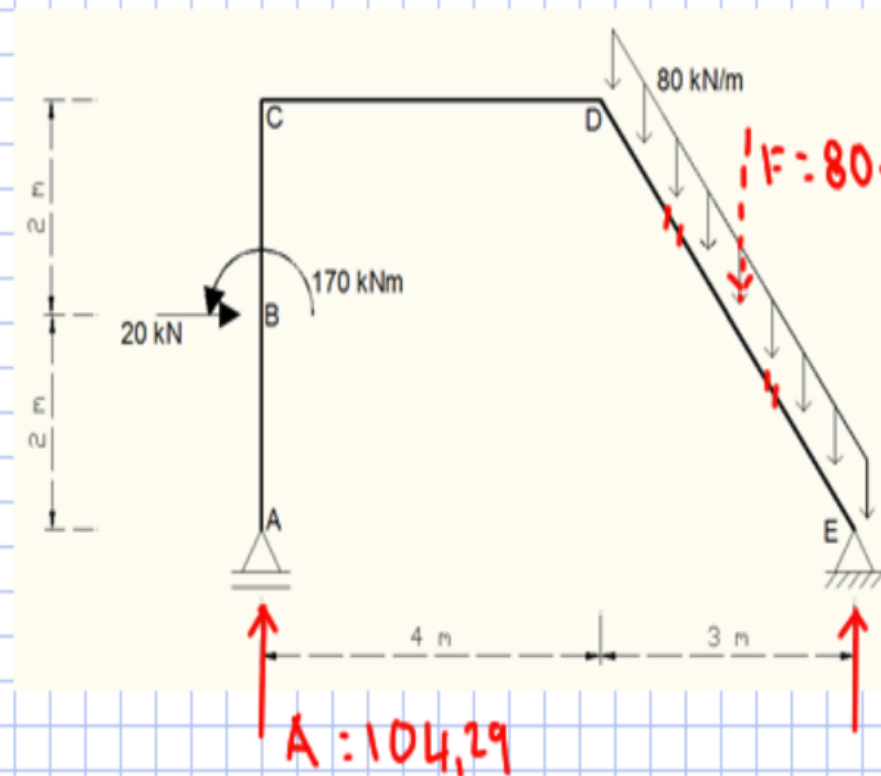


Usando as três equações de equilíbrio para determinar as reações:

$$\sum F_X = 0 : \rightarrow 20 - E_x = 0 \rightarrow E_x = 20 \text{ kN } (\leftarrow)$$

$$\sum M_A = 0 : \rightarrow 7 \cdot E_y + 170 = 80 \cdot 5 \cdot 5,5 + 20 \cdot 2 \rightarrow E_y = 295,71 \text{ kN } (\uparrow)$$

$$\sum F_y = 0 : \rightarrow A_y + E_y = 80 \cdot 5 \rightarrow A_y = 104,29 \text{ kN } (\uparrow)$$



$$\sum M_A = 0$$

$$7 E_y + 170 = 80 \cdot 5 \cdot 5,5 + 20 \cdot 2$$

$$E_y = 295,71 \text{ kN}$$

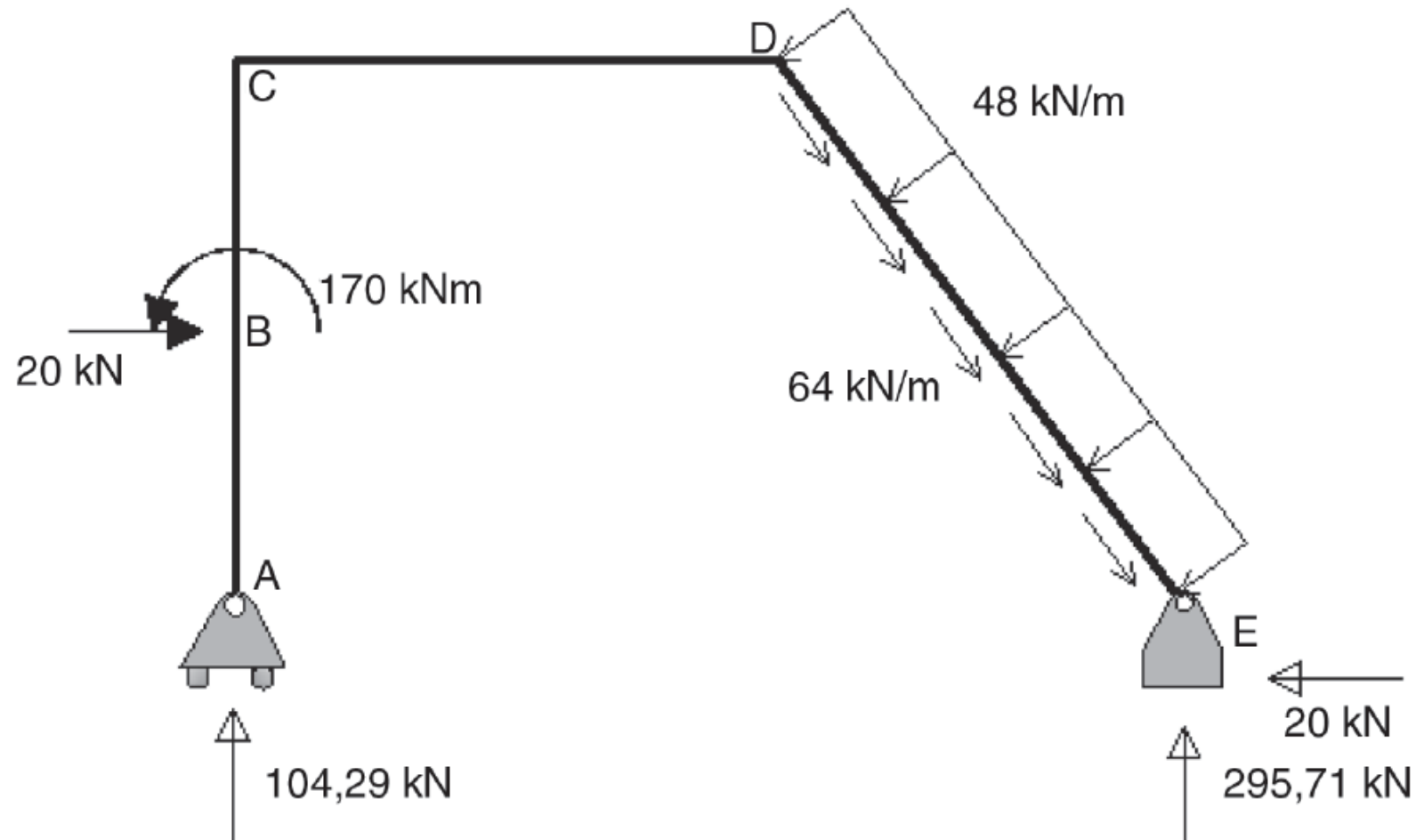
$$A = 104,29 \text{ kN}$$

$$\leftarrow 20$$

$$295,71$$

$$A : 104,29$$

Decompor a carga distribuída paralelo (x') e perpendicular (y') ao seu eixo. A sua resultante é de $80 \cdot 5 = 400$ kN, que a decompondo fica: $F_{x'} = 320$ kN, $F_{y'} = 240$ kN, e pode-se obter a força distribuída como: $q_{x'} = 320/5 = 64$ kN/m, $q_{y'} = 240/5 = 48$ kN/m.



Para os trechos: AB, BC, CD e DE, fazer cortes no início e fim, de modo a aplicar as três equações de equilíbrio, no sentido da convenção positiva, veja trecho AB:

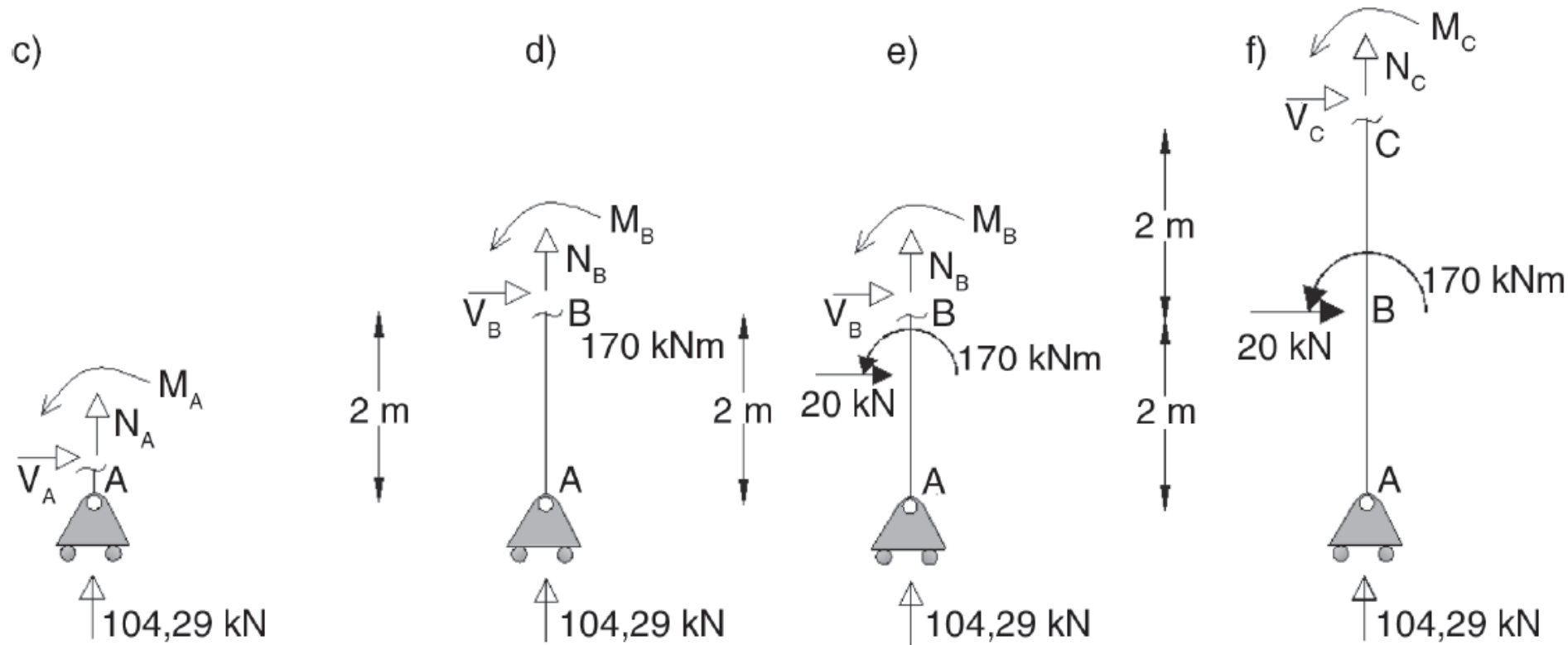


FIGURA 1.58C-F Esquema do corte: (c) início do trecho AB; (d) fim do trecho AB; (e) início do trecho BC; (f) fim do trecho BC.

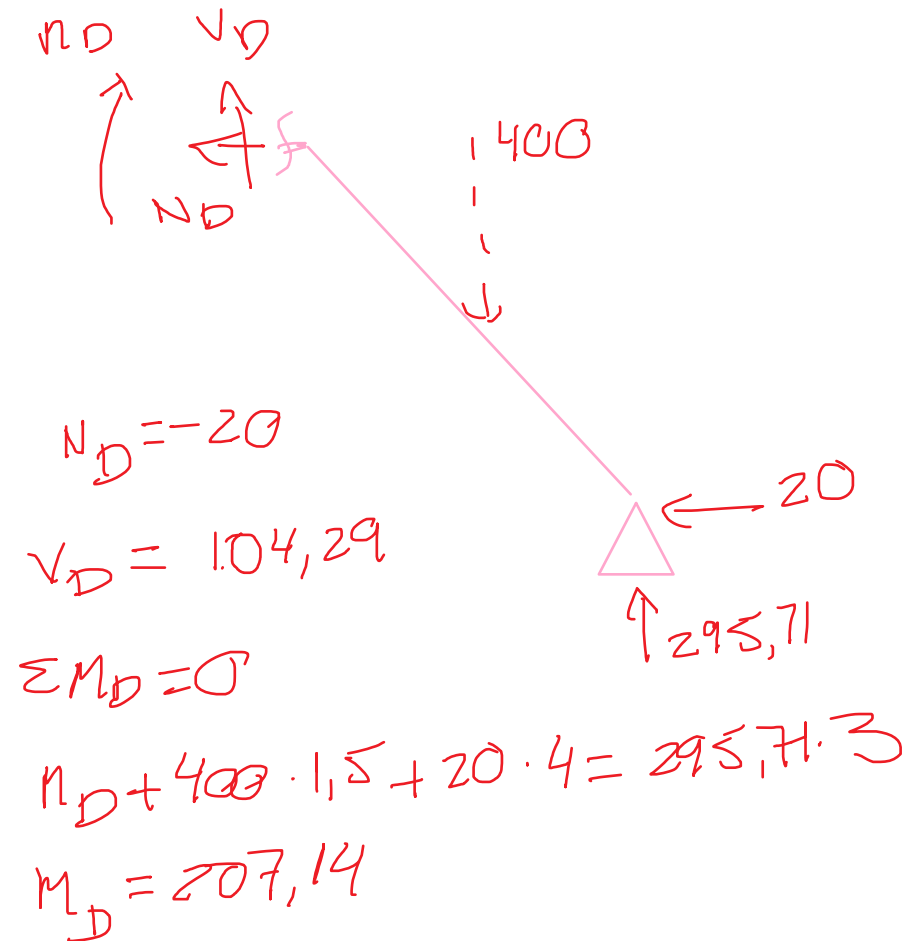
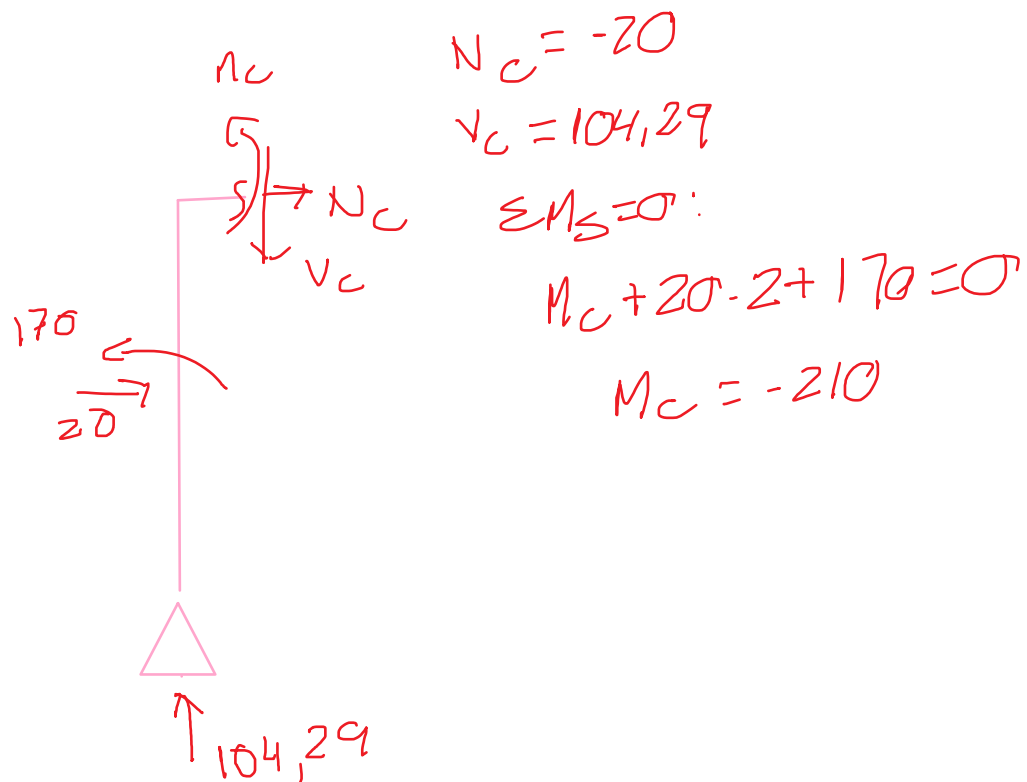
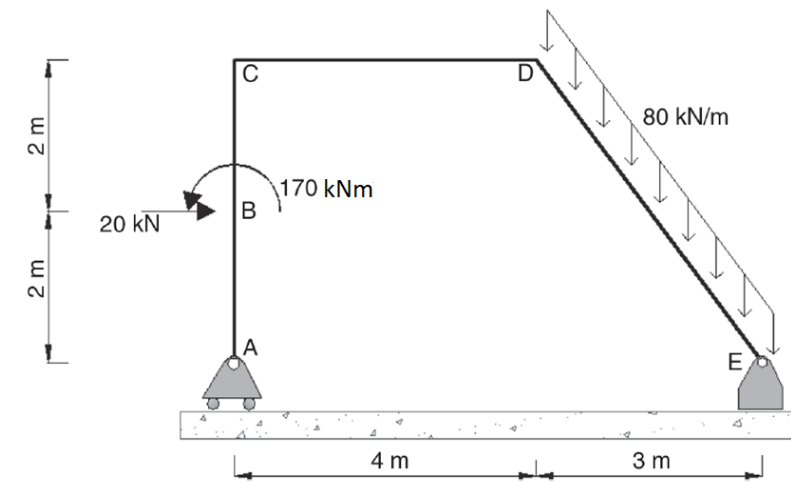
Conforme Figura 1.58C: $N_A = -104,29 \text{ kN}$; $V_A = 0$; $M_A = 0$

Conforme Figura 1.58D: $N_B = -104,29 \text{ kN}$; $V_B = 0$; $M_B = 0$

Trecho BC:

Conforme Figura 1.58E: $N_B = -104,29 \text{ kN}$; $V_B = -20 \text{ kN}$; $M_B = -170 \text{ kNm}$

Conforme Figura 1.58F: $N_C = -104,29 \text{ kN}$; $V_C = -20 \text{ kN}$; $M_C = -210 \text{ kNm}$



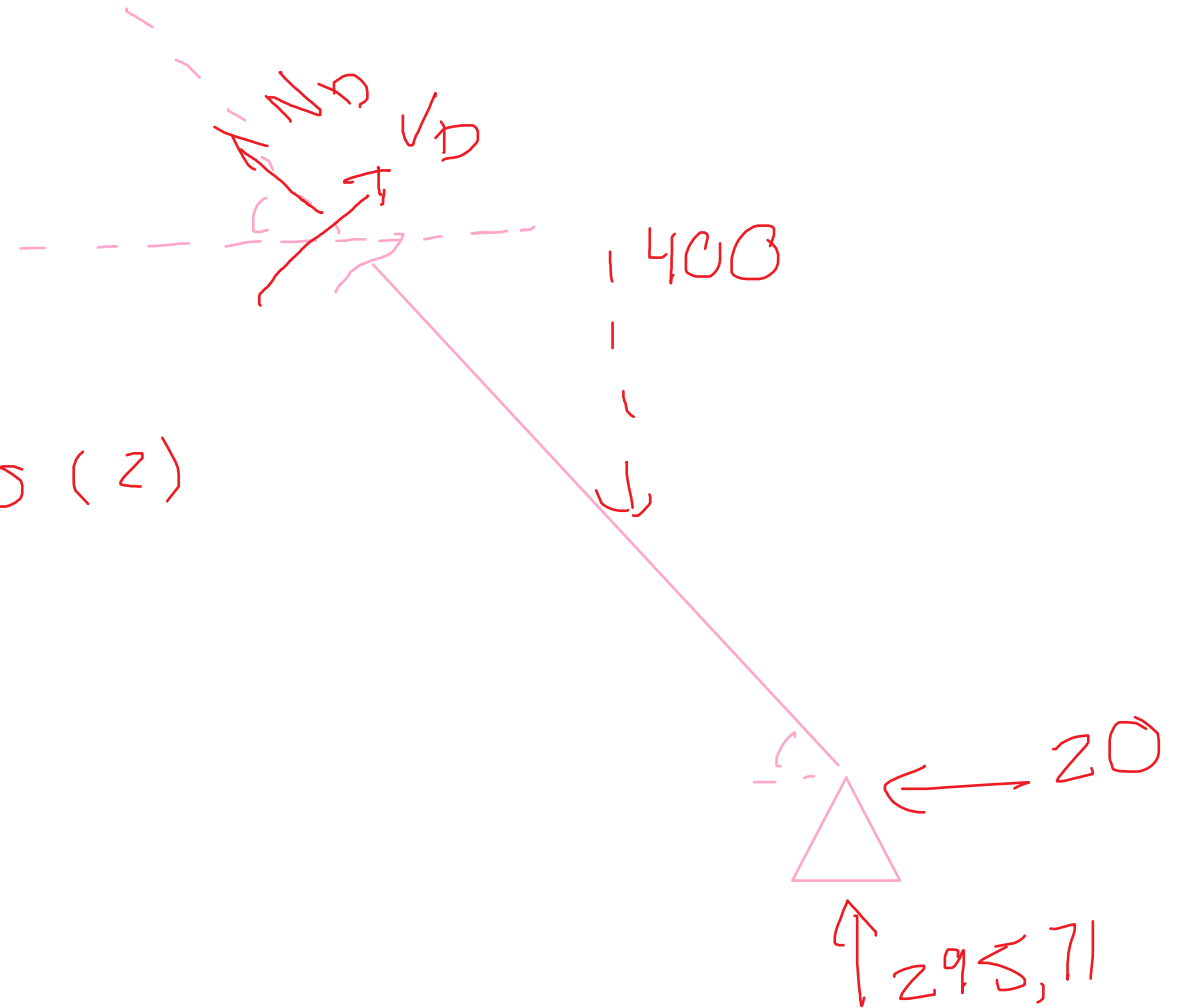
$$\sum F_x = 0$$

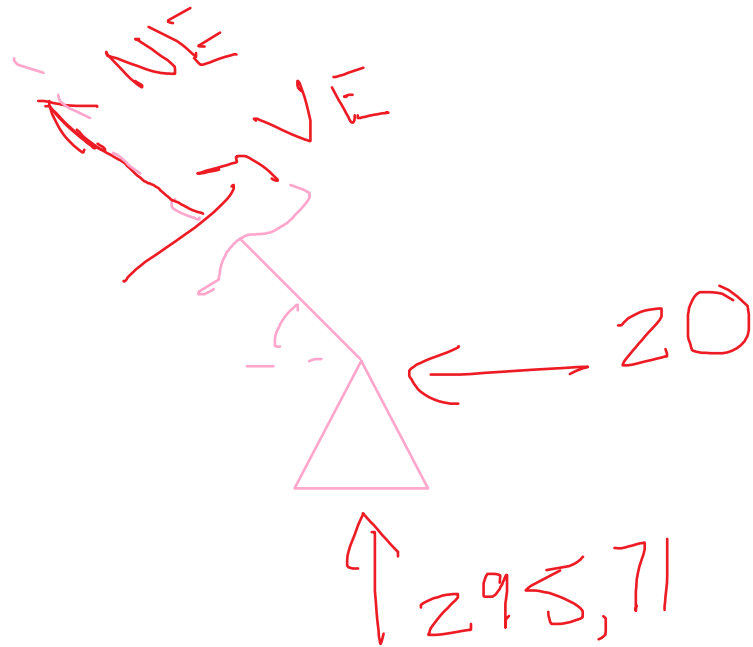
$$N_D \cos \alpha + 20 - V_D \sin \alpha = 0 \quad (1)$$

$$\sum F_y = 0$$

$$N_D \sin \alpha + V_D \cos \alpha + 295,71 - 400 = 0 \quad (2)$$

$$N_D = 71,43 ; V_D = 70,57$$



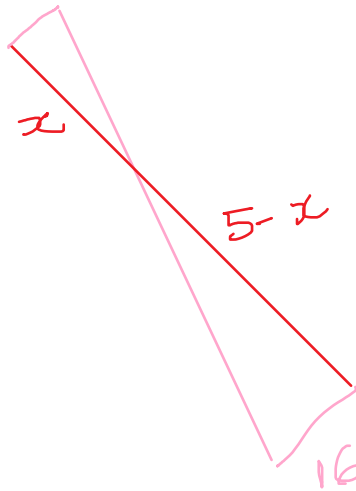


$$N_E = -248,57$$

$$V_E = -161,43$$

$$M_E = 0$$

78,57



$$\frac{78,57}{x} = \frac{161,43}{5-x}$$

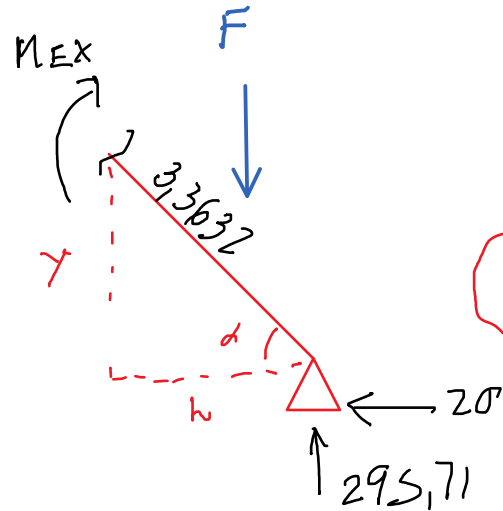
$$x = 1,6368 \text{ m} \quad (5-x = 3,3632 \text{ m})$$

$$F = 80 \cdot 3,3632$$

$$F = 269,06 \text{ kN}$$

$$\sin \alpha = 0,8 \rightarrow y = 2,6906 \text{ m}$$

$$\cos \alpha = 0,6 \rightarrow h = 2,0179 \text{ m}$$



$$\sum M_S = 0$$

$$M_{EX} + F \cdot \frac{h}{2} + 20 \cdot y = 295,71 \cdot h$$

$$M_{EX} = 271,44 \text{ kNm}$$

$$\frac{dM}{dx} = V(x) \Rightarrow \int_{x_1}^{x_2} \frac{dM}{dx} dx = \int_{x_1}^{x_2} V(x) dx$$

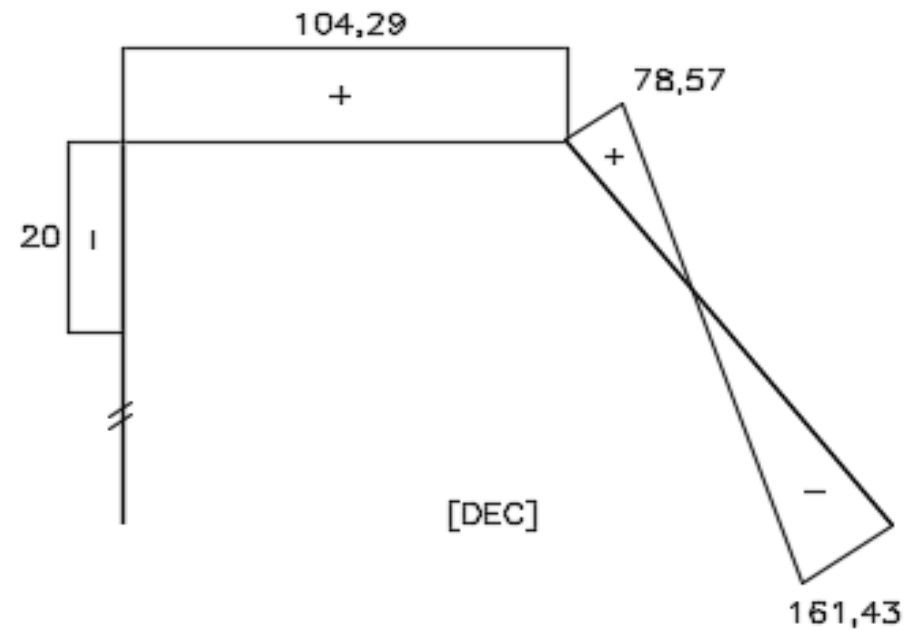
$$M_{x_2} = \int_{x_1}^{x_2} V(x) dx + C$$

$$M_{x_2} = \text{Area}(V)_{x_1}^{x_2} + \sum_{i=1}^{\text{conc.}} M_i$$

* $\text{Area}(V)$: ALGÉBRICO (tem sinal!!)

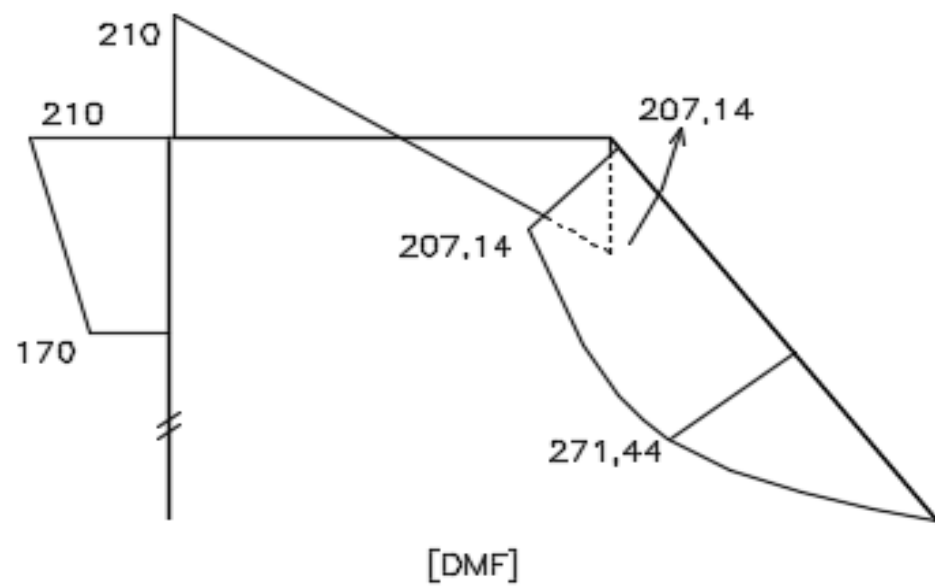
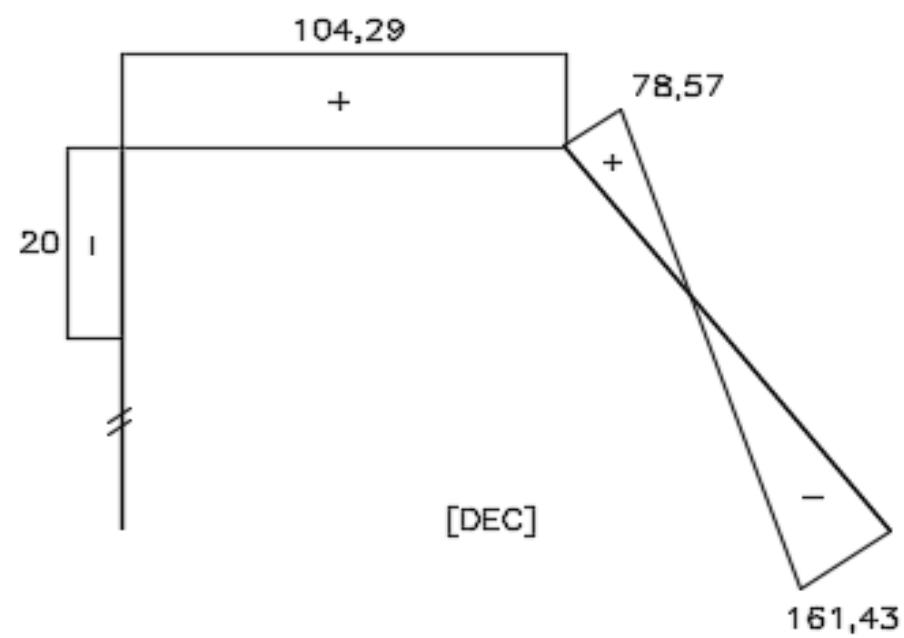
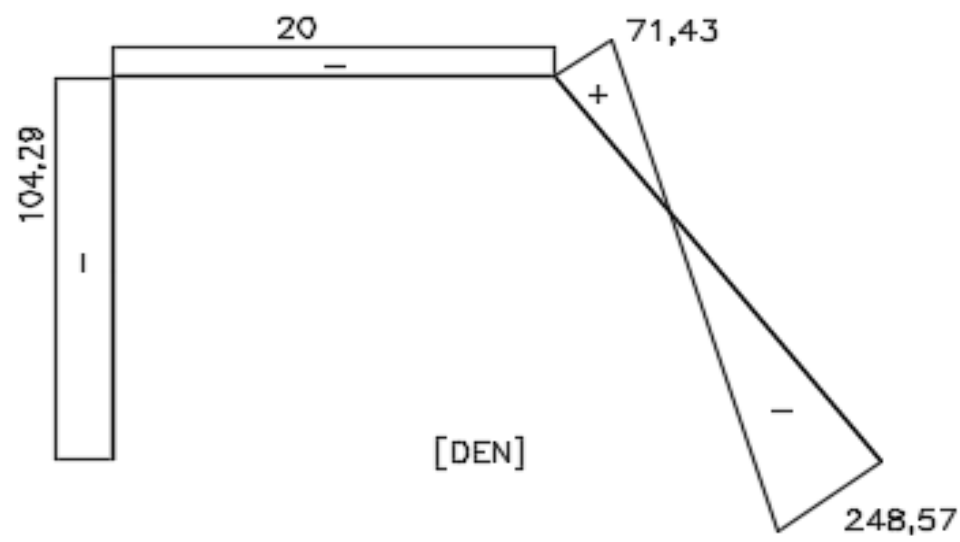
$$M_X = (-20 \cdot 2) + (104,29 \cdot 4) + \frac{78,57}{2} \cdot -170$$

$$M_X = 271,44 \text{ kNm}$$

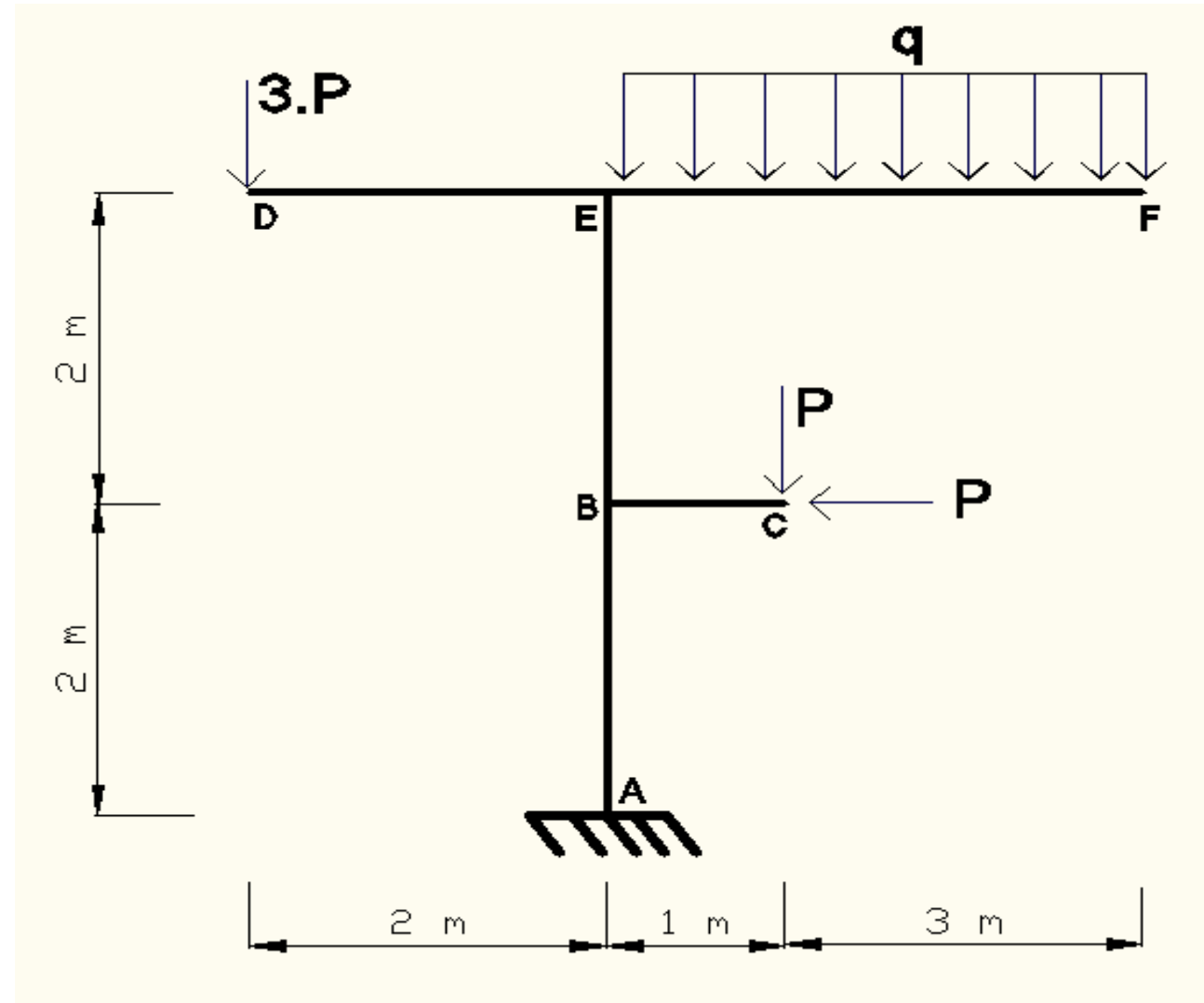


$$M_X = - \left[- \frac{161,43}{2} \cdot (5-x) \right] = 271,44$$

(-) : integrando da direita p/ esquerda)



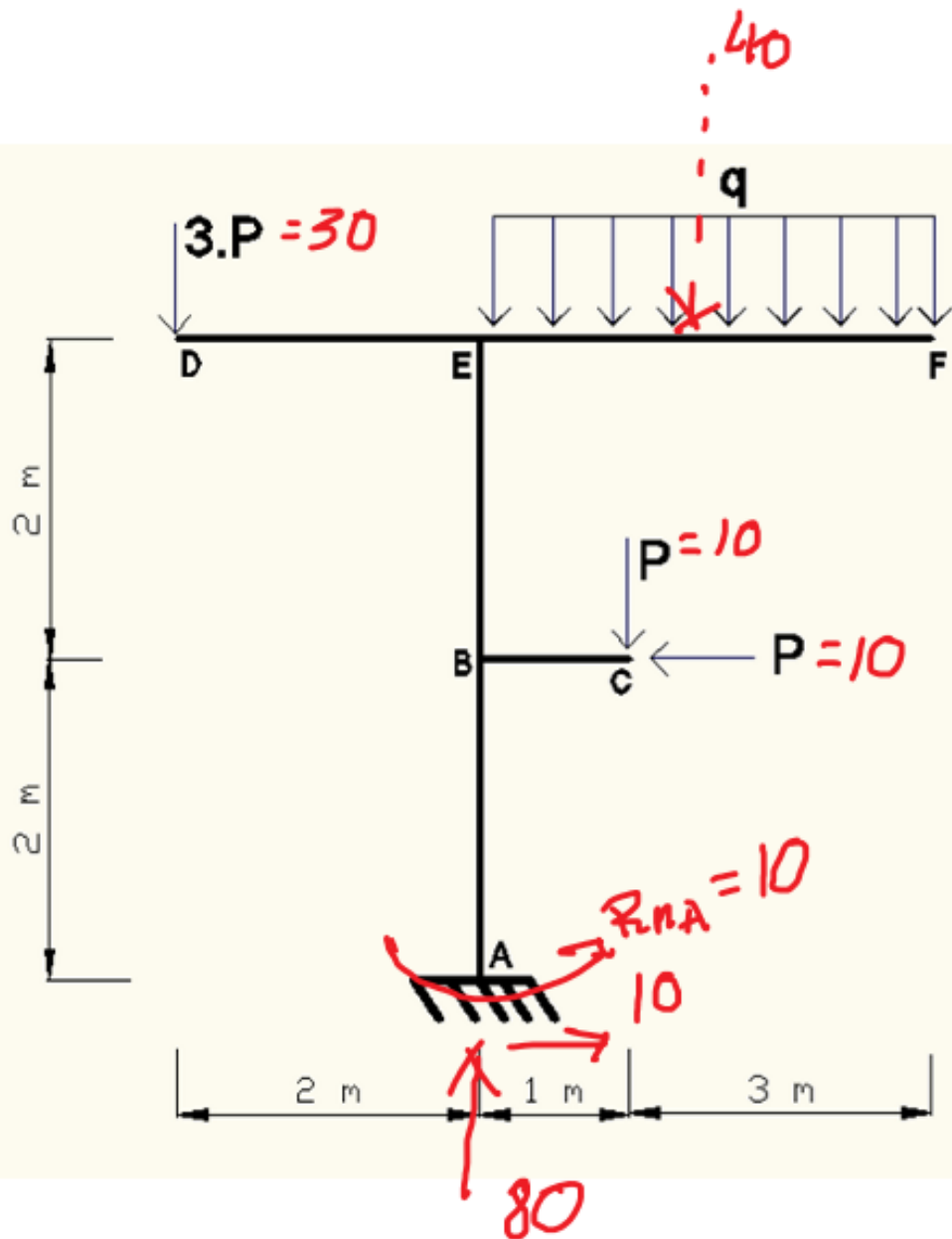
Exemplo 20: Para a estrutura plana a seguir, obtenha os esforços solicitantes para os trechos ABE e DEF. Sabe-se que $P = 10 \text{ kN}$ e $q = 10 \text{ kN/m}$. Indicar os diagramas nos desenhos em destaque.



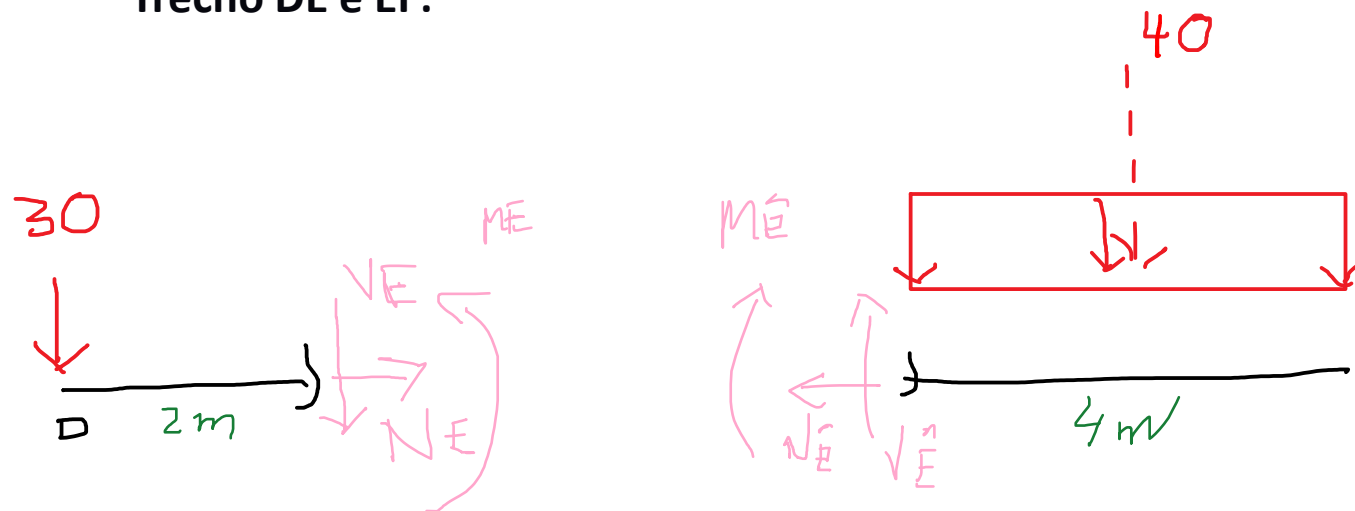
Reações:

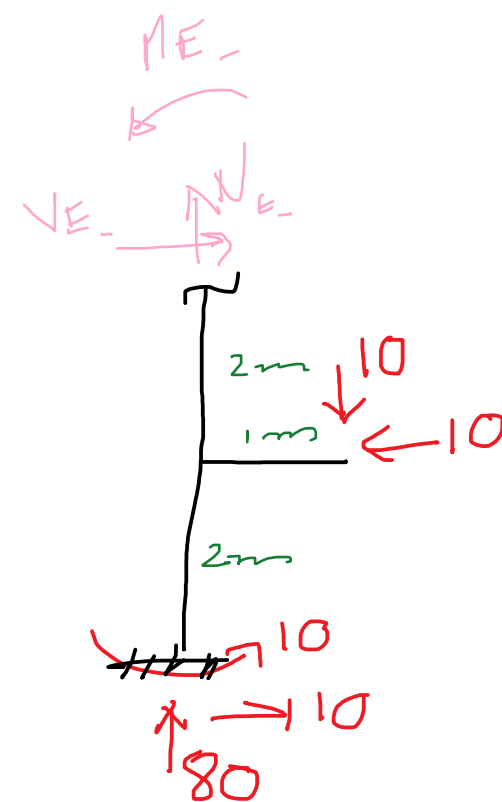
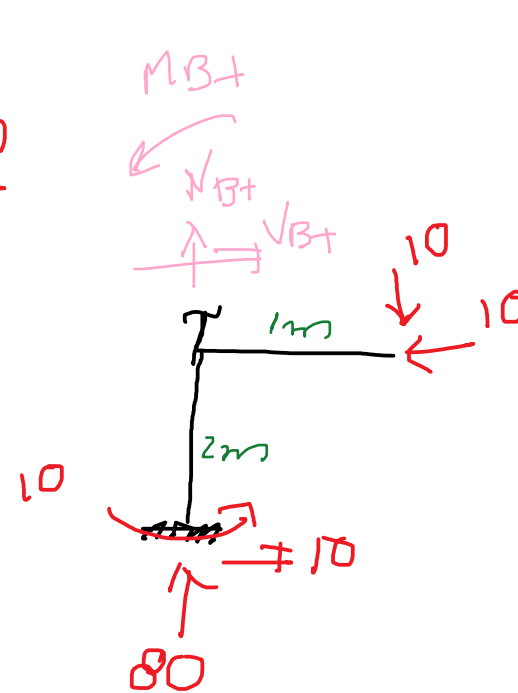
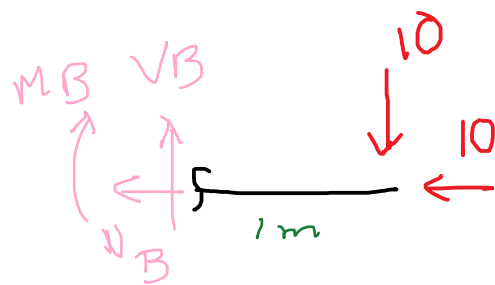
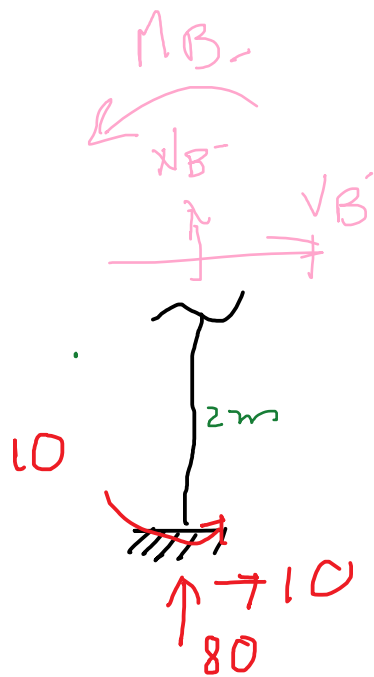
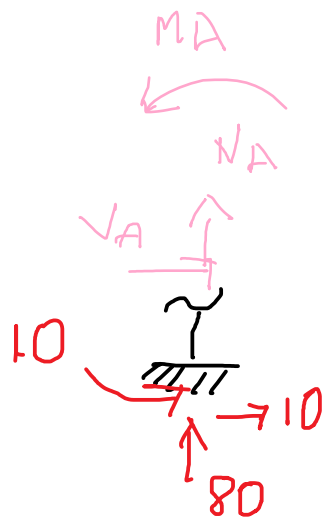
$$\sum M_A = 0 : R_{MA} + 10 \cdot 2 + 30 \cdot 2 = 40 \cdot 2 + 10 \cdot 1$$

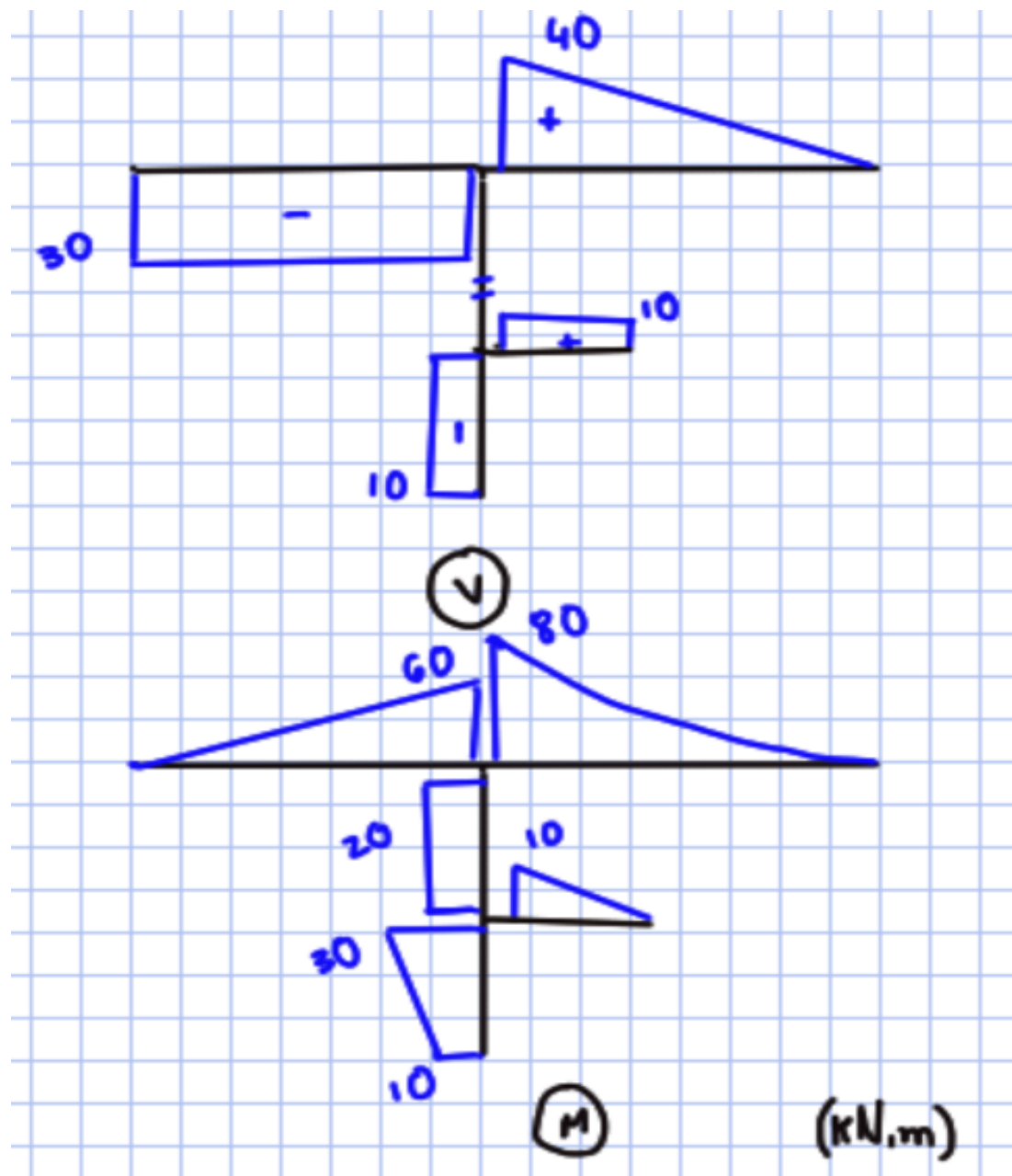
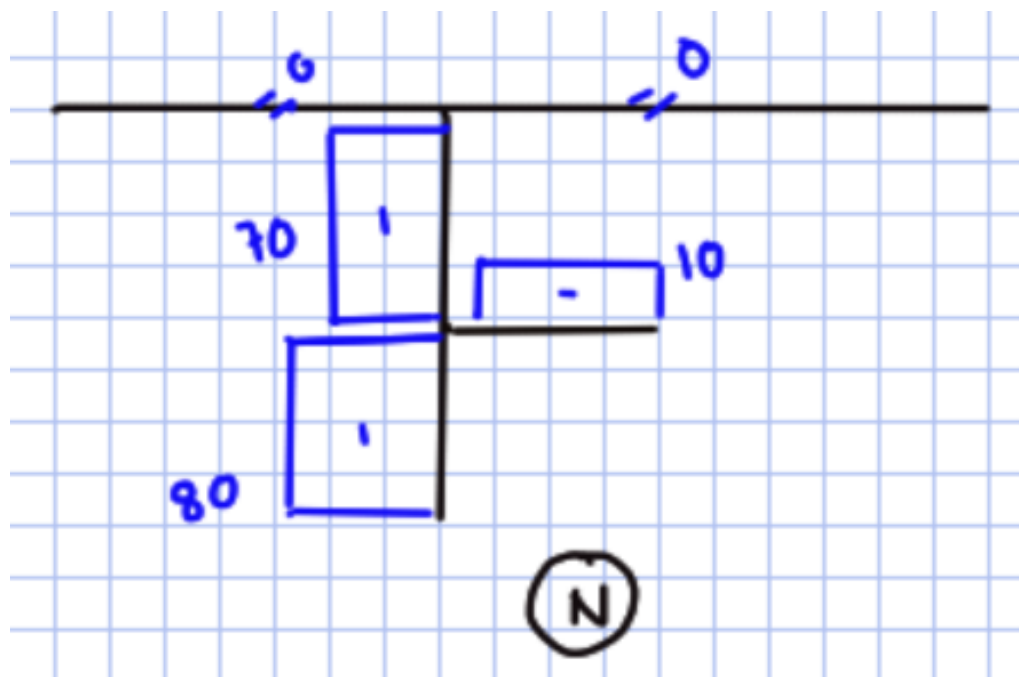
$$R_{MA} = 10 \text{ kNm}$$



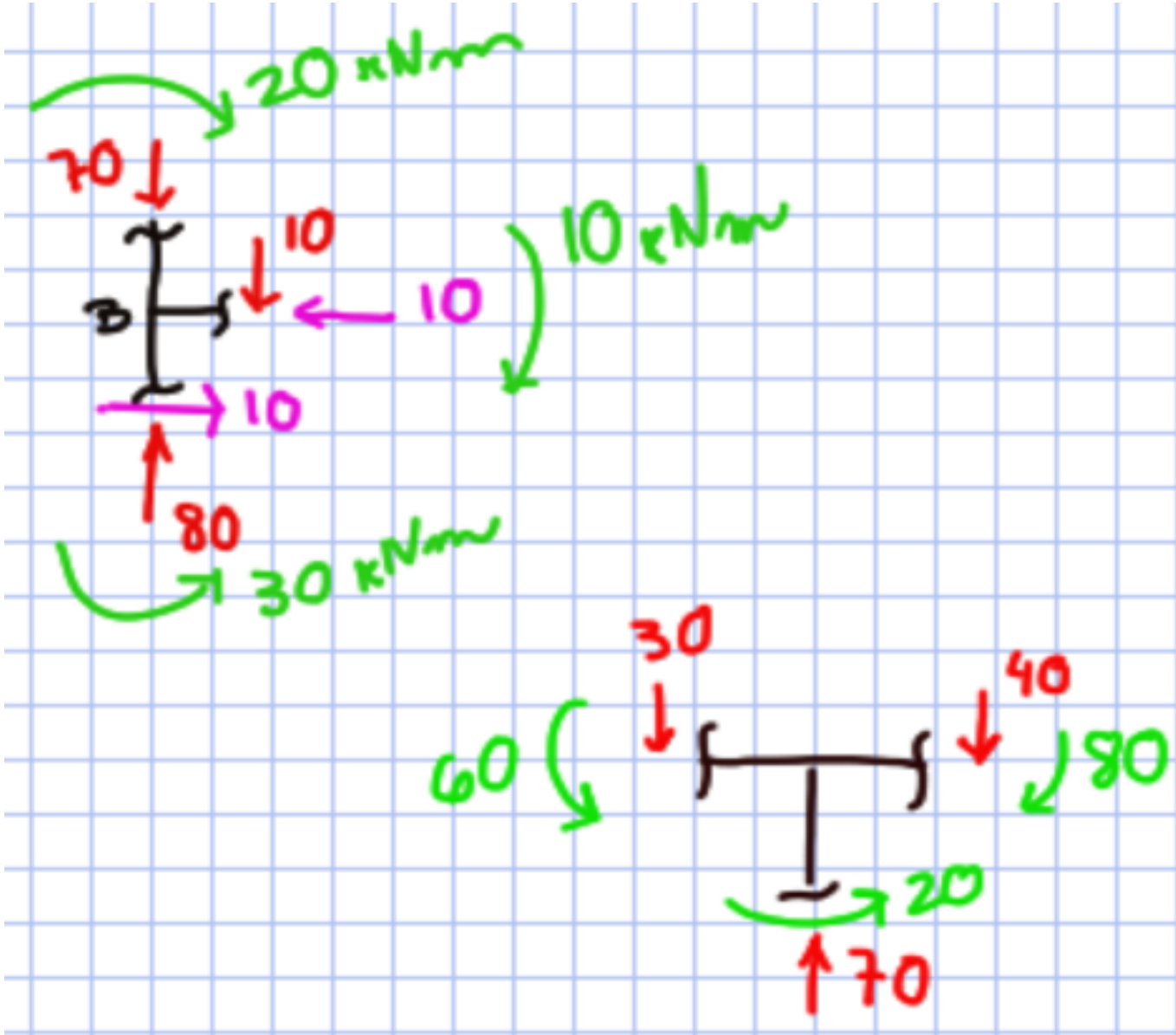
Trecho DE e EF:



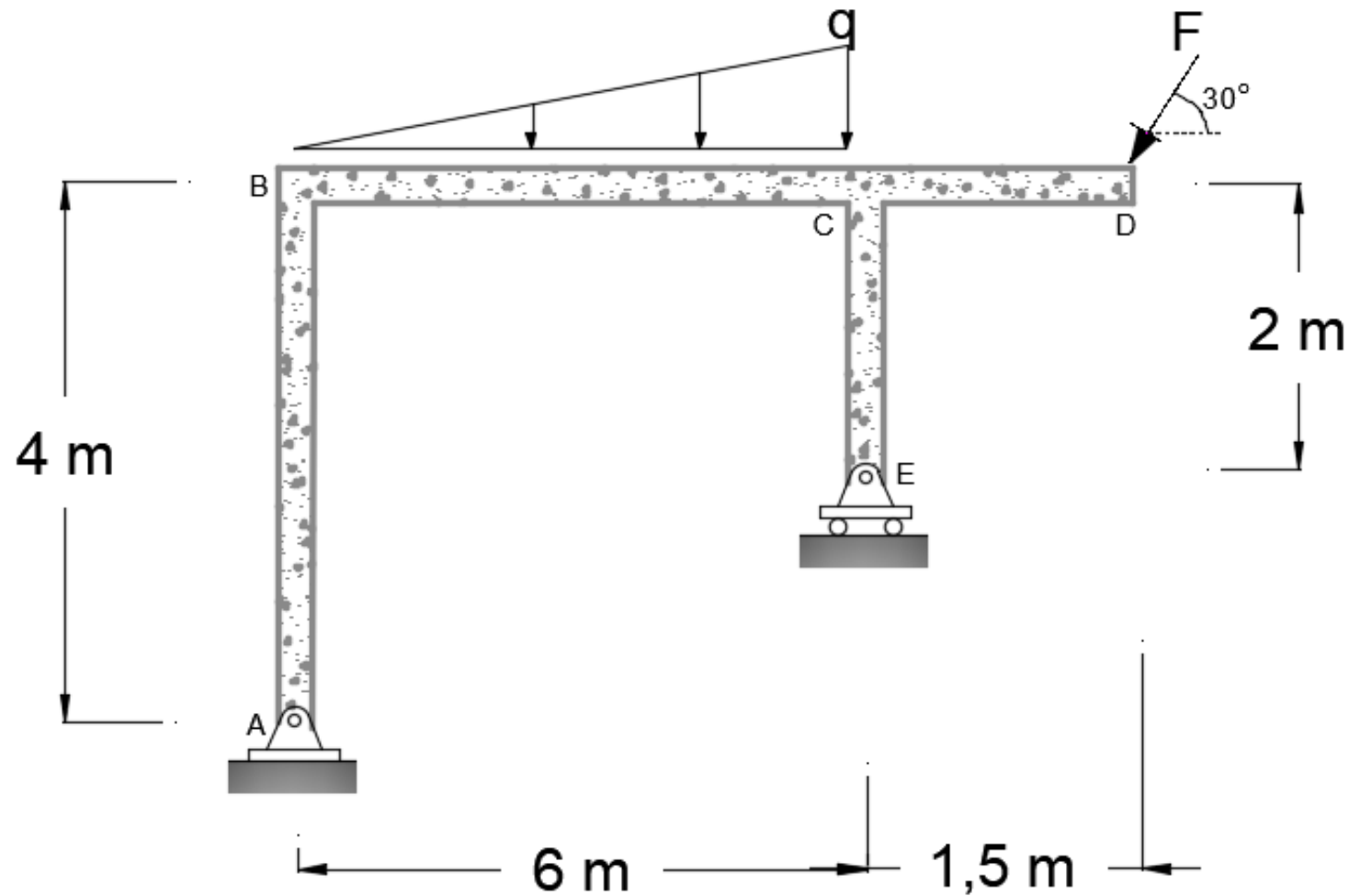




Verificação de equilíbrio nos pontos B e E.

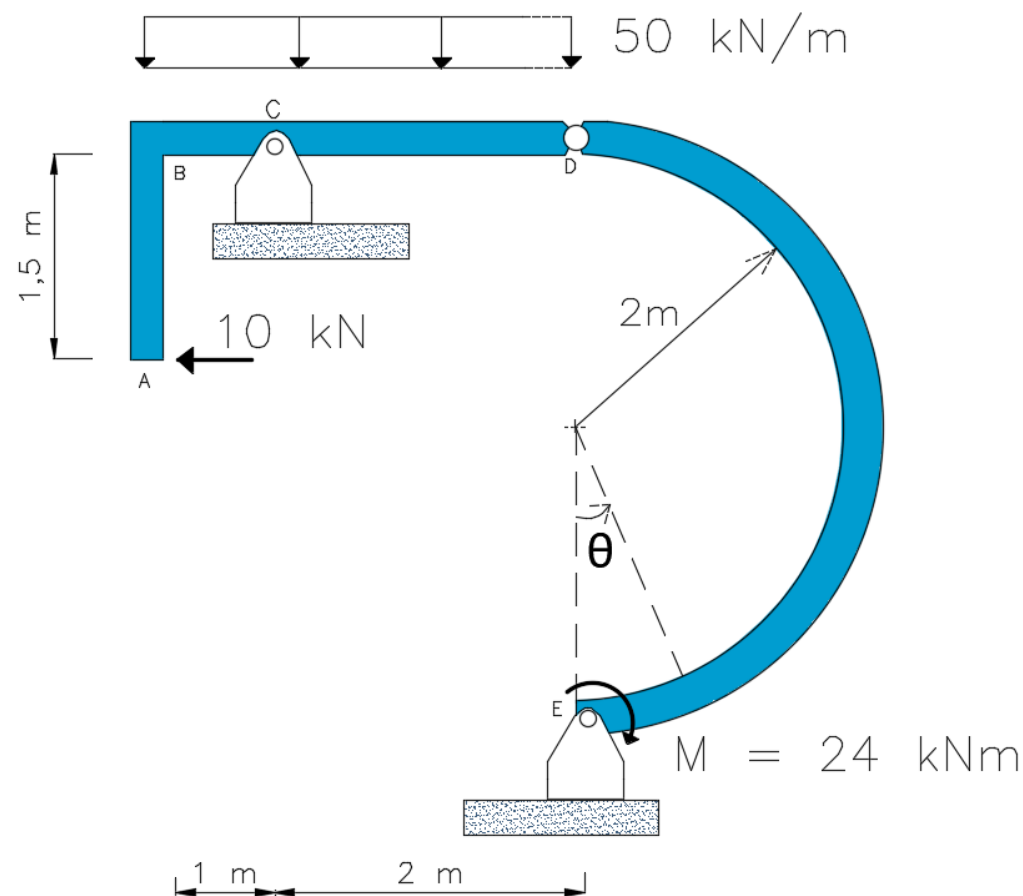


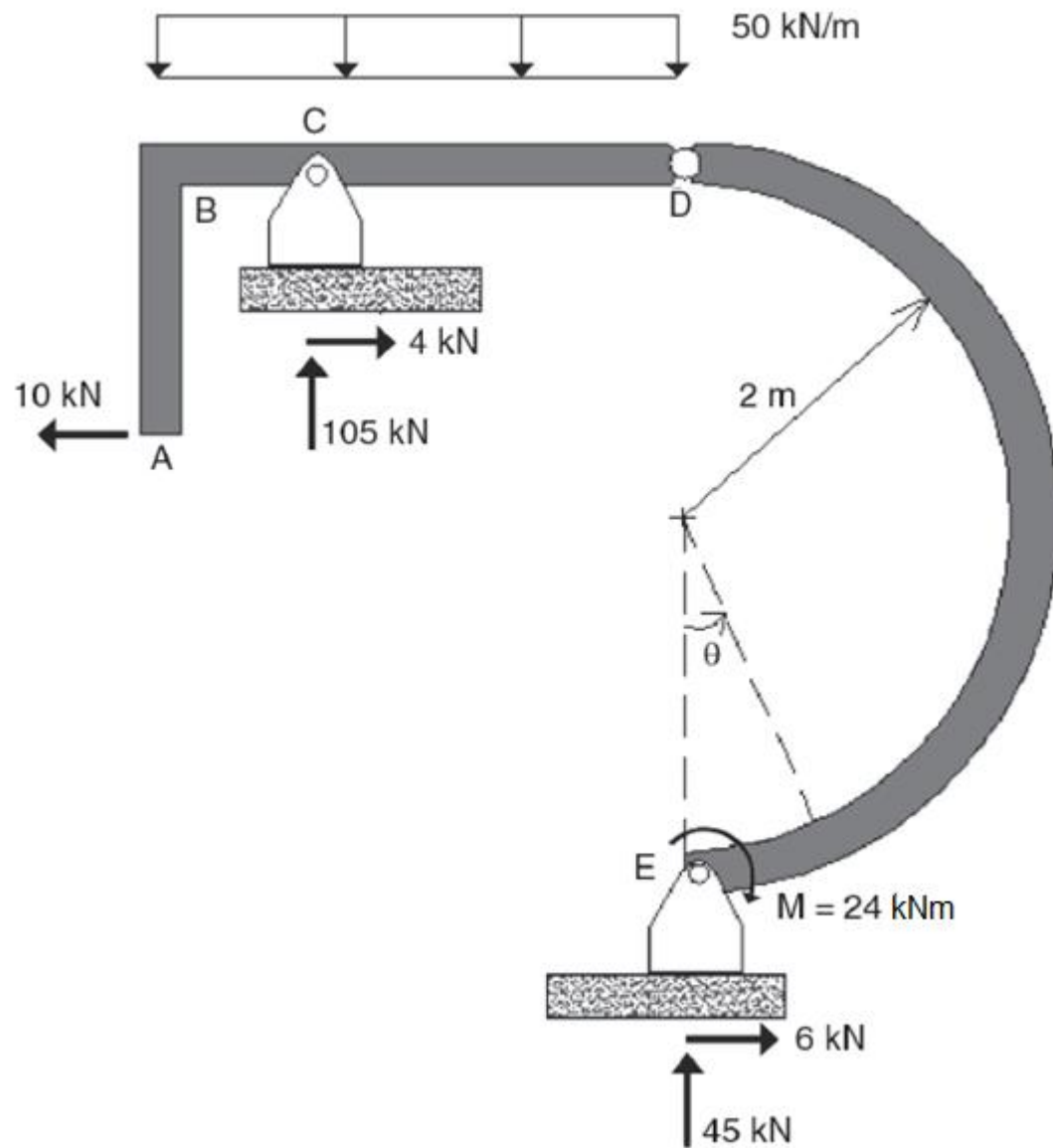
Para a estrutura a seguir, sabendo que $F = 10 \text{ kN}$ e $q = 10 \text{ kN/m}$, obtenha os diagramas de esforços solicitantes em todos os trechos, indicando os valores e posições dos extremos dos esforços em cada trecho.



****** A figura a seguir representa uma estrutura articulada (rotulada) em D e apoios fixos em C e E. Sob os carregamentos indicados, carga distribuída constante no trecho BD, força concentrada em A (10 kN) e momento concentrado ($M = 24$ kNm) em E, obtenha:

- a) Diagramas dos esforços solicitantes para os trechos ABCD;
- b) Para o trecho circular DE, obtenha o valor de momento fletor em kNm na seção em $\theta = 30^\circ$.
Explicite todas as passagens dos cálculos empregados na resolução, para melhor avaliação.



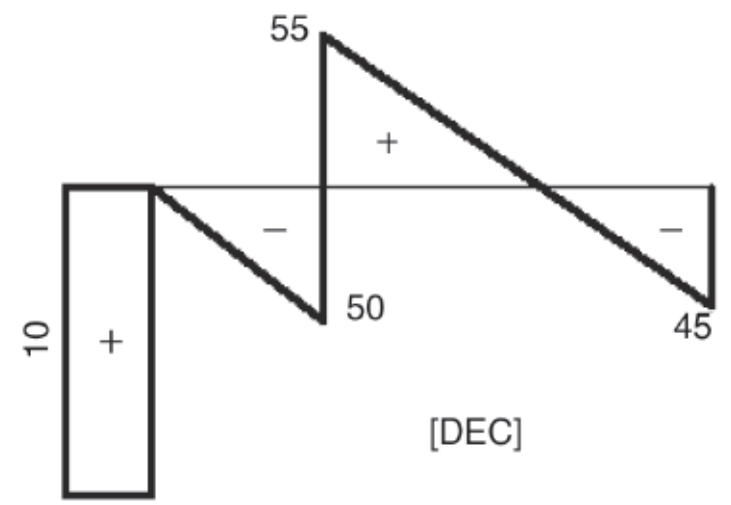
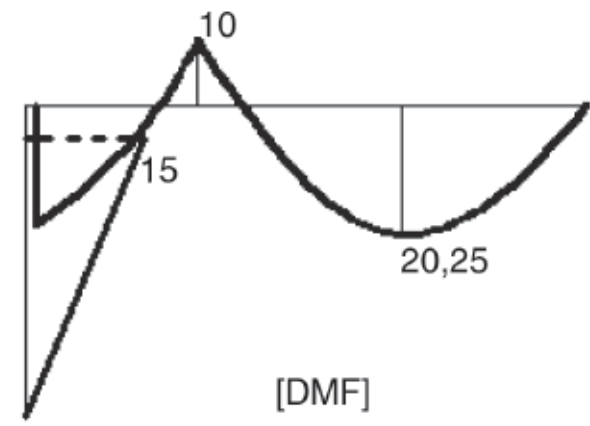
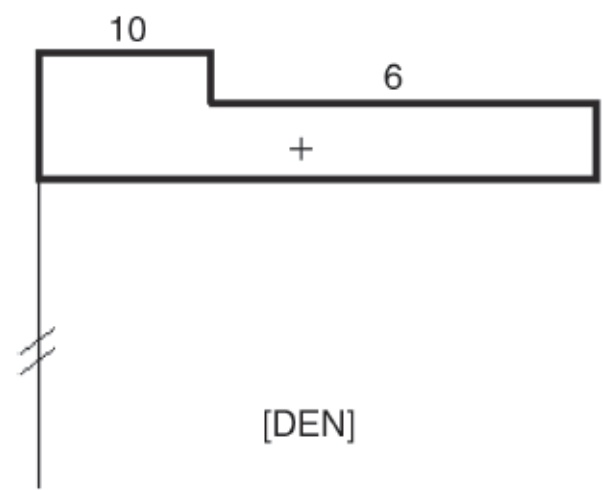
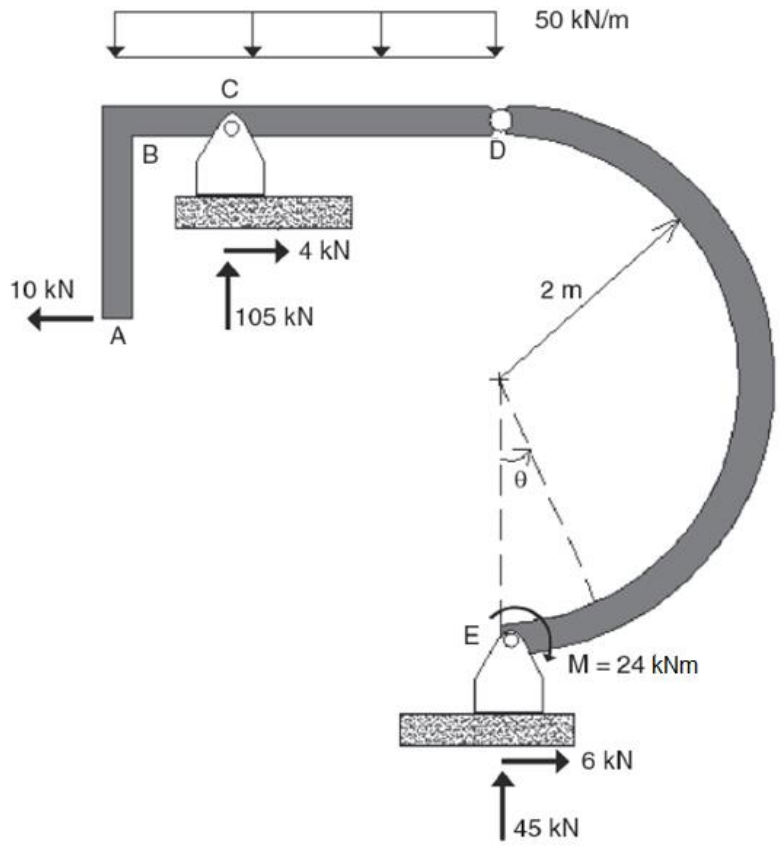


$$\sum M_D = 0 : \rightarrow 4 \cdot E_x = 24 \rightarrow E_x = 6 \text{ kN } (\rightarrow)$$

$$\sum F_x = 0 : \rightarrow E_x + C_x = 10 \rightarrow C_x = 4 \text{ kN } (\rightarrow)$$

$$\sum M_E = 0 : \rightarrow 2 \cdot C_y + 4 \cdot C_x + 24 = 50 \cdot 3 \cdot 1,5 + 10 \cdot 2,5 \rightarrow C_y = 105 \text{ kN } (\uparrow)$$

$$\sum F_y = 0 : \rightarrow C_y + E_y = 50 \cdot 3 \rightarrow E_y = 45 \text{ kN } (\uparrow)$$



(kN, m)

Para o trecho CD são obtidas as equações em termos do ângulo θ , conforme a Figura 1.59D: $N(\theta) = -[6 \cdot \cos(\theta) + 45 \cdot \sin(\theta)]$; $V(\theta) = -6 \cdot \sin(\theta) + 45 \cdot \cos(\theta)$

$$M(\theta) = 12 + 90 \cdot \sin(\theta) + 12 \cdot \cos(\theta)$$

