



METMAT

# TERMODINÂMICA DAS SOLUÇÕES

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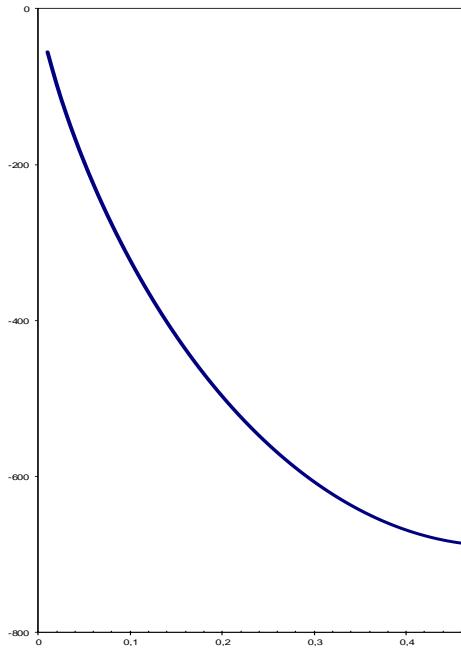


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## Solução Ideal

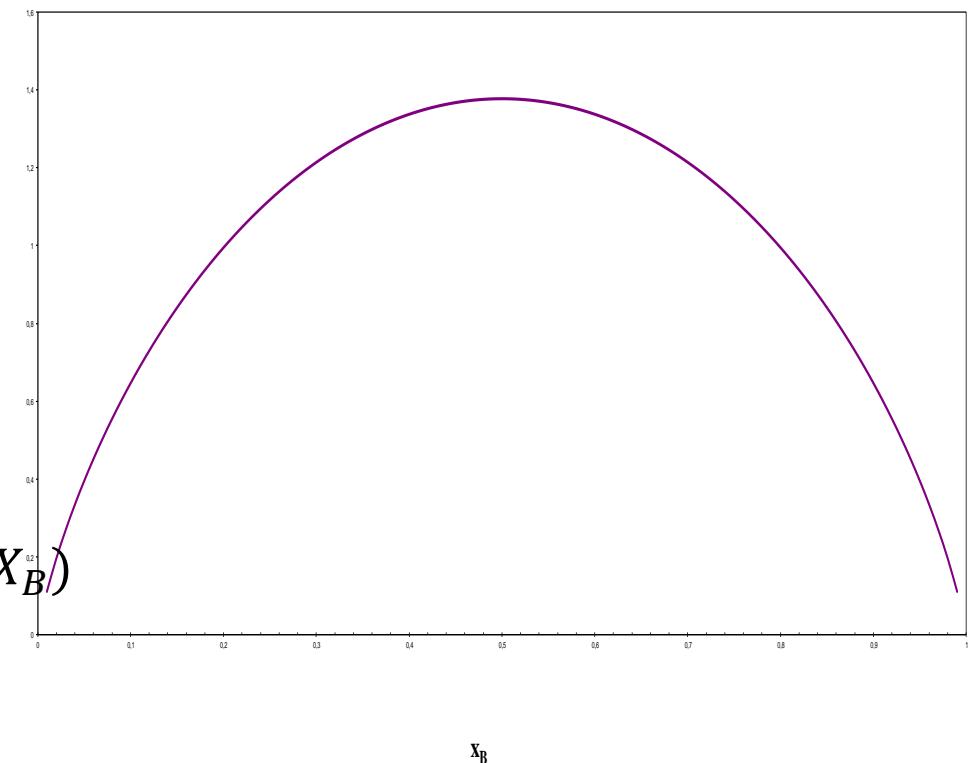
T=500K

$\Delta G^M$  (cal)



$$\Delta S^{M,ideal} = -R \cdot (X_A \cdot \ln X_A + X_B \cdot \ln X_B)$$

$\Delta S^M$  (cal)



$$\Delta G^{M,ideal} = R \cdot T \cdot (X_A \cdot \ln X_A + X_B \cdot \ln X_B)$$



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$$\Delta G^M = \Delta H^M - T\Delta S^M$$

Solução

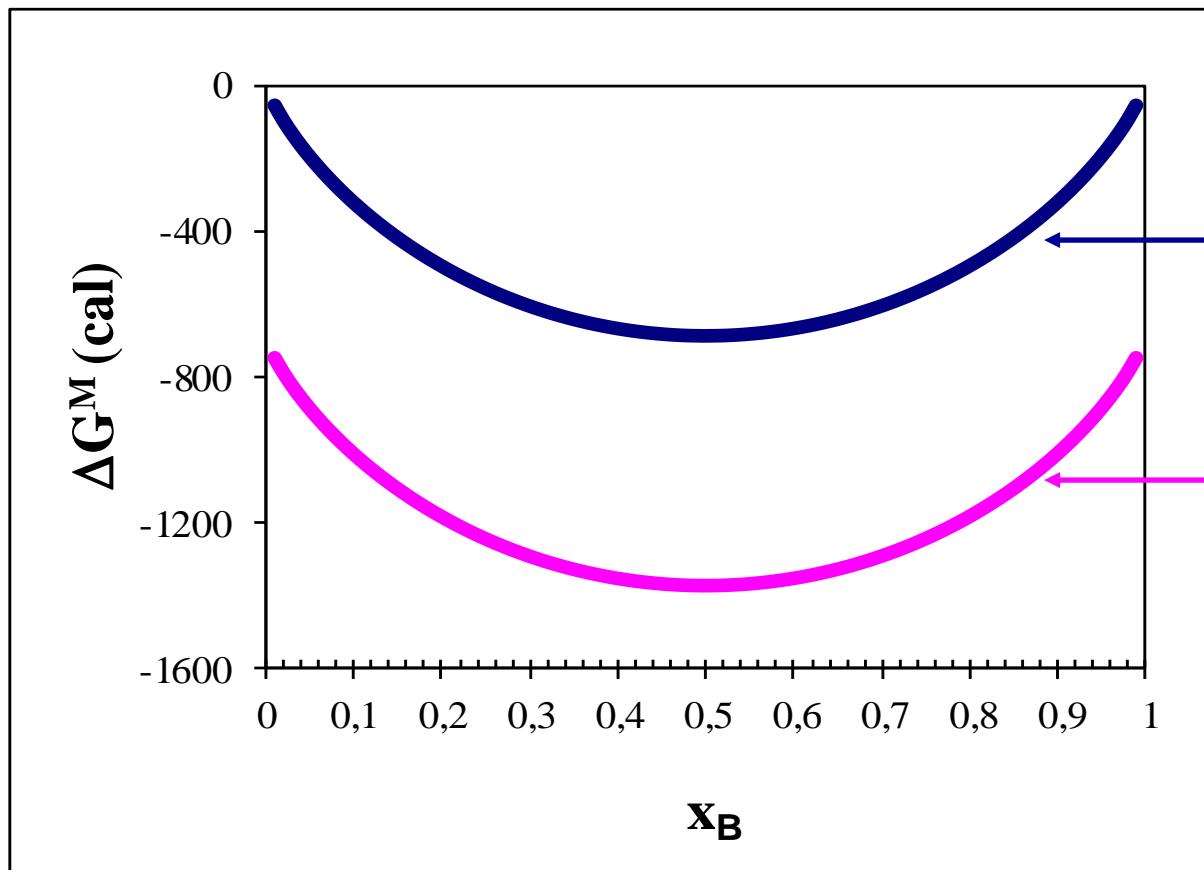
Non-Ideal: Desvio Negativo

$$\gamma=0,5$$

T=500K

Ideal

Desvio  
Negativo





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$$\Delta G^M = \Delta H^M - T\Delta S^M$$

Solução

Não-Ideal: Desvio Positivo

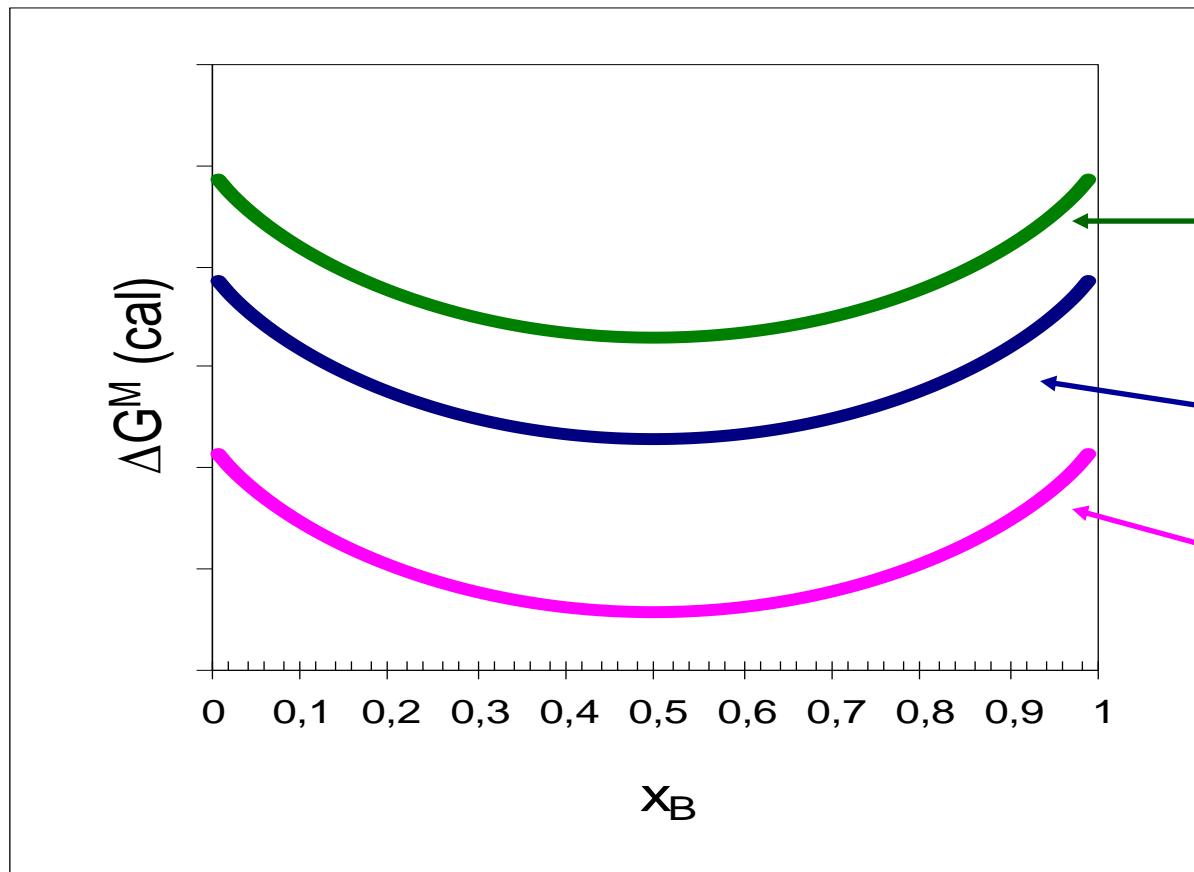
$$\gamma=1,5$$

T=500K

Desvio  
Positivo

Ideal

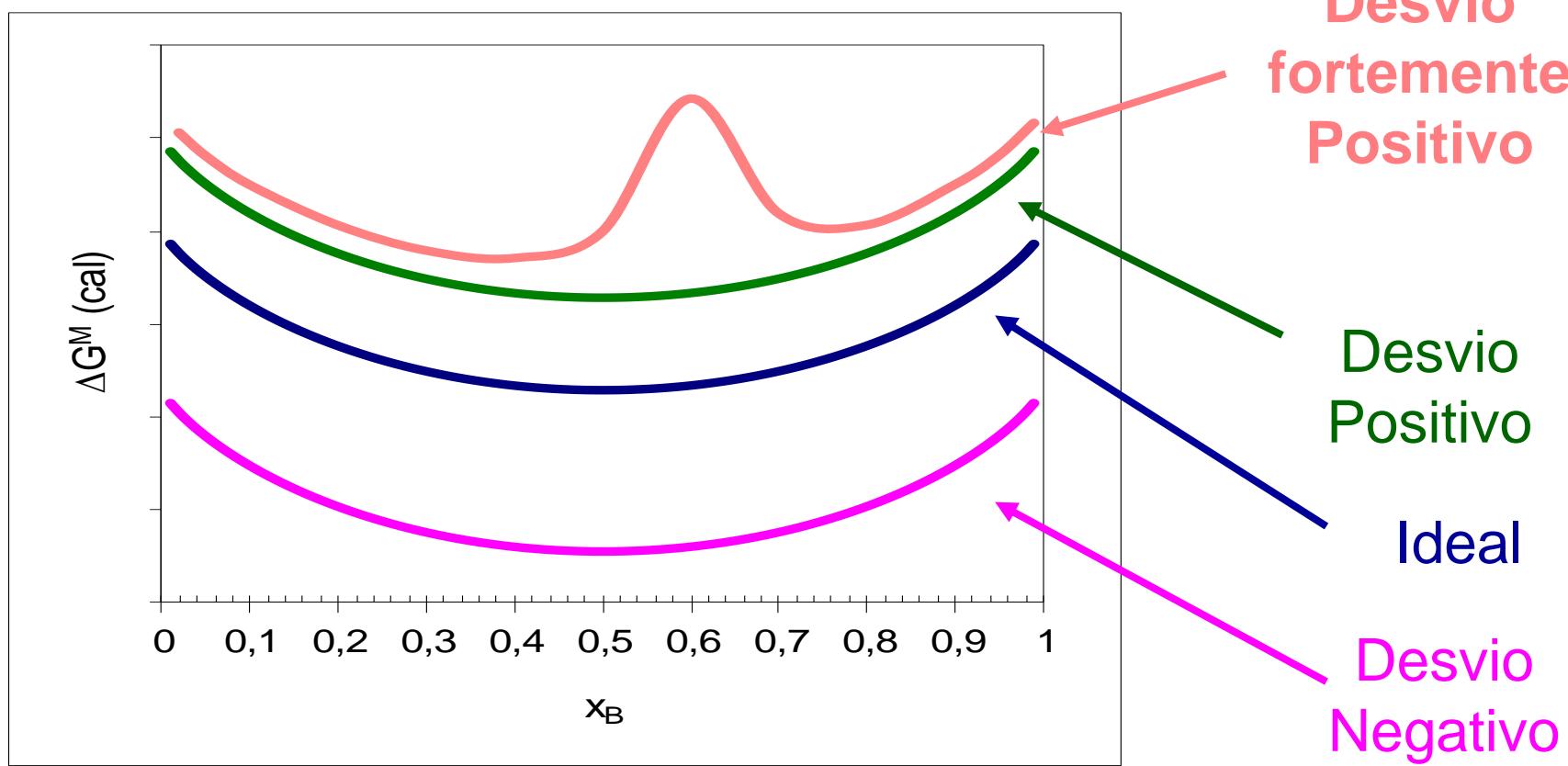
Desvio  
Negativo





$$\Delta G^M = \Delta H^M - T\Delta S^M$$

## Solução Não-Ideal: Desvio Positivo





O mesmo raciocínio pode ser feito para os valores de  $\mathbf{G}^{\text{M,fase}}$

Isto é, para a *energia livre de Gibbs das fases* presentes no sistema.

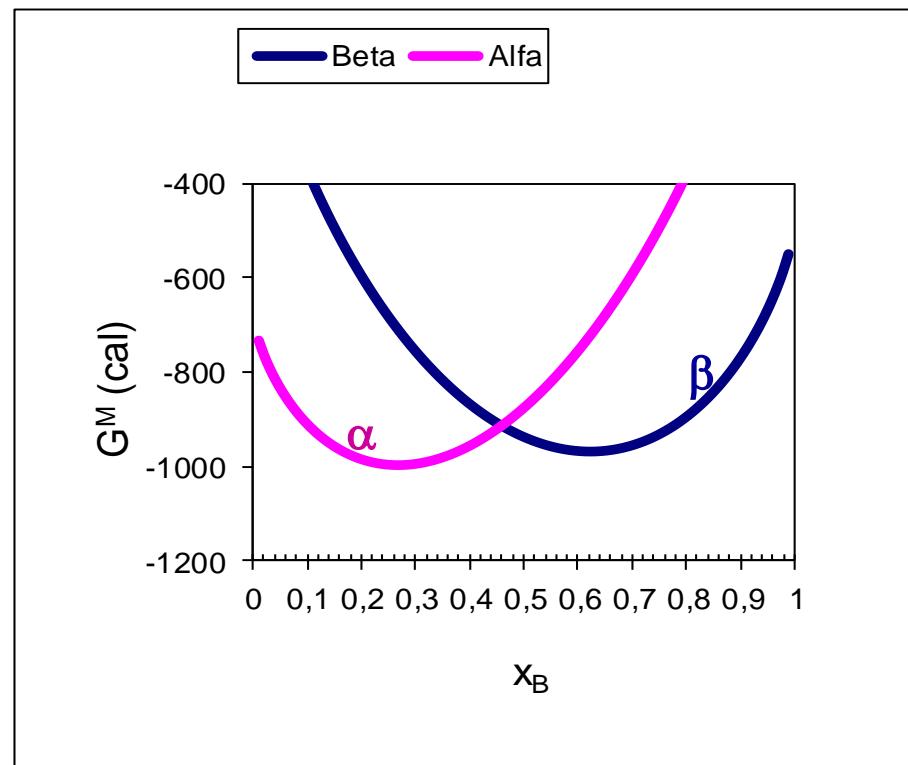
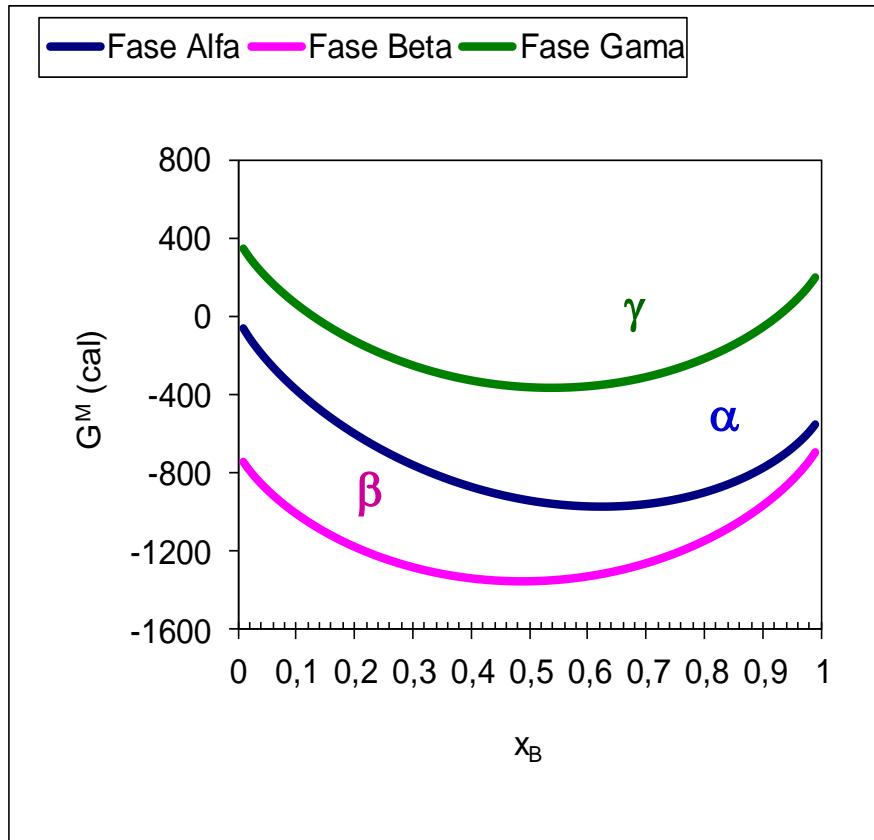
$$\text{IDEAL} - G^{\text{fase}} = X_A \cdot G_A^0 + X_B \cdot G_B^0 + R \cdot T \cdot (X_A \cdot \ln X_A + X_B \cdot \ln X_B)$$

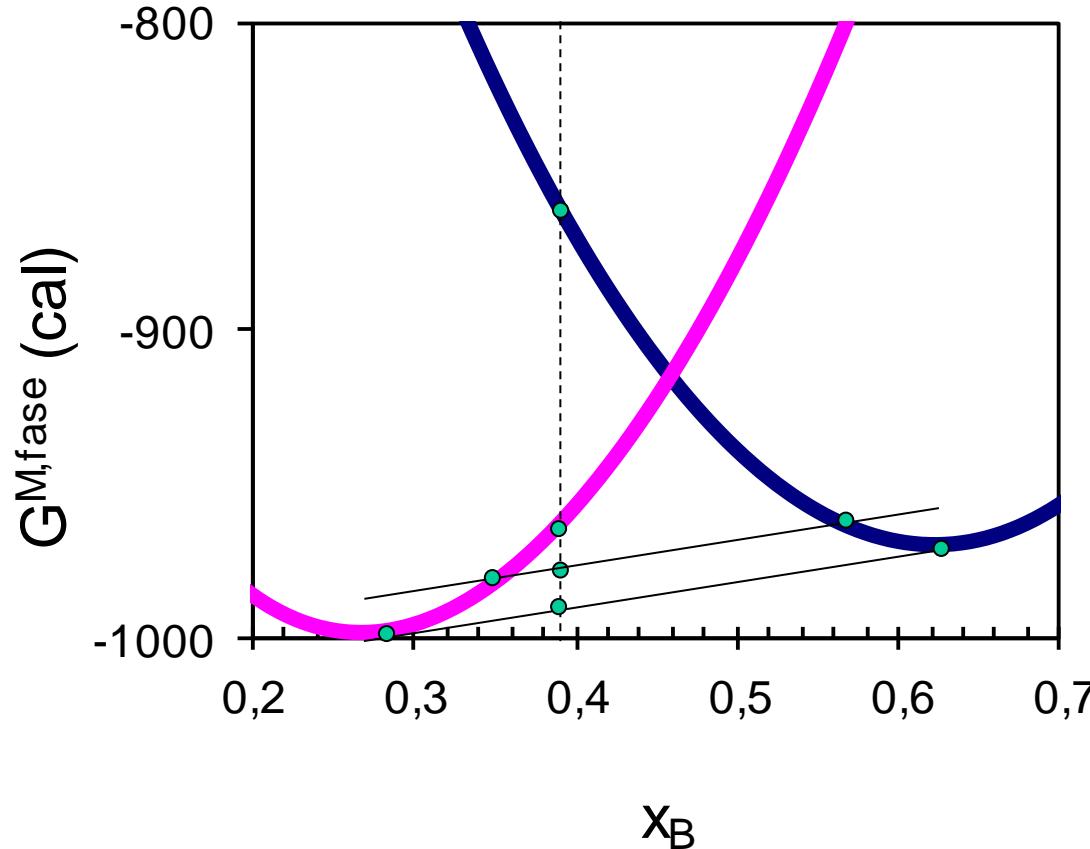
$$\text{REAL} - G^{\text{fase}} = X_A \cdot G_A^0 + X_B \cdot G_B^0 + R \cdot T \cdot (X_A \cdot \ln a_A + X_B \cdot \ln a_B)$$

$$\text{REGULAR} - G^{\text{fase}} = X_A \cdot G_A^0 + X_B \cdot G_B^0 + R \cdot T \cdot (X_A \cdot \ln X_A + X_B \cdot \ln X_B) + \Omega \cdot X_A \cdot X_B$$

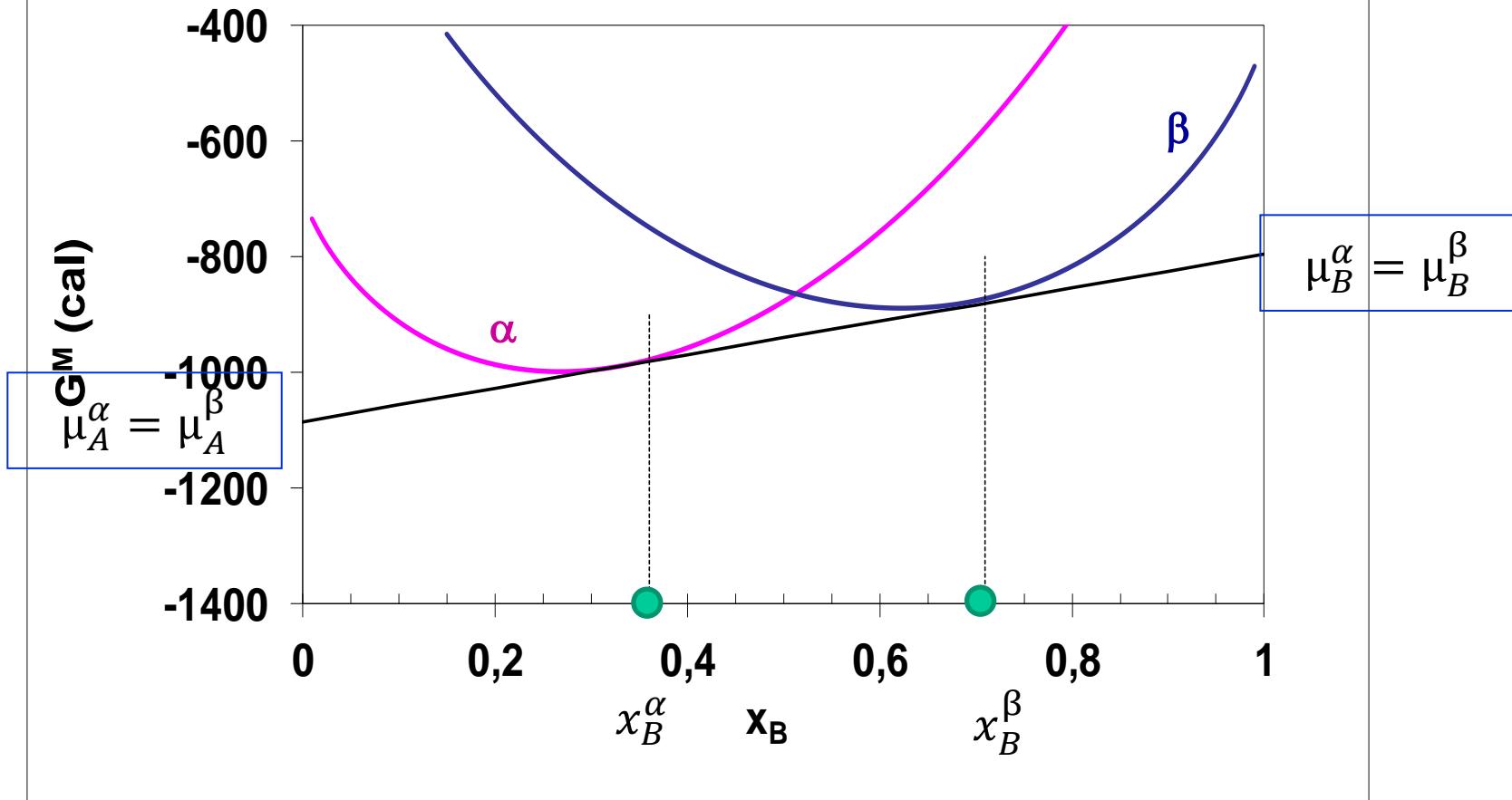


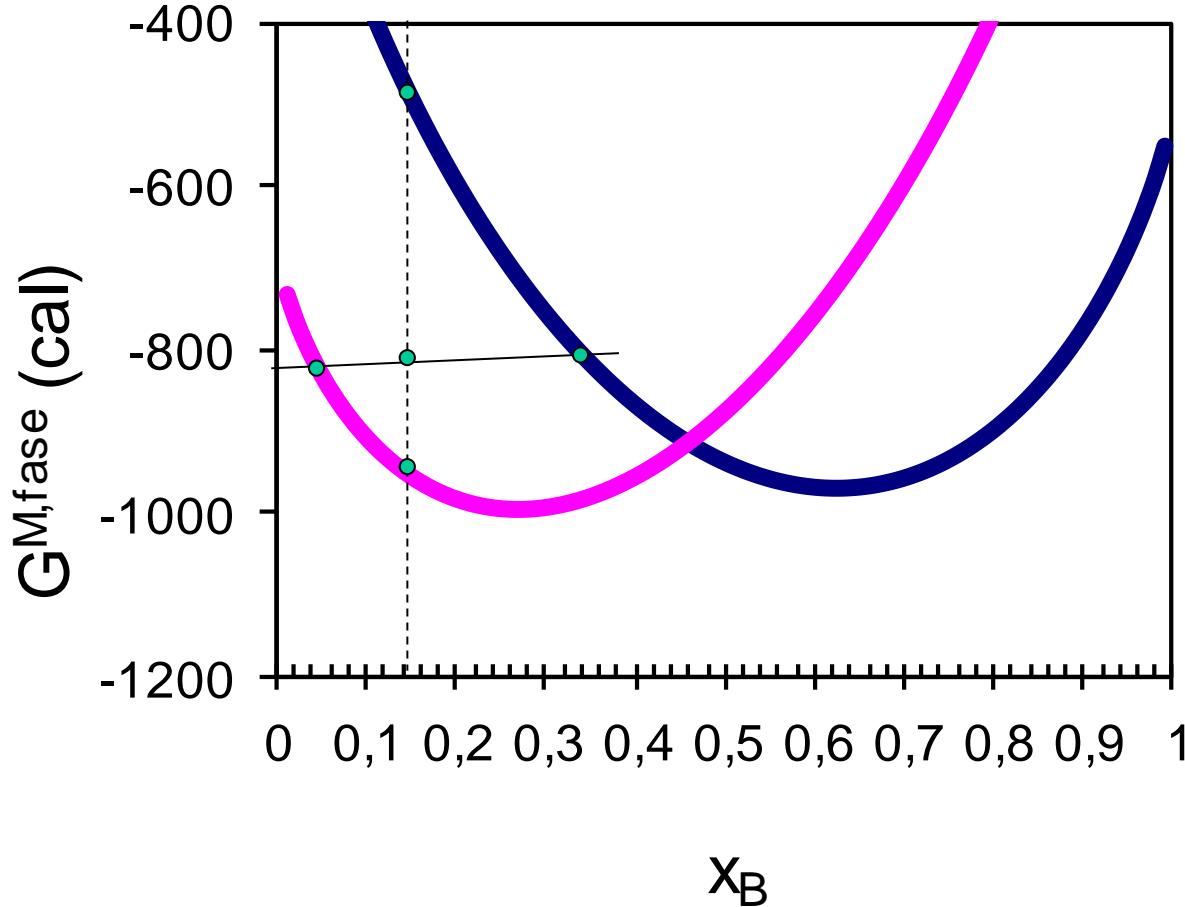
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1.  $\alpha$  seria mais estável
2. MM  $\alpha+\beta$  é mais estável
3. MM  $\alpha+\beta$  com os mesmos  $\mu$

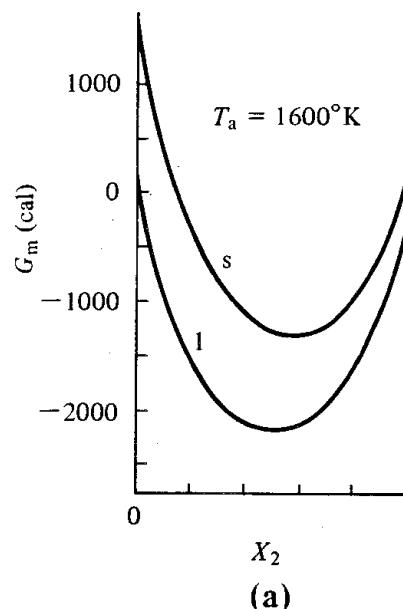




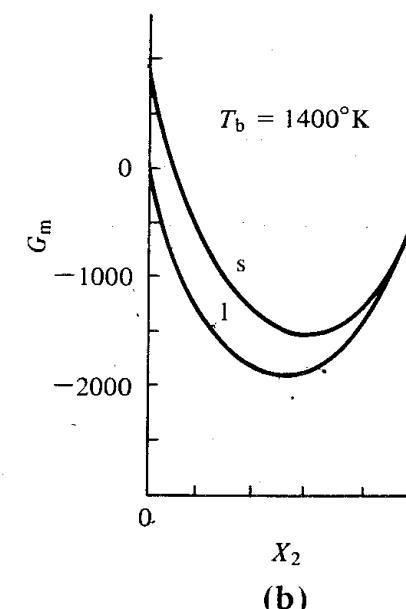
1.  $\alpha$  seria mais estável
2. MM  $\alpha+\beta$  é menos estável que  $\alpha$



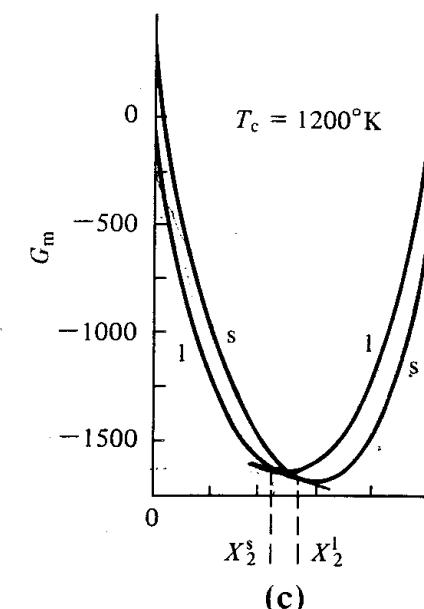
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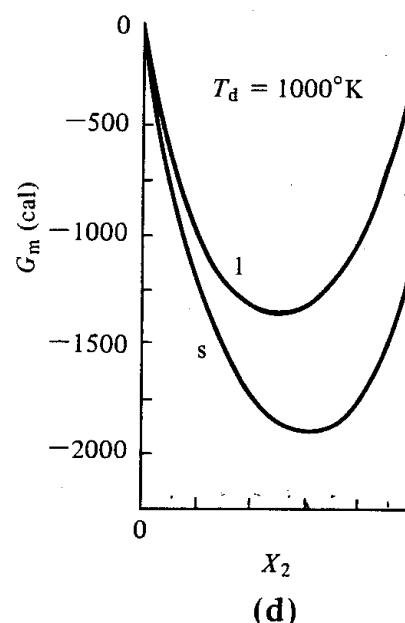
(a)



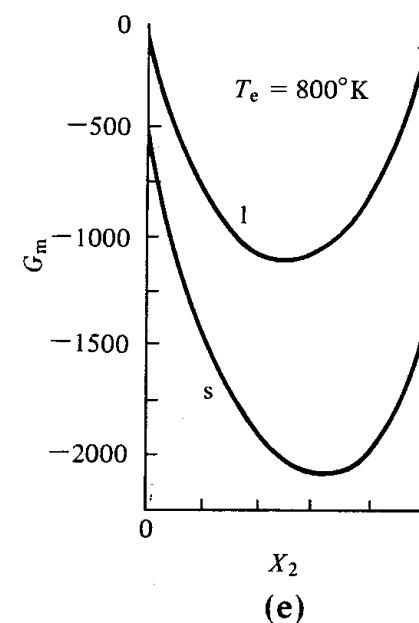
(b)



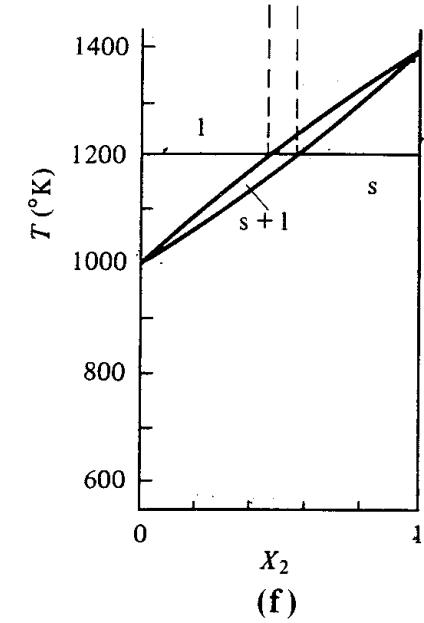
(c)



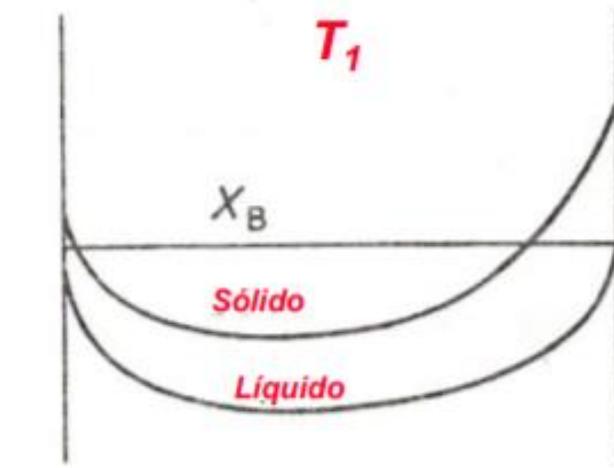
(d)



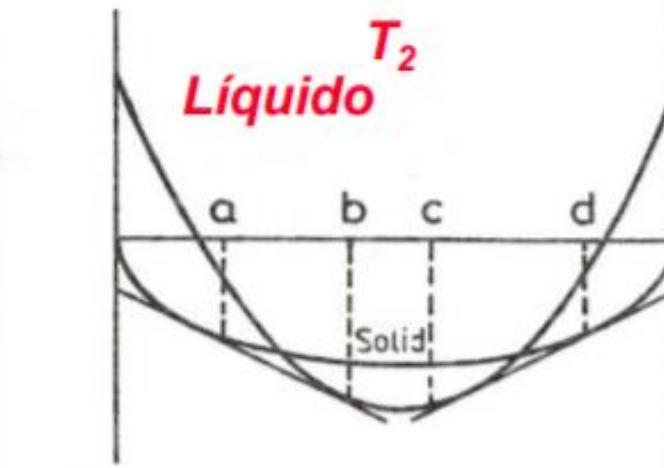
(e)



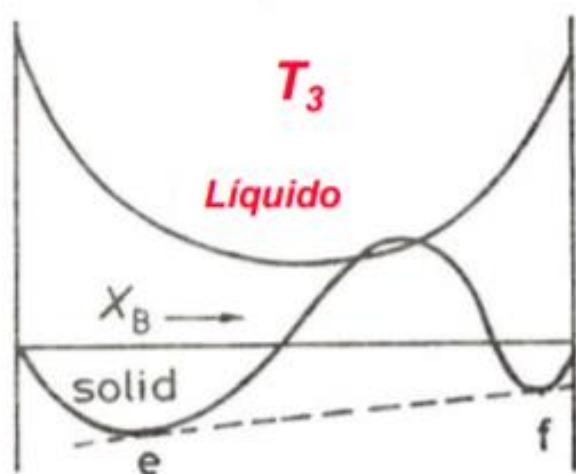
(f)



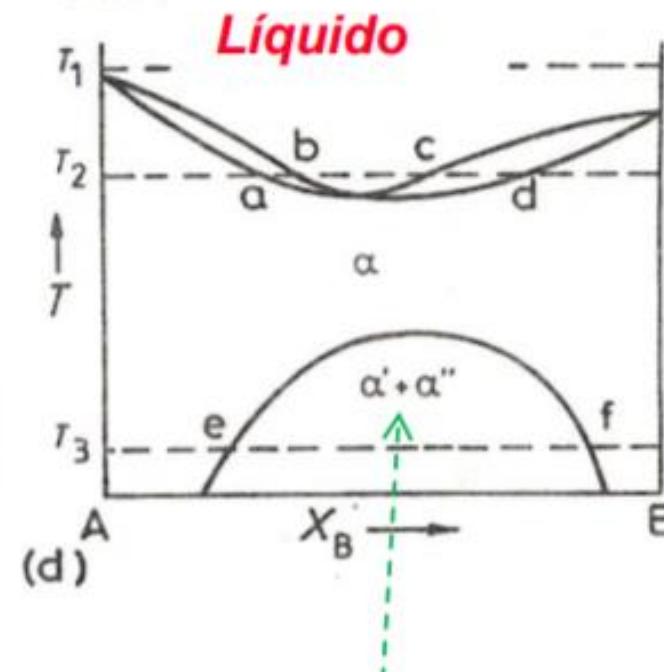
(a)

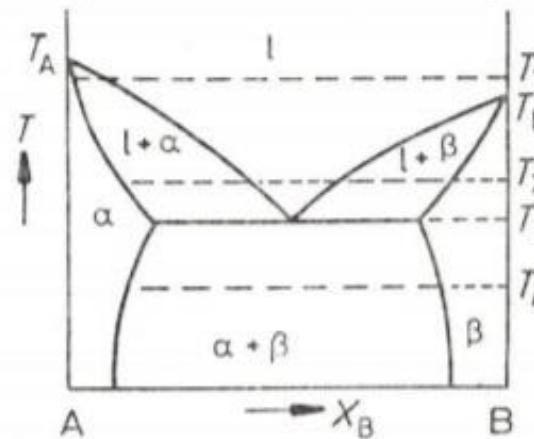
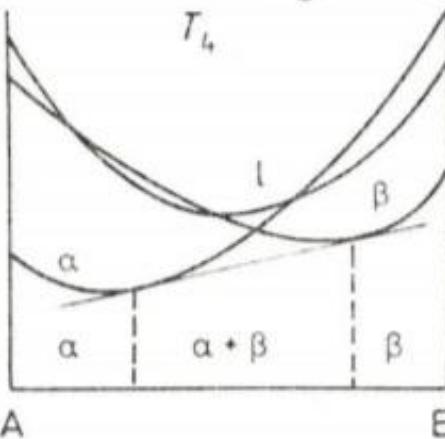
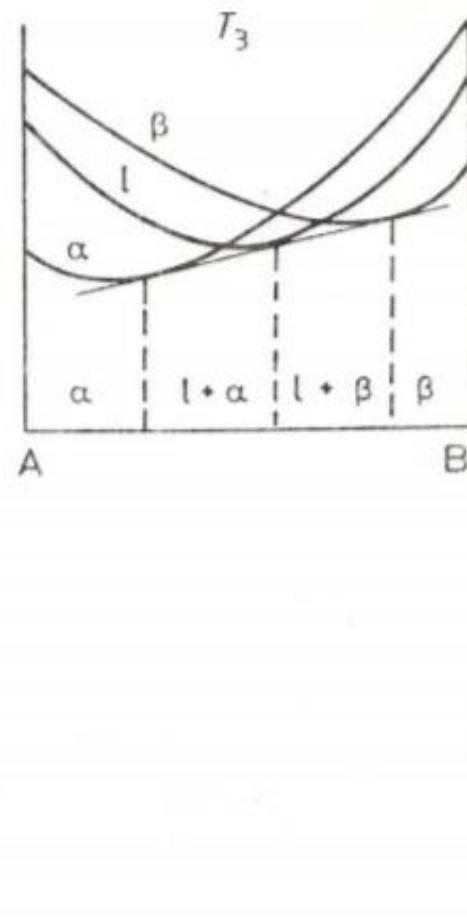
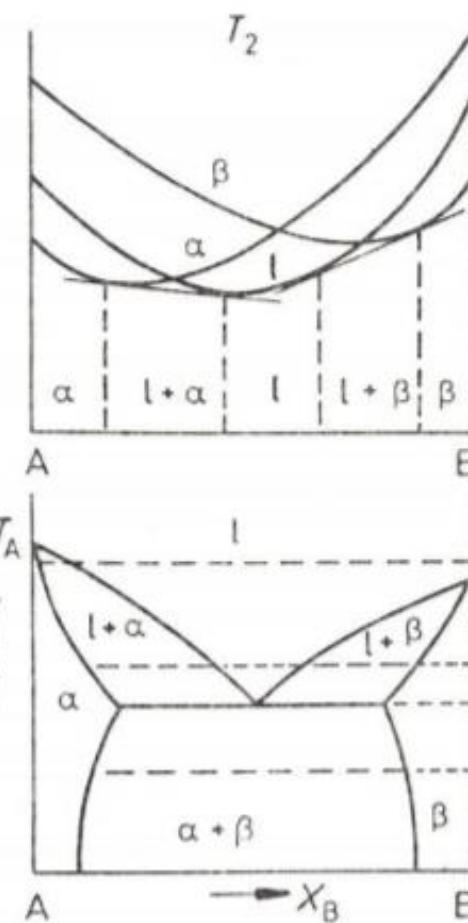
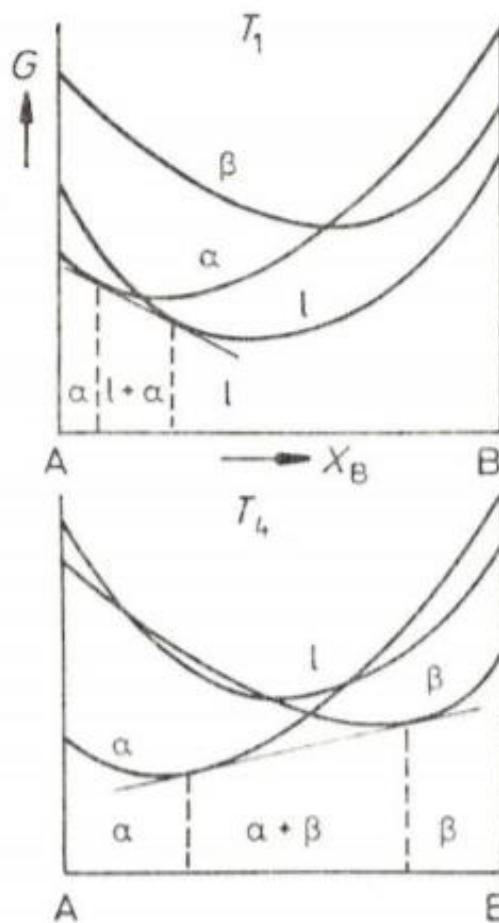


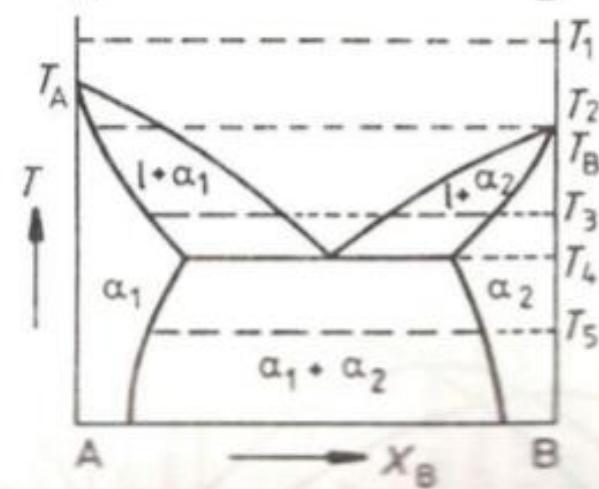
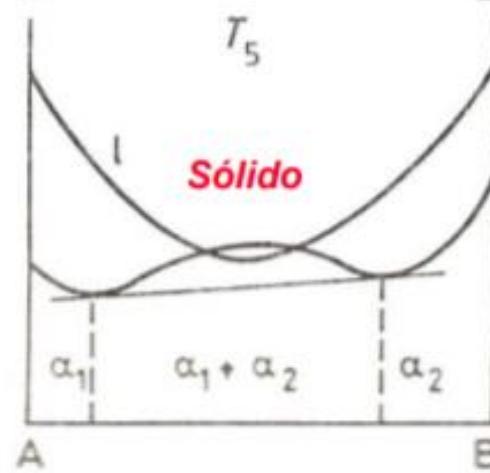
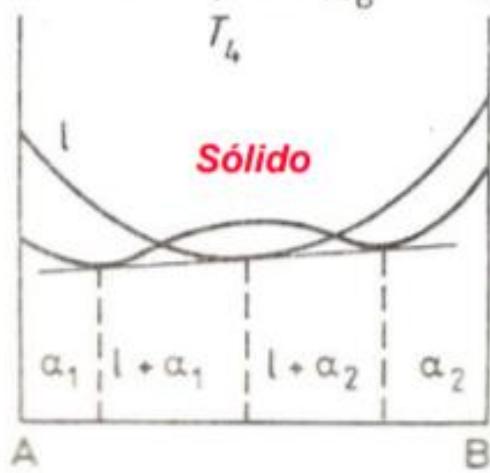
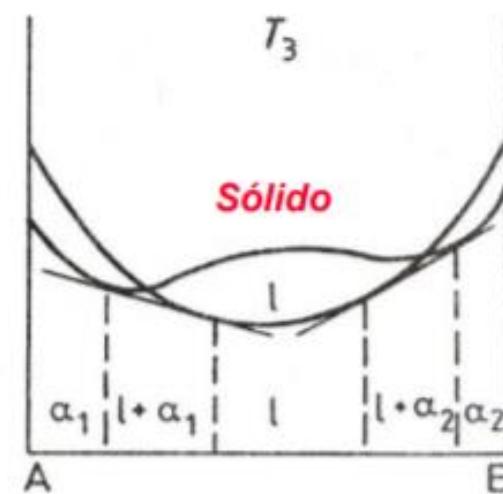
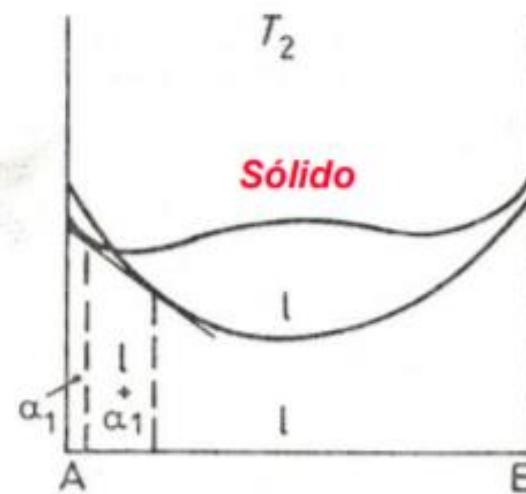
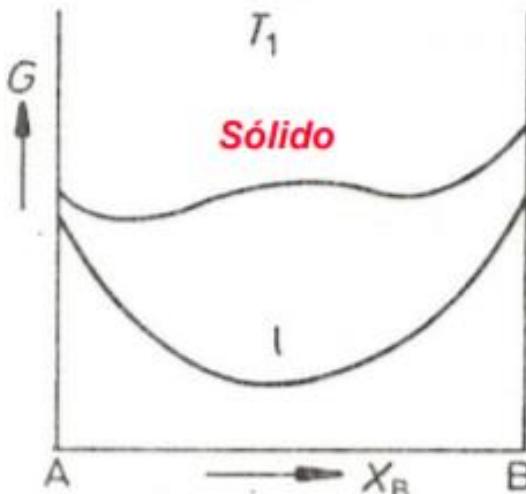
(b)



(c)









# CONSTRUÇÃO GRÁFICA DE DIAGRAMAS DE EQUILÍBRIO

- A precisão gráfica da tangente nas curvas de energia livre das fases é baixa – pode ser obtida analiticamente
- Sistemas Ideais
  - para as fases sólida e líquida: isomorfo
    - $\mu_i^{\text{sólido}} = \mu_i^{\text{líquido}}$
    - $\mu_i^{\text{fase}} = \mu_i^{o,\text{fase}} + R.T.\ln a_i^{\text{fase}}$
    - Para a reação  $\langle i \rangle = \{i\}$ 
      - $\ln\left(\frac{a_i^{\text{líquido}}}{a_i^{\text{sólido}}}\right) = -\frac{\Delta\mu_i^{o,\text{fusão}}}{R.T}$



# CONSTRUÇÃO GRÁFICA DE DIAGRAMAS DE EQUILÍBRIO

- $\Delta\mu_i^{o,fusão} = \mu_i^{o,líquido} - \mu_i^{o,sólido} = \Delta H_{f,i}^o - T \cdot \Delta S_{f,i}^o$
- $\Delta\mu_i^{o,fusão} = \Delta H_{f,i}^o \cdot \left(1 - \frac{T}{T_{f,i}}\right)$
- $\ln\left(\frac{a_i^{líquido}}{a_i^{sólido}}\right) = -\frac{\Delta H_{f,i}^o}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,i}}\right)$
- Como para as soluções ideais  $a_i = X_i$   
 $X_i^{líquido} = X_i^{sólido} \cdot \exp\left[-\frac{\Delta H_{f,i}^o}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,i}}\right)\right]$



# CONSTRUÇÃO GRÁFICA DE DIAGRAMAS DE EQUILÍBRIO

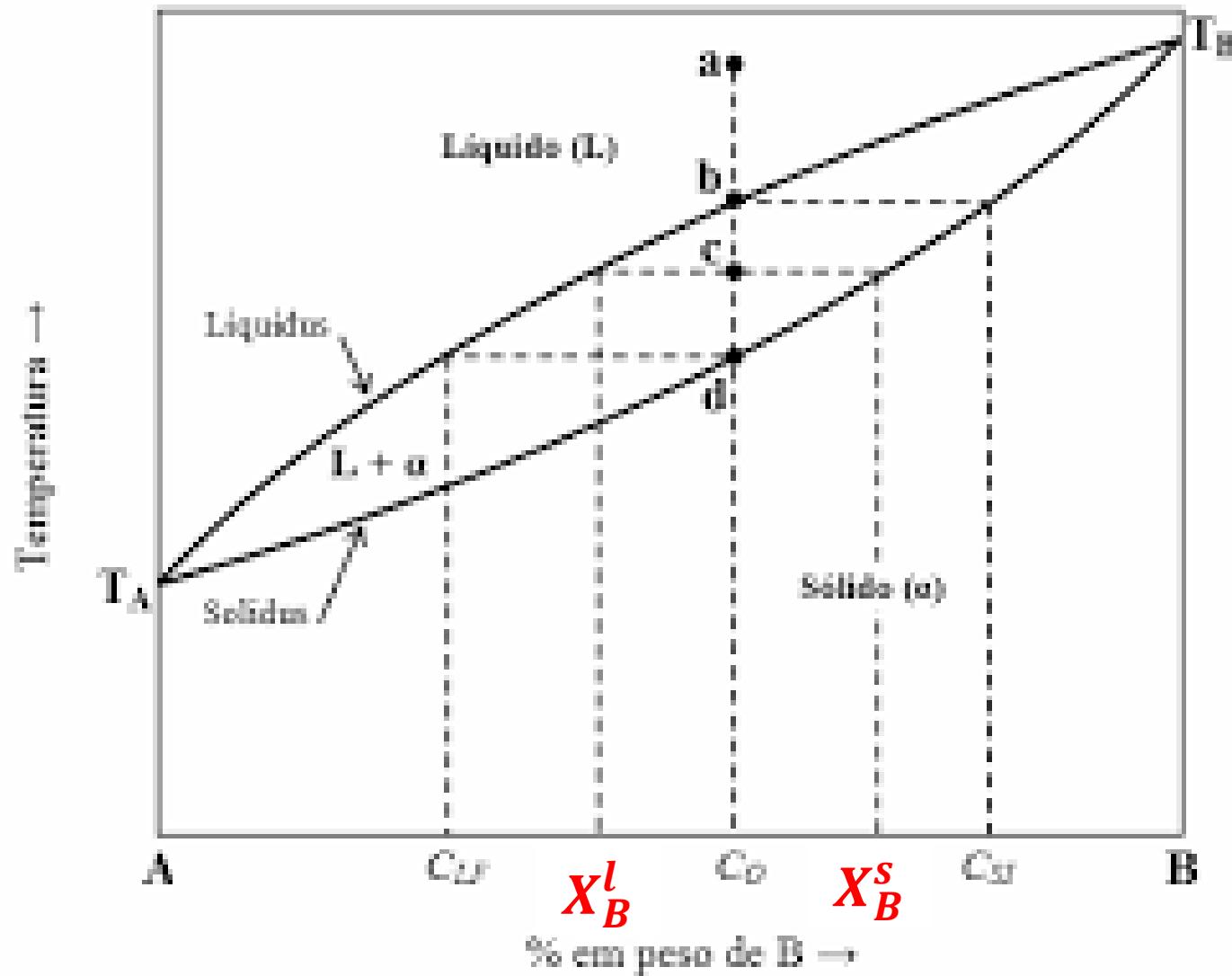
Mas, para um sistema binário A-B

$$\bullet X_A^{fase} + X_B^{fase} = 1$$

$$X_A^{\text{sólido}} \cdot \exp\left[-\frac{\Delta H_{f,A}^0}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,A}}\right)\right] + X_B^{\text{sólido}} \cdot \exp\left[-\frac{\Delta H_{f,B}^0}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,B}}\right)\right] = 1$$

e

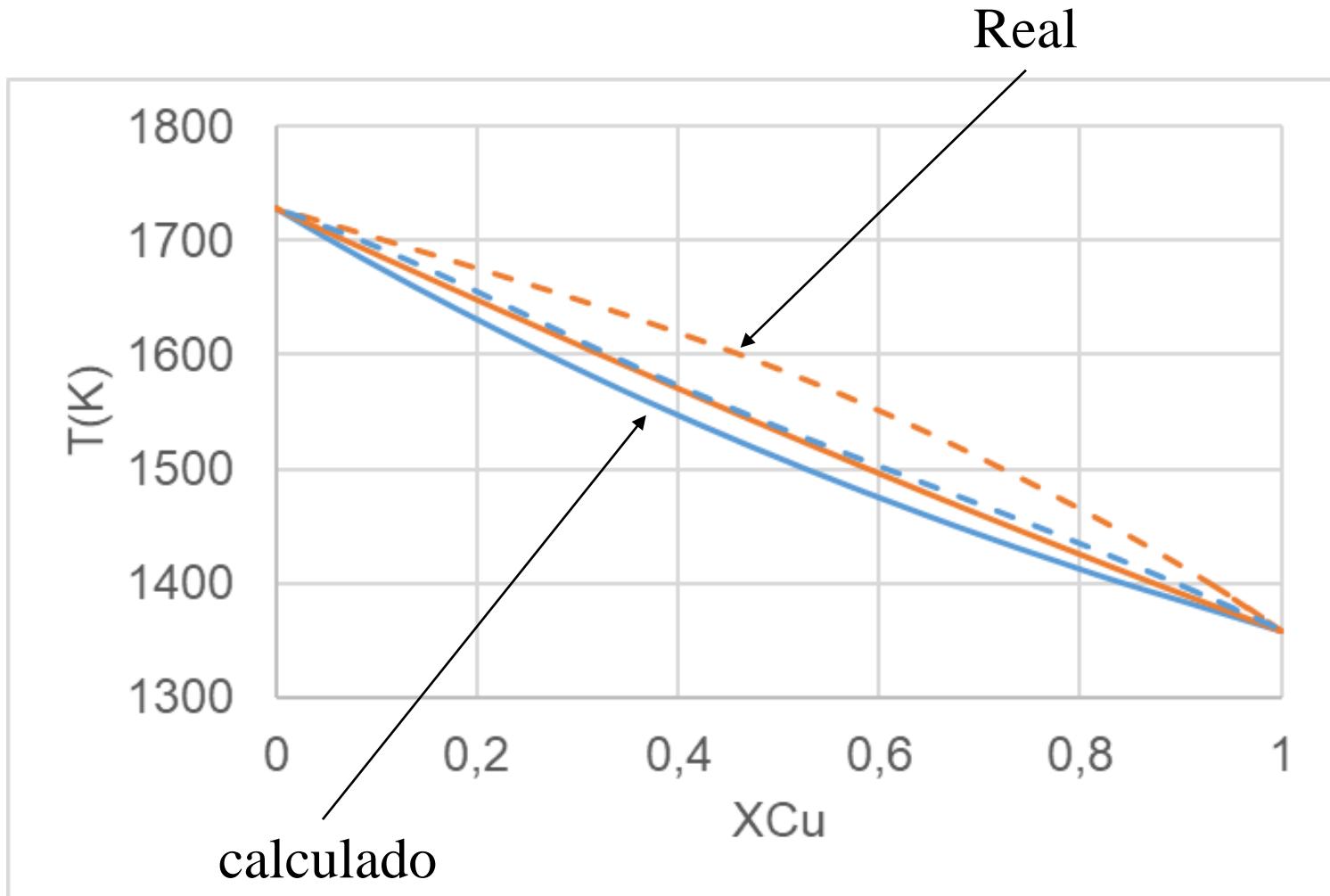
$$X_A^{\text{líquido}} \cdot \exp\left[\frac{\Delta H_{f,A}^0}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,A}}\right)\right] + X_B^{\text{líquido}} \cdot \exp\left[\frac{\Delta H_{f,B}^0}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_{f,B}}\right)\right] = 1$$





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# Cu-Ni





$$\Delta G^E = X_A \cdot X_B \cdot \sum L_v^T \cdot (X_A - X_B)^\nu \Rightarrow$$

```

LIQUID
EXCESS MODEL IS REDLICH-KISTER_MUGGIANU
CONSTITUENTS: CU,NI

G<LIQUID,CU;0>-H298<FCC_A1,CU;0> =
  298.15<T< 1358.02: +12964.84-9.510243*T-5.83932E-21*T**7+GHSERCU
  1358.02<T< 3200.00: +13495.4-9.920463*T-3.64643E+29*T**(-9)
  +GHSERCU
G<LIQUID,NI;0>-H298<FCC_A1,NI;0> =
  298.15<T< 1728.00: +11235.527+108.457*T-22.096*T*LN(T)
  -.0048407*T**2-3.82318E-21*T**7
  1728.00<T< 3000.00: -9549.775+268.598*T-43.1*T*LN(T)
L<LIQUID,CU,NI;0> = +11760+1.084*T
L<LIQUID,CU,NI;1> = -1671.8

```

$$G^{sol} = X_{Cu} \cdot G_{Cu}^0 + X_{Ni} \cdot G_{Ni}^0 + R.T. (X_{Cu} \cdot \ln X_{Cu} + X_{Ni} \cdot \ln X_{Ni}) + X_{Cu} \cdot X_{Ni} \cdot \sum L_v^T \cdot (X_{Cu} - X_{Ni})^\nu$$

$$G^{sol} = X_{Cu} \cdot G_{Cu}^0 + X_{Ni} \cdot G_{Ni}^0 + R.T. (X_{Cu} \cdot \ln X_{Cu} + X_{Ni} \cdot \ln X_{Ni}) + X_{Cu} \cdot X_{Ni} \cdot [L_0^T + L_1^T \cdot (X_{Cu} - X_{Ni})^1]$$



# Para casa

Considerando que os sistemas isomorfos são sistemas ideais, calcular os diagramas de equilíbrio de fases. Comparar com os diagramas experimentais e discutir diferenças:

1. Cu-Ni
2. FeO-MnO
3. Si-Ge
4. NiO-MgO
5. Ag-Au
6. Nb-Ta
7.  $\text{Al}_2\text{O}_3$ - $\text{Cr}_2\text{O}_3$
8. CaO-MnO
9. Ti-Ta
10. Ta-W
11. Bi-Sb
12. Ag-Pd
13. Pd-Rh