# What is Recreational Mathematics?

## *Definition by example: paradigms of topics, people and publications.*

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Mindful of the way Tom Sawyer got his fence whitewashed, I asked a number of mathematicians (including the speakers at the Miami University Recreational Mathematics Conference) to answer the question, "What is recreational mathematics?" The response was gratifying and illuminating. Many will recognize their brushmarks in the following discussion. For those who detect untouched areas on the fence, there are plenty of brushes available.

One thing is immediately clear: defining "recreational mathematics" is not recreational. The difficult task of defining "mathematics" is not simplified by the qualifying "recreational".

"Recreation" is defined in [1] as "a pastime, diversion, exercise, or other resource affording relaxation and enjoyment". One indulges in recreation to re-create oneself, to relax from work-a-day pursuits, and to clear and refresh the mind before returning it to its regular occupation. The closely related word "play" is defined in [1] as "exercise or action by way of amusement or recreation". Leonard [2] asserts that "all creative activity begins with play, to which will is then applied, as much in science and art as in sports. The Pythagorean theorem and the model of the double helix are at least as proportional, fit, elegant and admirable as the one-handed jump shot and the perfectly executed double play. Moreover, they are fun". Indeed, W. F. White [3] once observed that "amusement is one of the fields of applied mathematics".

"Mathematical work is highly satisfying. So is mathematical play. And, as most often is the case, one is apt to work much harder at any form of play, mental or physical, than one would for mere remuneration. Mathematical activity, more than any other, gives scope for the exercise of that faculty which has elevated man above other creatures" [4].

It is human nature to enjoy those things that can be done well. It is also human to resist mandatory tasks. At the University of Michigan the late Norman Anning used to take advantage of this human frailty by offering an intriguing non-credit problem at the end of each class section. The problem may or may not have been related to the course content, but it stuck in the long-time memories of many of his students. Some rather dry sounding problems can be recast into very nice recreational settings.

"Mathematics can provide enjoyment for a variety of reasons — meeting the needs of those who seek recreation, while giving satisfaction to those who are intrigued by solving problems, close reasoning and unexpected solutions" [5].

The allure of problem solving may be a matter of ego satisfaction. The joy of triumph over an opponent can be equalled or surpassed by the glow of finding a solution to a tough mathematics problem, even though it may turn out that the result had been published in an obscure 19th century journal. That many enjoy such challenges is attested to by the popularity of problem sections in various magazines. There the student who works all the problems in the textbooks just for the joy of it can further test his mathematical mettle. Decades ago Richard Bellman, by then an accomplished mathematician, told me that he had cut his mathematical eye-teeth on the challenge problems in *School Science and Mathematics.*

Challenge problems also can widen the mathematical horizons of those whose mathematical development has not been blocked by unfortunate circumstances, even though they do not follow

mathematics professionally. They have the tools to pursue an inexpensive and stimulating recreational activity. Their laboratory and shop consist of pencil and paper. At times, this recreation may re-create the work of others, but the possibility of discovering new relationships is always present. Howard Eves once compiled a long list of original articles that had been inspired by the problem departments of the *American Mathematical Monthly*.

Mathematics affords an ever-new never-boring avocation both before and after retirement where it is an effective weapon against vegetating. The human interest section of the Otto Dunkel Memorial Problem Book [**6**] lists the names of a nurseryman, a lawyer, a ballistics expert, a dentist, a steel works manager, a retired telephone engineer, an automobile dealer, an insurance inspector, a patent examiner, and a clergyman who were contributors to the problem departments of the *Monthly*. It is of more than passing interest that the problem departments of the journals of the two college level mathematical fraternities (Pi Mu Epsilon and Kappa Mu Epsilon) are edited by a practicing dentist and a practicing attorney — Dr. Leon Bankoff in the *Pi Mu Epsilon Journal* and Kenneth Wilke in *The Pentagon*.

In the less austere days of the problem departments of the *Monthly*, the late Norman Anning proposed the problem [**7**] to "find the element of likeness in: (a) simplifying a fraction, (b) powdering the nose, (c) building new steps on the church, (d) keeping emeritus professors on campus, (e) putting $B, C, D,$ in the determinant

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ a^3 & 1 & a & a^2 \\ B & a^3 & 1 & a \\ C & D & a^3 & 1 \end{vmatrix}$$

The published solution remarked that the value, $(1 - a^4)^3$, of the determinant was independent of the values of $B$, $C$, and $D$, so their insertion merely changes the appearance of the determinant and not its value. "Thus, the element of likeness in (a), (b), (c), (d), and (e) is that only the appearance of the principal entity is changed. The same element appears also in: (f) changing the name-label of a rose, (g) changing a decimal integer to the scale of 12, (h) gilding a lily, (i) whitewashing a politician, and (j) granting an honorary degree". Anning sent the solver a cartoon, clipped from the Saturday Review, of an Indian watching the cloud of an atomic blast. The caption: "I wish I had said that".

Other irreverent contributors to the problem departments are those with risible nom-de-plumes, such as ALICE MALICE, POLLY TOPE, NEV R. MIND, ZAZU KATZ, and ALFRED E. NEUMANN of MU ALPHA DELTA FRATERNITY. My favorites are NOSMO KING and BARBARA SEVILLE.

Still other recreational support of the tenet that mathematics is too important to take too seriously are: the sporadic appearances of Professor Euclide Paracelso Bombasto Umbugio, the priceless lyrics of Tom Lehrer, Leo Moser's verse [**8**, **9**], the Mathematical Swifties ("The angle is less than 90°, Tom noted acutely") in the 1964 *American Mathematical Monthly*, the Show Me's ("Show me a man who counts on his fingers and I'll show you a digital computer") in the *Journal of Recreational Mathematics*, and the varied offerings in that delightful new Canadian periodical, *Eureka* [**10**]. The disdainful may say that in mathematics a little humor goes the wrong way.

Few will disagree with the classification of mathematical humor, poems, anagrams, rebuses, word equations, cross-number puzzles, acrostics, and cryptarithms as purely recreational, although they have some educational use. Some may question whether they qualify as mathematics. However, to exclude the anecdotes of Eves' *In Mathematical Circles* would be to exclude a portion of history of mathematics as well. But what of other topics?

In the preamble to his excellent discussion of "Number Games and Other Mathematical Recreations", [**11**] William L. Schaaf remarks, "Mathematical recreations comprise puzzles and games that vary from naive amusements to sophisticated problems, some of which have never been solved. They may involve arithmetic, algebra, geometry, theory of numbers, graph theory, topology,

matrices, group theory, combinatorics (dealing with problems of arrangements or designs), set theory, symbolic logic, or probability theory. Any attempt to classify this colourful assortment of material is at best arbitrary". However, in his *Bibliography of Recreational Mathematics*, [12] Schaaf does impose a classification by means of the chapter headings: arithmetical recreations, number theory as recreation, geometric recreations, topological recreations, magic squares and related configurations, Pythagorean recreations, recreations in antiquity, combinatorial recreations, manipulative recreations, miscellaneous recreations, mathematics in related fields, and recreations in the classroom.

Most of the mathematics books with "Recreational", "Play", "Amusement", "Fun", "Diversions", "Pleasure", "Entertainment", or "Pastimes" in their titles are problem oriented. Predominant among the topics dealt with in such books are cryptarithms, magic squares, dissections and tessellations, decanting liquids, measuring, weighing, packing, shortest paths, calendars, the census, sliding movement, chess, dominoes, cards, river crossing, match arrangements, networks, permutations, combinations, diophantine equations, number properties, and various games.

In his detailed discussion of the first recreational mathematics book, that written by Bachet [13], Dudley [14] remarked that "problems of a recreational nature appear on Babylonian tablets 3500 years old" and "the Egyptian Rhind papyrus of about 1650 B.C. contains" a "problem that must have been made up for the fun of it". Dudley also reports the existence of other recreational problems in various works of the 16th century and earlier.

Those interested in the evolution of recreational mathematics through the ages will find an excellent historical account in [11], and a very good treatment of an international cross-section of recreational books in the Postscript of O'Bierne's *Puzzles and Paradoxes* [15]. The classic four volumes of Lucas [16] now appear in a French paperback edition. English translations are available of the recreational works of the Russians Domoryad [17] and Kordemsky [18], the Polish Steinhaus [19], the Dutch Schuh [20], and the Belgian Kraitchik [21].

Kraitchik was the editor of the defunct *Sphinx* (1931–1939), a magazine devoted to recreational mathematics. Its American counterpart, *Recreational Mathematics Magazine* (1961–1964), was also short-lived, although its successor, *Journal of Recreational Mathematics* (1968–), is flourishing.

The contemporaries Dudeney in England and Loyd in the United States were prolific inventors of mathematical puzzles which appeared in various periodicals. They were great rivals and did not hesitate to borrow from each other. Some of their efforts are preserved in [22, 23, 24]. Somewhat later Hubert Phillips under the *nom de plume* Caliban contributed problems to English periodicals, some of which are collected in [25].

One of the great stimuli of interest in recreational mathematics is Ball's comprehensive book [26] now in its 12th edition. Another great stimulus is the *Scientific American* column "Mathematical Games" written by the dean of contemporary recreational mathematics, Martin Gardner. A list, by title, of his skillfully written columns, beginning with "Flexagons" in December 1956, appears in [12]. Much of the column material has been assembled in his many books, such as [27, 28]. For a multitude of other worthy volumes see [12].

Some consider that for mathematics to be recreational there must be some element of play or games involved. But many who abhor games get great pleasure from other branches of mathematics. It may be a matter of definition of "games"; and if we accept Goodman's statement [29] that "Mathematics is the greatest game ever invented by man," we are back where we started from in search of a definition.

For many individuals, as they approach the limit of their abilities, mathematics loses its fun aspect. When a topic is undeveloped, it is recreational to many. As the theory is developed and becomes more abstract, fewer persons find it recreational. Of course, there are some who get their pleasure by concentrating on a single topic, pursuing it to the extent of current knowledge, and then trying to add something new. In many cases, what starts out to be purely recreational develops into an extensive discipline of serious mathematics such as now exists in number theory, topology, combinatorics, and game theory.

Many consider mathematics recreational if it is sufficiently elementary to be understandable by the non-mathematician. Recreational mathematics is "something you can explain to a business man sitting next to you on a flight from Chicago to Cincinnati". It is "a piece of mathematics that is both subtle enough to interest the professional mathematician and simple enough to be accessible to the man-in-the-street".

Others consider recreational mathematics from more of an academic slant: it is "a mathematician's holiday" that falls outside the bounds of standard school and university mathematics. "Learning what others have done is not recreational, doing it yourself is". Recreational mathematics is that which one works on "without thought of practicality, generality or academic rewards". "A recreational topic loses its standing when one squeezes from it an article to enhance the publication list."

Still others approach the topic from the standpoint of personal enjoyment. Surely the great amateurs Omar Khayyam, Leonardo Da Vinci, Blaise Pascal, and Pierre Fermat considered all mathematics to be fun. Indeed, many professional mathematicians consider all mathematics to be pleasurable. Richard Guy has said, "It always fascinates me that people are willing to pay me for doing what I would do for enjoyment in any case."

Recreational tastes are highly individualized, so no classification of particular mathematical topics as recreational or not is likely to gain universal acceptance.

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