CHAPTER 19

WELFARE ECONOMICS

19.1 SOCIAL WELFARE FUNCTIONS

Throughout this book it has been stressed repeatedly that the goal of any empirical science is the development of refutable propositions about some set of observable phenomena. Refutable propositions that survive repeated testing form the important principles on which the science is based. (It is easy, of course, to state refutable hypotheses that are in fact refuted.)

Parallel to the development of economics along the preceding lines has arisen a discipline called *welfare economics,* which seeks not to explain observable events but to *evaluate* the desirability of alternative institutions and the supposed resulting economic choices. For example, it is commonly alleged that "too many" fish are being caught in the oceans, that tariffs and other specific excise taxes cause an "inefficient" allocation of resources ("too little" production of the taxed item), that "too much" pollution and congestion occur in metropolitan areas, and the like. In this chapter we shall investigate the basis of these assertions and comment on the empirical content of such pronouncements.

It was common for classical economists to speak of "the benefits to society," the interest of the "working class," and other such phrases that implied a sufficient harmony of interests between members of the relevant class to permit speaking of them as a group. Today, we often hear of individuals representing "the interests of consumers" or of someone taking the position of "big business."

A difficulty in the concept of group preferences, or interests, was pointed out by Kenneth Arrow in his classic paper, "A Difficulty in the Concept of Social Welfare."† The use of such phrases implies that there is a well-defined function of individual preferences, or utility functions, representing the utility, or "welfare," of the group. Such a function was first posed explicitly by A. Bergson in 1938.‡ The social welfare function posited by Bergson had the form

$$W = f(U^1, \dots, U^m) \tag{19-1}$$

where $U^1, \dots, U^m$ were the utility functions of the m individuals in the group being considered, perhaps the whole economy. Bergson considered various first-order marginal conditions for the maximization of W subject to the resource constraints of the economy.

Arrow's discussion of these matters began with a 200-year-old example of the problem of construction of a group preference function. The example was based upon majority voting. Voting is a very common way for groups to reach decisions. Suppose one were to attempt to define collective preferences on the basis of what a majority of the community would vote for. Suppose there are three alternatives **a**, **b**, and **c** and three individuals in the group. Let P represent "is preferred to" so that $\mathbf{a}P\mathbf{b}$ means that **a** is preferred to **b**.

Suppose now that the three individuals have the following preferences:

Individual 1: $\mathbf{a}P\mathbf{b}$, $\mathbf{b}P\mathbf{c}$
Individual 2: $\mathbf{b}P\mathbf{c}$, $\mathbf{c}P\mathbf{a}$
Individual 3: $\mathbf{c}P\mathbf{a}$, $\mathbf{a}P\mathbf{b}$

Assume, in accordance with ordinary utility theory, that these consumers' preferences are *transitive*. That is, for individual 1, $\mathbf{a}P\mathbf{b}$ and $\mathbf{b}P\mathbf{c}$ means that $\mathbf{a}P\mathbf{c}$, etc. Then it can be quickly seen that a majority-rule social welfare function will have the unsatisfactory property of being *intransitive*. Consider, for example, alternative **a**. A majority of voters, namely voters 2 and 3, prefer **c** to **a**. Likewise, a majority of voters (1 and 3) prefer **a** to **b**, and another, different majority (1 and 2) prefer **b** to **c**. Whichever alternative is selected, a majority of voters will prefer some other

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† *The Journal of Political Economy,* **58:**328–346, 1950. This paper was part of a larger study, *Social Choice and Individual Values,* 2d ed., *Cowles Commission Monograph* 12, John Wiley & Sons, Inc., New York, 1963.

‡ Abram Bergson, "A Reformulation of Certain Aspects of Welfare Economics," *Quarterly Journal of Economics,* **52:**310–334, 1938.

alternative. Thus, the social welfare function based on what the majority wishes will exhibit the properties $\mathbf{a}P\mathbf{b}$, $\mathbf{b}P\mathbf{c}$, *and* $\mathbf{c}P\mathbf{a}$.[†]

Let us now summarize Arrow's theorem about social welfare functions. Arrow uses a weaker form of the preference relation: Let $\mathbf{a}\,R_i\,\mathbf{b}$ represent the statement "**a** is *preferred or indifferent* to **b**, according to individual *i*." Suppose there are *n* individuals in this society. Then, by a *social welfare function,* in this terminology, we mean a relation *R* that corresponds to the individual orderings, $R_1, \ldots, R_n$, of all social states by the *n* individuals in the society. That is, given the preference orderings of all people in the polity, there exists some social ordering *R* which denotes "society's" values and rankings of the alternatives being considered.

Arrow proceeded to list five conditions that he felt almost any reasonable social welfare function ought to contain. The first of these is that the social welfare function is in fact defined for all sets of individual orderings that obey some set of individualistic hypotheses about behavior, e.g., the usual economic postulates of convex indifference curves and the like.

Condition 1. The social welfare function is defined for every admissible pair of individual orderings R_1, R_2.

Second, the social ordering should describe welfare and not, in Arrow's word, "illfare." The social welfare function should react in the same direction, or at least not oppositely to, alterations in individual values.

Condition 2. If a social state **a** rises or does not fall in the ordering of each individual without any other change in those orderings, and if $\mathbf{a}R\mathbf{b}$ before the change, for any other alternative **b**, then $\mathbf{a}R\mathbf{b}$ after the change in individual orderings.

[†]This voting paradox illustrates one of the outstanding differences between *market* choices and *political* choices. In the former, the consumer has the option of expressing the *intensity* of a preference by the simple act of choosing to purchase differing amounts of goods. In political choice, however, ordinary voters get one and only one vote. The consumer under these circumstances is unable to express intensity of preference. In the above example, the three alternatives were merely *ranked.* The voters were not able to say, for example, that they preferred **a** a great deal more than **b** and **b** only slightly more than **c**. In legislative bodies, in which there are relatively few voters, the individuals can *trade* votes on successive issues. Suppose, for example, individual 1 has the above-stated intensities of preferences and individual 2 was almost indifferent between **a, b,** and **c**. Then voter 1 could make a contract or a deal to vote for some other issue which voter 2 felt strongly about (and which voter 1 had no strong preferences about) in exchange for an agreement from voter 2 to vote for alternative **a** in the text example. The paradox would be resolved through trade. However, more trade is not necessarily preferred to less trade for individuals, and voter 3 might end up worse off for such political trading. It is for these reasons that many people believe that special-interest legislation is more apt to be enacted by legislative bodies than by referendum vote. But such vote trading also protects minorities who feel intensely about some issue from the "tyranny of the majority." The gains-from-trade aspect of political trading is emphasized in James Buchanan and Gordon Tullock, *The Calculus of Consent,* University of Michigan Press, Ann Arbor, 1963.

The most controversial of Arrow's conditions is the third, the *independence of irrelevant alternatives*. Consider an election in which three candidates, **a**, **b**, and **c**, are running. Suppose an individual's preferences are $\mathbf{a}R_i\mathbf{b}R_i\mathbf{c}$. Suppose, before the election, candidate **b** dies. Then we would expect to observe $\mathbf{a}R_i\mathbf{c}$. In like manner, we expect the social welfare function's ranking of any two alternatives to be unaffected by the addition or removal of some other alternative.

Condition 3. Let R_1, R_2, and R'_1, R'_2 be two sets of individual orderings. Let S be the entire set of alternatives. Suppose, for both individuals and all alternatives **a**, **b** in S, that $\mathbf{a}R_i\mathbf{b}$ if and only if $\mathbf{a}R'_i\mathbf{b}$. Then the social choice made from S is the same whether the individual orderings are R_1 and R_2 or R'_1 and R'_2.

Conditions 4 and 5 imposed by Arrow amount to assertions that individual preferences *matter*. That is, individual values are to "count" in determining the social welfare function. Conditions 4 and 5 say that the social welfare function is not to be either *imposed* or *dictatorial*. A social welfare function is said to be *imposed* if, for some pair of alternatives **a** and **b**, $\mathbf{a}R\mathbf{b}$ for any set of individual orderings R_1, R_2, that is, irrespective of the individual orderings R_1, R_2, where R is the social ordering corresponding to R_1, R_2. Likewise, a social welfare function is said to be *dictatorial* if there exists an individual i such that for all **a** and **b**, $\mathbf{a}R_i\mathbf{b}$ implies $\mathbf{a}R\mathbf{b}$ regardless of the orderings of all individuals other than i, where R is the social preference ordering corresponding to the R_i's.

Condition 4. The social welfare function is not to be imposed.

Condition 5. The social welfare function is to be nondictatorial.

Arrow succeeded in showing that these five conditions could not all hold simultaneously. In particular, he showed that any social welfare function that satisfied the first three conditions was either imposed or dictatorial. This very strong result is called the *possibility theorem*.† It says that no matter how complicated a scheme might be constructed for determining a set of social preferences, social ordering R cannot meet all conditions 1 to 5. It will be impossible to construct *any* welfare function of the type described in Eq. (19-1), $W = f(U^1, \ldots, U^m)$, that is, some function of individual utility levels, obeying the preceding conditions.

Another interpretation of the possibility theorem is that interpersonal comparisons of social utility are ruled out. It is impossible to say that taking a dollar away from a rich person and giving it to a poor person will make society better off, in some nondictatorial or nonimposed sense. The problem of interpersonal comparisons of utility was a vehicle by which ordinal utility replaced the older cardinal utility idea.

On a less rigorous but more intuitive basis, the reason sensible social welfare functions cannot exist is that they conflict in a fundamental way with the notion that

†The authors would have called it the impossibility theorem.

more is preferred to less. At any given moment, there is a frontier of possibilities for the consumers in any society. Any movement *along* this frontier involves gains for some individuals and losses for others. Without a measure for comparing these gains and losses between individuals, there is no sense to the phrase "social welfare." (We shall explore these matters in more detail in Sec. 19.3.)

A rigorous proof of the possibility theorem is beyond the scope of this book. It can be found in the reference cited. We conclude this section by noting that in spite of this theorem, hundreds, perhaps thousands of articles have been written in economics journals using social welfare functions. Indeed, a whole new area of mathematical theology has arisen. However, to quote Samuelson,[†] "the theorems enunciated under the heading of welfare economics are not meaningful propositions of hypotheses in the technical sense. For they represent the deductive implications of assumptions which are not themselves meaningful refutable hypotheses about reality."

19.2 THE PARETO CONDITIONS

Faced with the impossibility of constructing a meaningful social welfare function, economists have opted for a weaker criterion by which to evaluate alternative situations. This criterion, known as the *Pareto condition,* after the Italian economist Vilfredo Pareto, states that a social state **a** is to be preferred to **b** if there is at least one person better off in **a** than in **b**, and no one is worse off in **a** than in **b**. This is a weaker value judgment only in the sense that more people would probably accept this judgment over more specific types of social orderings wherein some individuals lose and others gain. A state **a** that is preferred to **b** in the Paretian sense is said to be *Pareto-superior* to **b**. One can imagine some sort of frontier of possible states of the economy such that there are no Pareto-superior points. That is, along this frontier, any movement entails a loss for at least one individual. The points for which no Pareto-superior states exist are called *Pareto-optimal.*

In general, we shall find that the set of Pareto-optimal points is quite large. Whether or not these points are a useful guide to policy is debatable. Even so, to say that the economy *ought* to be at a Pareto-optimal state is a value judgment and therefore a part of moral philosophy and not part of the empirical science of economics. We can, however, as economists, investigate the conditions under which various ideal Pareto-optimal states will be obtained. In this section we shall investigate certain famous conditions that achieve Pareto optimality. It is useful, in these discussions, to maintain the perspective indicated in the preceding quotation from Samuelson.

Pure Exchange

Consider an economy containing two individuals who consume two commodities, x and y. Let x_i, y_i denote the amounts of x and y consumed by the ith person, whose

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[†] *Foundations of Economic Analysis,* Harvard University Press, Cambridge, MA, 1947, pp. 220–221.

utility function is $U^i(x_i, y_i)$. Suppose that the total amounts of x and y are fixed, that is, $x_1 + x_2 = x$, $y_1 + y_2 = y$, where x and y are constants. Under what circumstances will the allocation of x and y between the two individuals be Pareto-optimal?

This problem can be formulated mathematically as follows:

maximize

$$U^2(x_2, y_2)$$

subject to

$$\begin{gathered} U^1(x_1, y_1) = U_0^1 \\ x_1 + x_2 = x \qquad y_1 + y_2 = y \end{gathered} \tag{19-2}$$

It is meaningless to attempt to maximize both individual's utilities simultaneously.[†] Instead, we first fix either individual's utility at some arbitrary level; then, the other person's utility is maximized. In this way, a position is attained in which neither party can be made better off without lowering the other person's utility.

The Lagrangian for the preceding problem is

$$\mathcal{L} = U^2(x_2, y_2) + \lambda(U_0^1 - U^1(x_1, y_1)) + \lambda_x(x - x_1 - x_2) + \lambda_y(y - y_1 - y_2) \tag{19-3}$$

Differentiating with respect to x_1, y_1, x_2, y_2 and the Lagrange multipliers yields

$$\mathcal{L}_{x_2} = U_x^2 - \lambda_x = 0 \tag{19-4a}$$

$$\mathcal{L}_{y_2} = U_y^2 - \lambda_y = 0 \tag{19-4b}$$

$$\mathcal{L}_{x_1} = -\lambda U_x^1 - \lambda_x = 0 \tag{19-4c}$$

$$\mathcal{L}_{y_1} = -\lambda U_y^1 - \lambda_y = 0 \tag{19-4d}$$

and

$$\mathcal{L}_{\lambda} = U_0^1 - U^1(x_1, y_1) = 0 \tag{19-5a}$$

$$\mathcal{L}_{\lambda_x} = x - x_1 - x_2 = 0 \tag{19-5b}$$

$$\mathcal{L}_{\lambda_y} = y - y_1 - y_2 = 0 \tag{19-5c}$$

where $U_x^i = \partial U^i/\partial x_i$, etc. Combining Eqs. (19-4) gives

$$\frac{U_x^1}{U_y^1} = \frac{\lambda_x}{\lambda_y} = \frac{U_x^2}{U_y^2} \tag{19-6}$$

[†]We leave such constructions to those who aspire to find that economic system which seeks "the greatest good for the greatest number of people."

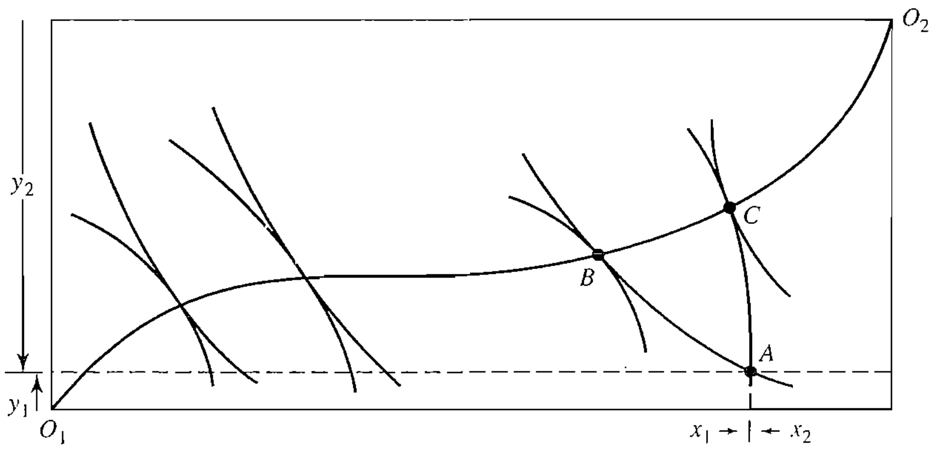

FIGURE 19-1
The Edgeworth box diagram is useful for depicting the set of Pareto-optimal points in a pure trade, zero transaction cost world. The dimensions of the box are the total amounts of each good available, x and y. Any point, such as A in the interior of the box, represents an allocation of x and y to the two individuals. Individual 1's utility function is plotted in the usual direction from the origin marked O_1. Individual 2's utility function is plotted opposite (right to left and down) from origin O_2. The set of points for which the slopes of U^1 and U^2 are identical at the same point, i.e., a level curve of U^1 is tangent to a level curve of U^2, is called the *contract curve,* designated $O_1 O_2$. This curve represents the set of points for which the gains from trade are exhausted. It is occasionally referred to as the *conflict curve* because movements *along* $O_1 O_2$ represent conflicts of interest: one individual gains and the other loses. For that reason, it is the set of Pareto-optimal points in this economy.

Equation (19-6) is the tangency condition that the consumers' indifference curves have the same slope. The marginal rate of substitution of x for y must be the same for both consumers. This is the familiar condition that must hold if the gains from trade are to be exhausted. The set of all points that satisfy (19-6) (and the constraints) is called the *contract curve,* as depicted in Fig. 19-1. This diagram is the Edgeworth box diagram first shown in the chapter on general equilibrium theory. (There, though, the axes were quantities of factors of production, not final goods as is the case here. The mathematics is, of course, formally identical.)

The set of Pareto-optimal points is the set of allocations for which the gains from exchange are exhausted. If the consumers were presented a different allocation, e.g., point A in Fig. 19-1, then with no cost of trading we should expect them to move to some point on the contract curve $O_1 O_2$. If the trade is voluntary, the final allocation must lie between (or on) the two original indifference curves, i.e., some point on the segment BC of the contract curve. Without a further specification of the constraints of the bargaining process, the theory is inadequate to determine the actual final point. But in the absence of transactions costs and coercion, self-seeking maximizers must wind up at *some* point along BC.

The problem as posed in (19-2) does not actually start at some particular point such as A and then move to the contract curve. As formulated in (19-2), the indifference level of individual 1 is fixed, say at the level that goes through point A. The resulting solution of the problem, i.e., solution to Eqs. (19-4) and (19-5), would place the economy at point B, where person 2 achieves maximum utility, leaving person 1 on the original indifference curve. Hence, the problem posed in (19-2) admits of a unique answer, even if a bargaining process that starts *both* individuals at A is unspecified.

The indirect utility function for individual 2 is obtained first by solving Eqs. (19-4) and (19-5) and substituting the chosen values of x_2 and y_2 into $U^2(x_2, y_2)$. Let the solutions to (19-4) and (19-5) be designated

$$x_i = x_i^*(U_0^1, x, y) \qquad y_i = y_i^*(U_0^1, x, y) \tag{19-7}$$

and likewise for the Lagrange multipliers:

$$\lambda = \lambda^*(U_0^1, x, y)$$
$$\lambda_x = \lambda_x^*(U_0^1, x, y) \qquad \lambda_y = \lambda_y^*(U_0^1, x, y) \tag{19-8}$$

Then

$$U^{2*} = U^2(x_2^*, y_2^*) = f(U_0^1, x, y) \tag{19-9}$$

Holding constant x and y, the total amounts of the goods, one can imagine a utility frontier, defined by Eq. (19-9). Starting with $U_0^1 = 0$, the maximum level of utility for person 2 is that which is achieved when person 2 consumes all of both goods, i.e.,

$$f(0, x, y) \equiv U^{2^*}(x, y)$$

Likewise some maximum level of U^1 exists, represented by the indifference curve for person 1 which passes through O_2, for which $U^2 = 0$.

This utility frontier is plotted as the curve UU in Fig. 19-2, where the subscript 0 on U^1 has been suppressed. Using the envelope theorem and Eqs. (19-4) leads to

$$\frac{\partial U^{2^*}}{\partial U^1} = \lambda = -\frac{U_x^2}{U_x^1} = -\frac{U_y^2}{U_y^1} < 0 \tag{19-10}$$

Assuming the tangencies defining the contract curve take place at positive marginal utilities (downward-sloping indifference curves), $\partial U^{2^*}/\partial U^1 < 0$, as indicated. The Pareto frontier could not very well exhibit $\partial U^{2^*}/\partial U^1 > 0$, since then movements along it in the northeast direction would imply gains for *both* individuals, contradicting the notion of Pareto optimality. It is *not* possible to infer that the Pareto frontier UU is concave to the origin; this follows from the ordinal nature of utility. A monotonic transformation of $U^1(x_1, y_1)$, say, could bend the frontier as desired, though keeping it downward-sloping.

Production

Suppose now we generalize the preceding discussion to the case where x and y are produced using two (or more) factors of production. In the preceding chapter on general equilibrium, an Edgeworth box diagram was constructed for the two-factor case. In order for consumers to be on the Pareto frontier in consumption,

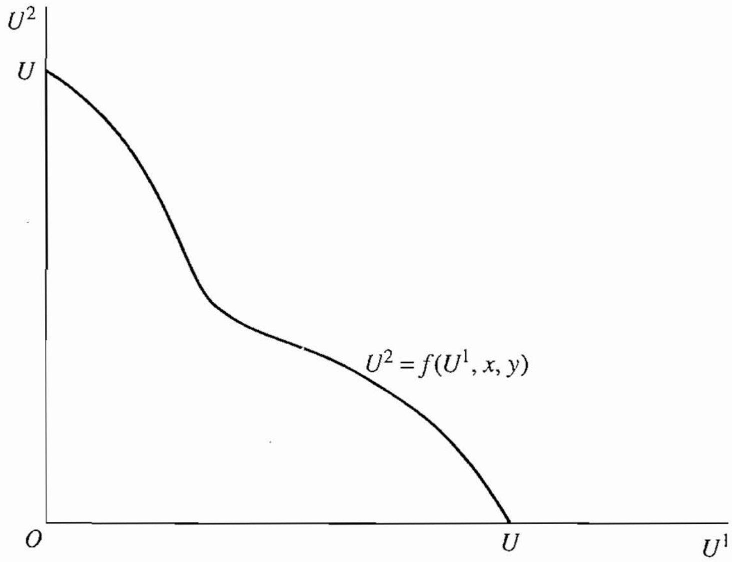

FIGURE 19-2
The Utility Frontier for Given Total Quantities of Goods.
For any given amount of the goods x and y there exists a whole set of points for which neither individual can gain without the other person's losing. This Pareto frontier consists of reading off the (ordinal) utility levels for each person at every point along the contract curve $O_1 O_2$. The frontier is necessarily downward-sloping, by definition of Pareto optimality.

the goods must be produced efficiently. That is, a production point interior to the production possibilities frontier cannot result in a Pareto-optimal state for consumers. The consumers could both (or all, in the n-person case) have more of all goods and hence higher utility if production were moved to the production possibilities frontier in the appropriate manner. Hence, the problem of defining the Pareto frontier for consumers in the case in which x and y are *produced,* and not fixed constants, begins with the problem of defining the production possibilities frontier. Points on the production frontier are called *efficient in production.*

The mathematics for the production case is formally identical to the preceding analysis of final goods. Let there be two factors of production, L and K, and let L_x denote the amount of labor used in producing x, etc. Then the problem of efficient production can be stated

maximize

$$y = f(L_y, K_y)$$

subject to

$$\begin{gathered} g(L_x, K_x) = x \\ L_x + L_y = L \qquad K_x + K_y = K \end{gathered} \tag{19-11}$$

where $f(L_y, K_y)$ and $g(L_x, K_x)$ are the production functions of y and x, respectively. The value x is taken as a parameter; it is *not* a decision variable.
The Lagrangian for the problem (19-11) is

$$\mathcal{L} = f(L_y, K_y) + \lambda(x - g(L_x, K_x)) + \lambda_L(L - L_x - L_y) + \lambda_K(K - K_x - K_y) \tag{19-12}$$

The resulting first-order relations are

$$f_L - \lambda_L = 0 \tag{19-13a}$$
$$f_K - \lambda_K = 0 \tag{19-13b}$$
$$-\lambda g_L - \lambda_L = 0 \tag{19-13c}$$
$$-\lambda g_K - \lambda_K = 0 \tag{19-13d}$$

and the constraints

$$x - g(L_x, K_x) = 0 \tag{19-14a}$$
$$L - L_x - L_y = 0 \tag{19-14b}$$
$$K - K_x - K_y = 0 \tag{19-14c}$$

From Eqs. (19-13),

$$\frac{f_L}{f_K} = \frac{\lambda_L}{\lambda_K} = \frac{g_L}{g_K} \tag{19-15}$$

The ratio of marginal products must be equal for both goods along the production contract curve. This is the tangency condition illustrated in Fig. 18-2. Solving Eqs. (19-13) and (19-14) simultaneously gives

$$L_y = L_y^*(x, L, K) \tag{19-16a}$$
$$K_y = K_y^*(x, L, K) \tag{19-16b}$$
$$L_x = L_x^*(x, L, K) \tag{19-16c}$$
$$K_x = K_x^*(x, L, K) \tag{19-16d}$$

and

$$\lambda = \lambda^*(x, L, K) \tag{19-17a}$$
$$\lambda_L = \lambda_L^*(x, L, K) \tag{19-17b}$$
$$\lambda_K = \lambda_K^*(x, L, K) \tag{19-17c}$$

Equations (19-16) give the chosen values of labor and capital in both industries. Substituting these values into the objective function gives the maximum y, y^* for any value of x:

$$y^* = f(L_y^*, K_y^*) = y^*(x, L, K) \tag{19-18}$$

Using the envelope theorem, we have

$$\frac{\partial y^*}{\partial x} = \frac{\partial \mathscr{L}}{\partial x} = \lambda^* \tag{19-19}$$

Hence, λ^* has the interpretation of the *marginal cost of* x, since it shows how much y^* must be given up in order to get an additional unit of x. The multiplier λ^* is the slope of the production possibility frontier by definition, since $\lambda^* = \partial y^*/\partial x$. Assuming the marginal products of the factors are positive, $\lambda^* < 0$; i.e., the production frontier is negatively sloped. As before, from Eqs. (19-13),

$$\lambda^* = -\frac{\lambda_L^*}{g_L} = -\frac{f_L}{g_L} = -\frac{f_K}{g_K} = -\frac{\lambda_K^*}{g_K} < 0 \tag{19-20}$$

This equation has the interesting interpretation that the marginal cost of x is the same if only labor is varied (the ratio f_L/g_L) or if only capital is varied (f_K/g_K) or if both are varied. In the partial equilibrium framework this phenomenon was encountered in the formula

$$\text{MC} = \frac{w_L}{f_L} = \frac{w_K}{f_K} \tag{19-21}$$

where the w's were the respective factor prices. Here, of course, λ_L and λ_K are the factor prices, *measured in terms of the physical output* y, i.e.,

$$\lambda_L^* = \frac{\partial y^*}{\partial L} \tag{19-22a}$$

$$\lambda_K^* = \frac{\partial y^*}{\partial K} \tag{19-22b}$$

This interpretation of λ_L and λ_K makes (19-21) and (19-20) equivalent except for units.

The production possibilities curve yields the set of "efficient" production plans. A necessary condition for overall Pareto optimality is to be on this frontier. However, that in itself is not sufficient. To exhaust all the gains from trade, the goods produced must be allocated to the consumers in an efficient manner. This requires at least that the previous analysis of the *consumer's* Edgeworth box diagram apply, i.e., the consumers must be on their contract curve, for any production levels (x, y). However, one more tangency condition must also apply: For each consumer, the marginal rates of substitution of x for y, that is, the marginal evaluation of x in terms of y forgone, must equal the marginal cost of producing x (in terms of y forgone). This condition implies that the consumers are on their contract curve, since *each* consumer's marginal evaluation of x must equal the marginal cost of x. Let us see how this last condition is derived.

The only difference between this last, and most general problem, and the first one posed in (19-2) is that instead of x and y being fixed, they are determined by the production possibilities frontier derived in the production model as Eq. (19-18). Thus, the locus of overall efficient (Pareto-optimal) points is defined by

maximize

$$U^2(x_2, y_2)$$

subject to

$$\begin{aligned} U^1(x_1, y_1) &= U_0^1 & x_1 + x_2 &= x \\ y_1 + y_2 &= y & y &= y^*(x, L, K) \end{aligned} \tag{19-23}$$

It will simplify the algebra to combine the last three constraints into one. These three equations define the production possibility curve, written in implicit form, as

$$h(x, y) = h(x_1 + x_2, y_1 + y_2) = 0$$

where the parameters L and K have been suppressed because they will not be used. The problem is then simply

maximize

$$U^2(x_2, y_2)$$

subject to

$$\begin{aligned} U^1(x_1, y_1) &= U_0^1 \\ h(x_1 + x_2, y_1 + y_2) &= 0 \end{aligned} \tag{19-24}$$

The Lagrangian for (19-24) is

$$\mathcal{L} = U^2(x_2, y_2) + \lambda_1\left(U_0^1 - U^1(x_1, y_1)\right) + \lambda h(x, y) \tag{19-25}$$

Noting that $\partial h/\partial x_i = (\partial h/\partial x)(\partial x/\partial x_i) = \partial h/\partial x$, etc., we see that the first-order conditions are

$$U_x^2 + \lambda h_x = 0 \tag{19-26a}$$

$$U_y^2 + \lambda h_y = 0 \tag{19-26b}$$

$$-\lambda_1 U_x^1 + \lambda h_x = 0 \tag{19-26c}$$

$$-\lambda_1 U_y^1 + \lambda h_y = 0 \tag{19-26d}$$

and the two constraints

$$U_0^1 - U^1(x_1, y_1) = 0 \tag{19-27a}$$

$$h(x, y) = 0 \tag{19-27b}$$

Eliminating the Lagrange multipliers from Eqs. (19-26), we find

$$\frac{U_x^1}{U_y^1} = \frac{U_x^2}{U_y^2} = \frac{h_x}{h_y} \tag{19-28}$$

The quantity h_x/h_y is the absolute slope of the production possibilities frontier; i.e., in explicit form, by the chain rule,

$$\frac{\partial y}{\partial x} = -\frac{h_x}{h_y}$$

Hence, Eq. (19-28) gives the marginal condition stated above: *For overall (production and consumption) Pareto optimality, the marginal evaluation of each commodity must be the same for all individuals, and that common marginal evaluation*

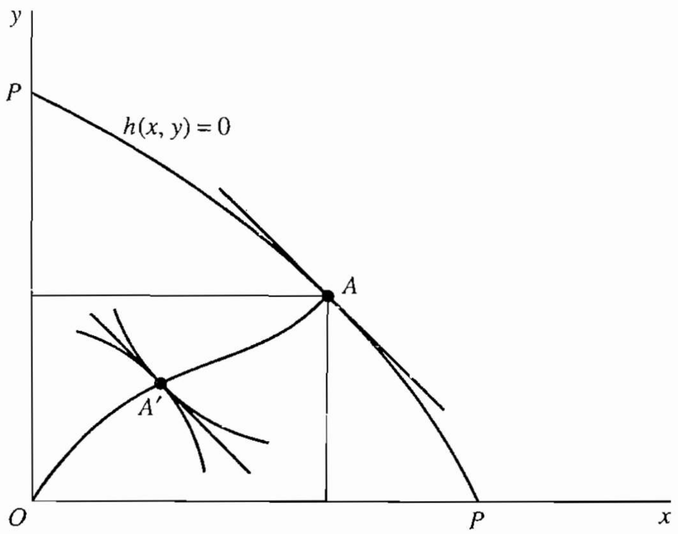

FIGURE 19-3
Overall Pareto Optimality. The curve *PP* represents the *production* possibilities frontier of the economy for given resource endowments. The slope of this frontier is the marginal cost of producing x, in terms of y forgone. At any point, say A, along the frontier, an Edgeworth box can be constructed as shown. The points in the box represent allocations of x and y to the two consumers. These consumers will presumably trade to the contract curve OA. At some point or points on OA, the slopes of the indifference curves will equal the slope of the transformation curve at A. This is an overall Pareto efficient point, since the MRSs of each consumer are equal and equal to marginal cost.

must equal the marginal cost of producing that good. (The words *all* and *each* have been used instead of *both*. The generalization of these results to n goods and m consumers is straightforward.)

The overall utility frontier is found by solving Eqs. (19-26) and (19-27) for $x_i = x_i^*(U_0^1)$, $y_i = y_i^*(U_0^1)$. Substituting these values into the objective function, we derive

$$U^2 = U^2(x_2^*, y_2^*) = U^{2*}(U_0^1) \tag{19-29}$$

This situation is shown geometrically in Fig. 19-3. The curve PP represents the production possibilities frontier for given resource endowments. At any point, say A, the slope of this frontier is the marginal cost of x. From this point, which represents a certain total amount of x and y, an Edgeworth box diagram is constructed. The points in the interior of the box represent the allocations of x and y to the two consumers. The curve OA represents the implied contract curve for the consumers. At some point (or points) along OA, say A', the marginal evaluations of x (the marginal rates of substitution) will equal the slope of the tangent line at A, the marginal cost of x. This is an overall Pareto-optimal allocation, i.e., efficient in production and consumption. The point A' represents one particular point on the implied *utility* frontier, as depicted in Fig. 19-2. It is a special point, however, in that marginal cost equals marginal benefits there.

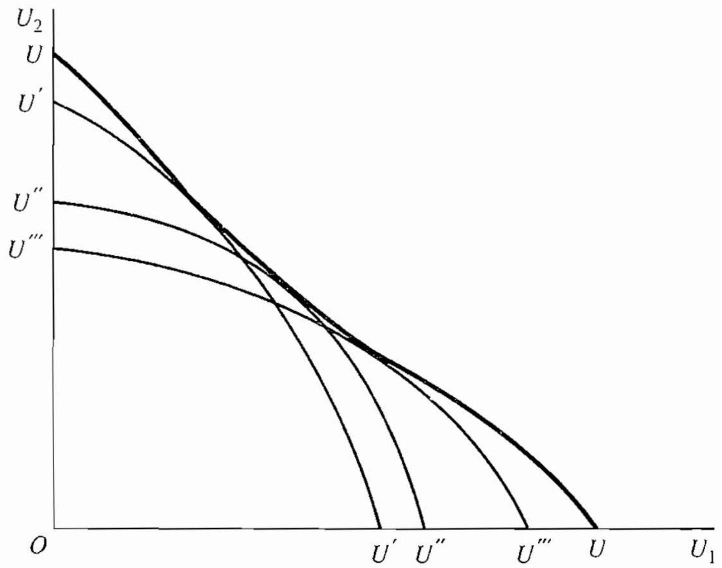

FIGURE 19-4
Partial and Overall Utility Frontiers. For any given x and y, that is, for some particular point on the production possibilities curve, some *utility* frontier is implied. Several of these are drawn: $U'U'$, $U''U''$, and $U'''U'''$. The *envelope* curve for all these partial frontiers is the overall, or *grand,* utility frontier UU. The frontier UU represents the maximum utility any one consumer can achieve for given level of the other person's utility. Each point on UU represents, in general, a different production point, though there is no reason why some partial frontier could not be tangent to UU at more than one point.

At each point along the *production* possibilities frontier, an Edgeworth box can be drawn and the overall efficient allocation(s) can be determined. In Fig. 19-4 the utility frontiers for several production points are drawn. The *envelope* curve for all these partial frontiers is Eq. (19-29), $U^2 = U^{2*}(U_0^1, x, y)$. The partial frontiers are those for specific values of x and y, that is, holding x and y constant. From general envelope considerations

$$\frac{\partial U^{2*}}{\partial U_0^1} = \left(\frac{\partial U^{2*}}{\partial U_0^1}\right)_{x,y} \tag{19-30}$$

That is, along the overall frontier, the slope of the frontier at any point is the same if x and y are held constant or allowed to vary.

The grand utility frontier UU represents the complete set of Pareto-optimal, or efficient, productions and distributions of the goods x and y. The choice of *which* Pareto-optimal point is somehow "best for society" is a value judgment and outside the scope of positive economics. If some social welfare function is posited (social welfare functions can exist, but not with all the properties outlined by Arrow), its indifference curves can be plotted in Fig. 19-4, and some optimal point along the frontier UU will be selected. There are some who believe that governments consciously seek some overall optimum as just described. It is difficult to explain political behavior with such a model.

19.3 THE CLASSICAL "THEOREMS" OF WELFARE ECONOMICS

In this section we shall present the classical "theorems" of welfare economics. The quotation marks are used because the propositions derived in what follows are not in fact refutable theorems. They represent generally unobservable first-order conditions for maximization, i.e., statements that at an optimum, marginal benefits equal marginal costs. As was indicated in the quotation from Samuelson's *Foundations of Economic Analysis,* these propositions represent the logical implications of propositions that are not themselves refutable.

The first "theorem" is that *perfect competition leads to a Pareto-optimal allocation of goods and services*. This proposition holds only under certain restrictive conditions. Specifically, the formulation of the problems posed in the previous section ruled out two major classes of phenomena: Interdependence of the consumer's utility functions and interdependence of the production functions. In the preceding presentation, there were no *externalities,* or side effects, present between any of the maximizing agents. Such interdependence would be indicated by writing, say,

$$y = f(L_y, K_y, x) \qquad (19\text{-}31a)$$

or

$$U^2 = U^2(x_2, y_2, U^1) \qquad (19\text{-}31b)$$

In the case of (19-31a), the output of y depends not only on the labor and capital inputs in the production function for y but also the level of x produced. In a later section we shall consider a particular example of this, where the output of a farm depends in part on a neighboring rancher's output of cattle, who trample some of the farmer's output. Similarly, (19-31b) indicates that another person's happiness is an influence on one's own utility.

In the absence of occurrènces (19-31a) and (19-31b) and in the absence of monopoly, the prices of goods and services offered in the economy will equal their respective marginal costs of production. The condition for profit maximization under competitive factor and output markets yields, for each industry h,

$$p_h f_i^k - w_i = 0 \qquad i = 1, \ldots, n \qquad (19\text{-}32)$$

where

$$\begin{aligned} f^k(x_1, \ldots, x_n) &= k\text{th firm's production function} \\ w_i &= \text{wage of } x_i \\ p_h &= \text{output price} \end{aligned}$$

Suppose there are m firms. The supply function of the firms is the solution of

$$p_h - \frac{\partial C_k^*}{\partial y_k} = 0 \qquad (19\text{-}33)$$

where $C_k^*(y_k, w_1, \ldots, w_n)$ is the firm's total cost function. From (19-32),

$$\frac{w_i}{w_j} = \frac{f_i^k}{f_j^k} \qquad k = 1, \ldots, m \tag{19-34}$$

This is precisely the condition that the economy be on the production possibilities frontier: The ratio of marginal products for all pairs of factors is the same for all firms, equal to the ratio of factor prices.

Moreover, utility-maximizing consumers with utility functions $U^k(y_1, \ldots, y_n)$ in the n output goods will set the ratios of marginal utilities equal to the price ratios; i.e.,

$$\frac{U_i^k}{U_j^k} = \frac{p_i}{p_j} \qquad \text{for all } i, j, k \tag{19-35}$$

Since all consumers will face the same prices, Eq. (19-35) says that all consumers' marginal evaluations of the good will be identical, the condition for efficient consumption for given outputs. Lastly, using Eq. (19-33),

$$\frac{U_i^k}{U_j^k} = \frac{p_i}{p_j} = \frac{\mathrm{MC}_i}{\mathrm{MC}_j} \qquad \text{for all } i, j, k \tag{19-36}$$

Hence, not only are all consumers' marginal evaluations equal, they are equal to the ratio of marginal costs of those goods, expressed in money terms. This ratio of money marginal costs is precisely the marginal cost of good i, in terms of good j forgone. That is, converting to units of good j makes $\mathrm{MC}_j \equiv 1$. [Note that the units of $\mathrm{MC}_i/\mathrm{MC}_j$ are $(\$y_i) \div (\$y_j) = y_j/y_i$, the amount of y_j forgone to produce another increment of y_i, or the *real* marginal cost of y_i.]

Thus, under perfect competition with no side effects (externalities), the Pareto conditions for overall efficiency hold. Therefore, in such a perfectly competitive economy, no individual will be able to improve himself or herself without making someone else worse off.

It does *not* follow from the preceding that it is desirable for the economy to be perfectly competitive. Consider Fig. 19-5, where the grand utility frontier *UU* has been plotted. Suppose, somehow, the economy has situated the two individuals at point *A*, a non-Paretian allocation. Any movement to the right or upward from *A*, resulting in a point on the utility frontier along the segment *BC*, is clearly Pareto-superior to *A*. However, a movement to *D*, a Pareto-optimal point, leaves consumer 2 worse off; it is not an improvement from consumer 2's standpoint. Hence, aside from being a value judgment, a move to the Pareto frontier may involve losses.

The second "theorem" of classical welfare economics is the statement that there is an allocation under perfect competition for any overall Pareto optimum. That is, starting now with a point on the Pareto frontier, there exists a competitive solution which achieves that optimum. The proof of this proposition, for general functional forms of utility and production functions, is a formidable mathematical

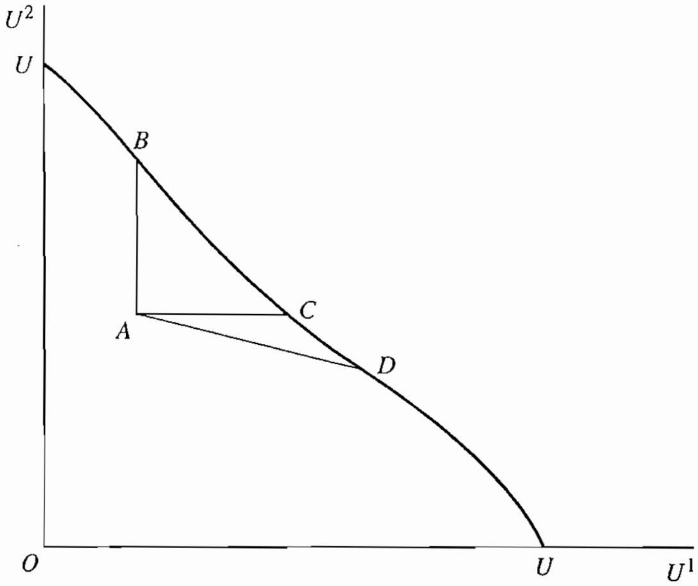

FIGURE 19-5

A Non-Pareto Move. Suppose the economy is at point A. Then any move northeast will be to a Pareto-superior position: Each consumer will gain. Any point along the segment BC of the Pareto frontier UU is Pareto-superior to A. However, not every point along UU is Pareto-superior to A. Point D, for example, leaves consumer 1 better off and consumer 2 worse off than at A. Consumer 2 will not advocate economic efficiency if it results in the economy's moving to point D. It is not possible to argue, even with the weak Paretian value judgment, that the economy "ought" to be at a Pareto-optimal point.

problem, which has been analyzed by K. Arrow,[†] G. Debreu,[‡] L. Hurwicz,[§] and others. A rigorous discussion is considerably beyond the scope of this book.

Note what this second "theorem" does *not* say: It does not say that in order to achieve a Pareto position the economy must be competitive. An omniscient dictator could mandate the correct prices and quantities so that the economy would reach the same position as a competitive economy would.

Two of the outstanding reasons why an economy might not be on the overall Pareto frontier are (1) excise taxes and (2) monopolistic raising of price over marginal cost. With regard to the latter, a *perfectly discriminating monopolist,* who extracts all the gains from trade via some sort of all-or-nothing pricing, does *not* disturb the Pareto conditions. The reason, fundamentally, is that all the gains from trade are exhausted. The only difference is that only the perfectly discriminating monopolist

[†]The principal investigation of both the above theorems is in K. Arrow, "An Extension of the Basic Theorems of Classical Welfare Economics," in J. Neyman (ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,* University of California Press, Berkeley, 1951.

[‡]Gerard Debreu, *Theory of Value,* John Wiley & Sons, Inc., New York, 1959.

[§]Leonid Hurwicz, "Optimality and Informational Efficiency in Resource Allocation Processes," in *Mathematical Methods in the Social Sciences 1959,* Stanford University Press, Stanford, CA, 1960.

gains, whereas with open markets the buyers and sellers both gain. But as long as all the gains from trade are exhausted, there can be no Pareto-*superior* moves.

19.4 A "NONTHEOREM" ABOUT TAXATION

A commonly stated proposition is that to raise any given amount of tax revenue it is best, from the standpoint of consumers' achieving the highest possible indifference curve, to collect those taxes via proportional excise taxes or income taxes. (With no savings in the economy, these taxes are equivalent.) The argument is loosely based on the observation that the Pareto conditions $p_j/p_i = \text{MRS}^k = \text{MC}_j/\text{MC}_i$ would not be disturbed if $p_j = (1+t)\text{MC}_j$, where the tax rate t is constant across all commodities. This, however, is a logical error, since these first-order marginal conditions for Pareto optimality, while necessary, are not sufficient. Other criteria may lead to the same conditions.

The "theorem" has been criticized on the empirical grounds that not all goods are easily taxed. A person's labor-leisure choice is affected by any tax on income. The price of leisure is the forgone wage; a tax on that wage income is a *subsidy* on leisure. In addition, many commodities, for more or less technological reasons, may be difficult to tax, e.g., services one provides for oneself or family. Under these conditions, a proportional tax on all taxable items is not a proportional tax on all items.

These empirical matters aside, however, a correct theorem is difficult to state. Even if one could tax all goods and services proportionately, this would not in general lead to a Pareto allocation, as we shall presently see. The most famous "proof" of this nontheorem was presented by Harold Hotelling in 1938.† Hotelling's proof went essentially as follows. Suppose a consumer currently consumes n goods, $q_i, i = 1, \ldots, n$, at prices $p_i = \text{MC}_i$. The consumer's *income* is taxed, however, and money income after taxes is $m = \sum p_i q_i$. Since the commodity bundle $\mathbf{q} = (q_1, \ldots, q_n)$ was chosen at prices $\mathbf{p} = (p_1, \ldots, p_n)$ and income m, any other bundle of goods $\mathbf{q}' = \mathbf{q} + \Delta\mathbf{q}$ that the consumer *could* have chosen must be inferior. Hotelling was asserting (without using the phraseology, which was not yet invented) that $\mathbf{q}$ was *revealed preferred* to $\mathbf{q}'$ if

$$\sum p_i q_i \geq \sum p_i q_i'$$

or

$$\sum p_i \Delta q_i \leq 0 \tag{19-37}$$

†H. Hotelling, "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates," *Econometrica,* **6**:242–269, 1938; reprinted in *A.E.A. Readings in Welfare Economics,* Richard D. Irwin, Homewood, IL, 1969.

Now suppose prices are changed by amounts Δp_i, representing excise taxes, but money income (tax) is also changed so that the consumer can have the same opportunities to purchase goods as before. By definition,

$$m + \Delta m = \sum (p_i + \Delta p_i)(q_i + \Delta q_i)$$

Subtracting $m = \sum p_i q_i$ gives

$$\Delta m = \sum p_i \Delta q_i + \sum \Delta p_i (q_i + \Delta q_i)$$

Rearranging terms, we have

$$\sum p_i \Delta q_i = \Delta m - \sum \Delta p_i (q_i + \Delta q_i) \tag{19-38}$$

Consider this last equation. The term $q_i + \Delta q_i$ represents the q_i's sold if taxed; hence, the last term represents the total tax revenue from the excise taxes, Δp_i, $i = 1, \ldots, n$. The term Δm represents the change in income taxes. Therefore, this expression says that if the change in excise taxes results in revenue absolutely greater than or equal to the income tax change, $\sum p_i \Delta q_i \leq 0$. In this case, it is argued, that since prices were set at marginal costs, replacing income taxes by excise taxes leads the consumer to purchase some bundle q' which was shown to be revealed inferior to q. Hence, to quote Hotelling,[†]

> *If government revenue is produced by any system of excise taxes there exists a possible distribution of personal levies among the individuals of the community such that the abolition of the excise taxes and their replacement by these levies will yield the same revenue while leaving each person in a state more satisfactory to himself than before.*

This "proof," however, seems to be merely a theorem about revealed *preferences*. Starting at *any* set of prices whatsoever, making the just stated changes in prices and income will leave the consumer worse off. Nowhere is the condition $p_i = \text{MC}_i$ used in this "proof." That marginal condition is irrelevant to the argument. No assumptions about *production* are contained in the argument; only assumptions concerning *preferences* are used. The same "proof" follows if initially $p_i \neq \text{MC}_i$ and the Δp_i's and m are changed so as to make $p_i = \text{MC}_i$ in the final position.

19.5 THE THEORY OF THE SECOND BEST[‡]

The problem of optimal excise taxation cannot be handled without considering the ends of this taxation. Suppose there are *three* goods—two private goods, x and y, and

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[†]Italics in the original. There is no apparent distinction in Hotelling's paper between income tax, proportional excise tax, and lump-sum or personal-levy tax.

[‡]R. G. Lipsey and K. Lancaster, "The General Theory of the Second Best," *Review of Economic Studies*, **24:**11–32, 1956.

government services, z. If these government services are services for which normal pricing is possible, e.g., postal services, the optimal taxes are zero. The government merely sells its services at marginal cost, which, together with selling x and y at their respective marginal costs, will yield a Pareto optimum. The question of optimal taxation makes sense only in the context that some good, say the services of the government, is not, for some reason, to be sold at marginal cost. In some cases, e.g., national defense, it would be difficult to do so. Also, an important class of goods exists, e.g., the so-called public goods discussed in the next section, for which marginal costs are less than average costs—the declining-AC industries. It is impossible to sell these goods at marginal cost without subsidies raised via taxation. The question thus becomes: Suppose some good z is not sold at marginal cost. Is it possible to infer that consumers will be on the highest indifference curves if the remaining goods are sold at prices proportional to their marginal costs, e.g., by proportional excise or income taxes? The answer is no, as the following argument shows.

Consider the simplest case of one consumer. The consumer maximizes utility subject to the production possibilities frontier, or

maximize

$$U(x, y, z)$$

subject to

$$g(x, y, z) = 0$$

The Lagrangian is

$$\mathcal{L} = U(x, y, z) + \lambda g(x, y, z)$$

producing the first-order conditions

$$U_x + \lambda g_x = 0 \qquad U_y + \lambda g_y = 0 \qquad U_z + \lambda g_z = 0$$

or

$$\frac{U_x}{U_y} = \frac{g_x}{g_y} \qquad \frac{U_z}{U_y} = \frac{g_z}{g_y} \tag{19-39}$$

The marginal rates of substitution equal the respective marginal costs. Suppose now that z is not sold at MC. A simple constraint which expresses this is $U_z = kg_z$, where $k \neq U_y/g_y$. Let us now maximize $U(x, y, z)$ subject to this new constraint also, in addition to the resource constraint $g(x, y, z) = 0$. The Lagrangian for this problem is

$$\mathcal{L} = U(x, y, z) + \lambda g(x, y, z) + \mu(U_z - kg_z)$$

The first-order conditions for this maximization are (excluding the constraints)

$$\mathcal{L}_x = U_x + \lambda g_x + \mu(U_{zx} - kg_{zx}) = 0$$

$$\mathcal{L}_y = U_y + \lambda g_y + \mu(U_{zy} - kg_{zy}) = 0$$

$$\mathcal{L}_z = U_z + \lambda g_z + \mu(U_{zz} - kg_{zz}) = 0$$

Since the constraint $U_z - kg_z$ is assumed to be binding, $\mu \neq 0$. Solving for the marginal rates of substitution,

$$\frac{U_x}{U_y} = \frac{-\lambda g_x - \mu(U_{zx} - kg_{zx})}{-\lambda g_y - \mu(U_{zy} - kg_{zy})} \tag{19-40}$$

with a similar expression for U_x/U_z or U_y/U_z.

The left-hand side of Eq. (19-40) is the MRS between x and y. It cannot be inferred that this MRS should be equal to $\text{MC}_x/\text{MC}_y = g_x/g_y$. For arbitrary values of the cross-partials U_{zx}, g_{zx}, U_{zy}, and g_{zy}, *nonproportional* excise taxes on x and y will in general satisfy (19-40). It might be noted that if these cross-partials are all 0, Hotelling's "theorem" holds, but this is a special case.

In general, therefore, it cannot be argued that if some distortion, that is, $p_j \neq \text{MC}_j$, is removed in the economy, consumers will move closer to the Pareto frontier if other distortions are present. If the industries involved are unrelated, a case might be made that the above cross-partials are 0. In that case, a more efficient allocation is implied by removal of the distortion.

Hotelling correctly argued for a nondistorting, or lump-sum, tax. As previously mentioned, an income tax is a subsidy on leisure and hence distorts the labor-leisure choice. A poll tax is cited as an example of a lump-sum tax. More precisely, an existence tax is advocated. Even with this type of tax, however, we shall find, in the long run, less existence, i.e., fewer children, less spent on lifesaving devices, etc. For all practical purposes, it is probably safe to conclude that there is no such thing as a lump-sum tax.

19.6 PUBLIC GOODS

There is an important class of goods that have the characteristic of being *jointly consumed* by more than one individual. These goods, known as *public goods,* are goods for which there is *no congestion*. Ordinary private goods are goods for which congestion is so severe that only one person can consume the good.

The most famous example of a public good is perhaps the service national defense. The protection afforded any individual by the nation's foreign policy and military prowess is substantially unaffected if additional recipients are added to that service flow. Similarly, driving on an uncrowded freeway, watching a movie or play in an uncrowded theater, or watching a television program are services for which the marginal cost, in terms of resources used up, of accommodating an additional consumer is essentially zero. These goods are the polar case of goods for which average costs are forever declining.

The problem such goods raise for welfare economic considerations is that the Pareto frontier is reached only if all goods and services are sold at their marginal cost of production. If public goods are sold at marginal cost, no revenues will be generated to finance the production of those goods. If production of the public good is financed by revenues derived from taxation of other goods, these other goods will be sold to consumers at prices other than marginal cost, thereby moving the economy off the Pareto frontier. The problems of second best, just discussed, apply to these goods.

Matters of financing aside, assuming that the public good is to be sold at marginal cost, that is, zero, what level of the good is to be produced in the first place, i.e., how many uncrowded highways, open-air concerts, etc., are to be produced? The *production* of public goods is not free; these goods are "free" only in the sense that the marginal cost of having an additional individual consume the good, once produced, is zero. In the case of private goods, this problem does not arise (except in the case of declining average costs). The goods are produced by profit-maximizing firms and sold at marginal cost. No private firm, however, could produce a public good and satisfy the Pareto condition $p = \mathrm{MC} = 0$.

Suppose there are two consumers with utility functions $U^1(x_1, y_1)$ and $U^2(x_2, y_2)$, where x is the public good and y is the ordinary private good. By definition of a public good, both consumers consume the total amount x of the good produced. Hence,

$$x_1 = x_2 = x \tag{19-41}$$

For the private good, as before, $y_1 + y_2 = y$. Suppose there is a transformation surface $g(x, y)$ defining the production possibilities frontier for the economy. The Pareto optimum is achieved by solving

maximize

$$U^2(x, y_2)$$

subject to

$$U^1(x, y_1) = U_0^1 \qquad g(x, y) = 0 \tag{19-42}$$

with $y = y_1 + y_2$. The Lagrangian is

$$\mathcal{L} = U^2(x, y_2) + \lambda_1\left(U_0^1 - U^1(x, y_1)\right) + \lambda g(x, y) \tag{19-43}$$

Differentiating $\mathcal{L}$ with respect to x, y_1, y_2 and the multipliers, noting that $g_{y_i} = g_y(\partial y/\partial y_i) = g_y$, $i = 1, 2$, we have, denoting $U_{x_j}^j = U_x^j$, etc.,

$$\mathcal{L}_x = U_x^2 - \lambda_1 U_x^1 + \lambda g_x = 0 \tag{19-44a}$$

$$\mathcal{L}_{y_1} = -\lambda_1 U_y^1 + \lambda g_y = 0 \tag{19-44b}$$

$$\mathcal{L}_{y_2} = U_y^2 + \lambda g_y = 0 \tag{19-44c}$$

with the constraints

$$\mathcal{L}_{\lambda_1} = U_0^1 - U^1(x, y_1) = 0 \tag{19-45a}$$

$$\mathcal{L}_\lambda = g(x, y) = 0 \tag{19-45b}$$

From (19-44*c*), $\lambda = -U_y^2/g_y$. Substituting this in (19-44*b*) gives $\lambda_1 = -U_y^2/U_y^1$. Using these two expressions in (19-44*a*) leads to

$$U_x^2 + \frac{U_y^2}{U_y^1}U_x^1 - \frac{U_y^2}{g_y}g_x = 0 \tag{19-46}$$

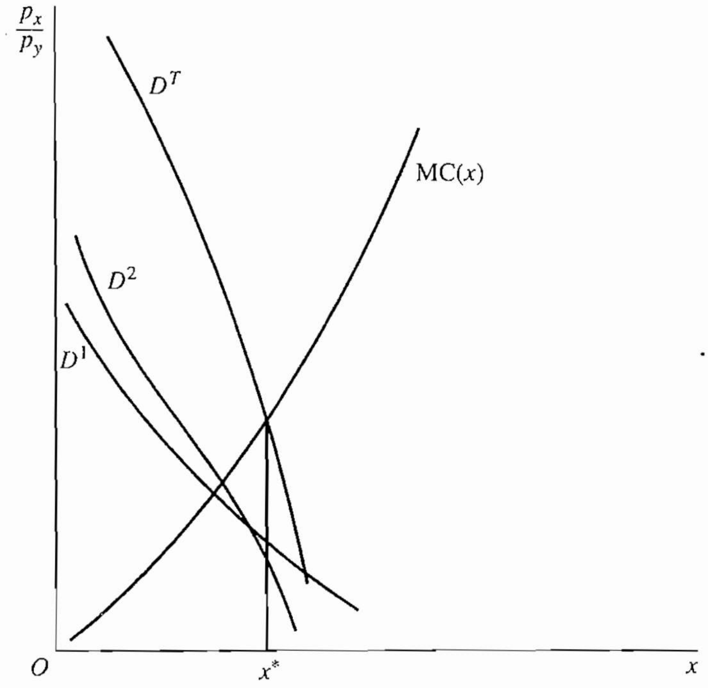

FIGURE 19-6
Market Demand for Public Good. If D^1 and D^2 are the two individual demand curves for the public good, the total demand D^T is the *vertical* sum of D^1 and D^2. That is, D^T represents the sum of each consumer's marginal evaluation of the public good. This vertical summation occurs because both consumers consume the total quantity of public good produced. The output x^* at which D^T intersects the marginal cost curve for producing x yields consumption on the Pareto frontier.

Dividing through by U_y^2 yields

$$\frac{U_x^2}{U_y^2} + \frac{U_x^1}{U_y^1} = \frac{g_x}{g_y} \tag{19-47}$$

Equation (19-47) admits of an interesting interpretation. U_x^1/U_y^1 and U_x^2/U_y^2 are, respectively, the marginal rates of substitutions, or the marginal evaluations, of the public good x. The expression g_x/g_y is the marginal rate of transformation of y into x or the marginal cost of the public good in terms of private good forgone. Since both consumers consume the *total* amount of x produced, the marginal benefits to society of the public good are the *sum* of each consumer's marginal benefits. Equation (19-47) therefore says that when the Pareto frontier is achieved, the total consumers' marginal benefits equal marginal cost. The usual reasoning of equating benefits and costs at the margin is preserved. The rule is adapted for goods with the characteristic of joint consumption.

Equation (19-47) says that to find the market demand curve for a public good, the individual demand curves are to be added *vertically,* as shown in Fig. 19-6. The market demand for ordinary goods is, of course, the *horizontal* sum of individual demands, because each consumer consumes a part of the total. For public goods, each consumer jointly consumes the total. The height of the individual demand curves, D^1 and D^2 in Fig. 19-6, are the marginal evaluations of the public good x.† The curve D^T

†The income being held constant in these demand curves is the total value of x and y given by the transformation surface $g(x, y) = 0$.

is the vertical sum of D^1 and D^2, representing the benefits of x at the margin to both consumers jointly. The quantity x^* where D^T intersects the marginal cost curve of producing x is the point that satisfies the Pareto conditions for production of a public good.

The preceding analysis generalizes in a straightforward manner to the case of K consumers. In that case, the Pareto conditions for public good production become

$$\sum_{i=1}^{K} \text{MRS}^i = \text{MC} \tag{19-48}$$

The problem of private production of public goods is that the ordinary market transactions are not likely to yield the Pareto allocation. In order to arrive at production of x at the level x^* where $\sum \text{MRS} = \text{MC}$, each consumer's differing marginal evaluations would have to be known. However, consumers will have no occasion to reveal these preferences. With private goods, consumers reveal their preferences by their choices in the market, purchasing additional units of a good until the marginal evaluation falls to the market price. There is no comparable mechanism for public goods. Each consumer consumes the total amount produced, and each has in general a different marginal evaluation of that good. Moreover, since the good is to be dispersed in total, it will pay consumers to understate their evaluation of the benefits of the good, lest the government attempt to allocate the good on the basis of fees based on each consumer's personal evaluations of benefits. Lastly, a fee charged for per unit use of the public good will result in "too little" consumption of the good. Consider the case of an uncrowded bridge. When a toll is charged, consumers will not cross the bridge if their marginal evaluation of the benefits is greater than zero but less than the toll. But since the resource cost to society for the consumer's use of the bridge is zero, the ideal Pareto optimum cannot be achieved. Thus, the ordinary contracting in the marketplace for public goods production is not likely to lead to an efficient allocation of resources in terms of the Pareto ideal.

19.7 CONSUMER'S SURPLUS AS A MEASURE OF WELFARE GAINS AND LOSSES

We have previously investigated the problems associated with defining, in units of money income, the gains from trade. One of the most prominent uses of these measures is the evaluation of costs and benefits of alternative tax schemes or the benefits of public good production. Let us briefly recapitulate these issues and apply the analysis to the problem of public good production.

Since the publication of Marshall's *Principles,* economists have attempted to measure the benefits of consumption by some sort of calculation based on the area beneath a consumer's demand curve. In Fig. 19-7, the height of the consumer's demand curve at each point represents the consumer's marginal evaluation of the good in terms of other goods forgone, measured in terms of money. It is therefore tempting to integrate, or add up, these marginal gains to arrive at the total gain received from consuming some positive level of the good rather than none at all.

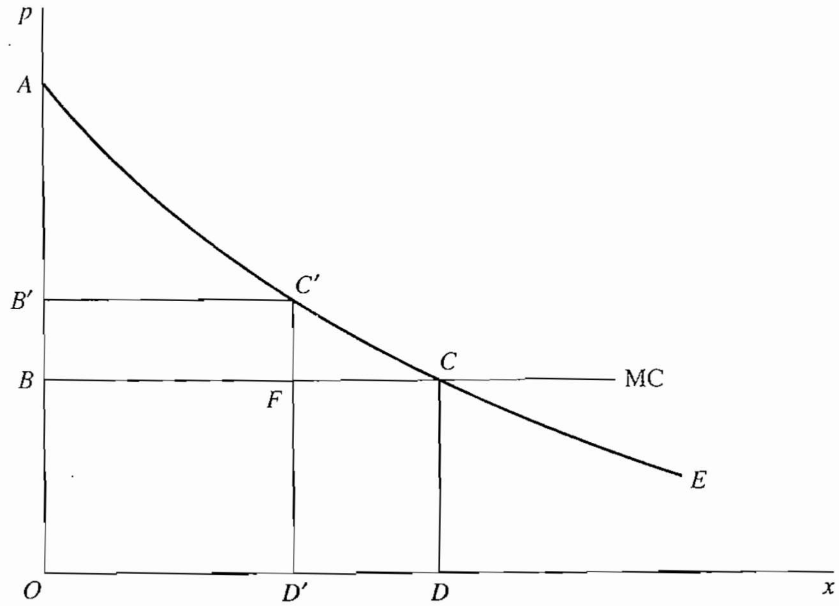

FIGURE 19-7
The Attempt to Measure Welfare Losses by Consumer's Surplus. The analysis of welfare losses is an attempt to have a money measure of the loss in utility incurred from selling commodities at prices other than marginal cost. Let OB = MC of x, and suppose $OD' = B'C'$ of x is sold at OB'. The traditional analysis asserts that the benefits from consuming OD is the trapezoidal area $OACD$. At price OB', total benefits are supposed to be $OAC'D'$. The difference, $D'C'\ CD$, is partitioned into $FC'C$ and $D'FCD$. The latter area is an amount of income spent on other commodities, presumed to be sold at marginal cost. The remaining area $FC'C$ is called *deadweight loss,* a money measure of the loss due to the price distortion BB'. This distortion is commonly attributed to excise taxation or monopolistic sale of x. If AE is a *real-income,* or *utility-held-constant,* demand curve, then although these areas represent well-defined measures of willingness to pay to face different prices, since these measures hold utility *constant,* they cannot very well measure utility *changes*.

However, we have seen in Chap. 11 that this is not possible. If the demand curve in Fig. 19-7 is a Hicksian, or *utility-held-constant* demand curve, the area $OACD$ represents the maximum dollar amount a consumer would pay to have OD units of x rather than none at all. It likewise follows that for these demand curves, ABC represents the maximum amount a consumer would pay for the *right* to consume x at unit price OB. If the license fee is actually paid, OD will be purchased and the consumer will remain on the same indifference level before and after the purchase, by definition of ABC as the *maximum* license fee the consumer would pay. This measure has the desirable property of being well defined and at least in principle observable.

If, on the other hand, the demand curve in Fig. 19-7 is a *money-income*-held-constant demand curve, the area ABC does *not* represent an observable quantity. The monetary value of gain in utility associated with the terminal prices OA and OB is generated by a line integral that is generally path-dependent; different adjustments of prices leading to the same initial and final price income vectors will generally lead to different monetary evaluations of the consumer's gain in utility. This is an inescapable index number problem for nonhomothetic utility functions. Only in the case of homothetic utility functions are changes in utility proportional to changes in income for any set of initial prices.

The quantity OD is special in the sense that the marginal benefits to consumers from x exactly equal the consumer's evaluation of the resources used to produce x in producing something else—the marginal opportunity cost of x. If there are no "distortions" of prices from marginal costs elsewhere in the economy, this occurrence is part of the Pareto conditions. However, if there are other goods whose prices differ from MC so that such efficient consumption levels do not occur, then, again, it is not possible to conclude that selling this good x at MC will lead the economy closer to the Pareto frontier. In general, if one good is sold at some price other than MC, say due to an excise tax on that good, then the set of excise taxes $(t_1, \ldots, t_n)$ on the n commodities in the economy which will lead to the Pareto frontier will not consist of zero tax rates on the other commodities, nor will they all necessarily be proportional to their respective marginal costs. The specification of such an optimal set of taxes $(t_1^*, \ldots, t_n^*)$, which leads the economy to the Pareto frontier for given deviations from MC of certain goods or for the purpose of financing government services, is too protracted a discussion to consider here.

Following the early French economist Dupuit, and stimulated greatly by Marshall's discussion of consumer's surplus, the monetary evaluation of the welfare loss associated with consuming OD' instead of OD units of x is usually given as the triangular area $FC'C$ in Fig. 19-7. The total benefits of consuming x are reduced by the trapezoidal area $D'C'CD$. However, the rectangular area $D'FCD$ represents income spent on other goods, *presumably at the marginal cost of those other goods,* eliminating this area as a part of welfare loss. The only remaining deadweight loss of the sale of x at price $OB' >$ MC is the area $FC'C$. Summing these areas over all commodities is commonly used to measure the welfare loss associated with a set of departures of price from marginal cost.

The compensating variation

$$M^*(\mathbf{p}, u) = -\int_{p_i+t_i}^{p_i} \sum x_i(\mathbf{p}, U)\, dp_i \tag{19-49}$$

represents the amount of money income *the consumer would be willing to pay* to face the prices p_i instead of $p_i + t_i, i = 1, \ldots, n$. (If some $t_i < 0$ and $M^* < 0$, M^* represents the amount a consumer *would have to be paid* to accept $p_i + t_i$ voluntarily instead of $p_i, i = 1, \ldots, n$.) The problem of using M^* as a measure of the benefits from increased utility is that M^* *depends only on one indifference level.* Utility is held constant in the integral (19-49). This may lead to inappropriate welfare rankings.

In Fig. 19-8, we set $p_y = 1$ arbitrarily. Since the vertical intercepts are $M/p_y = M$ in this case, changes in income can be read directly off the vertical axis. The consumer initially faces price p_x for x, producing the budget line emanating from A, with income OA. From the graph, the consumer is willing to pay an amount AB to have the price of x reduced to p_x' and willing to pay AC to have the price of x reduced to p''_x. Suppose $AB = \$10$ and $AC = \$20$. Suppose the consumer is actually going to have to pay \$5 (AB') to have p_x reduced

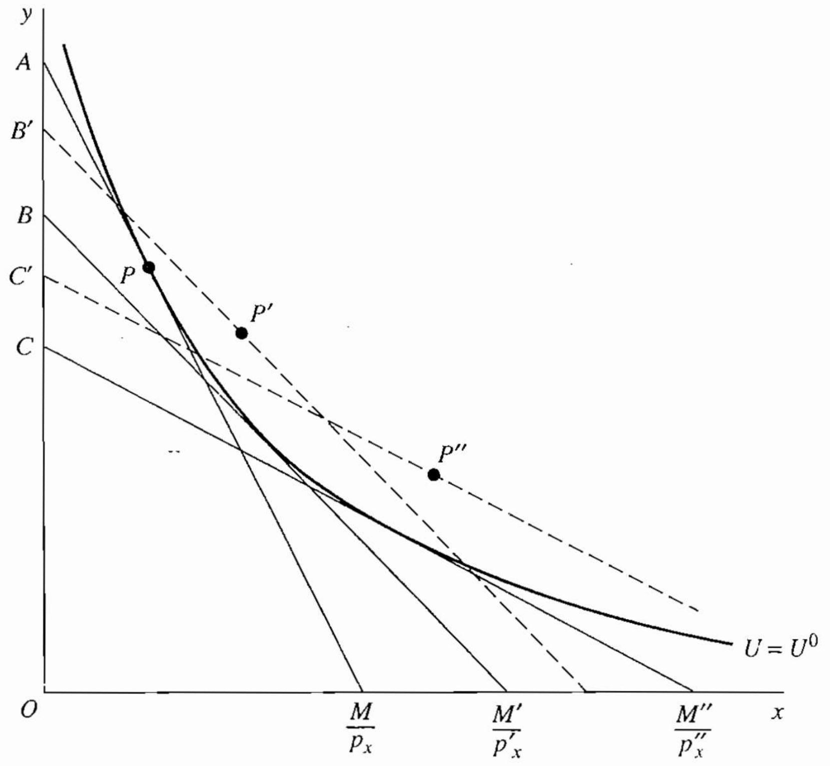

FIGURE 19-8

Measuring Gains from Trade by Compensating Variations in Income. The consumer has income M and faces prices p_x, $p_y = 1$. Money income is therefore measurable as distances along the vertical axis since the budget line intercepts that axis at $M/p_y = M$. At price p_x, the consumer consumes at point P on utility level U^0. The consumer is willing, according to this diagram, to pay amounts AB, AC to face the lower prices p'_x, p''_x, respectively. Suppose the consumer only *has* to pay AB', AC' to face those lower prices. Suppose the differences between what the consumer is *willing* to pay and what is actually paid, that is, BB' and CC', are not equal; e.g., suppose $BB' < CC'$. Can one infer that the second situation leaves the consumer on a higher indifference curve? Alas, no. For the first case, the consumer faces price p'_x with income OB', winding up at some point P'. In the second situation, the consumer faces price p''_x and income OC', winding up at some point P''. There is no way in general to tell which if either of P' and P'' is on a higher indifference level. The only indifference curve specified is $U = U^0$; no information is provided (except convexity) about where preferred indifference levels lie. Hence, the differences between compensating variations and actual costs of, say, two mutually exclusive projects may be unreliable measures of their ultimate benefits for consumers.

to p'_x or is actually going to pay \$14 (AC') to have p_x reduced to p''_x. Suppose AB' and AC' represent the cost of two alternative, mutually exclusive public works projects. Are these data sufficient to evaluate these projects in terms of answering which will place this consumer on a higher indifference level? Although the gain measured by the compensating variation minus the cost is greater for the second project, one *cannot* conclude that the consumer would be better off with it. With the first project, lowering p_x to p'_x, the consumer will wind up at some point P' on the budget line emanating from B' with slope p'_x. For the second project, the consumer will be at some point P'' on the budget line emanating from C'

with slope p_x''. Now within a broad range of price changes, there is no way to determine whether P' is more preferred or less preferred than P''. The reason is that nothing has been said of the properties of this consumer's utility function other than the one indifference curve $U = U^0$ from which all the compensating variations are derived. One must therefore conclude that integrals of the form (19-49) may not be reliable measures of gains from trade; they hold utility constant throughout.

19.8 PROPERTY RIGHTS AND TRANSACTIONS COSTS

The analysis of the Pareto conditions for economic efficiency has been presented in the absence of any institutional framework. We have assumed that production, exchange, and consumption take place without conflict. In actuality, production and exchange based on mutual benefit is not a universally admired principle; in many parts of the world such activities are severely proscribed by government edict. No society allows literally any mutually advantageous trade; but more importantly, the ability, or *cost* of engaging in trade can vary substantially from good to good, and from nation to nation. The extent to which trade takes place depends on the rights individuals have over the use of resources and the costs of exchange.

Robinson Crusoe will always achieve an efficient outcome given his preferences; he maximizes utility subject to his production constraint. The introduction of another individual, Friday, presents Crusoe (and Friday) with several more "margins" to consider. Gains through specialization are possible, but specialization requires agreement as to the terms of trade, and *enforcement* of the contract. Trade almost always involves "asymmetric information"; one usually knows better what one is giving up than what is about to be received. Crusoe and Friday will have to worry a bit about whether the other individual is living up to the terms of the contract. In modern societies, goods have many dimensions and are difficult to measure completely; production and exchange may involve many individuals, each with their own self-interest, and intruders, who would steal some of the goods, may be present. Whereas it is probably a useful first step to lay out the marginal conditions that must be satisfied in order for all gains from exchange to be exhausted, the empirical realization of such gains is subject to a society's laws and institutions that regulate commerce, and the transactions costs attendant upon production and exchange. Specialization could hardly take place, if, for example, stealing were rampant.

In recent years economists have taken renewed interest in the relationships between property rights and economic activity. While the Pareto conditions are generally unobservable, it is possible to show that certain institutions, or lack thereof, would make the achievement of the Pareto frontier very unlikely. The study of transactions costs, and how the structure of contracts changes to accommodate the realization of gains from trade under varying constraints, is an important new area of economics. Transactions costs are not the same as, say, a tax, which can be analyzed in the usual way by shifting a supply curve by the amount of the tax. Transactions costs are the lost gains from trade, due to imperfect monitoring of exchange, caused by the uncertainty of receiving what is bargained for.

A resource is "private" if it has three essential attributes:[†]

1. *Exclusivity*—an individual has the right to exclude others from use.
2. *Ownership of income*—an individual may derive (and keep) the income produced by the resource.
3. *Transferability*—an individual may transfer the resource to others at some mutually agreed upon price.

In modern societies, these rights may be varyingly enforced (and attenuated) by the government. These rights are almost never complete. Most land in the United States not held by the government is private in the preceding sense, but, for example, use and transferability may be restricted by zoning laws, rights to remove underground minerals may be restricted, etc. When the American west was settled in the nineteenth century, the "homestead acts" gave land title to individuals, but those individuals had to work that land themselves and could not resell the land, usually for about 10 years. Pollution from a factory is an attenuation of our right to breathe fresh air; rowdy neighbors infringe on our ability to enjoy the income produced by our homes, thus reducing the degree to which our homes are "private." Economic activity varies in important ways as the enforcement of private property varies.

An important polar case occurs when the right to exclude is completely absent. In that case, no one owns the resource; it is called *common property*. A prominent instance is provided by deep-sea fishing. Outside a country's territorial limits, varying from 3 to 200 miles offshore, unless covered by specific treaty, ocean resources, and specifically fish, are often not subject to effective ownership. Even when treaties are present, the ability to police limits on catches of fish in, say, the Pacific Ocean may be severely limited. In some countries, private land ownership is severely restricted or forbidden. In such cases we can usually predict the resource will be utilized beyond the level implied by the Pareto conditions.

Suppose the daily production of food takes place by combining labor, L, and land, K, according to some well-behaved production function $y = f(L, K)$; output y is sold competitively at price p. In Fig. 19-9, the (value of the) marginal and average product curves of labor are shown. Assume workers are available at daily wage w and that this wage represents the opportunity cost of labor in food production. That is, workers could produce nonagricultural output valued at w per day; for each worker in agriculture, nonagricultural output in the amount w is foregone per day.

Consider now two "stylized" systems of property rights.

[†]This categorization of private property was first presented by Steven N. S. Cheung in "A Theory of Price Control," *Journal of Law and Economics*, **17**(1): 53–72, April 1974. It is similar to the analysis presented in the first edition of Armen Alchian and William Allen's *Exchange and Production*, Wadsworth, Belmont, CA, 1964.

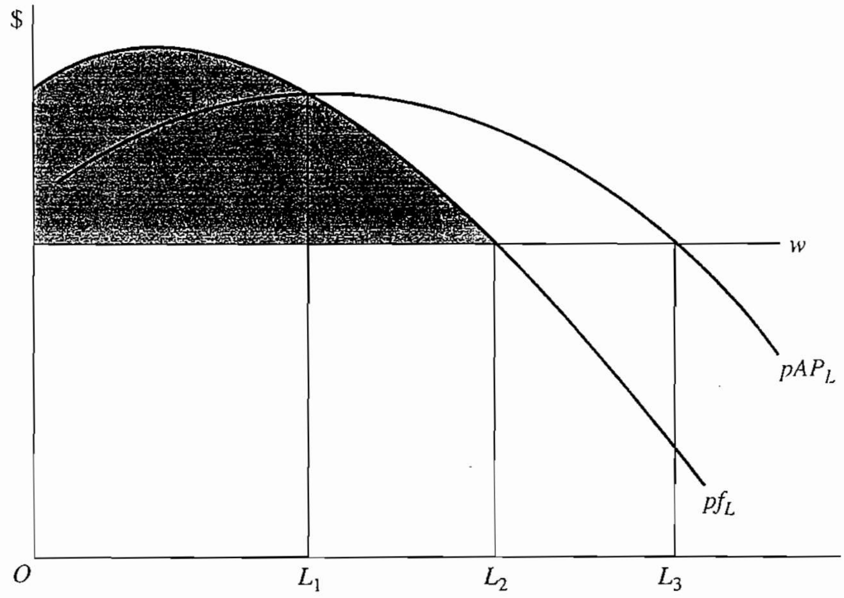

FIGURE 19-9
Allocation of Resources Under Private and Common Property. Faced with an opportunity cost of labor of w, a private owner of some resource, say land, will hire L_2 workers, where $pf_L = w$. This is an efficient allocation since the marginal value of goods produced in this firm equals the marginal value of labor elsewhere; no reallocation of labor could increase output. Under common property, workers crowd onto the land until *their* own return, which includes some share of the rents on the land, equals their opportunity cost elsewhere. Under this system, L_3 workers, where $pAP_L = w$, will work the land. This is an inefficient allocation since workers add only pf_L on the farm, less than their opportunity cost elsewhere.

PRIVATE PROPERTY. Suppose a fixed plot of land is privately owned by an individual. A private owner maximizes the rents on the land, i.e.,

$$\underset{L}{\text{maximize}} \quad R = pf(L, K) - wL$$

The first-order conditions for rent maximization are

$$pf_L = w$$

or input L_2 in Fig. 19-9. Since the area under the marginal product schedule is total product, the shaded area under pf_L and over the wage line w represents the maximum daily rent on the land.

The important aspect of this outcome is that the Pareto conditions are satisfied: the gains from trade are exhausted. For labor inputs $0 \leq L \leq L_2$, $pf_L \geq w$; thus the additional agricultural output generated exceeds the output lost in the other sector of the economy. Beyond L_2, the forgone nonagricultural output exceeds what the economy is getting in the way of food. No further mutual benefits can be realized by applying more workers to the land. The "invisible hand" is working: Private ownership leads to the greatest output gain to society, though the owner of this land neither knows nor intends that outcome.

COMMON PROPERTY. Suppose now that access to the land is unrestricted: Anyone can become a "squatter" on the land. For example, suppose agriculture is organized into "communes," with unrestricted entry. Anyone can join the commune and share equally in the output produced.† Since workers share equally in output, each receives the value of *average product*. In making their choice as to whether to join the commune, workers compare their alternative earnings w with their average product on the farm. At labor input levels less than L_3, workers earn more on the farm. This extra income derives from ownership of the land rents acquired when workers join the commune. With unrestricted entry, however, the rent on the land is *nonexclusive income*. Workers will compete with each other for ownership of this income, until, at the margin, it no longer exists (or exceeds the cost of acquiring it). In this example, workers will continue to join the commune until the marginal gain from joining (the average product) equals their alternative earnings w. When $pAP_L = w$, total product $= wL =$ total factor cost. The rents are *dissipated*.

This outcome is inefficient; i.e., further gains from trade are possible. At labor inputs greater than L_2, the marginal contribution to output when workers engage in farming, pf_L, is less than what workers could produce elsewhere, w. Resources are being directed to activities that lower, rather than increase, total output. If these extra workers could be induced to leave the farm, the resulting increment in output could in principle be shared, making everyone better off.‡ Nothing in the preceding argument depends on exhaustion of the land, as might especially be the case with ocean fish (though the problem exists with land also). In the case of deep-sea fishing, for example, preservation of the stock of fish for future harvest is an important margin. Increasing the catch this year may reduce the future stock of fish, raising the marginal cost of catching fish in the future. This is a separate and important issue. Under common property, valuable species may be depleted, perhaps to extinction, because no individual owns the right to any *future* income derived from preserving the resource. In that case, wealth maximization leads to shifting consumption to the present to a level where consumers' marginal value of present consumption of that good is less than its opportunity cost, in terms of the present value of future consumption forgone.

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†We ignore the problem of "shirking," which is perhaps the main reason this type of firm is not prevalent.

‡Other types of legal ownership can lead to different misallocations. For example, "socialist cooperative" firms, in which workers currently employed decide the labor input, and share, say, equally in the output, will maximize average product (at L_1 in Fig. 19-9), leading to *too little* agriculture production.

Freeway congestion is another common property problem. As was shown in Chap. 8 (Sec. 8.4), with no restrictions on access, cars enter the freeway until the average time cost equals the marginal (and average, if the "bad" roads are never congested) cost on the side streets. However, each car slows all the others; thus the sum of marginal time costs to all drivers will exceed the gain to any one driver who enters the freeway. As drivers compete for the rents received by access to the freeway (in terms of time saved), those rents are dissipated as all traffic slows down. Resources would be saved if some cars took the side streets. Under private ownership, a toll will be charged leading to the efficient outcome; under common property, the freeway is "overutilized."

Price controls typically create nonexclusive income. Suppose the market price of gasoline would be \$1.50 per gallon, but, in an attempt to transfer rents to consumers, the government fixes the price at \$1.00. If gas tanks hold 10 gallons, say, the price control would grant each driver a gift of \$5.00 per fill-up. However, this income is nonexclusive; it can be acquired only by the act of filling up one's gas tank. Car owners will compete for this gift. Though the exact form this competition will take depends upon the additional legal and economic restrictions attendant on the price control, the typical response, such as occurred in the 1970s (apart from some minor violence), is for drivers to compete by waiting in line for purchase. In so doing, the \$5.00 gain is at least partially dissipated by having to forego alternative, utility-increasing activities (including, perhaps, leisure). If consumers have identical alternative costs of time, given, say, by a marginal wage rate of \$5.00 per hour, the line will be 1 hour long, and the rent will be completely dissipated. The dissipation can be prevented by the issuance of freely tradeable ration coupons; in that case, the price of gasoline would again be \$1.50, \$1.00 in cash plus \$0.50 forgone by not selling the coupon to someone else. By giving exclusive title to the \$0.50 gain, the rents can actually be transferred to consumers.†

The Coase Theorem

The first systematic discussion of the role of transaction costs in relation to the allocation of resources was Ronald Coase's pathbreaking article, "The Problem of Social Cost."‡ The context of the misallocations were various "technological externalities"—the situation where production of one good was, in this case, a negative input in the production of some other good. The example first cited was the historically important case of straying cattle: A rancher-producer raises cattle who invariably trample some of a neighboring farmer's crop.

The classical welfare economic treatment of this problem, in the tradition of A. C. Pigou, took place as follows. Consider Fig. 19-10. The marginal private cost

†A less expensive procedure, however, is to simply tax the gasoline \$0.50 per gallon and return the receipts to consumers through some lump-sum tax not related to consumers' own purchases.

‡Ronald Coase, "The Problem of Social Cost," *Journal of Law and Economics*, pp. 1–44, October 1960.

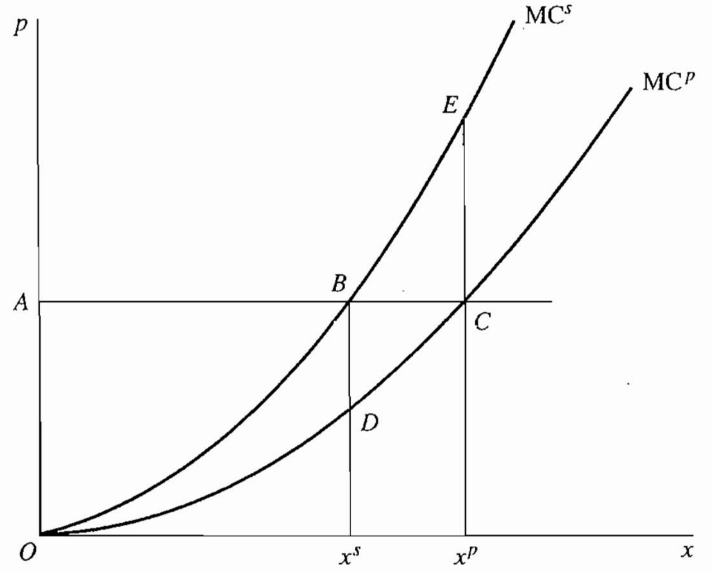

FIGURE 19-10
Private and Social Marginal Costs. Diagrams of this type have been used to indicate the difference between social and private costs. The curve MC^p misspecifies the marginal cost of producing cattle in the present example by excluding the cost of destroyed crops. The marginal destruction of crops, the side effect, or externality, is represented by the vertical difference between MC^s, social, i.e., actual marginal cost, and MC^p. It was formerly alleged that if the rancher was not legally liable for damaged crops, x^p would be produced. This analysis never came to grips with why individual maximizers would not reach the contract curve. In order to analyze where production will take place, the cost of contracting, i.e., the constraints on the rancher and farmer, must be specified. If these transactions costs are 0, the rancher and farmer are essentially one person. They will never allow x^p to be produced instead of x^s since the additional net profits gained *BCD* are less than the crop damage *BECD*. If the rancher is not liable for damage done to crops, the farmer will contract to pay the rancher more than *BCD* (but less than *BECD*) for the rancher to produce x^s instead of x^p. Since both parties will gain, such a contract is implied.

of cattle, disregarding the trampled crops, is labeled MC^p. This curve describes all the usual forgone opportunities of production, expressed in terms of costs of feed, land, shelter, fences, etc. However, an additional cost of production is also incurred by society. Each additional steer raised tramples some crops, lowering output of the adjacent farmer. With this damage treated as a cost of producing cattle, a marginal social cost curve MC^s is drawn, the difference $MC^s(x) - MC^p(x)$ being the marginal damage to the farmer's crops.

Suppose the price of cattle is *OA*. Traditional (Pigovian) welfare analysis held that unless the rancher were somehow made liable for the crop damage, the rancher would have no occasion to consider marginal *social* costs; cattle would be produced in the amount x^p, where $p = MC^p$. There would be a misallocation of resources in that too many cattle (and too little food) would be produced. The marginal conditions for Pareto optimality require that output x^s, where $p = MC^s$, be produced, since MC^s represents the actual marginal cost function for producing cattle.

If x^p is produced, the total private cost is the area OCx^p, the total damage to crops is the area between the two marginal cost curves *OEC*, and thus the total cost of production is the area OEx^p. At this level of output, resources are misallocated: At output greater than x^s, the marginal opportunity cost of producing cattle is greater than the marginal benefits to consumers, measured by the price OA. Producing x^p instead of x^s results in a deadweight loss in the amount of BEC.

Coase's contribution was to point out that the preceding argument could be valid only if the rancher and the farmer were somehow prevented from further contracting with each other. A misallocation of resources means that some mutual gains from trade or transacting are being lost. *If* the cost of transacting is zero (and no specific mention of transactions costs was presented), it cannot be that individual maximizers would arrive at some point *off* the contract curve. It would be a denial that more is preferred to less for two people to agree to a non-Pareto allocation or misallocation, of resources.

The assignment of legal liability for the wandering cattle constitutes a specification of *endowments* only. Any rancher who does not have to pay damages for trampled crops will be wealthier. It is an expansion of the rancher's property rights and an attenuation of the farmer's property rights. Likewise, a court ruling that the rancher *is* liable for crop damage is a transfer of assets only, from the rancher to the farmer, not a change in production possibilities or preferences. There is no reason why a change in endowments should foreclose a movement to the contract curve, i.e., the Pareto frontier. The classical theorems of welfare economics indicate that individuals will move to the contract curve irrespective of where the endowment point is placed in the Edgeworth box.

The error of assuming a non-Pareto solution hinged upon a failure to consider the range of contracting possibilities available to individuals, e.g., the rancher and farmer in the preceding case. If the rancher is liable for crop damage, no further contracting is necessary; the state enforces the contract that the rancher pay the farmer for damage. If the rancher is *not* liable, however, there are still options to consider. The farmer can contract with the rancher to reduce cattle production for some fee. Consider Fig. 19-10. The damage to the farmer's crops caused by producing x^p instead of x^s is the area $DBEC$. However, the net profit to the rancher derived from this extra production is only part of that area, DBC. Since the damage to the farmer is greater by the amount BEC than the gain to the rancher from producing x^p instead of x^s, the farmer will be able to offer the rancher more than DBC, the rancher's gain, but less than $DBEC$ to induce the rancher to reduce production to x^s. With no transactions cost, this contract is implied, since both the farmer and the rancher are better off. At any level of production beyond x^s, the damages to the farmer exceed the incremental gains to the rancher; both parties will gain by a contract wherein the farmer pays the rancher something in between these two amounts to reduce cattle production to x^s.

If transactions costs are not zero, forgone gains from trade may exist. To point this out, however, is to only begin the problem. The parties involved still have an incentive to consider various contracts to extract some of the mutual benefits. Different contracts have different negotiation and enforcement costs associated with them. Merger or outright purchase of one firm by another can be used to internalize side effects such as trampled crops. With merger or outright purchase, the rancher will produce x^s cattle, since it will now be the rancher's crops that are being trampled. We should expect to see individuals devising contracts that lead to the greatest extraction of mutual gains from exchange. In fact, this hypothesis is the basis for an emerging theory of contracts, based on maximizing behavior.†

The Theory of Share Tenancy: An Application of the Coase Theorem

Perhaps the first empirical application of Coase's analysis was the analysis of sharecropping by Steven N. S. Cheung.‡ Sharecropping is a form of rent payment in agriculture in which the landlord takes some share of the output, specified in advance, instead of a fixed amount, as payment for the use of the land (rent). This form of contract is somehow less enthusiastically regarded by many social reformers than the fixed-rent contract.

Sharecropping as a contractual form of rent payment came under attack by various economists on the grounds that it misallocated resources relative to the fixed-rent contract. In its neoclassical formulation, the rental share paid to the landlord was regarded as equivalent to an excise tax on the sharecropper's efforts, inducing sharecroppers to reduce output below the level where the marginal value product of the sharecropper equaled their alternative wage.

Consider Fig. 19-11. The top curve is the marginal product of labor. Under a fixed-rent or fixed-wage contract, labor input L_2 would be hired, where the marginal product of labor equals its alternative wage OM. Suppose, however, the tenant has contracted to pay r percent of the output to the landlord as payment for rent. Then the lower curve $(1-r)\mathrm{MP}_L$ represents the tenant's marginal product curve net of rental payments. It is tempting to conclude that the tenant, under these conditions, will produce at input level L_1, an inefficient point since there the true marginal product of labor is higher than its next best use, measured by the wage line w.§

The argument is correct up to this point. A *tax* on labor of r percent of the tenant's output would indeed lead the tenant to produce at L_1. The mistake is to apply this tax analysis to sharecropping, a situation in which a landlord and tenant voluntarily *contract* with each other. Again, the fundamental issue raised by Coase

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†Coase also showed that when transactions costs were not zero, it is not possible to deduce a priori which assignment of liability would reduce misallocation more. Consider the famous case of a railroad that occasionally sets fire to fields adjacent to the tracks because of sparks from the locomotive. If the railroad is made liable for all damage, the farmers lose an incentive to reduce the damage by not planting flammable crops too close to the tracks. The land close to the tracks may have as its highest value use a repository for sparks. On the other hand, if the railroad is not liable, it may run too many trains, i.e., produce beyond where $\mathrm{MC}^s = p$. One form of contract which may emerge is for the railroad to purchase land near the tracks, eliminating most, if not all, of the problems.

‡Steven N. S. Cheung, *The Theory of Share Tenancy,* University of Chicago Press, Chicago, 1969.

§Curiously enough, much social criticism of sharecropping appears to be based upon the landlord's working his tenants to an undue degree, perhaps, as we shall see, a more astute observation than the above economic argument.

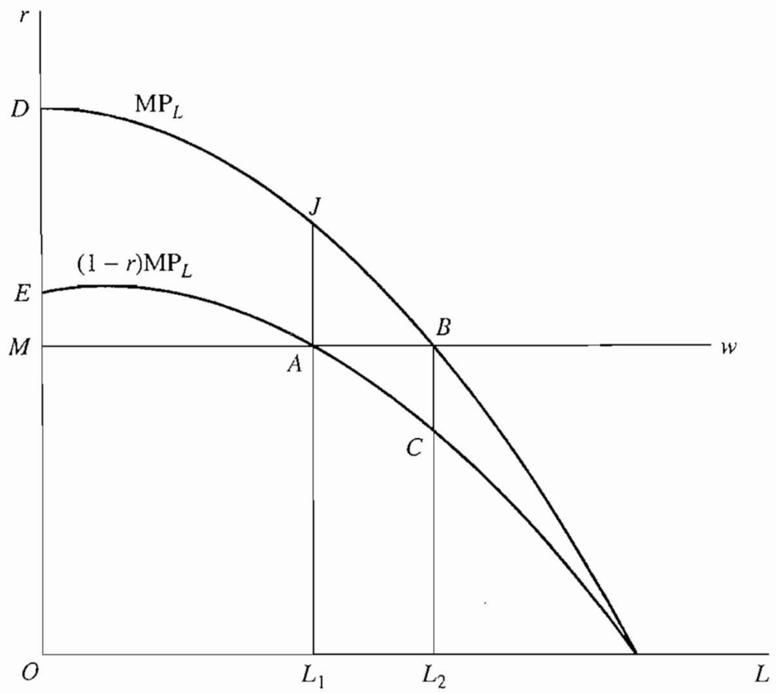

FIGURE 19-11
The Tax-Equivalent Approach to Sharecropping. This diagram has been used to show that sharecropping is an inefficient contract. Using a tax analogy, if MP_L is the marginal product of labor, and if r percent of the tenant's output is collected as rent, the net marginal product to the tenant is $(1 - r)MP_L$. With such a tax, the tenant would produce at L_1, where the actual marginal benefits MP_L exceed the opportunity cost of labor measured by its wage *OM*. This argument, while correct with regard to an excise tax on labor, cannot easily be extended to the case of sharecropping. In a share *contract,* many more variables are specified than the share itself. Farm size, nonlabor inputs in general, and labor inputs are negotiable. Under the postulate that the landlord maximizes the rent on the land subject to the constraint of competing for labor at labor's alternative cost, the Pareto condition $MP_L = w$ is implied. (From S. N. S. Cheung, *The Theory of Share Tenancy,* University of Chicago Press, Chicago, 1969, p. 43.)

is invoked: Why would utility maximizers get together and *not* exhaust the gains from trade? If L_1 instead of L_2 is used, the total output lost is L_1JBL_2, whereas the alternative cost to society of this labor differential, $L_2 - L_1$, is L_1ABL_2. Hence, mutual gains *JAB* are lost. Why should the landlord be willing to forgo this additional rental value on the land?

Applying the tax analysis to sharecropping amounts to assuming that the only variable that can be specified is the rental share or the wage rate. A contract, however, need not contain only one clause. It is possible to specify more than one variable in a contract. (Indeed, why else would contracts exist?) Even in the normal wage contract, often an informal agreement between employer and employee, the hourly wage is not the only thing specified. The employer expects the employee to show up on time, work a certain number of hours at some minimum level of intensity, etc. If only the wage were specified, maximizing behavior indicates that workers would show up and do no work at all. Real-world share contracts specify such things as amount of land to be cultivated, nonlabor inputs to be supplied by the tenant, "the droppings [of water buffalo] go to the [landowner's] soil," etc.† Under these conditions, the tax analysis is simply inapplicable. The test conditions of the experiment are entirely different.

That sharecropping as a contractual form is consistent with the Pareto conditions is shown by the following argument. Suppose the landlord owns an amount of land (capital) K. Labor is available at wage rate w, representing the alternative value of labor. The landlord can subdivide his land into m tenant farms, *where m is a choice variable*. Similarly, *the rental share r going to the landlord is not fixed but is also a choice variable*. Let the amount of labor supplied to each tenant farm be L. The amount of land supplied to each farm is $k = K/m$. The tenant's production function can therefore be written

$$y = f(L, k) = f\left(L, \frac{K}{m}\right)$$

The landlord will seek to maximize the rent on the land, $R = mry$. However, this is not an unconstrained maximization. Landlords must compete for tenants. Under this constraint of competition, the wage share to the tenant cannot be lower than the tenant's alternative earnings in wage labor. The model thus becomes

maximize
m, r, L

$$R = mrf(L, k)$$

subject to

$$wL = (1 - r)f(L, k) \tag{19-50}$$

From the constraint,

$$rf(L, k) = f(L, k) - wL$$

Hence, the problem can be posed in the unconstrained form when the variable r has been eliminated:

maximize
m, L

$$R = m[f(L, k) - wL] \tag{19-51}$$

Differentiating and remembering that $k = K/m$, we have

$$\frac{\partial R}{\partial m} = m\frac{\partial f}{\partial k}\left(-\frac{K}{m^2}\right) + [f(L, k) - wL] = 0 \tag{19-52a}$$

$$\frac{\partial R}{\partial L} = m\frac{\partial f}{\partial L} - mw = 0 \tag{19-52b}$$

†Cheung, op. cit.

From Eq. (19-52*b*), we immediately see that the landlord will contract with the tenant so as to set the (value of the) marginal product of labor equal to the alternative cost of labor. Thus, the labor input in Fig. 19-11 will be L_2, *not* L_1. The Pareto conditions will be satisfied. Substituting $w = \partial f/\partial L$ into Eq. (19-52*a*) and rearranging leads to

$$\frac{\partial f}{\partial k}k + \frac{\partial f}{\partial L}L = f(L, k) \tag{19-53}$$

Equation (19-53) is a statement of product exhaustion (*not* the Euler expression, which is an *identity*). The imputed value of land (capital) measured by its marginal product times the land input plus the same expression for labor equals the total output of the farm.

The share of output going to the landlord, $rf(L, k)$, from the original constraint in (19-50) and (19-53), is

$$rf(L, k) = f(L, k) - \frac{\partial f}{\partial L}L = \frac{\partial f}{\partial k}k \tag{19-54}$$

The landlord's share is precisely the imputed land value of the farm. In Fig. 19-11 this is the area MDB. *When r is chosen so as to maximize the rent of the land,* the landlord's share is also represented by the area $EDBC = MDB$. However, $EDBC$ is not the landlord's share for *any* arbitrary r, only for the rent-maximizing r. This rent-maximizing share, from (19-54), is

$$r^* = \frac{f_k k}{y}$$

This share is not determined by custom or tradition; it is a *contracted* amount. It varies with the fertility of the land, the cost of labor, and other variables specified in the share contract.

Showing that sharecropping is consistent with the Pareto conditions, however, is to merely state a normative condition. The interesting question of *positive* economic analysis is why the form of contract varies; i.e., why is it sometimes a fixed rent and other times a share contract? The reader is referred to Cheung for detailed answers to this question. We shall merely indicate here that some answers lie in the area of contracting cost and risk aversion. Share contracting is likely to be a more costly contract to enforce. However, to cite one example from agriculture, if the variance in output, due, say, to weather, is high, the landlord and tenant may *share the risk* of uncertain output by using a share contract. Indeed, empirical evidence from Taiwan indicates that share contracting is more prevalent in wheat than rice farming, wheat having a much higher coefficient of variation of output than rice. Other tests of these hypotheses are available.

It is generally uninteresting merely to pronounce some economic activity inefficient. The normative statements of welfare analysis are perhaps most useful if they are used to investigate why it is that certain ideal marginal conditions are being violated. The analysis then becomes positive rather than normative. Instead of labeling certain actions as irrational or inefficient, one asserts that the participants will seek to contract with each other to further exhaust the mutual gains from trade and one derives refutable propositions therefrom.

PROBLEMS

1. Explain why it is nonsense to seek the greatest good for the greatest number of people.
2. Suppose two consumers have the utility functions $U^1 = x_1^{1/3}y_1^{2/3}$, $U^2 = x_2^{2/3}y_2^{1/3}$. Suppose $x = x_1 + x_2$, $y = y_1 + y_2$ represent the total amount of goods available. Find the equation representing the contract curve for these consumers.
3. Suppose there are two goods x and y that are *both* public goods. There are two individuals whose entire consumption is made up of these two goods. There is a production possibilities frontier given by $g(x, y) = 0$. Find the marginal conditions for production levels of x and y that satisfy the Pareto conditions.
4. Suppose all firms except one in an economy are perfect competitors, the remaining firm being a perfectly discriminating monopolist. Explain why the Pareto conditions will still be satisfied. What differences in allocation and distribution of income result from that firm's not being a perfect competitor also?
5. Two farmers, A and B, live 8 and 12 miles, respectively, from a river and are separated by 15 miles along the river. The river is their only source of water. Pumphouses cost P dollars each and must be located on the river. Laying pipe costs $100 per mile. Once the pipe is laid and pumphouses installed, the water is available at no extra cost.
   (*a*) Do farmers have an incentive to minimize the total (to both farmers) cost of obtaining water?
   (*b*) If one pumphouse is used to supply both farmers, show that it will be located 6 miles from the point on the river closest to farm A. (Use either calculus or similar triangles.) What will the cost of water be for each farmer and totally in terms of P?
   (*c*) Suppose the farmers build their own pumps. What will the cost to each be and the total cost?
   (*d*) Show that in a certain range of pumphouse costs, one farmer will induce the other to share a pumphouse if transactions costs are low enough. (Assume for simplicity that pumphouse cost is shared equally. Then relax that assumption.)
6. Explain why the utility frontier must be downward-sloping and why it is not necessarily concave to the origin on the basis of the elementary properties of utility functions.
7. "Interdependencies in individuals' utility functions or in production functions will lead to non-Pareto allocations of resources." Evaluate.
8. The cases where markets allocate resources less efficiently than the Pareto ideal is often called *market failure.*
   (*a*) Why isn't the case where governments allocate resources less than the Pareto ideal called *government failure?*
   (*b*) Under what conditions will there be market failure?
   (*c*) Suppose, to cite a famous example, that in a certain region there is apple growing and beekeeping and that bees feed on apple blossoms. If the apple farmers increase their production of apples, they will allegedly increase honey production. The apple farmers acting alone will not, it is said, perceive the true marginal product of apple trees and hence will misallocate resources. Devise a model for this problem. Would the existence of actual contracts between beekeepers and apple farmers affect your conclusions as to whether market failure is a necessary consequence of production externalities, or interdependencies?

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