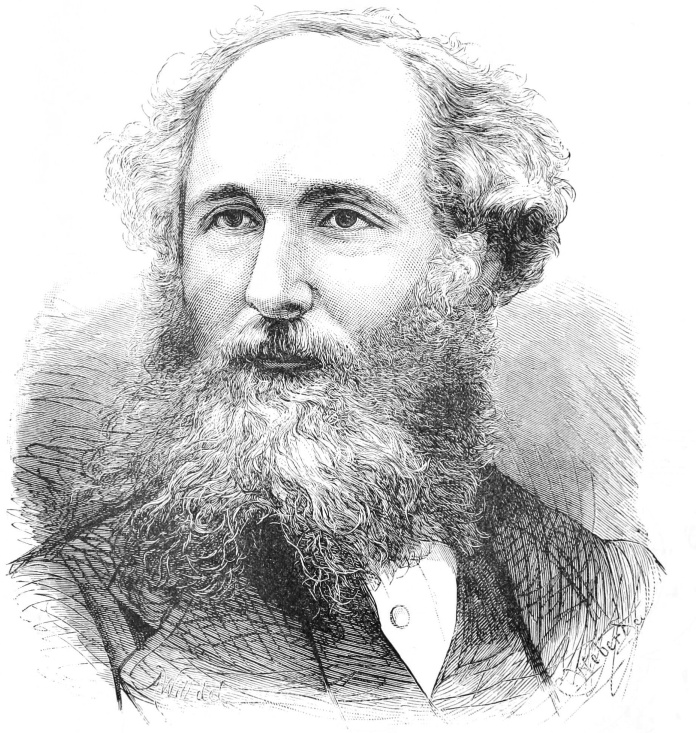
Eletromagnetismo Avançado

11 de agosto

Leis de conservação

Equações de Maxwell

Vácuo



$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = \mu_0 \left(\vec{\mathbf{J}} + \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right)$$

Ondas eletromagnéticas

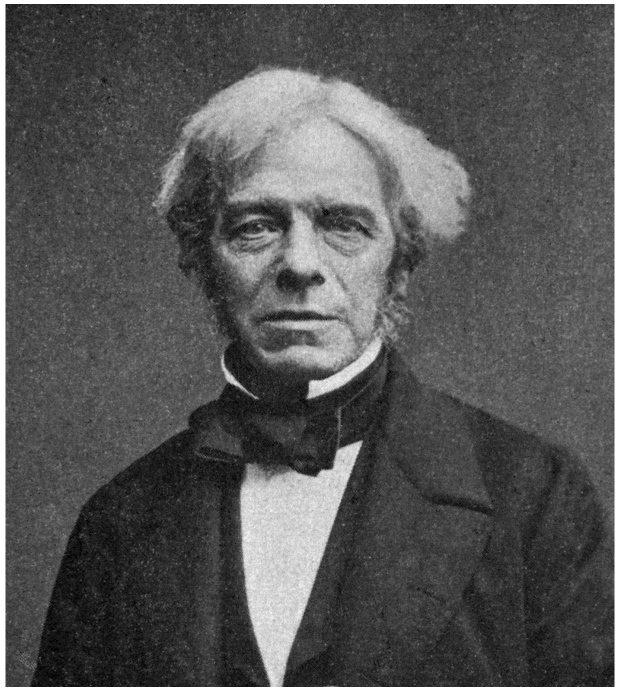
$$\nabla^2\vec{\mathbf{E}} = \frac{1}{c^2}\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2\vec{\mathbf{B}} = \frac{1}{c^2}\frac{\partial^2\vec{\mathbf{B}}}{\partial t^2}$$

Campo elétrico e potenciais

$$\vec{\nabla} \times \vec{\mathbf{E}} = 0 \quad \Rightarrow \quad \vec{\mathbf{E}} = -\vec{\nabla} V$$

Campo elétrico e potenciais



$$\boxed{\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}}$$

Transformações de gauge

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0 \quad \Rightarrow \quad \vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

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$$\vec{\mathbf{E}}' = -\vec{\boldsymbol{\nabla}}V - \frac{\partial \vec{\mathbf{A}}'}{\partial t} \quad \Rightarrow \quad \vec{\mathbf{E}}' = -\vec{\boldsymbol{\nabla}}V - \frac{\partial \vec{\mathbf{A}}}{\partial t} - \frac{\partial \vec{\boldsymbol{\nabla}}\lambda}{\partial t}$$

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$$V' = V - \frac{\partial \lambda}{\partial t} \quad \Rightarrow \quad \vec{\mathbf{E}}' = \underbrace{-\vec{\nabla} V' - \frac{\partial \vec{\mathbf{A}}'}{\partial t}}_{\vec{\mathbf{E}}}$$

Transformações de gauge

$$\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \vec{\boldsymbol{\nabla}}\lambda$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

Leis de conservação

Emmy Noether

(1882-1935)

Conservação da carga

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

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$$\boxed{\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{J}} + \frac{\partial \rho}{\partial t} = 0}$$

Conservação da energia

Capacitor carregado

$$W = \frac{q^2}{2C}$$

Meio condutor

- Carga decai
- Energia decai
- Como decai?

Conservação da energia

Capacitor carregado

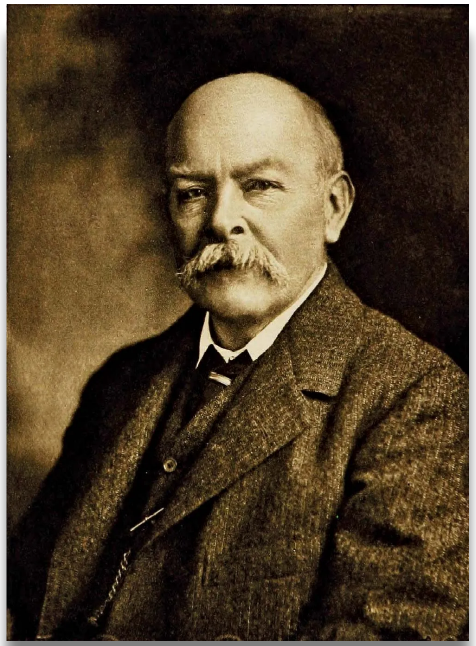
$$W = \frac{q^2}{2C}$$

Meio condutor

- Carga decai
- Energia decai

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau - \frac{1}{\mu_0}\int_{\mathcal{S}} \vec{\mathbf{E}} \times \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\,\mathrm{d}a$$

Conservação da energia



Vetor de Poynting

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$