

Ernst Křenek



STUDIES IN
COUNTERPOINT

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STUDIES IN COUNTERPOINT

Based on the Twelve-Tone Technique

By

ERNST KŘENEK



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CONTENTS

CHAPTER	PAGE
INTRODUCTION	vii
I. THE TWELVE-TONE SERIES	1
II. ONE-PART WRITING	3
III. TWO-PART WRITING	7
IV. DERIVATIVE FORMS OF THE SERIES	11
V. TWO-PART INVENTIONS USING O AND I	12
VI. TWO-PART WRITING USING ALL FOUR FORMS OF THE SERIES	15
VII. THREE-PART WRITING	19
VIII. EXTENSION OF THE RULES CONCERNING REPETITION OF TONES	26
IX. TRANSPOSITIONS OF THE SERIES-FORMS	28
X. DISPOSITION OF LARGER FORMS	32
APPENDIX—SPECIAL SERIES	36

INTRODUCTION

Tonality and Atonality Music organized technically by the device of major and minor keys is called "tonal" music. Music not organized by such means may be called "atonal", a term applied particularly to the music of the twentieth century in so far as this music lacks the criteria of tonality in the sense explained above.

It is undoubtedly possible to establish a broader definition of tonality. One might call tonality any method of setting up recognizable relationships between musical elements. In this sense, the system of major and minor keys, characteristic of a certain historical period, would represent but one out of many conceivable aspects of tonality, and music that does not comply with the postulates of this system should show some other system of elementary relationships, *i.e.* another type of tonality. Yet, as contemporary music written without keys is usually called "atonal" music, it simplifies the discussion if one confines the reference of the term "tonality" to the principle of major and minor keys. Moreover, this narrower definition is in accordance with the statements of the acknowledged dictionaries.

Arnold Schoenberg is quoted in the article on "Tonality" in *The International Cyclopaedia of Music and Musicians* (New York, 1939) as saying: "Music depends not only on acoustics but upon logic and upon those particular laws which result from the combination of tone and tune . . . Tonality, tending to render *harmonic facts* perceptible and to correlate them, is therefore not an *end* but a *means*." (italics mine)

If tonality is a means, what, then, is the end towards which tonality aims? It is obviously such a general organization of the musical material that musical formations may be perceived as logically coherent entities.

With the inevitable disintegration of tonality brought about by the evolution of music in the nineteenth century, there arose the question what new methods could be devised to create logically coherent forms in the atonal material.

The "Unifying Idea" Schoenberg proposed the twelve-tone technique as a means to this end. In a letter to Nicolas Slonimsky (published in Slonimsky's book *Music since 1900*, p. 574), Schoenberg writes concerning the origin of the twelve-tone technique: "I was always occupied with the aim to base the structure of my music consciously on a *unifying idea*, which produced not only all the other ideas but regulated also their accompaniment and the chords, the 'harmonies'." (italics mine)

Special care for creating unity within extended forms can indeed be traced in all of Schoenberg's works. Even in his early tonal compositions, he did not content himself with the elementary unity realized in key-relationships; but, upon these basic elements, he built a thematic superstructure of extraordinary compactness with regard to motif-relationships. His First String Quartet, for instance, a piece of unusual length and variety, is built on but a few basic thematic elements which appear again and again in manifold variations and combinations.

When key-consciousness vanished completely and music became "atonal", technical unity could no longer emerge from a solid harmonic groundwork. Quite logically, the

INTRODUCTION

attention was focused on the motif-relationships. Whereas they had formerly been a superstructure erected above the harmonic groundwork, they now became responsible for the consistency of the whole edifice.

Motif and Twelve-Tone Series Emphasis on motif-relationships of various kinds can be found in all of Schoenberg's "atonal" compositions, even before he developed the twelve-tone technique. Though he began to compose in the new idiom early in the twentieth century, it was not until 1923 that he published his first compositions in the twelve-tone technique. In this technique, the use of motif-relationships as the "Unifying Idea" for the new material is carried out with striking thoroughness.

The twelve-tone series—the fundamental feature of the twelve-tone technique—takes the place of the basic set of motifs out of which Schoenberg developed the various ideas of his tonal compositions. It can do so, because it comprehends the sum total of the available material—the twelve tones into which our octave is divided—and presents this material in a characteristic order.

Thus, the primary function of the series is that of a sort of "store of motifs" out of which all the individual elements of the composition are to be developed. By virtue of its ceaseless repetitions throughout the whole composition, however, the series accomplishes more than that: it assures the technical homogeneity of the work, by permeating its whole structure, like a red thread which, woven into a fabric, lends it a characteristic color shade, without ever becoming conspicuous as such.

Twelve-Tone Technique and Counterpoint The idea of tonality (as a means "tending to render harmonic facts perceptible") emanates from a basically *harmonic* conception of music. The essentials of tonality—such as the key, the dominant-tonic function, the tonal cadence—are harmonic phenomena. In so far as atonality depends for its organization upon motif-relationships, it apparently brings *melodic* phenomena to the fore.

Thus, the new idiom is based on an essentially polyphonic conception of music, very much related to the angle from which music was viewed in the Middle Ages, before tonality (in our sense of the term) had developed. Therefore it appears sound to approach atonality and twelve-tone technique by way of counterpoint. Harmonic facts in atonality have but a secondary significance, at least in the present stage of atonal development.

Purpose of the Present Book This book does not pretend to sum up or to codify the practice of the twelve-tone technique as it appears in the works of Schoenberg, his disciples Alban Berg and Anton Webern, and several other composers. This technique being still in the making and subject to change in every new work, the student would easily become confused should he begin by examining its manifold applications in the works of these composers.

The author wishes to set forth the elementary principles of the twelve-tone technique as he has applied it in a number of his own works, and in a way that has proved useful in teaching. The following pages cover approximately as much of the twelve-tone technique as can at present be unequivocally formulated in more or less definite rules. The talented student, working along these lines, will be able, after a relatively short period, to express

INTRODUCTION

himself logically and consistently in the atonal idiom. Then he may proceed to analyse the more involved applications of this technique and to get suggestions for his further attempts out of these studies.

It is the belief of the author that, in a later stage of development, atonal music may not need the strict regulations of the twelve-tone technique. He anticipates that the essentials of this technique will grow into a sort of second nature. This consummation, however, will materialize only if the twelve-tone technique is constantly used as a training for composing in the atonal idiom, just as the theory of classical harmony is taught as an introduction to "free" tonal composition.

As the twelve-tone technique is approached here from the viewpoint of counterpoint, the knowledge of strict (Palestrina) counterpoint is recommended as prerequisite, though not indispensable.

CHAPTER I

THE TWELVE-TONE SERIES

The twelve-tone series which are to be used in the following exercises are successions of twelve different tones. In the course of these exercises, the student will soon become aware of the importance of the intervals between the consecutive tones of the series, as their frequent recurrence will make them conspicuous melodic and harmonic landmarks. As the first exercises are rather short, this consideration is not yet paramount. Thus, in building up twelve-tone series, it is necessary to observe but a few essential rules:

1. Do not use series with too many equal intervals, because the repetition of the same interval will make it difficult to avoid monotony in the melodic development.
2. Avoid more than two major or minor triads formed by a group of three consecutive tones, as for instance:

Ex. 1



because the tonal implications emanating from a triad are incompatible with the principles of atonality.

As a series is not a theme, but only a basic pattern, it is advisable to write it down in even whole notes, without bar-lines.

The following is an example of a suitable series:

Ex. 2



All examples in the body of this book, in so far as they are related to the twelve-tone technique, will be based on the series of Ex. 2.

In the twelve-tone technique, only the order of succession of the tones in the twelve-tone series is relevant, regardless of their register. The following series are considered identical with Ex. 2:

Ex. 3



Ex. 4



Accordingly, in composition, the tones of the series can be used in any octave position, provided only the order of succession remains unchanged.

The use of # and ♭ is arbitrary. In tonal music, the meaning of

Ex. 5



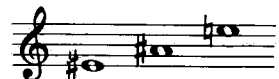
and

Ex. 6



and

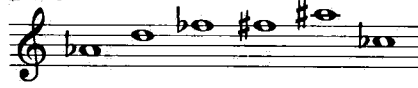
Ex. 7



is different, even when produced on keyboard instruments tuned in the tempered pitch, because of the coordination of these tones to different tonal centers. In atonal music, D♭,

for instance, is identical with C#, even when played on stringed instruments. A certain inadequacy of our notation for the writing of music without keys may be noticed; yet, for practical reasons, we have none the less to use this notation. If we write the following phrase

Ex. 8



rather in the form:

Ex. 9



we do so only from a desire for higher graphic clarity, not from considerations of tonal coordination.

CHAPTER II

ONE-PART WRITING

The student may begin immediately to write brief inventions, using his own twelve-tone series. Like exercises in strict counterpoint, these pieces may be conceived for voice; but it will be more useful and stimulating to write them for instruments (Violin, Viola, 'Cello, Flute, Clarinet, and the like), because of their greater mobility. The pieces should not, in general, exceed a length of twenty measures. Let us begin with a one-part invention.

A one-part composition in the twelve-tone technique consists of continuous repetitions of the basic series; in other words, when the twelve tones of the series have been used in the given succession, the last (twelfth) must be followed again by the first, and so on, until the piece ends with the twelfth tone of the last entry of the series. Once introduced, the series must be continued until its end.

In building melodies on the chosen twelve-tone series, it is essential to bear in mind the following considerations:

1. A theme is not necessarily identical with the series; on the contrary, that will only occasionally be the case. Therefore the cesuras between the themes (or, in general, between the articulated sections of the melodic line) should not coincide with the consecutive entrances of the series.
2. Repetition of tones is allowed *before* the following tone of the series is introduced, and within the same octave, as for instance:

Ex. 10



Repetition of a tone *after* the appearance of the forthcoming tone of the series is allowed in trills:

Ex. 11



tremolos:

Ex. 12



tremolo-like formations:

Ex. 13



and in groups where one note can be considered an auxiliary note:

Ex. 14



It is evident that repetitions of this kind can take place only within groups of two consecutive tones. Extensions of these rules will be given later (see Chapter VIII).

As to melodic, rhythmic, and metric shaping of the melody, there are no limitations other than those peculiar to the chosen instrument.

Since music written in the twelve-tone technique is not determined by tonal considerations, one-part inventions must show the highest degree of articulation in the melodic line itself. The plasticity of the melody will be greatly enhanced if the highest and the lowest tones occur only once. Rhythmic liveliness is a vital requirement of the style under consideration. The student will soon realize that the protracted use of unaltered rhythmic patterns results in a monotony less admissible in this style than in any other idiom. Symmetric periodicity produced by equal number of metric units (as in the four- and eight-bar periods of tonal music) is alien to the style, not only because the harmonic fundamentals of this periodicity are absent, but also because symmetric periods are not consistent with the contrapuntal character of this music.

Articulation being mainly a product of phrasing, the student should use expression marks to fit the metric, rhythmic, and agogic meaning of every note exactly.

The following example presents a short invention for Violin *solo*, built on the series of Ex. 2, which fulfils some of these requirements, but is inadequately phrased:

Ex. 15

Allegro moderato ♩ = 90

A single staff of music for Violin in 4/4 time. The tempo is *Allegro moderato* with a quarter note equal to 90 beats per minute. The piece consists of six measures, each numbered in a box above the staff. Measure 1 starts with a piano (*p*) dynamic. Measure 2 has a mezzo-forte (*mf*) dynamic. Measure 3 has a forte (*f*) dynamic. Measure 4 has a fortissimo (*ff*) dynamic. Measure 5 has a mezzo-forte (*mf*) dynamic. Measure 6 has a piano (*p*) dynamic. The piece ends with a fermata over the final note.

Discussion: The composer obviously intended to make a thematic statement in the first $1\frac{1}{2}$ measures, then to bring in a contrasting middle section (Measures 2-4), to reach a climax (Measure 4), and to end with a sort of reminiscence of the first theme.

Criticism: 1. The cesura between the theme and the middle section is not sufficiently pronounced. In a tonal invention, for instance:

Ex. 16

Allegro ♩ = 108

A single staff of music for Violin in 4/4 time. The tempo is *Allegro* with a quarter note equal to 108 beats per minute. The piece consists of eight measures, each numbered in a box above the staff. Measure 1 starts with a forte (*f*) dynamic. Measure 2 has a mezzo-forte (*mf*) dynamic. Measure 3 has a mezzo-forte (*mf*) dynamic. Measure 4 has a fortissimo (*ff*) dynamic. Measure 5 has a pianissimo (*pp*) dynamic. Measure 6 has a pianissimo (*pp*) dynamic. Measure 7 has a mezzo-forte (*mf*) dynamic. Measure 8 has a mezzo-forte (*mf*) dynamic. The piece ends with a fermata over the final note.

the cesura between Measures 4 and 5 (Ex. 16) might be sufficient, because the following contrasting section presents a new harmonic color (the subdominant) and the underlying metric scheme is constructed in four-measure symmetry. In atonal music, however, a break such as that occurring after the second beat of Measure 2 of Ex. 15 requires more emphasis in order to make the articulation perfectly clear.

2. The culminating point (Measure 4) is too closely followed by the coda, thus obscuring the formal design. The coda enters so suddenly that the middle section has no time to develop. Thus the formal structure of the whole invention becomes ill-balanced and indistinct.

Suggested improvements:

- 1.

Ex. 17

Allegro moderato ♩ = 90



The thematic element has more time to develop and is more clearly separated from the middle section.

2. The main purpose of the following improvement is to emphasize the culminating point (Measure 4 of Ex. 15). For this purpose, some minor changes in the middle section seem advisable although the original version is not definitely wrong. In order to make the climax more pronounced, it is desirable to quicken the motion in the vicinity of the culminating point rather than at the beginning of the section:

Ex. 18



It will be noted that the strong beats of the phrase (marked by $>$) do not coincide with the strong beats of the measure. In the first measure of Ex. 18, the strong beats are the first and the sixth notes, not the first and the fifth. In atonal music the musical phrase creates its own metre, independently of the time-signature which merely serves the purpose of facilitating the survey. Ex. 18 could also be written in this way:

Ex. 19



Both ways of writing the passage result in the same musical effect, and differ only in their metric arrangement.

STUDIES IN COUNTERPOINT

3. In order to prepare the coda it is necessary not to drop too abruptly the elements introduced in the middle section. One might suggest, for instance:

Ex. 20

In the element a) (Ex. 20), the culminating point (last measure of Ex. 18) is recalled as a sort of echo. The motion of the middle section slows down from sixteenth notes to triplets in eighths, and finally to the simple eighth notes, the sonority being reduced at the same time by means of the *pizzicato*.

The addition of these new thematic fragments makes it necessary to add one more entrance of the series. (The series ends with the last note of the triplet in the third measure of Ex. 20; the following two tones, D and G♯, belong to a new entrance of the series which, according to rule, must be continued to its end.)

By using the remaining ten tones of the series, we are able to write the coda as a recapitulation of the first thematic element (Ex. 17), in the form of a free inversion:

Ex. 21

and, for a finish:

Ex. 22

The whole invention, with all improvements from Ex. 17 to Ex. 22 incorporated into it, now reads thus:

Ex. 23

This example has been discussed in detail in order to demonstrate the considerations that should guide the student in working on further exercises.

CHAPTER III

TWO-PART WRITING

1. *Intervals*

In two-part writing, the following rules for the intervals between the parts should be observed:

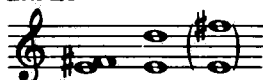
1. The octave is not allowed. The fact that one tone of the octave interval is felt as almost identical with the other (without, however, being actually identical, as is the case with the unison) creates the impression of a complete standstill of the musical flow, thus interfering with the principle of continuous tension and motion essential to atonal music.
2. The two parts may come together occasionally in the unison, but they must not move in parallel unisons.
3. The following intervals are regarded as consonances:

Ex. 24



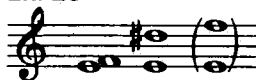
4. Dissonances are distinguished by their degree of tension.
 - a. Dissonances of lower tension ("mild" dissonances):

Ex. 25



- b. Dissonances of higher tension ("sharp" dissonances):

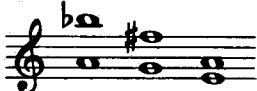
Ex. 26



The degree of the tension may be explained by vibration-ratios, combination-tones, or other acoustical phenomena; yet, the decision of what shall be considered a dissonance and how it should be handled is an arbitrary assumption inherent in a particular musical style, for it depends exclusively on aesthetic concepts.

5. Whether the interval of five semitones (perfect fourth) should be considered a consonance or a dissonance depends on the context. For instance, in the progression:

Ex. 27



the character of the fourth is rather consonant, because of the preceding intervals

of higher tension, whereas in the following passage the fourth (represented by an eleventh) appears dissonant:

Ex. 28



6. The interval of six semitones (diminished fifth) is a neutral interval, dividing the octave in two equal parts.

As the harmonic relationships of tonal music play no part in atonal music, terms such as "major third", "perfect fourth", "diminished fifth", and so on, have no meaning. In this book, we shall use them only occasionally, for the convenience of the reader. In fact, it is only the number of semitones in each interval that matters in atonality. Thus, the intervals may be marked by numerals:

Ex. 29



2. Counterpoint

To compose a two-part invention, the best method at first is to add a second part to some of the one-part inventions written previously, using the same twelve-tone series as in the first part.

In the resulting counterpoint, parallel motion should be avoided, as a rule, because it tends to neutralize the individuality of each part. Crossing of the voices is quite permissible.

The second part should support and bring out the formal elements of the original part. Among these elements, culmination-points are most important. It is well to introduce them by accelerated motion and increasing sharpness of dissonances. Where the composition, however, tends to decrease in intensity, a slowing-down of the motion, milder dissonances, and consonances will be adequate.

In the second part, the series may recur any number of times, regardless of the number of series-entries used in the first part. Thus, in the following example, the Violin part (taken from Ex. 23) contains three complete series, while the 'Cello part, moving at a quicker pace, contains four complete series:

Ex. 30

Allegro moderato ♩ = 90

Violin part: *p*, *f*, *f*, *accel.*

'Cello part: *pp*, *mf*, *f*

The example shows two staves: Violin (top) and 'Cello (bottom). The Violin part is marked with dynamics *p*, *f*, *f*, and *accel.* and contains three complete series. The 'Cello part is marked with dynamics *pp*, *mf*, and *f* and contains four complete series. The tempo is Allegro moderato with a quarter note equal to 90 beats per minute.

3. Imitation

Especially when more extended compositions are planned, it is necessary to establish pronounced thematic unity between the two parts (besides the structural homogeneity that is guaranteed by the use of the same basic series). This purpose will be served best by presenting in the second part some of the characteristic motifs set forth in the original part.

When a motif of the first part is repeated by the second part, either while its presentation in the first part is still in progress, or soon after it is brought to an end in the first part, we call this procedure *imitation*.

When imitation is carried through the whole composition, we speak of a *canon*.

When the imitation concerns only the opening tones of the motif, whereupon the imitating part continues differently, we may call the phenomenon *deceptive imitation*.

So long as our second part uses only the same series as the original part, imitations will mostly be possible only at the unison or octave.

Ex. 31

Allegro moderato $\text{♩} = 90$

STUDIES IN COUNTERPOINT

In Ex. 31, strict imitations are used at a) and b). At c), a deceptive imitation is obtained by exploiting the fact that the descending "major third" between the third and fourth tones of the fundamental series occurs once more between the tenth and eleventh tones of that series. The continuation of the 'Cello melody after the E in Measure 8 (Ex. 31) cannot be carried out as a strict imitation of the Violin motif after the D \flat in Measure 8, for the succession of the tones of the series after E is different from the succession after D \flat . Yet, the rhythmic pattern of the Violin motif is conserved. (The fact that, in the finished composition, the Violin part in Measure 8 seems to imitate the 'Cello motif beginning at the last beat of Measure 7, instead of *vice versa*, is, of course, without relevance to the character of the imitation.)

Similar circumstances make the 'Cello motif in Measures 10 and 11 a deceptive imitation of the opening motif in the Violin part (Measures 1 and 2).

Now, the student may proceed to write two-part inventions in which the second part is not added to a previously written first part, but where both parts are conceived simultaneously. The rules and principles to be followed in these exercises are the same as before.

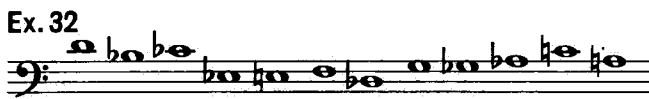
CHAPTER IV

DERIVATIVE FORMS OF THE SERIES

1. *Inversion*

From every twelve-tone series another series can be derived by changing the ascending intervals of the original successively to equivalent descending intervals, and *vice versa*. We shall call the original form of the series "O", and the inversion "I", from the initial letters of the words "Original" and "Inversion".

The Inversion of the series of Ex. 2 is:



The ascending interval D-Gb of the original form is replaced by the equivalent descending interval D-Bb, the descending interval Gb-F by the ascending interval Bb-Cb, and so on.

2. *Retrograde Form*

The retrograde form of the series is obtained by reading the original backwards, proceeding from the last tone to the first. This form will be called "R".



3. *Retrograde Inversion*

The Retrograde Inversion may be derived either from R by applying the method by which I was derived from O (change of the direction of intervals), or from I by reading it backwards. This form will be called "RI", from the initial letters of the words "Retrograde" and "Inversion".

Writing RI of our series by changing the direction of the intervals of R (Ex. 33), we obtain the succession:



Reading I (Ex. 32) backwards results in the following series:



The two forms of the series in Ex. 34 and Ex. 35 have the same intervals, the second being merely a transposition of the first, ten semitones down.

CHAPTER V

TWO-PART INVENTIONS USING O AND I

1. Relations between O and I

As the coherence of a composition (especially of an atonal composition, as indicated in the Introduction) depends largely on the compactness of motif-relationships, it will be wise to examine the series and its derivative forms from the point of view of interval-relations and the possibilities of their development. In the O-form of our series (Ex. 2), there are two ascending and two descending "major thirds":

Ex. 36

Ex. 36 shows a series of notes on a staff with four intervals labeled a, b, c, and d. Above the notes, 'asc.' is written above a) and c), and 'desc.' is written above b) and d). The notes are: a) D4, b) Ab4, c) G4, d) Ab4.

In the I-form, these "thirds" are represented as follows:

Ex. 37

Ex. 37 shows a series of notes on a staff with four intervals labeled a, b, c, and d. Below the notes, 'desc.' is written below a) and c), and 'asc.' is written below b) and d). The notes are: a) D4, b) Ab4, c) G4, d) Ab4.

Both "major thirds" at a) and d) in both forms of the series have their first tones in common (D and Ab). The descending "third" b) of O (Ex. 36) is identical with c) of I (Ex. 37); on the other hand, c) of O (Ex. 36) is identical with b) of I (Ex. 37). Of course, these interval-relations are peculiar to our series. Other series will show different characteristics. It is clear that conscious exploitation of such peculiarities will enhance the coherence of a composition.

The student may now write two-part inventions, by using O for one part, I for the other, as for instance:

Ex. 38
Allegretto ♩ = 108

Ex. 38 is a two-part invention for Flute and Clarinet. The top staff is for Flute and the bottom staff is for Clarinet. The music is in 4/4 time and marked Allegretto with a tempo of 108. The score includes dynamic markings (p, mf, f) and articulation marks (accents, slurs). There are four numbered boxes (1, 2, 3, 4) highlighting specific musical phrases. In the Clarinet part, there are labels 'a)' and 'b)' above some notes.

For the sake of simplicity, in all the examples in this book, music for transposing instruments (such as clarinets) is written as it sounds.

In Ex. 38, the Flute part uses only O (Ex. 2), the Clarinet part I (Ex. 32). As to the exploitation of characteristic intervals ("major thirds", in this series), one may notice the relationship between group $\boxed{\text{a}}$ of the Clarinet part and group $\boxed{\text{b}}$ of the Flute part. The fact that the "thirds" d) of Exs. 36 and 37 have their first tones in common is used for obtaining the feature c) of Ex. 38, with its cadence-like appearance:

Ex. 39

The way for it is prepared by the analogous use of the same peculiarity of the "thirds" a) of Exs. 36 and 37 in Measure 5 of Ex. 38:

2. Alternate Use of O and I

In the next exercises, the student may make alternate use of O and I within the same part so that, in general, both forms are presented simultaneously. It is, however, neither necessary nor advisable to insist that the sections occupied by a complete series in each of the two parts should end at the same time.

Ex. 41

Musical score for Ex. 41, featuring a series that crosses between the Violin and Cello parts. The score is in 4/4 time and includes dynamic markings such as *mf*, *f*, and *p*. The series is numbered 5 through 9, with measure 5 starting in the Violin and measure 9 ending in the Cello.

3. Crossing-Over of the Series from One Part to the Other

The series may also cross over from one part to the other before being presented completely by the part that has introduced it.

Ex. 42

Andante ♩ = 72

Musical score for Ex. 42, featuring a series that crosses frequently between the Violin and Cello parts. The score is in 4/4 time and includes dynamic markings such as *p*, *mp*, *mf*, *f*, and *pp*. The series is numbered 1 through 8, with measure 1 starting in the Violin and measure 8 ending in the Cello. Performance instructions include *pizz.* and *arco*.

If the series is not brought to its end by the part to which it has crossed over, it must cross back to the original part. The following example presents a modification of Ex. 42, showing these more frequent crossings of the series:

Ex. 43

Andante ♩ = 72

Musical score for Ex. 43, featuring a series that crosses frequently between the Violin and Cello parts. The score is in 4/4 time and includes dynamic markings such as *p* and *pp*. The series is numbered 1 through 3, with measure 1 starting in the Violin and measure 3 ending in the Cello. Performance instructions include *pizz.* and *arco*, and the score concludes with *etc.*

CHAPTER VI

TWO-PART WRITING USING ALL FOUR FORMS OF THE SERIES

It is advisable to derive the formal scheme and the principal motif-ideas of the composition from the interval-relations peculiar to the four forms of the series. Devices for linking different sections of the piece may be developed, for instance, from the obvious fact that **R** begins with the same interval as that with which **O** ends, presenting the retrograde form of this interval (which, at the same time, is a transposition of its inversion). The same is true of **I** and **RI**. The recurrence of similar or identical intervals in different forms of the series may be used in building up characteristic motifs. In our series, several combinations of ascending and descending "major thirds" are conspicuous, as indicated in the first section of the preceding chapter (p. 12). Thus, an invention making use of this contingency may read as follows:

Ex. 44
Andante sostenuto $\text{♩} = 48$

Clar.
Cello

(The continuity of the different forms of the series can be traced by following the different lines connecting the single notes; the last note of each series is marked by an arrow.)

Notice the relationship of the motifs in the Clarinet part, Measures 2-3 and 7-8.

Ex. 44 shows an important *addendum* to the rules concerning the *crossing-over* of series-forms from one part to another: it is not necessary that the continuation of an interrupted series-presentation take place immediately after the point where the interruption occurred. In Ex. 44, the presentation of **I** by the Clarinet part is interrupted after the first two tones (D and B \flat) on the fourth beat of Measure 1. It is taken over by the Cello part two beats later, on the third beat of Measure 2. Likewise, the presentation of a series-form in the same part may be interrupted by interpolation of fragments of another series-form, as happens again and again in Ex. 44.

According to the succession of the tones in our series, the B (Clarinet part, Measure 5, fifth note) should appear before the Eb (Cello part, Measure 5, fourth note). In Ex. 44, the Eb is anticipated by one eighth. Such slight *anticipations* are permissible when required by the logical progression of the parts.

Another idea of using all four forms of the series would be to construct the second half of the composition as a retrogression of its first half (*i.e.*, in "crab motion"). In this case, for instance, we may decide to use O and I in the first half, R and RI in the second. In the following example, this idea is carried out by using continuous imitations (see p. 9). Thus, the piece may be called a crab canon.

Ex. 45
 Allegro risoluto ♩ = 144

The score is written for Violin (top staff) and Viola (bottom staff) in 3/4 time. It consists of 16 measures, numbered 1 through 16. The key signature has one flat (B-flat). The tempo is marked 'Allegro risoluto' with a quarter note equal to 144 beats per minute. The score is divided into two halves of 8 measures each. The first half (measures 1-8) uses forms O and I. The second half (measures 9-16) uses forms R and RI. Dynamics include *f*, *mf*, *ff*, and *p*. Performance markings include *poco rit.* and *poco sostenuto*. The piece ends with a *mf* dynamic in measure 16.

Musical score for measures 17-21. The Violin part (top staff) has measures 17, 18, 19, 20, and 21. The Viola part (bottom staff) has measures 17, 18, 19, 20, and 21. Dynamics include *f*, *mf*, *p*, and *ff*. A circled 'RI' label is placed above measure 19. Measure 21 features a triplet of eighth notes.

The turning-point of the canon lies between Measures 10 and 11. From there on, the Violin part repeats Measures 1 to 8 of the Viola part in the retrograde form. The Viola repeats Measures 2 to 10 of the Violin part in the retrograde form, commencing at the third beat of Measure 12. According to the arrangement in the opening of the piece, this imitation occurs five beats after the corresponding entrance of the Violin (first beat of Measure 11).

To fill the gap between the end of the canon theme in the Viola, Measure 8, and the entrance of the retrograde imitation (third beat of Measure 12), the Viola presents one more I-form (Measure 9 to the second beat of Measure 12). This element is repeated as a coda by the Violin, eventually in the form of RI (third beat of Measure 19 through Measure 21). As the space to be filled here is smaller than it was when this element appeared the first time, it is presented in diminution.

In Measure 11 (Viola) two consecutive tones of the series are played simultaneously (as a double stop) in order to emphasize the culmination-point. In the present exercises, this process should be applied but exceptionally. Its application as a rule will be explained later (p. 24).

In Measure 18 (Violin) the D_b appears before the C, instead of *vice versa*. This is due to the retrogression of the repetition figure in Measure 2 (Viola) and justified by the principle of strict imitation. The same applies to the corresponding figure in the Viola, Measure 19, in regard to the succession of E_b and F_b .

The following example shows a variant of the canon of Ex. 45 in that the Viola starts the retrogression five beats *before* the Violin (Measure 9 of Ex. 46, second beat) instead of five beats *after* the Violin, as in Ex. 45. Consequently, the coda element appears in Ex. 46 only once, at the third beat of Measure 17 (Viola).

Ex. 46. Musical score for measures 8-11. The Violin part (top staff) has measures 8, 9, 10, and 11. The Viola part (bottom staff) has measures 8, 9, 10, and 11. Dynamics include *f*, *mf*, and *ff*. A circled 'RI' label is placed above measure 11. Measure 11 features a triplet of eighth notes.

Musical score for Studies in Counterpoint, measures 12-19. The score is in two systems, each with a treble and bass staff. Measures 12-15 are in the first system, and measures 16-19 are in the second. The key signature has two flats (B-flat and E-flat). Dynamics include *f*, *mf*, and *p*. A circled "RI" indicates a retrograde section starting at measure 13. Measure 13 features a triplet of eighth notes. Measure 15 has a circled "R" above it. Measure 16 starts with a piano (*p*) dynamic. Measure 18 has a circled "RI" below it. Measure 19 has a circled "R" above it.

Musical liveliness and expressive quality of compositions of this type are, of course, by no means guaranteed by the application of the twelve-tone technique. In writing melodies that are intended for presentation later in the retrograde form, the composer must always see to it that the retrogression appears reasonable, no matter whether the piece is based on a twelve-tone series or not.

CHAPTER VII

THREE-PART WRITING

1. *Tension-Degrees of Chords*

In three-part writing the harmonic factor naturally stands out more conspicuously than in two-part, for three parts are more likely to form what is felt as a *chord*.

The chords whose emergence from the interlacing of independent voices is legalized by the rules of strict (Palestrina) counterpoint without any restrictions are triads and, to a certain extent, their inversions. All chords containing dissonances may appear only under definite conditions stated by the rules for the use of dissonances. Harmonic evaluation of chords, such as is set forth in the theory of harmony regulating the system of major and minor tonalities, is limited in the realm of strict counterpoint to a few cadence formulae.

Atonality has neither rules for a special treatment of dissonances nor does it formulate a harmonic theory comparable with that of tonality. The only characteristic of a chord that has to be taken into consideration is the degree of tension that the chord shows by virtue of its constituent intervals. One cannot expect that rules resulting from such considerations would form as definite a system as the rules of either strict counterpoint or tonal harmony.

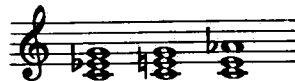
The rigidity that the twelve-tone technique forces to a certain extent upon melodic and contrapuntal construction is compensated by the freedom that it allows in the field of harmony. Bearing in mind this state of things, the student may realize that music written in the twelve-tone technique as well as music organized by any other principle rests, in the final analysis, upon imagination and inspiration.

According to the principles outlined in the paragraph dealing with the tension-degrees of intervals (p. 7f), we can make a survey of three-tone chords in regard to their tension-degrees. It is evident that these tension-degrees depend on what kinds of intervals are formed by the constituent tones of the different chords.

Chords may consist of

1. three consonances:
(the third chord, for instance, has the consonances C-E, C-A \flat , and E-A \flat)
2. two consonances and one mild dissonance:
(the second chord, for instance, has the consonances C-G and C-A, and the mild dissonance G-A)
3. one consonance and two mild dissonances:
(the first chord has the consonance C-E and the mild dissonances C-D and D-E)
4. two consonances and one sharp dissonance:

Ex. 47



Ex. 48



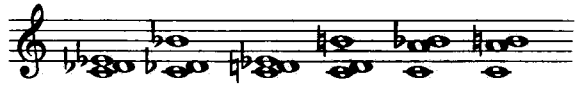
Ex. 49



Ex. 50



5. one consonance, one mild dissonance, and one sharp dissonance: **Ex. 51**



6. one mild and two sharp dissonances:

Ex. 52



Whether, in a three-tone chord, the intervals of five semitones (perfect fourth) and six semitones (diminished fifth) will assume the character of a consonance or of a dissonance depends on the third tone added. In the following table the chords containing the mentioned intervals are classified as "consonant", "mild", or "sharp", according to the influence exerted by the additional third tone.

Chords containing "perfect fourths":

Ex. 53



Chords containing "diminished fifths":

Ex. 54



The above classification of chords does not involve any evaluation of their conformity to traditional ideas of "beauty" or "ugliness", or of their admissibility or usefulness in composition. From this catalogue of chords the student may learn nothing more than certain criteria by which to determine tension-degrees of chords in general. He should bear in mind that in practical composition the tension-degrees are subject to manifold variations, resulting from the position of the intervals, dynamics, instrumentation, etc.

The following three positions of the second chord in the sixth category of the tabulation above (Ex. 52) obviously show different nuances of the same tension-degree:

Ex. 55



Under otherwise equal conditions of dynamics, instrumentation, etc., the last of these three chords will most probably appear as the sharpest.

The first of the three combinations in Ex. 55 will produce different effects in each of the three following manners of orchestration:

Ex. 56

Violin I *con sord.*
pp

Violin II *con sord.*
pp

Ex. 57

Clar. I *mf*

Clar. II *mf*

Horn *on mute*
p

Ex. 58

Tpt. *f*

Trb. I *f*

Trb. II *f*

In spite of being objectively one of our "sharpest" chords, its aspect in Ex. 56 certainly will appear milder than does that of the following chord:

Ex. 59

Fl., Cl., Ob. *ff*

Tpt. *ff*

Trb. *ff*

which in turn is objectively milder than the first one (Ex. 56), as far as intervals are concerned.

It is necessary to beware of pedantry in applying in practical composition the distinctions of tension-degrees here explained. More than in any other respect, the student must be guided in this field by his musicality, taste, and imagination.

2. Counterpoint

In using three-tone combinations of different tension-degrees, the student should follow the same principles as indicated on p. 8 in regard to two-part writing: sharper dissonances will introduce and stress culmination-points, while decreasing intensity of the musical flow will be characterized by milder chords.

Chords of class 1 (Ex. 47), as well as chords of the type classified as consonant chords in Ex. 53 and Ex. 54, should be used with great caution, for the same reason that excludes the use of the octave interval (see p. 7). It is, however, admissible to use such consonant chords occasionally if the context does not obtrude their latent tonal implications.

In the following example, we have added a third ('Cello) part to Ex. 41, using the R-form of our series:

Ex. 60
Andantino $\text{♩} = 90$

Violin

Viola

'Cello

Alternative version:

Measures 1-4: Violin (1, 2, 3, 4), Viola (0), Cello (R, p). Measure 3 includes a circled 'I'.

Measures 5-7: Violin (5, 6, 7), Viola (I, mf), Cello (mf, R, 3). Measure 6 includes a circled '0'.

Measures 8-11: Violin (8, 9, 10, 11), Viola (f, p), Cello (f, p, R, 3). Measure 8 includes a circled 'R'. Measure 10 includes a circled '0'.

A triad (sixth chord) appears on the first beat of Measure 4. It is admissible because no tonal implications (of D major or otherwise) show up in the context. The same is true of the triads in Measures 5 and 7.

From Measures 6 to 11, two alternatives are given. The version in the upper line of the 'Cello part creates appropriate strong interval-tensions at the culminating point of Measure 8. The tension in Measure 7, however, may be considered too mild, in view of the increasing intensity of the musical flow, hinted at by the melodic development in the Violin part. This is particularly objectionable in comparison with the sharp tension-degrees of Measure 6, at a relatively less accented point of the composition. Therefore, the version of the lower line of the 'Cello part seems preferable, in spite of the almost too rapid decrease of tension at the last beat of Measure 8.

In the lower line, it is not necessary that the G \flat in Measure 6 and the A in Measure 9 be actually played by the 'Cello. These tones of the R-form can be considered represented in Measure 6 by the G \flat of the Violin part (belonging to the O-form) and in Measure 9 by the A of the Viola (belonging to the I-form). Common tones related in this way to different series-forms will occur often in polyphonic writing.

In three-part inventions, it is not advisable to use crossings-over of series-forms from one part to another too frequently lest the composition become too involved. The following example shows a three-part invention using all four forms of the series:

Ex. 61
Andante $\text{♩} = 60$

Flute

Clar.

Bass Cl.

1 2 3

4 5 6

pp *p* *mf* *f* *espr.*

p *espr.* *f*

p *mf* *f*

R *RI* *I* *O*

3. Harmonic Elements

In order to establish a more complete unity within a composition, we may wish to use harmonic elements as well as motif-elements, harmony being called to our attention more obtrusively in three-part than in two-part writing. The exceptional feature of Measure 11 of Ex. 45 (the double stop in the Viola part) will now become a rule: *several consecutive tones of a series-form can be telescoped into a chord.*

Let us suppose that we decide to use the chord: Ex. 62

built up of the first three tones of our series (O) as a constructive element of a piece. We should like to present this chord on different steps. In our particular series we are fortunate in that it allows us to build up this chord twice more out of three consecutive tones. The fifth, sixth, and seventh tones form the following chord:

Ex. 63

the tenth, eleventh, and twelfth tones produce the following chord:

Ex. 64

both chords being transpositions of the first chord (Ex. 62).

The corresponding chords in I will be:

Ex. 65

R, of course will furnish the same chords as O, and RI the same as I.

Sometimes it may be neither possible nor desirable to form the chosen characteristic chords by telescoping consecutive tones of the same series-form. In this case, one has to

combine different series-forms for the desired effect. The following example shows both methods:

Ex. 66
Allegretto energico $\text{♩} = 60$

The musical score for Ex. 66 is arranged in three systems. The first system includes the Oboe, Viola, and Cello parts. The Oboe part starts with a *mf* dynamic and includes markings for first (I) and right-hand (RH) positions. The Viola part includes *pizz.* (pizzicato) and *arco* (arco) markings. The Cello part includes *pizz.* and *arco* markings. The second system continues the Oboe, Viola, and Cello parts, with dynamics ranging from *f* to *p* and *mf*. The Oboe part includes a *p espr.* marking. The third system includes measures 8, 9, 10, and 11, with dynamics ranging from *mf* to *f*. The Cello part includes a *poco rit.* marking in measure 9 and a *a tempo, veloce* marking in measure 10. The score is heavily annotated with slurs, ties, and other musical symbols.

On the fifth beat of Measure 4 and the first beat of Measure 6, the characteristic chord (Exs. 62-64) comprises tones of different series-forms, instead of consecutive tones of a single form of the series.

CHAPTER VIII

EXTENSION OF THE RULES CONCERNING REPETITION OF TONES

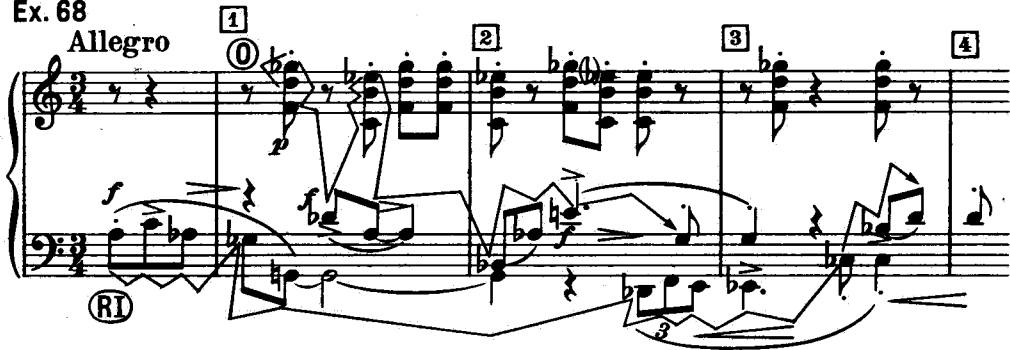
The rules concerning repetition, as given on p. 3f, apply similarly to chords. Thus, effects like the following are correct:

Ex. 67

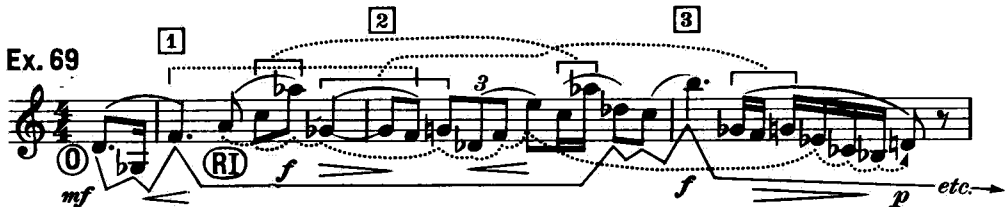


Repetitions of this sort are also admissible if other parts present the forthcoming tones of the series between the first statement of the repeated element and its actual repetition. For instance:

Ex. 68



Occasionally, forthcoming tones of the series may also be interpolated between a tone and its repetition in the same part. Generally speaking, *interpolation of new material between an element (tone or chord) and its repetition is permissible in so far as the repetition can reasonably be expected to be felt as such.* The following example, in which repetitions of elements have been marked by dotted lines, is objectionable because it does not meet this requirement:

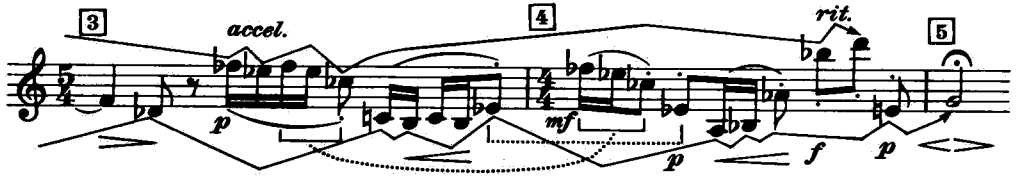


Neither can the F in Measure 2 be felt as a repetition of the F in Measure 1, nor can a relationship of repetition be realized between the double statements of the groups C-A \flat (Measures 1 and 2) and G \flat -F-G (Measures 1-2 and 3).

The following arrangement, however, would be correct:

Ex. 70





It would again seem pedantic to set up rigid rules on the extent to which a composer might go in using repetition of elements without infringing upon the principles of a technique to which he professed allegiance. Putting licenses into regulations would contradict their very essence.

CHAPTER IX

TRANSPPOSITIONS OF THE SERIES-FORMS

Every one of the four forms of the series can be transposed eleven times, to the different steps of the chromatic scale; in other words, it can begin with twelve different tones. If one wishes to use those transpositions in composing, he will find it useful to write down all the possible forty-eight patterns of the series, as in the following example:

Ex. 71

Ex. 71 displays twelve staves of musical notation, each representing a different transposition of a series form. The staves are numbered 1 through 12. Above the first four staves are circled labels: 'O' above staff 1, 'I' above staff 2, 'R' above staff 3, and 'RI' above staff 4. Each staff contains a sequence of notes and rests, representing a specific transposition of the series form.

The composer may dispose of these patterns at his convenience. It will, however, be advisable not to use them promiscuously, but rather according to a certain plan that emerges from definite musical purposes.

Let us suppose, for instance, that we want to use the following phrase as a principal thematic element for a composition:

Ex. 72

Ex. 72 shows a single staff of music in treble clef. It contains a four-note phrase: G4, Bb4, D5, and F#5. The notes are connected by a slur, and a bracket is placed underneath the entire phrase.

We postulate that the phrase (as well as the whole composition) should be constructed from our series and its derivative forms and transpositions. The opening four tones apparently belong to the untransposed O-form of our series (third to sixth tone of O); the following four notes, being the inversion of the opening group, can be identified as the third to sixth tones of the I-form, transposed to begin on C:

Ex. 73

Ex. 73 shows a single staff of music in bass clef. It contains a four-note phrase: C3, E3, G3, and Bb3. The notes are connected by a slur, and a bracket is placed underneath the entire phrase.

Another idea for the proposed composition might be to continue the thematic element of Ex. 72 in such a way that the opening of the second phrase is like that of the first one, but the development different. When the second phrase, accordingly, begins with:



(like the first one), but does not proceed to A (as the first phrase did), we have to take this second succession of D \flat -C-B from some form of the series other than the untransposed O.

This opportunity is furnished by some of the RI-forms, where the needed succession of two descending semitones appears from the seventh to the ninth tone:




The succession of D \flat -C-B which we need for our specific problem appears eventually in the transposition of RI beginning on F:

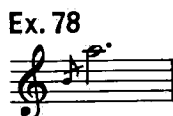


By using this transposition, we are indeed able to start the second phrase of our theme exactly like the first phrase (Ex. 72) and yet to continue it differently:



As this phrase is the retrogression of the second link of the first phrase ( in Ex. 72), we may now wish to conclude the theme with a retrogression of the opening four-tone group, by using between the first and second links of the second phrase the same interval-relationship as that between the first and second links of the first phrase—in retrograde order, of course.

Consequently, the skip of ten semitones up (in Ex. 72):




must be answered by a skip of ten semitones down (in our continuation of Ex. 77):



Thus, the second phrase of the theme would read (according to our plan of constructing its closing link as the retrogression of the first link of the first phrase):

Ex. 80



The last four tones ( of Ex. 80) must eventually belong to an R-form of the series because they represent the retrogression of the opening four tones (of Ex. 72) which came from an O-form. And, in fact, they appear in the transposition of R beginning on F:

Ex. 81



Thus, for the complete theme:

Ex. 82



we have used the O-form beginning on D, the I-form beginning on C, the RI-form beginning on F, and the R-form beginning on F.

Locating the remaining tones of these four series-forms, we may obtain the following complete picture of our theme:

Ex. 83

Allegro $\text{♩} = 144$

At the x-mark in Measure 6, B-F is considered a repetition of B-F in Measure 5, according to the rules given in Chapter VIII. Likewise B \flat , the last eighth in Measure 6, is a repetition of the B \flat on the first beat of that measure; and the F \sharp on the first beat of Measure 7 is a repetition of the F \sharp on the first beat of Measure 6.

It will be noticed that, of the four forms used here, **R** and **RI** start from the same tone (F) while **O** and **I** end on the same tone (G). These circumstances may be exploited in further using these four forms of the series.

If this theme were used for a more extended composition, it would be wise to use, in the section dominated by the theme, no more than the four transpositions that form the theme. New transpositions should be introduced only when new, contrasting thematic material is to be presented.

CHAPTER X

DISPOSITION OF LARGER FORMS

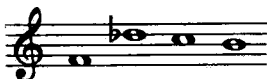
Since this subject exceeds the frame of mere contrapuntal work, we shall limit ourselves to a few hints on how formal ideas can be deduced from the structural concept of the twelve-tone technique.

1. *Thematic Material*

In order to build an extended composition on the forty-eight patterns of a twelve-tone series, it is necessary first to develop as many different thematic elements as possible.

In the theme of Ex. 82, we put the group:

Ex. 84



and its derivative forms in the foreground. A contrasting theme could well be developed, for instance, from the idea:

Ex. 85



which is built on O. The tones Db-C-B, missing between F and Eb, will appear in the accompaniment. In view of our intention of creating a theme to contrast with Ex. 82, this arrangement is very fortunate because it was just this chromatic three-tone succession that stood out conspicuously in the first theme (Ex. 82).

The theme of Ex. 85 is furthermore characterized by the fact that the descending skips (D-Gb, Eb-A, Ab-E) become successively smaller while the ascending skips (Gb-F, A-Bb, E-G) grow proportionally larger. Regularities of this sort are very much in the spirit of the twelve-tone technique.

Combining the theme of Ex. 85 with an I-form from which we select and arrange three groups of three tones each by the method applied in developing Ex. 85 out of O, we may obtain the following:

Ex. 86



The groups indicated by $\underline{\quad}$ are taken from the I-form of the series, transposed to begin on F. The tones Gb, G, and Ab, missing between D and E of this form, would again appear somewhere in the accompaniment. Notice the regularity emerging from the fact that the groups a) and b) are linked together by the common tone F, while the groups c) and d) have the same outer tones (A and Bb), differing only in their first tones (Eb and E).

Combination of Ex. 85 with other forms and transpositions of the series may yield other interesting and useful thematic elements.

Another arrangement that might be useful for melodically neutral transition-passages can be derived from O by reducing it, as far as possible, to scale-like progressions:

Ex. 87



Similar and different thematic ideas can, of course, be derived from other forms of the series. By virtue of the relationships of the four series-forms, all these thematic elements can be transformed into each other.

Interesting results may be obtained if one takes a thematic element characteristically related to one of the series-forms and tries to represent it by means of another form. Thus, if we try to imitate the outline of the motif:

Ex. 88



as far as possible by means of I, R, and RI, we should obtain:

Ex. 89



Ex. 90



Ex. 91



Some of the relationships to be noted in the examples from 88 to 91 are as follows:

1. Ex. 88 and Ex. 90 end with the same skip (thirteen semitones down); likewise, Ex. 89 and Ex. 91 end with the same skip (eleven semitones down).
2. Ex. 88 begins with a "major third" up, Ex. 91 with a "minor third"; Ex. 89 begins with a "minor sixth", Ex. 90 with a "major sixth".
3. The characteristic interval between the third and fourth note is, in Ex. 88, a "major third" down, in Ex. 89 a "minor sixth" down (harmonic inversion of the major third); in Ex. 90 it is a "minor seventh" down, in Ex. 91 a "major second" down (harmonic inversion of the minor seventh).

STUDIES IN COUNTERPOINT

2. *Distribution of the Forty-Eight Patterns*

In my Variations for Piano, Op. 79, I used the following scheme:

Variation	Transpositions of			
	O	I	RI	R
	beginning on			
1	F# and G			
2	F and G#			
3	E and A	C# and C		
4	E \flat and B \flat	D and B		
5	D and B	E \flat and B \flat	G and F#	
6	C# and C	E and A	G# and F	
7		F and A \flat	A and E	C# and C
8		F# and G	B \flat and E \flat	D and B
9			B and D	E \flat and B \flat
10			C and C#	E and A
11				F and A \flat
12				F# and G

The symmetry of the arrangement is evident. Paralleling the O-forms beginning on F# and G in the first variation, the last one uses the R-forms beginning on F# and G.

The regularity of distribution of the transpositions is shown even more clearly thus:

Ex. 92

Var. 1 2 3 4 5 6 7 8 9 10 11 12

The recurrence of similar combinations of steps on which the different transpositions begin offers the opportunity of recurrent harmonic elements.

Furthermore, it will be noticed that the compactness of the structure increases towards the middle of the piece and decreases from there on. Since the sixth and seventh variations are welded together into one comprehensive Adagio movement, this middle part is the only section where all four forms of the series are in use simultaneously. The symmetry of the composition in relation to an imaginary axis between the sixth and seventh variations is stressed by the fact that this Adagio is developed as a four-part crab canon which, after a certain point, returns step by step to its beginning.

One may easily imagine that many different formal conceptions can be thought up, based on similar arrangements.

In my Sixth String Quartet, Op. 78 (Universal Edition, Vienna), I used the twelve transpositions of each series-form for each of the first four movements; only the fifth movement, a fugue with four subjects, each derived from the main theme of one of the previous movements, has all forty-eight patterns simultaneously. A more elementary application of the twelve-tone technique is shown in my Twelve Short Piano Pieces, Op. 83 (G. Schirmer, Inc., New York), the analysis of which is strongly recommended to the beginner. The plan of distribution of the series-forms is given in the preface to that composition.

APPENDIX—SPECIAL SERIES

1. *Symmetrical Series*

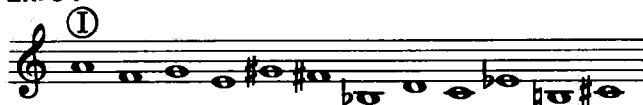
It is possible to build twelve-tone series that can be called symmetrical by virtue of the relationships existing between their two halves. For instance:

Ex. 93



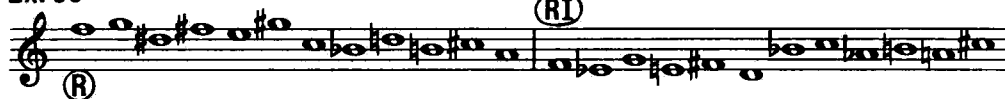
Here, the second half of the series is the inversion of the first half, transposed. Eventually, the second half of I will be identical with a transposition of the first half of O (Ex. 93). This can be shown by forming I of Ex. 93:

Ex. 94



The same relationships exist necessarily between R and RI of this series:

Ex. 95



In the following series, the second half of O is the retrograde form of the first half, transposed. Therefore, R of the whole series is identical with O, transposed; and I is identical with RI, transposed:

Ex. 96



As the second half of the next series is the retrograde inversion of its first half, transposed, RI of the whole series is identical with O, transposed; and I is identical with R, transposed:

Ex. 97



Many series of these kinds can be established. In using them one has to keep in mind that, since twenty-four patterns will coincide with their respective derivative forms, there will be only twenty-four different patterns available, instead of forty-eight.

2. *All-Interval Series*

In order to obtain series containing all the eleven intervals possible between the twelve different tones, it is necessary to arrange the tones of the series in one direction, either ascending or descending, as for instance:

Ex. 98

Ex. 99

(The numerals between the notes show the number of semitones forming the different intervals.)

The sum total of the numbers of semitones represented by the eleven intervals ($1+2+3+4+5+6+7+8+9+10+11$) being 66, every all-interval series must involve 66 semitones, or cover $5\frac{1}{2}$ octaves. Half an octave being a diminished fifth, the last tone of every all-interval series must be the "diminished fifth" of its first tone.

3. *Symmetrical All-Interval Series*

These are series of the following type:

Ex. 100

The arrangement of the lower line shows that the second half of the series is the retrograde form of its first half, transposed. The symmetry is further shown by the fact that the sum of each two interval-figures equidistant from the center (6) is 12 ($1+11$, $2+10$, $7+5$, $4+8$, $3+9$). All series of this kind show two triad combinations, one in each half (B-D#-F# and C-A-F in Ex. 100) and have the neutral interval of six semitones ("diminished fifth") in the middle, between the sixth and the seventh tones. The last tone of every such series must, of course, again be the "diminished fifth" of the first.

