Chapter 3
DISTRIBUTED LOADS ON THE SURFACE OF A SEMI-INFINITE MASS

3.1 Loading on an Infinite Strip

3.1.1 UNIFORM VERTICAL LOADING (Fig.3.1)

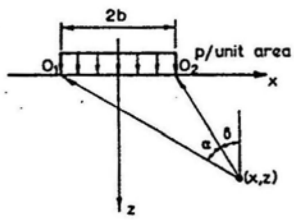


FIG.3.1

$$\sigma_z = \frac{p}{\pi}\left[\alpha + \sin\alpha \cos(\alpha+2\delta)\right] \qquad \ldots (3.1a)$$

$$\sigma_x = \frac{p}{\pi}\left[\alpha - \sin\alpha \cos(\alpha+2\delta)\right] \qquad \ldots (3.1b)$$

$$\sigma_y = \frac{2p}{\pi}\nu\alpha \qquad \ldots (3.1c)$$

$$\tau_{xz} = \frac{p}{\pi}\sin\alpha \sin(\alpha+2\delta) \qquad \ldots (3.1d)$$

$$\sigma_1 = \frac{p}{\pi}\left[\alpha + \sin\alpha\right] \qquad \ldots (3.1e)$$

$$\sigma_3 = \frac{p}{\pi}\left[\alpha - \sin\alpha\right] \qquad \ldots (3.1f)$$

$$\tau_{max} = \frac{p}{\pi}\sin\alpha \qquad \ldots (3.1g)$$

Loci of constant σ_1 and σ_2 are circles passing through O_1 and O_2.

Loci of constant τ_{max} are circles passing through O_1 and O_2.

Trajectories of σ_1 are a family of confocal hyperbolas, foci at O_1 and O_2. These curves bisect the angle, α, at all points.

Trajectories of σ_2 are a family of confocal ellipses, foci at O_1 and O_2.

Trajectories of τ_{max} are two orthogonal families: equiangular spirals intersecting the ellipses under 45°.

Maximum $\tau_{max} = p/\pi$, occurs at all points of the semi-circle through O_1 and O_2.

Maximum $\sigma_1 = p$, occurs at points $(x,0)$, $-b<x<b$.

Minimum $\sigma_3 = 0$, " " " $(x,0)$, $-b>x>b$.

Values of σ_x, σ_z, τ_{xz}, σ_1 σ_3 and τ_{max} are tabulated in Table 3.1, and contours of σ_z and τ_{max} are given in Fig.3.2 (Jurgenson, 1934).

As for line loading, displacements due to strip loading on or in a semi-infinite mass are only meaningful if evaluated as the displacement of one point relative to another point, neither point being located at infinity. The vertical displacement at the surface, relative to the centre of the strip, is given by

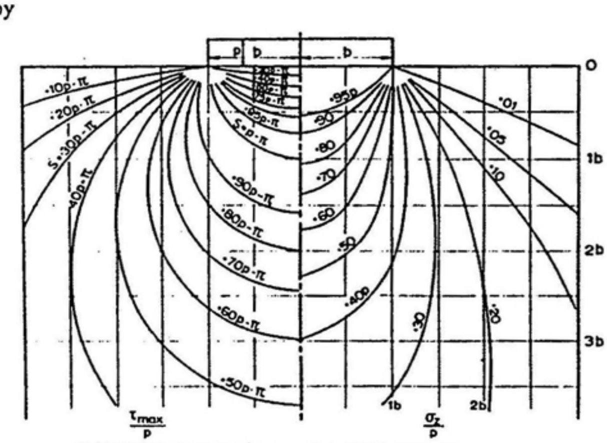


FIG.3.2 Stresses beneath a strip (Jurgenson,1934).

$$\rho_z(x,0) - \rho_z(0,0) = \frac{2p(1-\nu^2)}{\pi E}\{(x-b)\ell n|x-b| - (x+b)\ell n|x+b| + 2b\ell nb\} \quad \ldots (3.2)$$

(See plot in Fig.9.2a, Chapter 9)

TABLE 3.1

STRESSES BENEATH A UNIFORMLY LOADED STRIP
(Jurgenson, 1934)

x/b	z/b	σ_z/p	σ_x/p	τ_{zx}/p	β	τ_{max}/p	σ_1/p	σ_3/p
0	0	1.0000	1.0000	0	0	0	1.0000	1.0000
	.5	.9594	.4498	0	0	.2548	.9594	.4498
	1	.8183	.1817	0	0	.3183	.8183	.1817
	1.5	.6678	.0803	0	0	.2937	.6678	.0803
	2	.5508	.0410	0	0	.2546	.5508	.0410
	2.5	.4617	.0228	0	0	.2195	.4617	.0228
	3	.3954	.0138	0	0	.1908	.3954	.0138
	3.5	.3457	.0091	0	0	.1683	.3457	.0091
	4	.3050	.0061	0	0	.1499	.3050	.0061
0.5	0	1.0000	1.0000	0	0	0	1.0000	1.0000
	.25	.9787	.6214	.0522	8°35'	.1871	.9871	.6129
	.5	.9028	.3920	.1274	13°17'	.2848	.9323	.3629
	1	.7352	.1863	.1590	14°52'	.3158	.7763	.1446
	1.5	.6078	.0994	.1275	13°18'	.2847	.6370	.0677
	2	.5107	.0542	.0959	11°25'	.2470	.5298	.0357
	2.5	.4372	.0334	.0721	9°49'	.2143	.4693	.0206
1	.25	.4996	.4208	.3134	41°25'	.3158	.7760	.1444
	.5	.4969	.3472	.2996	37°59'	.3088	.7308	.1133
	1	.4797	.2250	.2546	31°43'	.2847	.6371	.0677
	1.5	.4480	.1424	.2037	26°34'	.2546	.5498	.0406
	2	.4095	.0908	.1592	22°30'	.2251	.4751	.0249
	2.5	.3701	.0595	.1243	19°20'	.1989	.4137	.0159
1.5	.25	.0177	.2079	.0606	73°47'	.1128	.2281	.0025
	.5	.0892	.2850	.1466	61°50'	.1765	.3636	.0106
	1	.2137	.2488	.2101	47°23'	.2115	.4428	.0198
	1.5	.2704	.1807	.2022	38°44'	.2071	.4327	.0184
	2	.2876	.1268	.1754	32°41'	.1929	.4007	.0143
	2.5	.2851	.0892	.1469	28°09'	.1765	.3637	.0106
2	.25	.0027	.0987	.0164	80°35'	.0507	.1014	.0002
	.5	.0194	.1714	.0552	71°59'	.0940	.1893	.0014
	1	.0776	.2021	.1305	58°17'	.1424	.2834	.0052
	1.5	.1458	.1847	.1568	48°32'	.1578	.3232	.0074
	2	.1847	.1456	.1567	41°27'	.1579	.3232	.0073
	2.5	.2045	.1256	.1442	36°02'	.1515	.3094	.0064
2.5	.5	.0068	.1104	.0254	76°43'	.0569	.1141	.0003
	1	.0357	.1615	.0739	65°12'	.0970	.1957	.0016
	1.5	.0771	.1645	.1096	55°52'	.1180	.2388	.0029
	2	.1139	.1447	.1258	48°32'	.1265	.2556	.0036
	2.5	.1409	.1205	.1266	42°45'	.1269	.2575	.0036
3	.5	.0026	.0741	.0137	79°25'	.0379	.0758	.0001
	1	.0171	.1221	.0449	69°42'	.0690	.1384	.0005
	1.5	.0427	.1388	.0757	61°15'	.0895	.1803	.0012
	2	.0705	.1341	.0954	54°12'	.1006	.2029	.0018
	2.5	.0952	.1196	.1036	48°20'	.1054	.2128	.0020
	3	.1139	.1019	.1057	43°22'	.1058	.2137	.0020

β is angle between direction of σ_1 and the vertical.

3.1.2 UNIFORM HORIZONTAL LOADING (Fig.3.3).

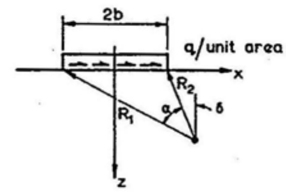


FIG.3.3

$$\sigma_x = \frac{q}{\pi}\left[\log_e \frac{R_1^2}{R_2^2} - \sin\alpha \sin(\alpha+2\delta)\right] \quad \ldots (3.3a)$$

$$\sigma_z = \frac{q}{\pi}\left[\sin\alpha \sin(\alpha+2\delta)\right] \quad \ldots (3.3b)$$

$$\tau_{xz} = \frac{q}{\pi}\left[\alpha - \sin\alpha \cos(\alpha+2\delta)\right] \quad \ldots (3.3c)$$

Values of σ_x/q have been tabulated and are given in Table 3.2. It should be noted that σ_z/q values are equal to the corresponding values of τ_{xz}/p for uniform vertical loading.

TABLE 3.2

VALUES OF σ_x/q FOR UNIFORM HORIZONTAL LOADING OVER STRIP

(Scott, 1963)

z/b \ x/b	0	0.5	1.0	2.0	4.0
0.0	0.00	0.70	-	0.60	0.33
0.2	0.00	0.62	1.16	0.68	0.32
0.5	0.00	0.39	0.60	0.57	0.32
1.0	0.00	0.13	0.26	0.39	0.28
1.5	0.00	0.06	0.12	0.24	0.25
2.0	0.00	0.03	0.06	0.15	0.20
3.0	0.00	0.01	0.02	0.06	0.14
5.0	0.00	0.00	0.00	0.01	0.05

The expression for the horizontal displacement of a point on the surface, relative to the centre of the strip, $\rho_x(x,0)-\rho_x(0,0)$, is identical with the expression for the relative vertical displacement due to uniform vertical load in equation (3.2).

3.1.3 VERTICAL LOADING INCREASING LINEARLY (Fig.3.4)

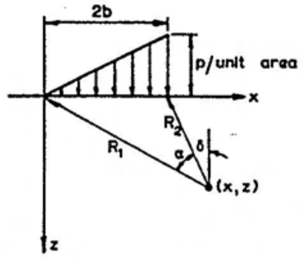


FIG.3.4

$$\sigma_z = \frac{p}{2\pi}\left[\frac{x}{b}\alpha - \sin 2\delta\right] \quad \ldots (3.4a)$$

$$\sigma_x = \frac{p}{2\pi}\left[\frac{x}{b}\alpha - \frac{z}{b}\log_e\frac{R_1^2}{R_2^2} + \sin 2\delta\right] \quad \ldots (3.4b)$$

$$\tau_{xz} = \frac{p}{2\pi}\left[1 + \cos 2\delta - \frac{z\alpha}{b}\right] \quad \ldots (3.4c)$$

Values of σ_z/p have been tabulated by Scott (1963) and are given in Table 3.3.

TABLE 3.3

VALUES OF σ_z/p FOR LINEARLY INCREASING LOAD ON STRIP (Scott, 1963)

z/b \ x/b	-3.0	-2.0	-1.0	0	1.0	2.0	3.0	5.0
0.0	0.00	0.00	0.00	0.00	0.50	0.50	0.00	0.00
0.5	0.00	0.00	0.00	0.08	0.48	0.42	0.02	0.00
1.0	0.00	0.00	0.02	0.13	0.41	0.35	0.06	0.00
2.0	0.01	0.03	0.06	0.16	0.28	0.25	0.13	0.01
3.0	0.02	0.05	0.10	0.15	0.20	0.19	0.12	0.04
4.0	0.03	0.06	0.09	0.13	0.16	0.15	0.11	0.05

The vertical displacement of a point on the surface, relative to the value at $x=0$, is given by

$$\rho_z(x,0)-\rho_z(0,0) = \frac{p(1-\nu^2)}{\pi bE}\{2b^2 \ell n\ 2b - \frac{x^2}{2}\ell n x + (\frac{x^2}{2} - 2b^2)\ell n|2b-x| + bx\} \quad \ldots (3.5)$$

3.1.4 HORIZONTAL LOADING LINEARLY INCREASING (Max. loading = q, Fig.3.4)

$$\sigma_x = \frac{q}{2\pi}\left[\frac{3z\alpha}{b} + \frac{x}{b}\log_e\frac{R_1^2}{R_2^2} - \cos 2\delta - 5\right] \quad \ldots (3.6a)$$

$$\sigma_z = \frac{q}{2\pi}\left[1 + \cos 2\delta - \frac{z}{b}\alpha\right] \quad \ldots (3.6b)$$

$$\tau_{xz} = \frac{q}{2\pi}\left[\frac{x}{b}\cdot\alpha - \frac{z}{b}\log_e\frac{R_1^2}{R_2^2} + \sin 2\delta\right] \quad \ldots (3.6c)$$

The expression for the horizontal displacement of a point on the surface, relative to $x=0$, is identical with the expression for relative vertical movement due to vertical loading (equation 3.5).

3.1.5 SYMMETRICAL VERTICAL TRIANGULAR LOADING (Fig.3.5, Gray, 1936)

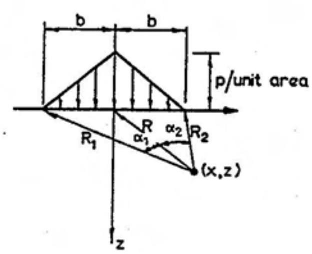


FIG.3.5

$$\sigma_z = \frac{p}{\pi}\left[(\alpha_1+\alpha_2) + \frac{x}{b}(\alpha_1-\alpha_2)\right] \quad \ldots (3.7a)$$

$$\sigma_x = \frac{p}{\pi}\left[(\alpha_1+\alpha_2) + \frac{x}{b}(\alpha_1-\alpha_2) - 2\frac{z}{b}\log_e\frac{R_1R_2}{R_0^2}\right] \quad \ldots (3.7b)$$

$$\tau_{xz} = -\frac{pz}{\pi b}(\alpha_1-\alpha_2) \quad \ldots (3.7c)$$

$$\sigma_1 = \frac{p}{\pi}\left[(\alpha_1+\alpha_2) + \frac{x}{b}(\alpha_1-\alpha_2) - \frac{z}{b}\log_e\frac{R_1R_2}{R_0^2}\right] + \frac{pz}{\pi b}\left(\log_e^2\frac{R_1R_2}{R_0^2} + (\alpha_1-\alpha_2)^2\right)^{\frac{1}{2}} \quad \ldots (3.7d)$$

$$\sigma_3 = \frac{p}{\pi}\left[(\alpha_1+\alpha_2) + \frac{x}{b}(\alpha_1-\alpha_2) - \frac{z}{b}\log_e\frac{R_1R_2}{R_0^2}\right] - \frac{pz}{\pi b}\left(\log_e^2\frac{R_1R_2}{R_0^2} + (\alpha_1-\alpha_2)^2\right)^{\frac{1}{2}} \quad \ldots (3.7e)$$

$$\tau_{max} = \frac{pz}{\pi b} \left(\log_e^2 \frac{R_1 R_2}{R_0^2} + (\alpha_1-\alpha_2)^2\right)^{\frac{1}{2}} \quad \ldots (3.7e)$$

Stresses have been tabulated by Jurgenson (1934) and are given in TABLE 3.4.

Contours of σ_z and τ_{max} are shown in Fig.3.6 (Jurgenson, 1934).

TABLE 3.4

STRESSES BENEATH A STRIP WITH SYMMETRICAL TRIANGULAR LOADING

(Jurgenson, 1934)

x/b	z/b	σ_z/p	σ_x/p	τ_{zx}/p	β	τ_{max}/p	σ_1/p	σ_3/p
0	θ	1.0000	1.0000	0	0	0	1.0000	1.0000
	.25	.8440	.3931	0	0	.2255	.8440	.3931
	.5	.7048	1925	0	0	.2562	.7048	.1925
	.75	.5904	.1025	0	0	.2440	.5904	.1025
	1.0	.5000	.0588	0	0	.2206	.5000	.0588
	1.25	.4296	.0359	0	0	.1969	.4296	.0359
	1.5	.3744	.0234	0	0	.1755	.3744	.0234
	1.75	.3305	.0158	0	0	.1574	.3305	.0158
	2.0	.2952	.0111	0	0	.1421	.2952	.0111
	2.5	.2422	.0062	0	0	0	.2422	.0062
.25	0	.7500	.7500	0	-	0	.7500	.7500
	.25	.7196	.3874	.1151	17°22'	.2021	.7556	.3514
	.5	.6344	.2026	.1146	13°59'	.2444	.6629	.1741
	.75	.5462	.1138	.0951	11°53'	.2361	.5661	.0939
	1.0	.4711	.0681	.0756	10°17'	.2152	.4848	.0544
	1.25	.4101	.0432	.0597	9°01'	.1930	.4197	.0337
0.5	0	.5000	.5000	0	-	0	.5000	.5000
	.25	.4949	.3357	.1525	31°13'	.1720	.5873	.2433
	.5	.4714	.2152	.1762	27°00'	.2178	.5611	.1255
	.75	.4350	.1385	.1570	23°19'	.2160	.5028	.0708
	1.0	.3955	.0913	.1299	20°15'	.2000	.4434	.0434
	1.25	.3577	.0617	.1055	17°45'	.1817	.3914	.0280
	1.5	.3238	.0433	.0858	15°44'	.1644	.3480	.0192
	2.0	.2682	.0229	.0582	12°42'	.1358	.2814	.0098
	2.5	.2266	.0130	.0415	10°37'	.1146	.2344	.0052
0.75	0	.2500	.2500	0	-	0	.2500	.2500
	.25	.2620	.2620	.1476	45°00'	.1476	.4096	.1144
	.5	.2875	.2162	.1810	39°26'	.1845	.4364	.0674
	.75	.3000	.1611	.1735	34°05'	.1869	.4175	.0437
	1.0	.2980	.1167	.1528	29°39'	.1777	.3851	.0297
	1.25	.2869	.0847	.1309	26°10'	.1654	.3512	.0204
1.0	0	0	0	0	-	0	0	0
	.25	.0766	.1956	.0959	60°54'	.1129	.2490	.0232
	.5	.1393	.2005	.1414	51°06'	.1447	.3146	.0252
	.75	.1813	.1693	.1534	43°53'	.1535	.3288	.0218

β is angle between vertical and direction σ_1.

TABLE 3.4 (Cont.)

x/b	z/b	σ_z/p	σ_x/p	τ_{zx}/p	β	τ_{max}/p	σ_1/p	σ_3/p
1.0	1.0	.2048	.1338	.1476	38°14'	.1518	.3211	.0175
	1.25	.2148	.1033	.1343	33°43'	.1454	.3045	.0137
	1.5	.2159	.0794	.1189	30°04'	.1371	.2848	.0106
	1.75	.2048	.0471	.0903	29°25'	.1199	.2459	.0061
	2.5	.1874	.0298	.0685	20°30'	.1044	.2130	.0042
1.25	.25	.0155	.1278	.0399	72°18'	.0689	.1406	.0028
	.5	.0580	.1668	.0899	60°35'	.1051	.2175	.0073
	.75	.1002	.1599	.1169	52°10'	.1207	.2508	.0094
	1.0	.1319	.1379	.1256	45°41'	.1256	.2605	.0093
	1.25	.1526	.1137	.1232	40°30'	.1247	.2579	.0085
1.5	.25	.0046	.0840	.0186	77°27'	.0439	.0882	.0004
	.5	.0250	.1294	.0540	67°01'	.0751	.1523	.0021
	.75	.0545	.1396	.0828	58°37'	.0934	.1905	.0037
	1.0	.0824	.1315	.0992	51°58'	.1022	.2092	.0048
	1.25	.1049	.1156	.1053	46°28'	.1054	.2157	.0049
	1.5	.1211	.0981	.1044	41°51'	.1050	.2146	.0046
	2.0	.1375	.0681	.0929	34°45'	.0992	.2020	.0036
	2.5	.1403	.0470	.0781	29°33'	.0910	.1847	.0027
2.0	.5	.0064	.7773	.0222	74°02'	.0420	.0000	.0841
	1.0	.0332	.1041	.0572	60°54'	.0673	.1360	.0014
	1.5	.0636	.0955	.0763	50°55'	.0780	.1576	.0016
	2.0	.0862	.0761	.0791	43°10'	.0793	.0019	.1605
	2.5	.0985	.0581	.0741	37°23'	.0768	.0015	.1551

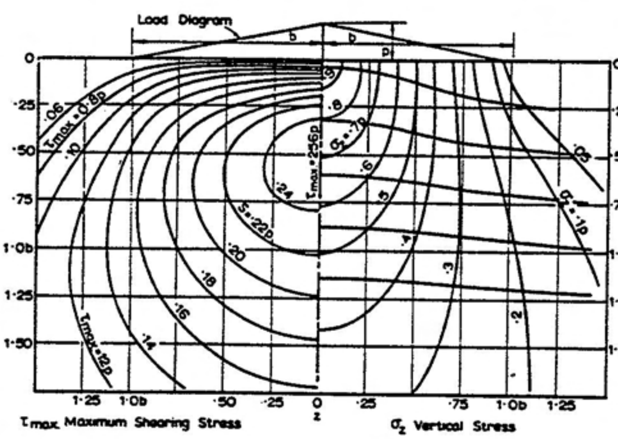


FIG.3.6 Stresses beneath strip with triangular loading (Jurgenson, 1934).

3.1.6 ASYMMETRICAL VERTICAL TRIANGULAR LOADING (Fig.3.7, Gray, 1936)

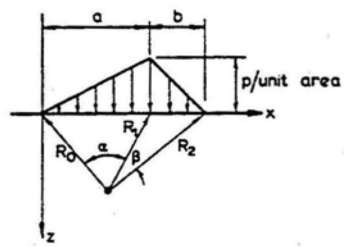


FIG.3.7

$$\sigma_z = \frac{p}{\pi}\left[\frac{x\alpha}{a} + \frac{a+b-x}{b}\beta\right] \quad \ldots (3.8a)$$

$$\sigma_x = \frac{p}{\pi}\left[\frac{x\alpha}{a} + \frac{a+b-x}{b}\beta + \frac{2z}{a}\log_e\frac{R_1}{R_0} + \frac{2z}{b}\log_e\frac{R_1}{R_2}\right] \quad \ldots (3.8b)$$

$$\tau_{xz} = \frac{pz}{\pi}\left[\frac{\alpha}{a} - \frac{\beta}{b}\right] \quad \ldots (3.8c)$$

$$\sigma_1 = \frac{p}{\pi}\left[\frac{x}{a}\alpha + \frac{a+b-x}{b}\beta + \frac{z}{a}\log_e\frac{R_1}{R_0} + \frac{z}{b}\log_e\frac{R_1}{R_2}\right] + \frac{pz}{\pi}\left(\left(\frac{1}{a}\log_e\frac{R_1}{R_0} + \frac{1}{b}\log_e\frac{R_1}{R_2}\right)^2 + \left(\frac{\alpha}{a} - \frac{\beta}{b}\right)^2\right)^{\frac{1}{2}} \quad \ldots (3.8d)$$

$$\sigma_3 = \frac{p}{\pi}\left[\frac{x\alpha}{a} + \frac{a+b-x}{b}\beta + \frac{z}{a}\log_e\frac{R_1}{R_0} + \frac{z}{b}\log_e\frac{R_1}{R_2}\right] - \frac{pz}{\pi}\left(\left(\frac{1}{a}\log_e\frac{R_1}{R_0} + \frac{1}{b}\log_e\frac{R_1}{R_2}\right)^2 + \left(\frac{\alpha}{a} - \frac{\beta}{b}\right)^2\right)^{\frac{1}{2}} \quad \ldots (3.8e)$$

$$\tau_{max} = \frac{pz}{\pi}\left(\left(\frac{1}{a}\log_e\frac{R_1}{R_0} + \frac{1}{b}\log_e\frac{R_1}{R_2}\right)^2 + \left(\frac{\alpha}{a} - \frac{\beta}{b}\right)^2\right)^{\frac{1}{2}} \quad \ldots (3.8f)$$

3.1.7 VERTICAL "EMBANKMENT" LOADING (Fig.3.8, Gray, 1936)

$$\sigma_z = \frac{p}{\pi}\left[\beta + \frac{x\alpha}{a} - \frac{z}{R_2^2}(x-b)\right] \quad \ldots (3.9a)$$

$$\sigma_x = \frac{p}{\pi}\left[\beta + \frac{x\alpha}{a} + \frac{z}{R_2^2}(x-b) + \frac{2z}{a}\log_e\frac{R_1}{R_0}\right] \quad \ldots (3.9b)$$

$$\tau_{xz} = -\frac{p}{\pi}\left[\frac{z\alpha}{a} - \frac{z^2}{R_2^2}\right] \quad \ldots (3.9c)$$

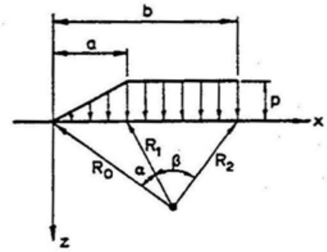


FIG.3.8

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{p}{\pi}\left[\beta + \frac{x\alpha}{a} + \frac{z}{a}\log_e\frac{R_1}{R_2} \pm \frac{z}{a}\left(\left\{\frac{a}{R_2^2}(x-b) + \log_e\frac{R_1}{R_0}\right\}^2 + \left(\alpha - \frac{az}{R_2^2}\right)^2\right)^{\frac{1}{2}}\right] \quad \ldots (3.9d)$$

Influence factors for σ_z beneath the edge of the loading have been published by Osterberg (1957) and are shown in Fig.3.9.

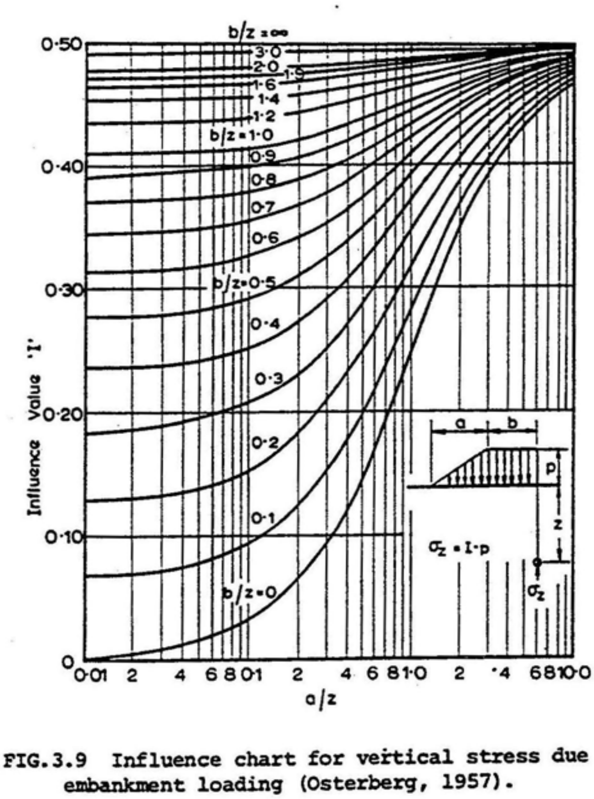


FIG.3.9 Influence chart for vertical stress due to embankment loading (Osterberg, 1957).

3.2 Loading over Half the Infinite Surface

3.2.1 UNIFORM VERTICAL LOADING (Fig.3.10, Gray, 1936).

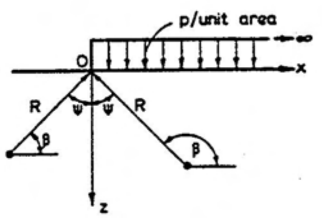


FIG.3.10

$$\sigma_z = \frac{p}{\pi}\left(\beta + \frac{xz}{R^2}\right) \quad \ldots (3.10a)$$

$$\sigma_x = \frac{p}{\pi}\left(\beta - \frac{xz}{R^2}\right) \quad \ldots (3.10b)$$

$$\tau_{xz} = -\frac{p}{\pi} \sin^2\beta \quad \ldots (3.10c)$$

$$\sigma_1 = \frac{p}{\pi}(\beta + \sin\beta) \quad \ldots (3.10d)$$

$$\sigma_3 = \frac{p}{\pi}(\beta - \sin\beta) \quad \ldots (3.10e)$$

$$\tau_{max} = \frac{p}{\pi}\sin\beta \quad \ldots (3.10f)$$

Loci of constant σ_1 and σ_3 are radial lines, OR.

Loci of constant τ_{max} are radial lines, OR.

Trajectories of σ_1 are a family of confocal parabolas*. Focus at 0. Horizontal axis.

Trajectories of σ_3 are a family of confocal parabolas*. Focus at 0. Horizontal axis.

The two families are orthogonal.

Trajectories of τ_{max} are two orthogonal families of parabolas. Focus at 0. Vertical axis.

Maximum $\tau_{max} = p/\pi$, occurs at points $(0,z)$, $z>0$.

Maximum $\sigma_1 = p$, " " " $(x,0)$, $x>0$.

Minimum $\sigma_3 = 0$, " " " $(x,0)$, $x<0$.

*These curves bisect the angle, β, at all points.

3.2.2 LINEARLY INCREASING VERTICAL LOADING (Fig.3.11, Gray, 1936).

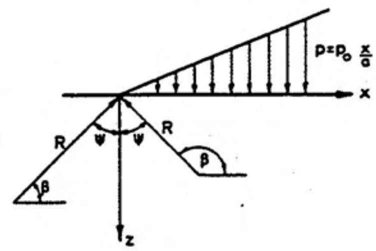


FIG.3.11

$$\sigma_z = \frac{p_o}{\pi a}[x\beta + z] \quad \ldots (3.11a)$$

$$\sigma_x = \frac{p_o}{\pi a}[x\beta - z - 2z \log_e R] \quad \ldots (3.11b)$$

$$\tau_{xz} = \frac{p_o}{\pi a} z\beta \quad \ldots (3.11c)$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{p_o}{\pi a}(x\beta - z \log_e R) \pm \frac{p_o z}{\pi a}\left[(1+\log_e R)^2 + \beta^2\right]^{\frac{1}{2}} \quad \ldots (3.11d)$$

$$\tau_{max} = \frac{p_o z}{\pi a}\left[(1+\log_e R)^2 + \beta^2\right]^{\frac{1}{2}} \quad \ldots (3.11e)$$

3.2.3 VERTICAL "EMBANKMENT" LOADING (Fig.3.12, Gray, 1936).

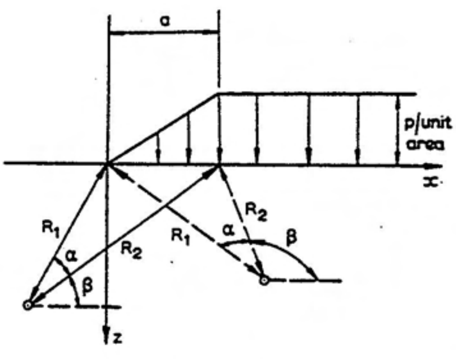


FIG.3.12

$$\sigma_z = \frac{p}{\pi a}[a\beta + x\alpha] \quad \ldots (3.12a)$$

$$\sigma_x = \frac{p}{\pi a}\left[a\beta + x\alpha + 2z \log_e \frac{R_2}{R_1}\right] \quad \ldots (3.12b)$$

$$\tau_{xz} = -\frac{p}{\pi a} z \alpha \qquad \ldots (3.12c)$$

$$\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = \frac{p}{\pi a}\left[\left(a\beta + x\alpha + z \log_e \frac{R_2}{R_1}\right) \pm z \left(\log_e^2 \frac{R_2}{R_1} + \alpha^2\right)^{\frac{1}{2}}\right] \qquad \ldots (3.12d)$$

$$\tau_{max} = \frac{pz}{\pi a}\left(\log_e^2 \frac{R_2}{R_1} + \alpha^2\right)^{\frac{1}{2}} \qquad \ldots (3.12e)$$

Values of stresses, presented originally by Jurgenson (1934), are tabulated in TABLE 3.5.

TABLE 3.5

STRESSES DUE TO SEMI-INFINITE EMBANKMENT LOADING (Jurgenson, 1934)

x/a	z/a	σ_z/p	σ_x/p	τ_{zx}/p	90°-β	τ_{max}/p	σ_1/p	σ_3/p
0	0	0	0	0	-	0	0	0
	.25	.0780	.3034	.1055	21°33'	.1544	.3451	.0363
	.5	.1476	.4038	.1762	27°00'	.2178	.4935	.0579
	.75	.2048	.4487	.2214	30°35'	.2528	.5796	.0740
	1.0	.2500	.4706	.2500	33°06'	.2732	.6335	.0871
	1.25	.2852	.4821	.2685	34°56'	.2860	.6697	.0977
	1.5	.3129	.4883	.2807	36°20'	.2941	.6947	.1065
	2.0	.3524	.4946	.2952	38°14'	.3036	.7271	.1199
	2.5	.3789	.4968	.3028	39°30'	.3085	.7464	.1294
0.25	0	.2500	.2500	0	-	0	.2500	.2500
	.25	.2643	.3924	.1619	34°13'	.1741	.5025	.1543
	.5	.3023	.4544	.2302	35°52'	.2424	.6208	.1360
	.75	.3381	.4784	.2643	37°34'	.2734	.6817	.1349
	1.0	.3659	.4885	.2828	38°53'	.2894	.7166	.1378
	1.25	.3867	.4935	.2936	39°51'	.2984	.7385	.1417
-0.25	0	0	0	0	-	0	0	0
	.25	.0161	.2201	.0468	12°19'	.1123	.2304	.0058
	.5	.0633	.3430	.1157	19°48'	.1816	.3848	.0216
	.75	.1156	.4077	.1692	24°36'	.2235	.4852	.0382
	1.0	.1630	.4431	.2072	27°59'	.2501	.5532	.0530
	1.25	.2032	.4634	.2339	30°28'	.2677	.6010	.0656
0.5	0	.5000	.5000	0	-	0	.5000	.5000
	.25	.5000	.5000	.1762	45°00'	.1762	.6762	.3238
	.5	.5000	.5000	.2500	45°00'	.2500	.7500	.2500
	.75	.5000	.5000	.2807	45°00'	.2807	.7807	.2193
	1.0	.5000	.5000	.2952	45°00'	.2952	.7952	.2048
	1.25	.5000	.5000	.3028	45°00'	.3028	.8028	.1972
	1.5	.5000	.5000	.3072	45°00'	.3072	.8072	.1928
	2.0	.5000	.5000	.3119	45°00'	.3119	.8119	.1881
	2.5	.5000	.5000	.3144	45°00'	.3144	.8144	.1856
-0.5	0	0	0	0	-	0	0	0
	.25	.0051	.1642	.0237	8°19'	.0829	.1676	.0018
	.5	.0286	.2847	.0738	14°59'	.1478	.3045	.0089
	.75	.0650	.3614	.1239	19°57'	.1932	.4064	.0200
	1.0	.1045	.4086	.1653	23°42'	.2246	.4812	.0320
	1.25	.1422	.4381	.1972	26°41'	.2465	.5367	.0437
	1.5	.1762	.4566	.2214	28°50'	.2621	.5785	.0543
	2.0	.2317	.4771	.2537	32°06'	.2818	.6362	.0726
	2.5	.2734	.4869	.2729	34°19'	.2930	.6732	.0872

TABLE 3.5 Continued

x/a	z/a	σ_z/p	σ_x/p	τ_{zx}/p	90°-β	τ_{max}/p	σ_1/p	σ_3/p
0.75	0	.7500	.7500	0	-	0	.7500	.7500
	.25	.7357	.6076	.1619	55°47'	.1741	.8458	.4976
	.5	.6978	.5457	.2302	54°08'	.2424	.8642	.3794
	.75	.6619	.5217	.2643	52°26'	.2734	.8652	.3184
	1.0	.6341	.5114	.2828	51°07'	.2894	.8622	.2834
	1.25	.6133	.5065	.2936	50°09'	.2984	.8583	.2615
-0.75	0	0	0	0	0	0	0	0
	.25	.0023	.1302	.0143	6°18'	.0656	.1319	.0007
	.5	.0147	.2382	.0493	11°54'	.1222	.2487	.0043
1.0	0	1.0000	1.0000	0	90°00'	0	1.0000	1.0000
	.25	.9220	.6973	.1055	68°24'	.1541	.9638	.6556
	.5	.8524	.5962	.1762	63°01'	.2178	.9421	.5065
	.75	.7951	.5512	.2214	59°26'	.2528	.9260	.4204
	1.0	.7500	.5294	.2500	56°54'	.2732	.9129	.3665
	1.25	.7148	.5178	.2685	55°05'	.2860	.9023	.3303
	1.5	.6871	.5117	.2807	53°41'	.2941	.8935	.3053
	2.0	.6476	.5054	.2952	51°47'	.3036	.8801	.2729
	2.5	.6211	.5032	.3028	50°31'	.3085	.8707	.2537
-1.0	0	0	0	0	0	0	0	0
	.5	.0083	.2031	.0348	9°50'	.1034	.2091	.0023
	1.0	.0452	.3369	.1024	17°33'	.1782	.3693	.0129
	1.5	.0967	.4092	.1621	23°02'	.2252	.4782	.0278
	2.0	.1475	.4465	.2049	26°57'	.2536	.5506	.0434
	2.5	.1915	.4674	.2343	29°45'	.2719	.6014	.0576
1.25	0	1.0000	1.0000	0	90°00'	0	1.0000	1.0000
	.25	.9803	.7763	.0461	77°51'	.1120	.9903	.7663
	.5	.9367	.6570	.1157	70°13'	.1816	.9785	.6153
	.75	.8844	.5923	.1692	65°24'	.2235	.9619	.5149
	1.0	.8370	.5569	.2072	62°02'	.2501	.9471	.4469
	1.5	.7632	.5247	.2528	57°38'	.2795	.9235	.3645
	2.0	.7123	.5124	.2763	54°57'	.2938	.9062	.3186
	2.5	.6765	.5069	.2896	53°10'	.3018	.8935	.2899
1.5	0	1.0000	1.0000	0	90°00'	0	1.0000	1.0000
	.25	.9949	.8357	.0237	81°42'	.0829	.9982	.8234
	.5	.9714	.7152	.0738	75°02'	.1478	.9911	.6955
	.75	.9350	.6385	.1239	70°03'	.1932	.9800	.5936
	1.0	.8955	.5913	.1653	66°19'	.2246	.9680	.5188
	1.5	.8238	.5433	.2214	61°11'	.2621	.9457	.4215
	2.0	.7682	.5229	.2537	57°54'	.2818	.9274	.3638
	2.5	.7266	.5130	.2729	55°41'	.2930	.9128	.3268
2.0	0	1.0000	1.0000	0	90°00'	0	1.0000	1.0000
	.25	.9989	.8919	.0096	84°55'	.0543	.9997	.8911
	.5	.9916	.7968	.0348	80°10'	.1034	.9976	.7908
	1.0	.9548	.6631	.1024	72°28'	.1781	.9871	.6309
	1.5	.9032	.5908	.1621	66°59'	.2252	.9722	.5218
	2.0	.8524	.5534	.2049	63°04'	.2537	.9566	.4492
	2.5	.8085	.5325	.2343	60°15'	.2719	.9424	.3986

3.3 Loading on a Circular Area

3.3.1 UNIFORM VERTICAL LOADING (Fig.3.13)

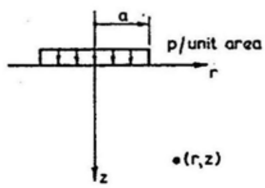


FIG.3.13

On the axis $(r=o)$,

$$\sigma_z = p\left[1 - \left\{\frac{1}{1+(a/z)^2}\right\}^{3/2}\right] \qquad \ldots (3.13a)$$

$$\sigma_r = \sigma_\theta = \frac{p}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{\frac{1}{2}}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right] \qquad \ldots (3.13b)$$

$$\Theta = 2(1+\nu)p\left[1 - \frac{z}{(a^2+z^2)^{\frac{1}{2}}}\right] \qquad \ldots (3.13c)$$

$$\rho_z = \frac{2pa(1-\nu^2)}{E}\left(\sqrt{1+(z/a)^2} - z/a\right) \cdot \left[1 + \frac{z/a}{2(1-\nu)\sqrt{1+(z/a)^2}}\right] \qquad \ldots (3.13d)$$

General expressions and tabulated values for all stresses, strains and displacements are given in Appendix B, as a special case of the values for an anisotropic material.

Values of σ_z, σ_r, σ_θ, τ_{rz} and ρ_z for $\nu=0.5$ have been presented in graphical form by Foster and Ahlvin (1954) and are reproduced in Figures 3.14 to 3.18.

A complete tabulation of stresses, strains and deflections for all values of ν has been presented by by Ahlvin and Ulery (1962) and is given in Tables 3.6 to 3.13. The key to the use of these Tables is shown below.

Bulk stress

$$\Theta = 2p(1+\nu)A = \sigma_z+\sigma_r+\sigma_\theta = e\,\frac{E}{1-2\nu} \qquad \ldots (3.14a)$$

Vertical stress

$$\sigma = p[A + B] \qquad \ldots (3.14b)$$

Radial horizontal stress

$$\sigma_r = p[2\nu A + C + (1-2\nu)F] \qquad \ldots (3.14c)$$

Tangential horizontal stress

$$\sigma_\theta = p[2\nu A - D + (1-2\nu)E_1] \qquad \ldots (3.14d)$$

Vertical-radial shear stress

$$\tau_{rz} = \tau_{zr} = pG_1 \qquad \ldots (3.14e)$$

Bulk strain

$$e = p\,\frac{2(1+\nu)}{E}\,(1-2\nu)A = \varepsilon_z+\varepsilon_r+\varepsilon_\theta = \Theta\,\frac{1-2\nu}{E} \qquad \ldots (3.14f)$$

Vertical strain

$$\varepsilon_z = p\,\frac{1+\nu}{E}[(1-2\nu)A + B] \qquad \ldots (3.14g)$$

Radial horizontal strain

$$\varepsilon_r = p\,\frac{1+\nu}{E}\,[(1-2\nu)F + C] \qquad \ldots (3.14h)$$

Tangential horizontal strain

$$\varepsilon_\theta = p\,\frac{1+\nu}{E}\,[(1-2\nu)E_1 - D] \qquad \ldots (3.14i)$$

Vertical-radial shear strain

$$\gamma_{rz} = \gamma_{zr} = p\,\frac{2(1+\nu)}{E}\,G_1 = \frac{2(1+\nu)}{E}\,\tau_{rz} \qquad \ldots (3.14j)$$

Vertical deflection

$$\rho_z = p\,\frac{1+\nu}{E}\,a[\tfrac{z}{a}A + (1-\nu)H] \qquad \ldots (3.14k)$$

Radial horizontal deflection

$$\rho_r = p\,\frac{1+\nu}{E}\,(-r)\;[(1-2\nu)E_1 - D] = -r\,\varepsilon_\theta \qquad \ldots (3.14l)$$

Tangential horizontal deflection

$$\rho_\theta = 0 \qquad \ldots (3.14m)$$

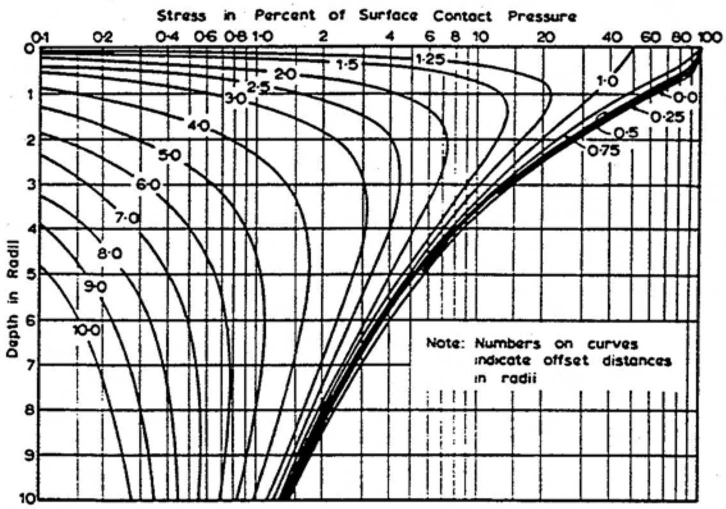


FIG. 3.14 Vertical stress σ_z beneath uniform circular load (Foster and Ahlvin, 1954).

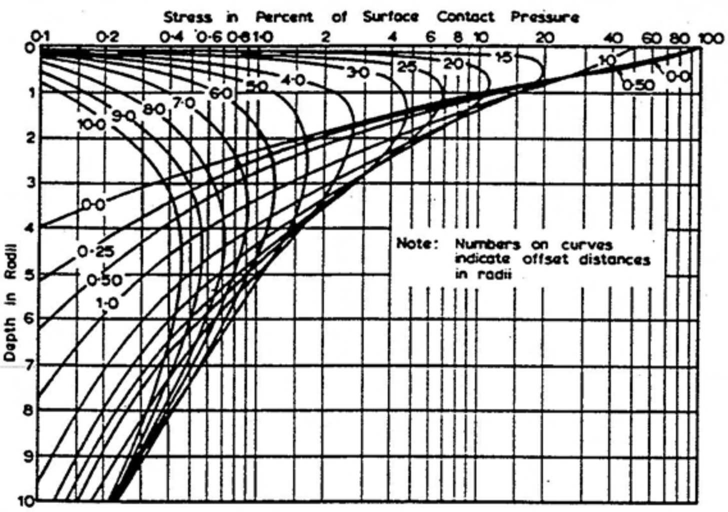


FIG. 3.15 Horizontal stress σ_r beneath uniform circular load (ν=0.5) (Foster and Ahlvin, 1954).

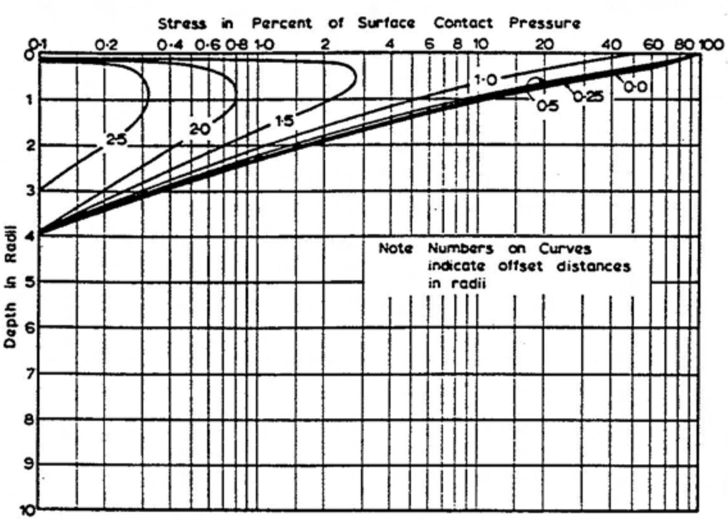


FIG. 3.16 Horizontal stress σ_θ beneath uniform circular load (ν=0.5) (Foster and Ahlvin, 1954).

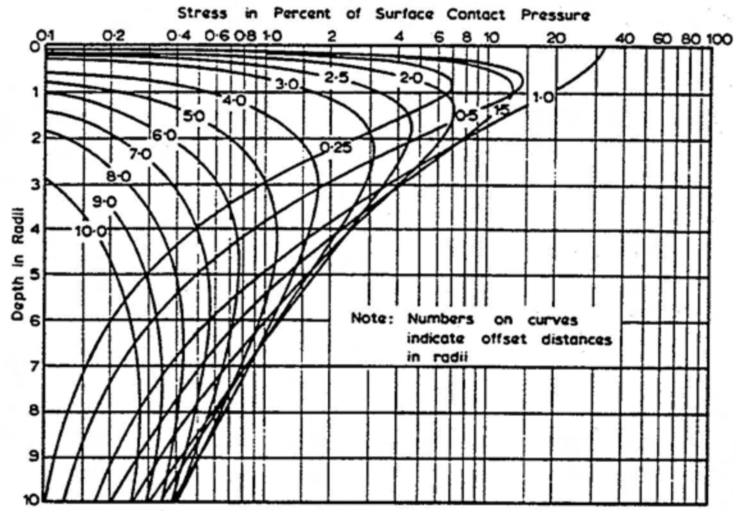


FIG.3.17 Shear stress τ_{rz} beneath uniform circular load (Foster and Ahlvin, 1954).

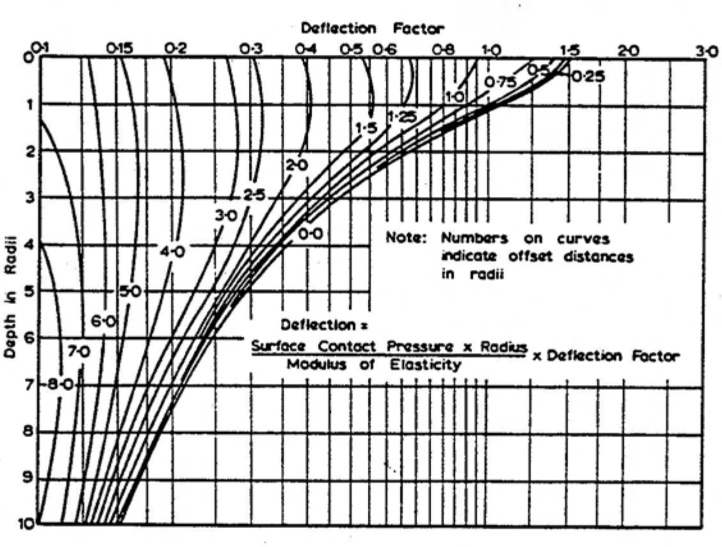


FIG.3.18 Vertical deflection ρ_z beneath uniform circular load (Foster and Ahlvin, 1954).

TABLE 3.6

FUNCTION "A"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	1.0	1.0	1.0	1.0	1.0	.5	0	0	0	0	0	0	0	0	0	0	0	
0.1	.90050	.89748	.88679	.86126	.78797	.43015	.09645	.02787	.00856	.00211	.00084	.00042						
0.2	.80388	.79824	.77884	.73483	.63014	.38269	.15433	.05251	.01680	.00419	.00167	.00083	.00048	.00030	.00020			
0.3	.71265	.70518	.68316	.62690	.52081	.34375	.17964	.07199	.02440	.00622	.00250							
0.4	.62861	.62015	.59241	.53767	.44329	.31048	.18709	.08593	.03118									
0.5	.55279	.54403	.51622	.46448	.38390	.28156	.18556	.09499	.03701	.01013	.00407	.00209	.00118	.00071	.00053	.00025	.00014	.00009
0.6	.48550	.47691	.45078	.40427	.33676	.25588	.17952	.10010										
0.7	.42654	.41874	.39491	.35428	.29833	.21727	.17124	.10228	.04558									
0.8	.37531	.36832	.34729	.31243	.26581	.21297	.16206	.10236										
0.9	.33104	.32492	.30669	.27707	.23832	.19488	.15253	.10094										
1	.29289	.28763	.27005	.24697	.21468	.17868	.14329	.09849	.05185	.01742	.00761	.00393	.00226	.00143	.00097	.00050	.00029	.00018
1.2	.23178	.22795	.21662	.19890	.17626	.15101	.12570	.09192	.05260	.01935	.00871	.00459	.00269	.00171	.00115			
1.5	.16795	.16552	.15877	.14804	.13436	.11892	.10296	.08048	.05116	.02142	.01013	.00548	.00325	.00210	.00141	.00073	.00043	.00027
2	.10557	.10453	.10140	.09647	.09011	.08269	.07471	.06275	.04496	.02221	.01160	.00659	.00399	.00264	.00180	.00094	.00056	.00036
2.5	.07152	.07098	.06947	.06698	.06373	.05974	.05555	.04880	.03787	.02143	.01221	.00732	.00463	.00308	.00214	.00115	.00068	.00043
3	.05132	.05101	.05022	.04886	.04707	.04487	.04241	.03839	.03150	.01980	.01220	.00770	.00505	.00346	.00242	.00132	.00079	.00051
4	.02986	.02976	.02907	.02802	.02832	.02749	.02651	.02490	.02193	.01592	.01109	.00768	.00536	.00384	.00282	.00160	.00099	.00065
5	.01942	.01938				.01835			.01573	.01249	.00949	.00708	.00527	.00394	.00298	.00179	.00113	.00075
6	.01361					.01307			.01168	.00983	.00795	.00628	.00492	.00384	.00299	.00188	.00124	.00084
7	.01005					.00976			.00894	.00784	.00661	.00548	.00445	.00360	.00291	.00193	.00130	.00091
8	.00772					.00755			.00703	.00635	.00554	.00472	.00398	.00332	.00276	.00189	.00134	.00094
9	.00612					.00600			.00566	.00520	.00466	.00409	.00353	.00301	.00256	.00184	.00133	.00096
10								.00477	.00465	.00438	.00397	.00352	.00326	.00273	.00241			

TABLE 3.7

FUNCTION "B"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
0.1	.09852	.10140	.11138	.13424	.18796	.05388	-.07899	-.02672	-.00845	-.00210	-.00084	-.00042						
0.2	.18857	.19306	.20772	.23524	.25983	.08513	-.07759	-.04448	-.01593	-.00412	-.00166	-.00083	-.00024	-.00015	-.00010			
0.3	.26362	.26787	.28018	.29483	.27257	.10757	-.04316	-.04999	-.02166	-.00599	-.00245							
0.4	.32016	.32259	.32748	.32273	.26925	.12404	-.00766	-.04535	-.02522									
0.5	.35777	.35752	.35323	.33106	.26236	.13591	.02165	-.03455	-.02651	-.00991	-.00388	-.00199	-.00116	-.00073	-.00049	-.00025	-.00014	-.00009
0.6	.37831	.37531	.36308	.32822	.25411	.14440	.04457	-.02101										
0.7	.38487	.37962	.36072	.31929	.24638	.14986	.06209	-.00702	-.02329									
0.8	.38091	.37408	.35133	.30699	.23779	.15292	.07530	.00614										
0.9	.36962	.36275	.33734	.29299	.22891	.15404	.08507	.01795										
1	.35355	.34553	.32075	.27819	.21978	.15355	.09210	.02814	-.01005	-.01115	-.00608	-.00344	-.00210	-.00135	-.00092	-.00048	-.00028	-.00018
1.2	.31485	.30730	.28481	.24836	.20113	.14915	.10002	.04378	.00023	-.00995	-.00632	-.00378	-.00236	-.00156	-.00107			
1.5	.25602	.25025	.23338	.20694	.17368	.13732	.10193	.05745	.01385	-.00669	-.00600	-.00401	-.00265	-.00181	-.00126	-.00068	-.00040	-.00026
2	.17889	.18144	.16644	.15198	.13375	.11331	.09254	.06371	.02836	.00028	-.00410	-.00371	-.00278	-.00202	-.00148	-.00084	-.00050	-.00033
2.5	.12807	.12633	.12126	.11327	.10298	.09130	.07869	.06022	.03429	.00661	-.00130	-.00271	-.00250	-.00201	-.00156	-.00094	-.00059	-.00039
3	.09487	.09394	.09099	.08635	.08033	.07325	.06551	.05354	.03511	.01112	.00157	-.00134	-.00192	-.00179	-.00151	-.00099	-.00065	-.00046
4	.05707	.05666	.05562	.05383	.05145	.04773	.04532	.03995	.03066	.01515	.00595	.00155	-.00029	-.00094	-.00109	-.00094	-.00068	-.00050
5	.03772	.03760				.03384			.02474	.01522	.00810	.00371	.00132	.00013	-.00043	-.00070	-.00061	-.00049
6	.02666					.02468			.01968	.01380	.00867	.00496	.00254	.00110	.00028	-.00037	-.00047	-.00045
7	.01980					.01868			.01577	.01204	.00842	.00547	.00332	.00185	.00093	-.00002	-.00029	-.00037
8	.01526					.01459			.01279	.01034	.00779	.00554	.00372	.00236	.00141	.00035	-.00008	-.00025
9	.01212					.01170			.01054	.00888	.00705	.00533	.00386	.00265	.00178	.00066	.00012	-.00012
10								.00924	.00879	.00764	.00631	.00501	.00382	.00281	.00199			

TABLE 3.8

FUNCTION "C"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0												
0.1	-.04926	-.05142	-.05903	-.07708	-.12108	.02247	.12007	.04475	.01536	.00403	.00164	.00082						
0.2	-.09429	-.09755	-.10872	-.12977	-.14552	.02419	.14896	.07892	.02951	.00796	.00325	.00164	.00094	.00059	.00039			
0.3	-.13181	-.13484	-.14415	-.15023	-.12990	.01988	.13394	.09816	.04148	.01169	.00483							
0.4	-.16008	-.16188	-.16519	-.15985	-.11168	.01292	.11014	.10422	.05067									
0.5	-.17889	-.17835	-.17497	-.15625	-.09833	.00483	.08730	.10125	.05690	.01824	.00778	.00399	.00231	.00146	.00098	.00050	.00029	.00018
0.6	-.18915	-.18664	-.17336	-.14934	-.08967	-.00304	.06731	.09313										
0.7	-.19244	-.18831	-.17393	-.14147	-.08409	-.01061	.05028	.08253	.06129									
0.8	-.19046	-.18481	-.16784	-.13393	-.08066	-.01744	.03582	.07114										
0.9	-.18481	-.17841	-.16024	-.12664	-.07828	-.02337	.02359	.05993										
1	-.17678	-.17050	-.15188	-.11995	-.07634	-.02843	.01331	.04939	.05429	.02726	.01333	.00726	.00433	.00278	.00188	.00098	.00057	.00036
1.2	-.15742	-.15117	-.13467	-.10763	-.07289	-.03575	-.00245	.03107	.04522	.02791	.01467	.00824	.00501	.00324	.00221			
1.5	-.12801	-.12277	-.11101	-.09145	-.06711	-.04124	-.01702	.01088	.03154	.02652	.01570	.00933	.00585	.00386	.00266	.00141	.00083	.00039
2	-.08944	-.08491	-.07976	-.06925	-.05560	-.04144	-.02687	-.00782	.01267	.02070	.01527	.01013	.00671	.00462	.00327	.00179	.00107	.00069
2.5	-.06403	-.06068	-.05839	-.05259	-.04522	-.03605	-.02800	-.01536	.00103	.01384	.01314	.00987	.00707	.00506	.00369	.00209	.00128	.00083
3	-.04744	-.04560	-.04339	-.04089	-.03642	-.03130	-.02587	-.01748	-.00528	.00792	.01030	.00888	.00689	.00520	.00392	.00232	.00145	.00096
4	-.02854	-.02737	-.02562	-.02585	-.02421	-.02112	-.01964	-.01586	-.00956	.00038	.00492	.00602	.00561	.00476	.00389	.00254	.00168	.00115
5	-.01886	-.01810				-.01568			-.00939	-.00293	-.00128	.00329	.00391	.00380	.00341	.00250	.00177	.00127
6	-.01333					-.01118			-.00819	-.00405	-.00079	.00129	.00234	.00272	.00272	.00227	.00173	.00130
7	-.00990					-.00902			-.00678	-.00417	-.00180	-.00004	.00113	.00174	.00200	.00193	.00161	.00128
8	-.00763					-.00699			-.00552	-.00393	-.00225	-.00077	.00029	.00096	.00134	.00157	.00143	.00120
9	-.00607					-.00423			-.00452	-.00353	-.00235	-.00118	-.00027	.00037	.00082	.00124	.00122	.00110
10								-.00381	-.00373	-.00314	-.00233	-.00137	-.00063	.00030	.00040			

TABLE 3.9

FUNCTION "D"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0						
0.1	.04926	.04998	.05235	.05716	.06687	.07635	.04108	.01803	.00691	.00193	.00080	.00041						
0.2	.09429	.09552	.09900	.10546	.11431	.10932	.07139	.03444	.01359	.00384	.00159	.00081	.00047	.00029	.00020			
0.3	.13181	.13305	.14051	.14062	.14267	.12745	.09078	.04817	.01982	.00927	.00238							
0.4	.16008	.16070	.16229	.16288	.15756	.13696	.10248	.05887	.02545									
0.5	.17889	.17917	.17826	.17481	.16403	.14074	.10894	.06670	.03039	.00921	.00390	.00200	.00116	.00073	.00049	.00025	.00015	.00009
0.6	.18915	.18867	.18573	.17887	.16489	.14137	.11186	.07212										
0.7	.19244	.19132	.18679	.17782	.16229	.13926	.11237	.07551	.03801									
0.8	.19046	.18927	.18348	.17306	.15714	.13548	.11115	.07728										
0.9	.18481	.18349	.17709	.16635	.15063	.13067	.10866	.07788										
1	.17678	.17503	.16886	.15824	.14344	.12513	.10540	.07753	.04456	.01611	.00725	.00382	.00224	.00142	.00096	.00050	.00029	.00018
1.2	.15742	.15618	.15014	.14073	.12823	.11340	.09757	.07484	.04575	.01796	.00835	.00446	.00264	.00169	.00114			
1.5	.12801	.12754	.12237	.11549	.10657	.09608	.08491	.06833	.04539	.01983	.00970	.00532	.00320	.00205	.00140	.00073	.00043	.00027
2	.08944	.09080	.08668	.08273	.07814	.07187	.06566	.05589	.04103	.02098	.01117	.00643	.00398	.00260	.00179	.00095	.00056	.00036
2.5	.06403	.06565	.06284	.06068	.05777	.05525	.05069	.04486	.03532	.02045	.01183	.00717	.00457	.00306	.00213	.00115	.00068	.00044
3	.04744	.04834	.04760	.04548	.04391	.04195	.03963	.03606	.02983	.01904	.01187	.00755	.00497	.00341	.00242	.00133	.00080	.00052
4	.02854	.02928	.02996	.02798	.02724	.02661	.02568	.02408	.02110	.01552	.01087	.00757	.00533	.00382	.00280	.00160	.00100	.00065
5	.01886	.01950				.01816			.01535	.01230	.00939	.00700	.00523	.00392	.00299	.00180	.00114	.00077
6	.01333					.01351			.01149	.00976	.00788	.00625	.00488	.00381	.00301	.00190	.00124	.00086
7	.00990					.00966			.00899	.00787	.00662	.00542	.00445	.00360	.00292	.00192	.00130	.00092
8	.00763					.00759			.00727	.00641	.00554	.00477	.00402	.00332	.00275	.00192	.00131	.00096
9	.00607					.00746			.00601	.00533	.00470	.00415	.00358	.00303	.00260	.00187	.00133	.00099
10								.00542	.00506	.00450	.00398	.00364	.00319	.00278	.00239			

TABLE 3.10

FUNCTION "E_1"

(Ahlvin and Ulery, 1962)

r/a z/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	.5	.5	.5	.5	.5	.5	.34722	.22222	.12500	.05556	.03125	.02000	.01389	.01020	.00781	.00500	.00347	.00255
0.1	.45025	.44949	.44698	.44173	.43008	.39198	.30445	.20399	.11806	.05362	.03045	.01959						
0.2	.40194	.40043	.39591	.38660	.36798	.32802	.26598	.18633	.11121	.05170	.02965	.01919	.01342	.00991	.00762			
0.3	.35633	.35428	.33809	.33674	.31578	.28003	.23311	.16967	.10450	.04979	.02886							
0.4	.31431	.31214	.30541	.29298	.27243	.24200	.20526	.15428	.09801									
0.5	.27639	.27407	.26732	.25511	.23639	.21119	.18168	.14028	.09180	.04608	.02727	.01800	.01272	.00946	.00734	.00475	.00332	.00246
0.6	.24275	.24247	.23411	.22289	.20634	.18520	.16155	.12759										
0.7	.21327	.21112	.20535	.19525	.18093	.16356	.14421	.11620	.08027									
0.8	.18765	.18550	.18049	.17190	.15977	.14523	.12928	.10602										
0.9	.16552	.16337	.15921	.15179	.14168	.12954	.11634	.09686										
1	.14645	.14483	.14610	.13472	.12618	.11611	.10510	.08865	.06552	.03736	.02352	.01602	.01157	.00874	.00683	.00450	.00318	.00237
1.2	.11589	.11435	.11201	.10741	.10140	.09431	.08657	.07476	.05728	.03425	.02208	.01527	.01113	.00847	.00664			
1.5	.08398	.08356	.08159	.07885	.07517	.07088	.06611	.05871	.04703	.03003	.02008	.01419	.01049	.00806	.00636	.00425	.00304	.00228
2	.05279	.05105	.05146	.05034	.04850	.04675	.04442	.04078	.03454	.02410	.01706	.01248	.00943	.00738	.00590	.00401	.00290	.00219
2.5	.03576	.03426	.03489	.03435	.03360	.03211	.03150	.02953	.02599	.01945	.01447	.01096	.00850	.00674	.00546	.00378	.00276	.00210
3	.02566	.02519	.02470	.02491	.02444	.02389	.02330	.02216	.02007	.01585	.01230	.00962	.00763	.00617	.00505	.00355	.00263	.00201
4	.01493	.01452	.01495	.01526	.01446	.01418	.01395	.01356	.01281	.01084	.00900	.00742	.00612	.00511	.00431	.00313	.00237	.00185
5	.00971	.00927				.00929			.00873	.00774	.00673	.00579	.00495	.00425	.00364	.00275	.00213	.00168
6	.00680					.00632			.00629	.00574	.00517	.00457	.00404	.00354	.00309	.00241	.00192	.00154
7	.00503					.00493			.00466	.00438	.00404	.00370	.00330	.00296	.00264	.00213	.00172	.00140
8	.00386					.00377			.00354	.00344	.00325	.00297	.00273	.00250	.00228	.00185	.00155	.00127
9	.00306					.00227			.00275	.00273	.00264	.00246	.00229	.00212	.00194	.00163	.00139	.00116
10								.00210	.00220	.00225	.00221	.00203	.00200	.00181	.00171			

TABLE 3.11

FUNCTION "F"

(Ahlvin and Ulery, 1962)

r/a z/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	.5	.5	.5	.5	.5	0	-.34722	-.22222	-.12500	-.05556	-.03125	-.02000	-.01389	-.01020	-.00781	-.00500	-.00347	-.00255
0.1	.45025	.44794	.43981	.41954	.35789	.03817	-.20800	-.17612	-.10950	-.05151	-.02961	-.01917						
0.2	.40194	.39781	.38294	.34823	.26215	.05466	-.11165	-.13381	-.09441	-.04750	-.02798	-.01835	-.01295	-.00961	-.00742			
0.3	.35633	.35094	.34508	.29016	.20503	.06372	-.05346	-.09768	-.08010	-.04356	-.02636							
0.4	.31431	.30801	.28681	.24469	.17086	.06848	-.01818	-.06835	-.06684									
0.5	.27639	.26997	.24890	.20937	.14752	.07037	.00388	-.04529	-.05479	-.03595	-.02320	-.01590	-.01154	-.00875	-.00681	-.00450	-.00318	-.00237
0.6	.24275	.23444	.21667	.18138	.13042	.07068	.01797	-.02749										
0.7	.21327	.20762	.18956	.15903	.11740	.06963	.02704	-.01392	-.03469									
0.8	.18765	.18287	.16679	.14053	.10604	.06774	.03277	-.00365										
0.9	.16552	.16158	.14747	.12528	.09664	.06533	.03619	.00408										
1	.14645	.14280	.12395	.11225	.08850	.06256	.03819	.00984	-.01367	-.01994	-.01591	-.01209	-.00931	-.00731	-.00587	-.00400	-.00289	-.00219
1.2	.11589	.11360	.10460	.09449	.07486	.05670	.03913	.01716	-.00452	-.01491	-.01337	-.01068	-.00844	-.00676	-.00550			
1.5	.08398	.08196	.07719	.06918	.05919	.04804	.03686	.02177	.00413	-.00879	-.00995	-.00870	-.00723	-.00596	-.00495	-.00353	-.00261	-.00201
2	.05279	.05348	.04994	.04614	.04162	.03593	.03029	.02197	.01043	-.00189	-.00546	-.00589	-.00544	-.00474	-.00410	-.00307	-.00233	-.00183
2.5	.03576	.03673	.03459	.03263	.03014	.02762	.02406	.01927	.01188	.00198	-.00226	-.00364	-.00386	-.00366	-.00332	-.00263	-.00208	-.00166
3	.02566	.02586	.02255	.02395	.02263	.02097	.01911	.01623	.01144	.00396	-.00010	-.00192	-.00258	-.00271	-.00263	-.00223	-.00183	-.00150
4	.01493	.01536	.01412	.01259	.01386	.01331	.01256	.01134	.00912	.00508	.00209	.00026	-.00076	-.00127	-.00148	-.00153	-.00137	-.00120
5	.00971	.01011				.00905			.00700	.00475	.00277	.00129	.00031	-.00030	-.00066	-.00096	-.00099	-.00093
6	.00680					.00675			.00538	.00409	.00278	.00170	.00088	.00030	-.00010	-.00053	-.00066	-.00070
7	.00503					.00483			.00428	.00346	.00258	.00178	.00114	.00064	.00027	-.00020	-.00041	-.00049
8	.00386					.00380			.00350	.00291	.00229	.00174	.00125	.00082	.00048	.00003	-.00020	-.00033
9	.00306					.00374			.00291	.00247	.00203	.00163	.00124	.00089	.00062	.00020	-.00005	-.00019
10								.00267	.00246	.00213	.00176	.00149	.00126	.00092	.00070			

TABLE 3.12

FUNCTION "G_1"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	.31831	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	.00315	.00802	.01951	.06682	.31405	.05555	.00865	.00159	.00023	.00007	.00003						
0.2	0	.01163	.02877	.06441	.16214	.30474	.13592	.03060	.00614	.00091	.00026	.00010	.00005	.00003	.00002			
0.3	0	.02301	.05475	.11072	.21465	.29228	.18216	.05747	.01302	.00201	.00059							
0.4	0	.03460	.07883	.14477	.23442	.27779	.20195	.08233	.02138									
0.5	0	.04429	.09618	.16426	.23652	.26216	.20731	.10185	.03033	.00528	.00158	.00063	.00030	.00016	.00009	.00004	.00002	.00001
0.6	0	.04966	.10729	.17192	.22949	.24574	.20496	.11541										
0.7	0	.05484	.11256	.17126	.21772	.22924	.19840	.12373	.04718									
0.8	0	.05590	.11225	.16534	.20381	.21295	.18953	.12855										
0.9	0	.05496	.10856	.15628	.18904	.19712	.17945	.28881										
1	0	.05266	.10274	.14566	.17419	.18198	.16884	.12745	.06434	.01646	.00555	.00233	.00113	.00062	.00036	.00015	.00007	.00004
1.2	0	.04585	.08831	.12323	.14615	.15408	.14755	.12038	.06967	.02077	.00743	.00320	.00159	.00087	.00051			
1.5	0	.03483	.06688	.09293	.11071	.11904	.11830	.10477	.07075	.02599	.01021	.00460	.00233	.00130	.00078	.00033	.00016	.00009
2	0	.02102	.04069	.05721	.06948	.07738	.08067	.07804	.06275	.03062	.01409	.00692	.00369	.00212	.00129	.00055	.00027	.00015
2.5	0	.01293	.02534	.03611	.04484	.05119	.05509	.05668	.05117	.03099	.01650	.00886	.00499	.00296	.00185	.00082	.00041	.00023
3	0	.00840	.01638	.02376	.02994	.03485	.03843	.04124	.04039	.02886	.01745	.01022	.00610	.00376	.00241	.00110	.00057	.00032
4	0	.00382	.00772	.01149	.01480	.01764	.02004	.02271	.02475	.02215	.01639	.01118	.00745	.00499	.00340	.00167	.00090	.00052
5	0	.00214				.00992		.01343	.01551	.01601	.01364	.01105	.00782	.00560	.00404	.00216	.00122	.00073
6	0					.00602		.00845	.01014	.01148	.01082	.00917	.00733	.00567	.00432	.00243	.00150	.00092
7	0					.00396			.00687	.00830	.00842	.00770	.00656	.00539	.00432	.00272	.00171	.00110
8	0					.00270			.00481	.00612	.00656	.00631	.00568	.00492	.00413	.00278	.00185	.00124
9	0					.00177			.00347	.00459	.00513	.00515	.00485	.00438	.00381	.00274	.00192	.00133
10	0							.00199	.00258	.00351	.00407	.00420	.00411	.00382	.00346			

TABLE 3.13

FUNCTION "H"

(Ahlvin and Ulery, 1962)

z/a \ r/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	7	8	10	12	14
0	2.0	1.97987	1.91751	1.80575	1.62553	1.27319	.93676	.71185	.51671	.33815	.25200	.20045	.16626	.14315	.12576	.09918	.08346	.07023
0.1	1.80998	1.79018	1.72886	1.61961	1.44711	1.18107	.92670	.70888	.51627	.33794	.25184	.20081						
0.2	1.63961	1.62068	1.56242	1.46001	1.30614	1.09996	.90098	.70074	.51382	.33726	.25162	.20072	.16688	.14288	.12512			
0.3	1.48806	1.47044	1.40979	1.32442	1.19210	1.02740	.86726	.68823	.50966	.33638	.25124							
0.4	1.35407	1.33802	1.28963	1.20822	1.09555	.96202	.83042	.67238	.50412									
0.5	1.23607	1.22176	1.17894	1.10830	1.01312	.90298	.79308	.65429	.49728	.33293	.24996	.19982	.16668	.14273	.12493	.09996	.08295	.07123
0.6	1.13238	1.11998	1.08350	1.02154	.94120	.84917	.75653	.63469										
0.7	1.04131	1.03037	.99794	.91049	.87742	.80030	.72143	.61442	.48061									
0.8	.96125	.95175	.92386	.87928	.82136	.75571	.68809	.59398										
0.9	.89072	.88251	.85856	.82616	.77950	.71495	.65677	.57361										
1	.82843	.85005	.80465	.76809	.72587	.67769	.62701	.55364	.45122	.31877	.24386	.19673	.16516	.14182	.12394	.09952	.08292	.07104
1.2	.72410	.71882	.70370	.67937	.64814	.61187	.57329	.51552	.43013	.31162	.24070	.19520	.16369	.14099	.12350			
1.5	.60555	.60233	.57246	.57633	.55559	.53138	.50496	.46379	.39872	.29945	.23495	.19053	.16199	.14058	.12281	.09876	.08270	.07064
2	.47214	.47022	.44512	.45656	.44502	.43202	.41702	.39242	.35054	.27740	.22418	.18618	.15846	.13762	.12124	.09792	.08196	.07026
2.5	.38518	.38403	.38098	.37608	.36940	.36155	.35243	.33698	.30913	.25550	.21208	.17898	.15395	.13463	.11928	.09700	.08115	.06980
3	.32457	.32403	.32184	.31887	.31464	.30969	.30381	.29364	.27453	.23487	.19977	.17154	.14919	.13119	.11694	.09558	.08061	.06897
4	.24620	.24588	.24820	.25128	.24168	.23932	.23668	.23164	.22188	.19908	.17640	.15596	.13864	.12396	.11172	.09300	.07864	.06848
5	.19805	.19785				.19455			.18450	.17080	.15575	.14130	.12785	.11615	.10585	.08915	.07675	.06695
6	.16554					.16326			.15750	.14868	.13842	.12792	.11778	.10836	.09990	.08562	.07452	.06522
7	.14217					.14077			.13699	.13097	.12404	.11620	.10843	.10101	.09387	.08197	.07210	.06377
8	.12448					.12352			.12112	.11680	.11176	.10600	.09976	.09400	.08848	.07800	.06928	.06200
9	.11079					.10989			.10854	.10548	.10161	.09702	.09234	.08784	.08298	.07407	.06678	.05976
10								.09900	.09820	.09510	.09290	.08980	.08300	.08180	.07710			

3.3.2 CONICAL LOADING

Distributions of σ_z down the axis (Harr and Lovell, 1963) and σ_r along the axis (Schiffman, 1963) are shown in Figs. 3.19 and 3.20.

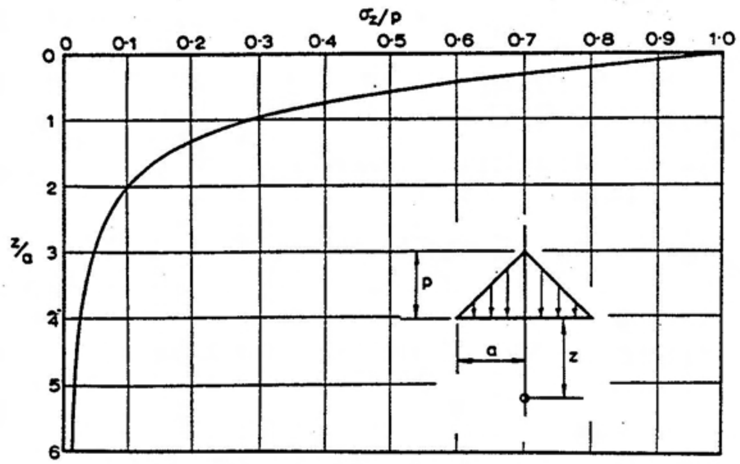


FIG.3.19 Vertical stress σ_z on axis due to conical loading (Harr and Lovell, 1963).

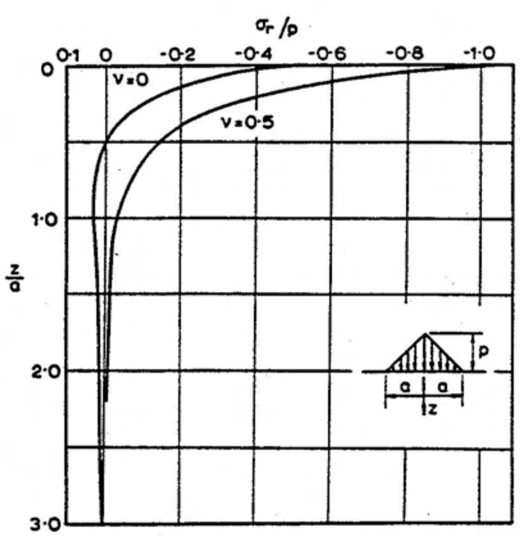


FIG.3.20 Radial stress σ_r on axis due to conical loading (Schiffman, 1963).

3.3.3 UNIFORM HORIZONTAL LOADING

As pointed out by Barber (1966), the vertical stress due to a uniform horizontal load is the same as the shear stress due to a uniform vertical load. Thus, these stresses may be evaluated from the values in Section 3.3.1.

Vertical and horizontal normal stresses presented by Barber (1963) are reproduced in Figs. 3.21 and 3.22.

Charts for determining the normal stresses at any point due to horizontal loading on any shape of loaded area are given in Section 3.6.

General expressions and tabulated values for all stresses, strains and displacements are given in Appendix B.

3.3.4 INWARD SHEAR LOADINGS

For uniform inward shear on a circle, values of σ_z presented by Barber (1963) are shown in Fig.3.23. General expressions and tabulated values for all stresses, strains and displacements are given in Appendix B.

For non-uniform inward shear on a circle, values of σ_z presented by Barber (1963) are shown in Figure 3.24.

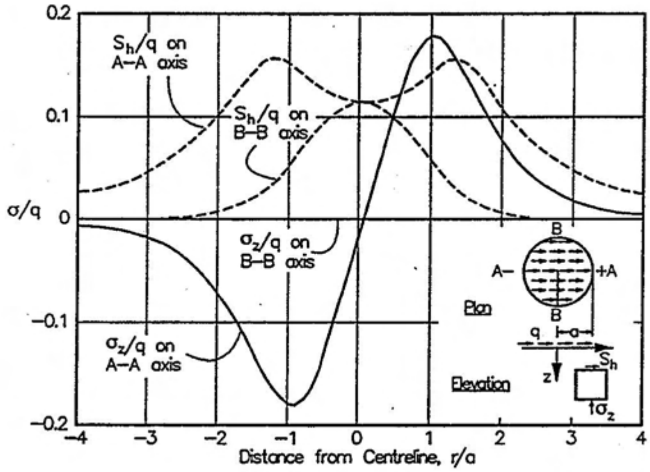


FIG.3.21 Stresses due to horizontal loading on circle. z/a=1, ν=0.5. (Barber, 1963).

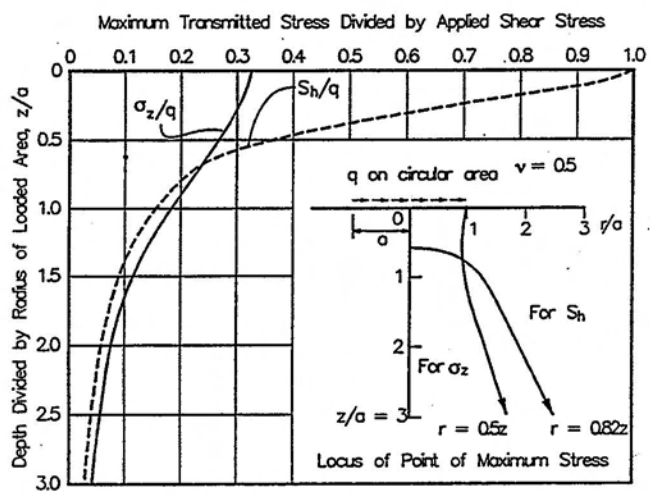


FIG.3.22 Maximum stress at different depths due to horizontal loading on circle (Barber, 1963).

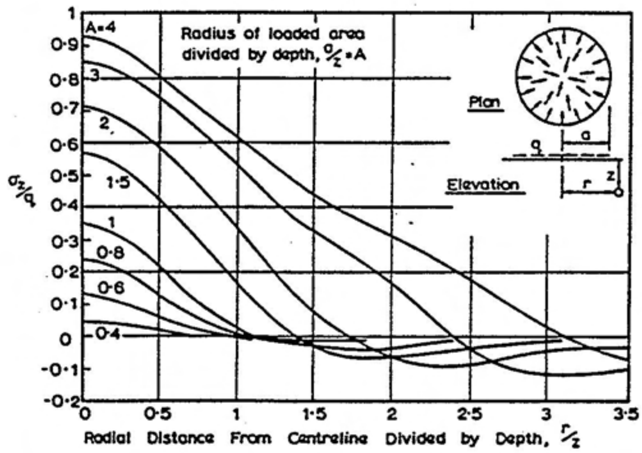


FIG.3.23 Vertical stress σ_z due to uniform inward shear on circle (Barber, 1963).

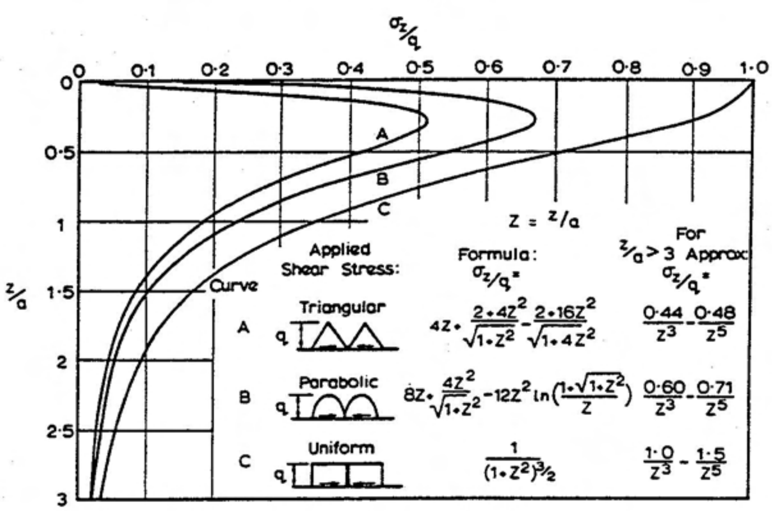


FIG.3.24 Vertical stress σ_z on axis due to various shear loads on circle (Barber, 1963).

3.3.5 UNIFORM VERTICAL LOADING ON A PERFECTLY ROUGH CIRCULAR AREA (Fig.3.13)

This problem was considered by Schiffman (1968), who obtained general solutions for stresses and displacements.

Along the axis, the stresses are as follows:

$$\sigma_z = p\left[1 - \left\{\frac{1}{1+(\frac{a}{z})^2}\right\}^{3/2} - \eta\frac{z}{a}K_o^2(0,\tfrac{z}{a})\right] \quad \ldots (3.15a)$$

$$\sigma_\theta = \sigma_r = p\left[\frac{(1+2\nu)}{2} - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right.$$

$$\left. -\eta\{(1+\nu)K_o^1(0,\tfrac{z}{a}) - \frac{z}{2a}K_o^2(0,\tfrac{z}{a})\}\right] \quad \ldots (3.15b)$$

$$\tau_{rz} = 0 \quad \ldots (3.15c)$$

where $K_o^1(0,z/a) = \sqrt{2/\pi}\left[\tan^{-1}(a/z) - \dfrac{z/a}{(1+(\frac{z}{a})^2)}\right]$

$$K_o^2(0,z/a) = \sqrt{8/\pi}\,/\,(1+(z/a)^2)^2$$

The distributions of σ_r and σ_z along the axis are shown in Figs. 3.25a and 3.25b for $\nu=0$ and 0.5. Also shown are the corresponding distributions for a frictionless or smooth surface. For $\nu=0.5$, friction has no influence on the stresses.

Along the surface $(z=0)$, the shear stress is

$$\tau_{rz} = \frac{(1-2\nu)\,p\,r}{(1-\nu)\pi\sqrt{a^2-r^2}} \qquad (r/a < 1) \quad \ldots (3.16a)$$

$$\tau_{rz} = 0 \qquad (r/a > 1) \quad \ldots (3.16b)$$

The radial distribution of τ_{rz} beneath the surface of the circle is shown in Fig.3.26 for various values of ν.

The surface displacements are as follows:

$$\rho_z = \frac{pa(1+\nu)}{E}\left[4(1-\nu)E(r/a) - \frac{(1-2\nu)^2}{1-\nu}\cdot\sqrt{1-(\tfrac{r}{a})^2}\right] \qquad (r/a < 1) \quad \ldots (3.17a)$$

$$\rho_z = \frac{4p(1-\nu^2)r}{\pi E}\left[E(\tfrac{a}{r}) - \frac{(r^2-a^2)}{r^2}K(\tfrac{a}{r})\right] \qquad (r/a > 1) \quad \ldots (3.17b)$$

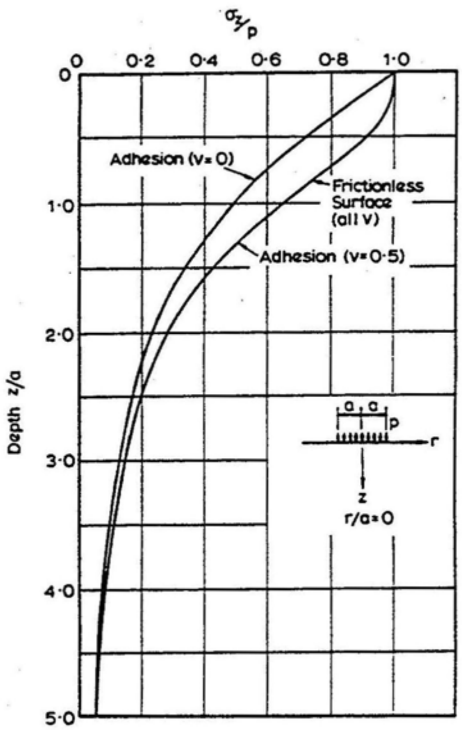
$$\rho_r = 0 \qquad (r/a < 1) \quad \ldots (3.17c)$$

$$\rho_r = \frac{(1+\nu)(1-2\nu)pa}{E\sqrt{\pi}}\left[\frac{r}{a}\sin^{-1}(\tfrac{a}{r}) + \left(\frac{1+\sqrt{r^2-a^2}}{r}\right)\right] \qquad (r/a > 1) \quad \ldots (3.17d)$$

where $K(k)$ is the complete elliptic integral of the first kind,
$E(k)$ is the complete elliptic integral of the second kind.

Surface displacement profiles are shown in Fig. 3.27 for both a frictionless and fully rough circular areas. For $\nu=0.5$, friction has no influence on displacement.

The influence of ν on the central vertical surface displacement is shown in Fig.3.28.



(a) Vertical stress σ_z

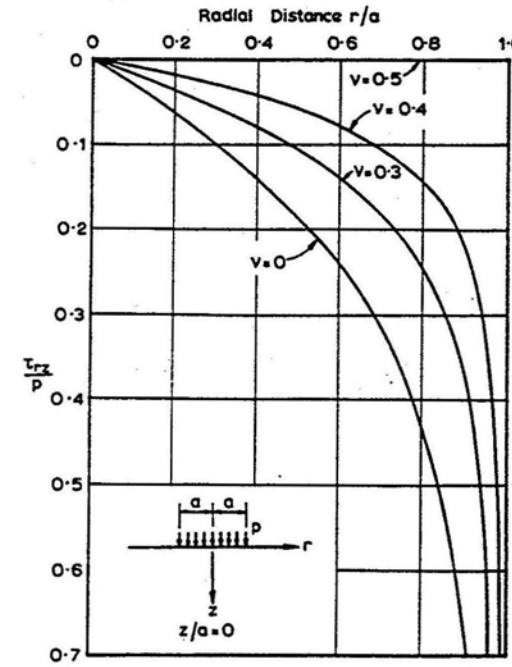
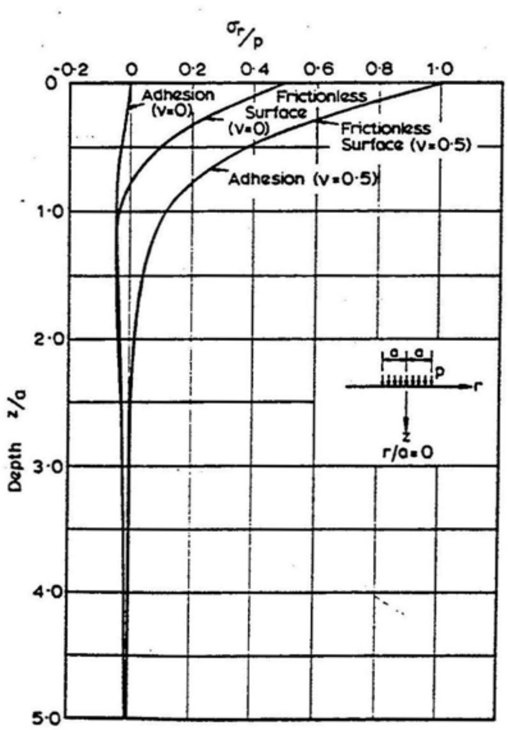


FIG.3.26 Distribution of shear stress τ_{rz} along surface (Schiffman, 1968).



(b) Radial stress σ_r

FIG.3.25 Distribution of stress on axis of circle. (Schiffman, 1968).

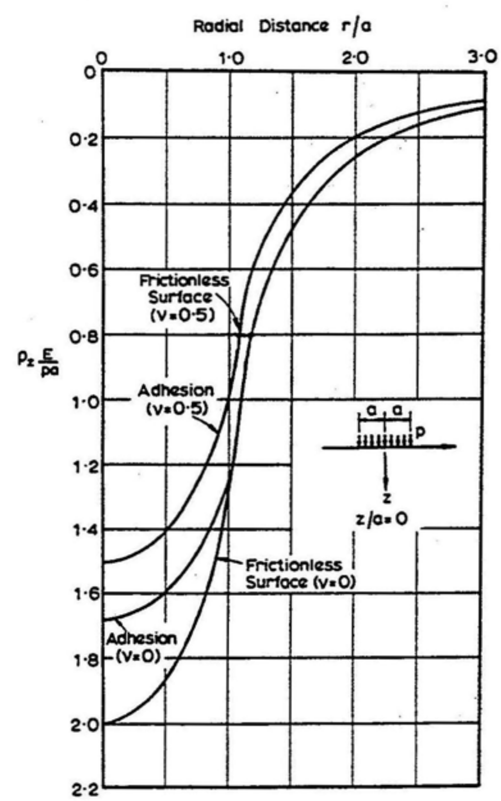


FIG.3.27 Vertical displacement profile along surface. (Schiffman, 1968).

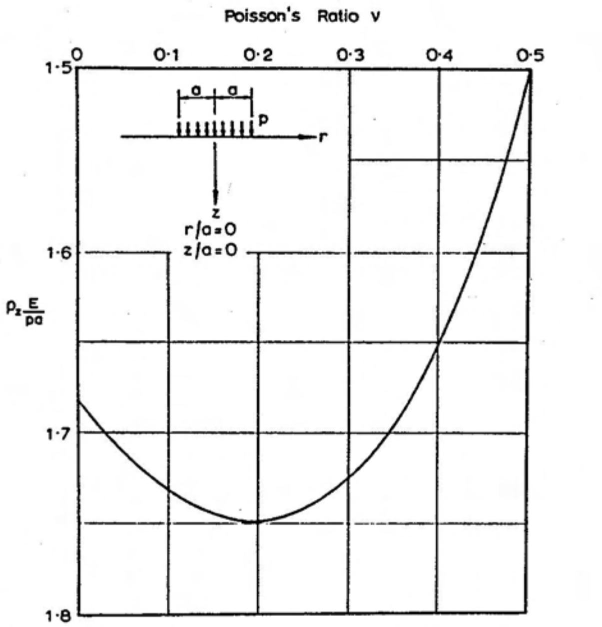


FIG.3.28 Influence of ν on ρ_z at surface on axis. (Schiffman, 1968).

3.3.6 OTHER TYPES OF LOADING

(i) Parabolic loading - see Harr and Lovell (1963) and Schiffman (1963).

(ii) Linearly varying vertical stress - see Appendix B.

(iii) Linearly varying torsional stress - see Appendix B.

3.4 Loading on a Rectangular Area

3.4.1 UNIFORM VERTICAL LOADING

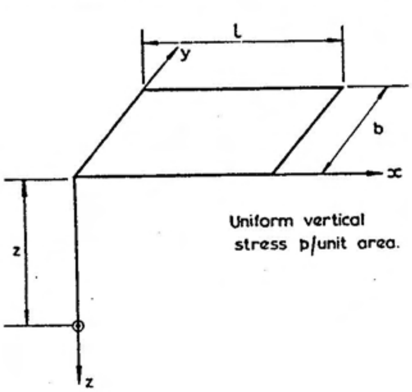


FIG. 3.29

Beneath the corner of the rectangle (see Fig. 3.29), Holl (1940) gives the following expressions for stresses for $\nu = 0.5$:

$$\sigma_z = \frac{p}{2\pi}\left[\tan^{-1}\frac{\ell b}{zR_3} + \frac{\ell b z}{R_3}\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\right] \quad \ldots (3.18a)$$

$$\sigma_x = \frac{p}{2\pi}\left[\tan^{-1}\frac{\ell b}{zR_3} - \frac{\ell b z}{R_1^2 R_3}\right] \quad \ldots (3.18b)$$

$$\sigma_y = \frac{p}{2\pi}\left[\tan^{-1}\frac{\ell b}{zR_3} - \frac{\ell b z}{R_2^2 R_3}\right] \quad \ldots (3.18c)$$

$$\tau_{xz} = \frac{p}{2\pi}\left[\frac{b}{R_2} - \frac{z^2 b}{R_1^2 R_3}\right] \quad \ldots (3.18d)$$

$$\tau_{yz} = \frac{p}{2\pi}\left[\frac{\ell}{R_1} - \frac{z^2 \ell}{R_2^2 R_3}\right] \quad \ldots (3.18e)$$

$$\tau_{xy} = \frac{p}{2\pi}\left[1 + \frac{z}{R_3} - z\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right] \quad \ldots (3.18f)$$

where $R_1 = (\ell^2+z^2)^{\frac{1}{2}}$

$R_2 = (b^2+z^2)^{\frac{1}{2}}$

$R_3 = (\ell^2+b^2+z^2)^{\frac{1}{2}}$

Influence factors for the normal stresses have been presented by Giroud (1970). These stresses are expressed as follows:

Under the corners:

$$\sigma_z = p\,K_o \quad \ldots (3.19a)$$

$$\sigma_x = p\,[K_2 - (1-2\nu)K_2^1] \quad \ldots (3.19b)$$

$$\sigma_y = p\,[L_2 - (1-2\nu)L_2^1] \quad \ldots (3.19c)$$

The influence factors K_o($\equiv I$ in Fig.3.30), K_2, K_2^1, L_2, L_2^1 are reproduced in Tables 3.14 to 3.18.

Influence factors for σ_z beneath the corner are shown in Fig.3.30(Fadum, 1948). For points other than the corner, the principle of superposition may be employed.

Beneath the corner of a rectangle (see Fig.3.29), Harr (1966) quotes the following expression for vertical displacement at depth z:

$$\rho_z = \frac{pb}{E}(1-\nu^2)\left(A - \frac{1-2\nu}{1-\nu}B\right) \quad \ldots (3.20)$$

where $$A = \frac{1}{2\pi}\left(\ell n\,\frac{\sqrt{1+m_1^2+n_1^2}+m_1}{\sqrt{1+m_1^2+n_1^2}-m_1} + m_1\,\ell n\,\frac{\sqrt{1+m_1^2+n_1^2}+1}{\sqrt{1+m_1^2+n_1^2}-1}\right)$$

TABLE 3.14

VALUES OF K_o

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.137	0.204	0.234	0.240	0.244	0.247	0.249	0.249	0.249	0.249	0.249	0.249	0.249	0.249
0.4	0.000	0.076	0.136	0.187	0.202	0.218	0.231	0.240	0.243	0.244	0.244	0.244	0.244	0.244	0.244
0.5	0.000	0.061	0.113	0.164	0.181	0.200	0.218	0.232	0.238	0.239	0.240	0.240	0.240	0.240	0.240
0.6	0.000	0.051	0.096	0.143	0.161	0.182	0.204	0.223	0.231	0.233	0.234	0.234	0.234	0.234	0.234
0.8	0.000	0.037	0.071	0.111	0.127	0.148	0.173	0.200	0.214	0.218	0.219	0.220	0.220	0.220	0.220
1	0.000	0.028	0.055	0.087	0.101	0.120	0.145	0.175	0.194	0.200	0.202	0.203	0.204	0.205	0.205
1.2	0.000	0.022	0.043	0.069	0.081	0.098	0.121	0.152	0.173	0.182	0.185	0.187	0.189	0.189	0.189
1.4	0.000	0.018	0.035	0.056	0.066	0.080	0.101	0.131	0.154	0.164	0.169	0.171	0.174	0.174	0.174
1.5	0.000	0.016	0.031	0.051	0.060	0.073	0.092	0.121	0.145	0.156	0.161	0.164	0.166	0.167	0.167
1.6	0.000	0.014	0.028	0.046	0.055	0.067	0.085	0.112	0.136	0.148	0.154	0.157	0.160	0.160	0.160
1.8	0.000	0.012	0.024	0.039	0.046	0.056	0.072	0.097	0.121	0.133	0.140	0.143	0.147	0.148	0.148
2	0.000	0.010	0.020	0.033	0.039	0.048	0.061	0.084	0.107	0.120	0.127	0.131	0.136	0.137	0.137
2.5	0.000	0.007	0.013	0.022	0.027	0.033	0.043	0.060	0.080	0.093	0.101	0.106	0.113	0.115	0.115
3	0.000	0.005	0.010	0.016	0.019	0.024	0.031	0.045	0.061	0.073	0.081	0.087	0.096	0.099	0.099
4	0.000	0.003	0.006	0.009	0.011	0.014	0.019	0.027	0.038	0.048	0.055	0.060	0.071	0.076	0.076
5	0.000	0.002	0.004	0.006	0.007	0.009	0.012	0.018	0.026	0.033	0.039	0.043	0.055	0.061	0.062
10	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.005	0.007	0.009	0.011	0.013	0.020	0.028	0.032
15	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.004	0.005	0.006	0.010	0.016	0.021
20	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.004	0.006	0.010	0.016
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.006

TABLE 3.15

VALUES OF K_2

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.069	0.116	0.149	0.159	0.169	0.177	0.184	0.187	0.188	0.188	0.188	0.188	0.188	0.188
0.4	0.000	0.031	0.058	0.085	0.095	0.106	0.118	0.128	0.133	0.134	0.134	0.134	0.135	0.135	0.135
0.5	0.000	0.022	0.043	0.064	0.073	0.083	0.094	0.105	0.110	0.112	0.112	0.112	0.113	0.113	0.113
0.6	0.000	0.017	0.032	0.049	0.056	0.065	0.075	0.086	0.091	0.093	0.093	0.094	0.094	0.094	0.094
0.8	0.000	0.009	0.018	0.029	0.034	0.040	0.047	0.057	0.062	0.064	0.064	0.065	0.065	0.065	0.065
1	0.000	0.006	0.011	0.018	0.021	0.025	0.030	0.037	0.042	0.044	0.045	0.045	0.045	0.045	0.045
1.2	0.000	0.003	0.007	0.011	0.013	0.016	0.020	0.025	0.029	0.031	0.032	0.032	0.032	0.032	0.032
1.4	0.000	0.002	0.004	0.007	0.008	0.010	0.013	0.017	0.020	0.022	0.023	0.023	0.023	0.023	0.023
1.5	0.000	0.002	0.004	0.006	0.007	0.008	0.011	0.014	0.017	0.019	0.019	0.020	0.020	0.020	0.020
1.6	0.000	0.001	0.003	0.005	0.006	0.007	0.009	0.012	0.015	0.016	0.017	0.017	0.017	0.017	0.017
1.8	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.008	0.011	0.012	0.012	0.013	0.013	0.013	0.013
2	0.000	0.001	0.001	0.002	0.003	0.003	0.004	0.006	0.008	0.009	0.009	0.010	0.010	0.010	0.010
2.5	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.005	0.005	0.006	0.006	0.006
3	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.003	0.003	0.003	0.003
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.16

VALUES OF K_2'

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.234	0.219	0.199	0.189	0.176	0.156	0.125	0.094	0.074	0.061	0.051	0.031	0.016	0.000
0.2	0.000	0.059	0.097	0.118	0.121	0.122	0.118	0.103	0.082	0.067	0.056	0.048	0.030	0.016	0.000
0.4	0.000	0.026	0.048	0.069	0.075	0.082	0.086	0.083	0.071	0.060	0.051	0.045	0.029	0.015	0.000
0.5	0.000	0.019	0.036	0.054	0.060	0.067	0.073	0.074	0.066	0.056	0.049	0.043	0.028	0.015	0.000
0.6	0.000	0.015	0.028	0.043	0.049	0.056	0.062	0.066	0.061	0.053	0.047	0.041	0.028	0.015	0.000
0.8	0.000	0.009	0.018	0.029	0.033	0.039	0.046	0.052	0.052	0.047	0.043	0.038	0.026	0.015	0.000
1	0.000	0.007	0.013	0.021	0.024	0.029	0.035	0.042	0.044	0.042	0.039	0.035	0.025	0.014	0.000
1.2	0.000	0.005	0.009	0.015	0.018	0.022	0.027	0.034	0.037	0.037	0.035	0.032	0.024	0.014	0.000
1.4	0.000	0.004	0.007	0.012	0.014	0.017	0.021	0.027	0.032	0.033	0.032	0.030	0.023	0.014	0.000
1.5	0.000	0.003	0.006	0.010	0.012	0.015	0.019	0.025	0.029	0.031	0.030	0.029	0.022	0.014	0.000
1.6	0.000	0.003	0.006	0.009	0.011	0.013	0.017	0.023	0.027	0.029	0.028	0.027	0.022	0.013	0.000
1.8	0.000	0.002	0.005	0.007	0.009	0.011	0.014	0.019	0.024	0.025	0.026	0.025	0.021	0.013	0.000
2	0.000	0.002	0.004	0.006	0.007	0.009	0.012	0.016	0.020	0.023	0.023	0.023	0.020	0.013	0.000
2.5	0.000	0.001	0.002	0.004	0.005	0.006	0.008	0.011	0.015	0.017	0.018	0.019	0.017	0.012	0.000
3	0.000	0.001	0.002	0.003	0.003	0.004	0.006	0.008	0.011	0.013	0.015	0.015	0.015	0.011	0.000
4	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.005	0.007	0.008	0.010	0.011	0.012	0.010	0.000
5	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.004	0.006	0.007	0.008	0.009	0.009	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.005	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.17

VALUES OF L_2

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.010	0.045	0.094	0.112	0.134	0.158	0.184	0.201	0.208	0.211	0.214	0.217	0.218	0.219
0.4	0.000	0.002	0.010	0.032	0.045	0.064	0.091	0.128	0.156	0.169	0.176	0.179	0.186	0.188	0.189
0.5	0.000	0.001	0.006	0.020	0.029	0.044	0.068	0.105	0.136	0.151	0.159	0.164	0.172	0.175	0.176
0.6	0.000	0.000	0.003	0.013	0.019	0.031	0.051	0.086	0.118	0.134	0.144	0.149	0.158	0.163	0.164
0.8	0.000	0.000	0.001	0.006	0.009	0.016	0.029	0.057	0.087	0.106	0.117	0.124	0.135	0.141	0.143
1	0.000	0.000	0.001	0.003	0.005	0.009	0.017	0.037	0.064	0.083	0.095	0.103	0.116	0.123	0.125
1.2	0.000	0.000	0.000	0.002	0.003	0.005	0.011	0.025	0.047	0.065	0.077	0.085	0.100	0.108	0.111
1.4	0.000	0.000	0.000	0.001	0.002	0.003	0.007	0.017	0.035	0.051	0.062	0.071	0.087	0.095	0.099
1.5	0.000	0.000	0.000	0.001	0.001	0.003	0.005	0.014	0.030	0.045	0.056	0.064	0.081	0.090	0.094
1.6	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.012	0.026	0.040	0.051	0.059	0.076	0.085	0.089
1.8	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.008	0.020	0.031	0.041	0.049	0.066	0.077	0.081
2	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.006	0.015	0.025	0.034	0.041	0.058	0.069	0.074
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.014	0.021	0.027	0.043	0.055	0.061
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.008	0.013	0.018	0.032	0.045	0.051
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.006	0.008	0.018	0.031	0.039
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.004	0.011	0.022	0.031
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.016
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.011
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.008
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003

TABLE 3.18

VALUES OF L_2'

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.016	0.031	0.051	0.061	0.074	0.094	0.125	0.156	0.176	0.189	0.199	0.219	0.234	0.250
0.2	0.000	0.013	0.025	0.041	0.049	0.060	0.076	0.103	0.130	0.148	0.160	0.169	0.188	0.203	0.219
0.4	0.000	0.010	0.020	0.032	0.039	0.047	0.061	0.083	0.106	0.122	0.133	0.141	0.159	0.174	0.189
0.5	0.000	0.009	0.017	0.029	0.034	0.042	0.054	0.074	0.096	0.111	0.121	0.129	0.146	0.161	0.176
0.6	0.000	0.008	0.015	0.025	0.030	0.037	0.048	0.066	0.086	0.100	0.110	0.118	0.134	0.149	0.164
0.8	0.000	0.006	0.012	0.020	0.023	0.029	0.037	0.052	0.069	0.082	0.091	0.098	0.114	0.127	0.143
1	0.000	0.005	0.009	0.015	0.018	0.023	0.029	0.042	0.056	0.067	0.075	0.082	0.097	0.110	0.125
1.2	0.000	0.004	0.007	0.012	0.015	0.018	0.024	0.034	0.046	0.056	0.063	0.069	0.083	0.096	0.111
1.4	0.000	0.003	0.006	0.010	0.012	0.015	0.019	0.027	0.038	0.046	0.053	0.058	0.072	0.084	0.099
1.5	0.000	0.003	0.005	0.009	0.011	0.013	0.017	0.025	0.035	0.043	0.049	0.054	0.067	0.079	0.094
1.6	0.000	0.002	0.005	0.008	0.010	0.012	0.016	0.023	0.032	0.039	0.045	0.050	0.062	0.074	0.089
1.8	0.000	0.002	0.004	0.007	0.008	0.010	0.013	0.019	0.027	0.033	0.039	0.043	0.055	0.066	0.081
2	0.000	0.002	0.003	0.006	0.007	0.008	0.011	0.016	0.023	0.029	0.033	0.038	0.048	0.059	0.074
2.5	0.000	0.001	0.002	0.004	0.005	0.006	0.007	0.011	0.016	0.020	0.024	0.027	0.036	0.047	0.061
3	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.008	0.012	0.015	0.018	0.021	0.028	0.038	0.051
4	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.005	0.007	0.009	0.011	0.013	0.018	0.026	0.039
5	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.006	0.007	0.009	0.013	0.019	0.031
10	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.004	0.007	0.016
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.011
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.008
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003

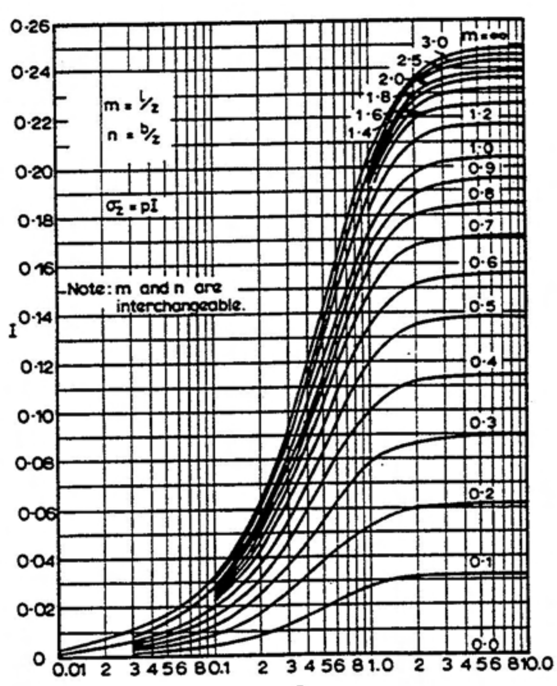


FIG.3.30 Vertical stress beneath corner of uniformly loaded rectangle. (Fadum, 1948).

$$B = \frac{n_1}{2\pi} \tan^{-1} \frac{m_1}{n_1 \sqrt{1 + m_1^2 + n_1^2}}$$

$$m_1 = \ell/b$$

$$n_1 = z/b$$

Explicit expressions and influence factors for the vertical displacement at the surface $(z=0)$ have been evaluated for four points beneath the rectangle, and for the mean displacement ρ_m, by Giroud (1968). These influence factors are shown in Fig.3.31 and are tabulated in Table 3.19.

For all points,

$$\rho_z = \frac{(1-\nu^2)}{E} p\, b\, I \qquad \ldots (3.21)$$

where b is the length of the shorter side
M is the centre of the longer side
N the centre of the shorter side
C the corner
O the centre of the rectangle
m the mean

TABLE 3.19

INFLUENCE FACTORS FOR VERTICAL SURFACE DISPLACEMENT BENEATH RECTANGLE (Giroud, 1968)

$\alpha = \frac{\ell}{b}$	I_C	I_M	I_N	I_O	I_m	$\alpha = \frac{\ell}{b}$	I_C	I_M	I_N	I_O	I_m
1	0.561	0.766	0.766	1.122	0.946	15	1.401	2.362	1.621	2.802	2.498
1.1	0.588	0.810	0.795	1.176	0.992	20	1.493	2.544	1.713	2.985	2.677
1.2	0.613	0.852	0.822	1.226	1.035	25	1.564	2.686	1.784	3.127	2.817
1.3	0.636	0.892	0.847	1.273	1.075	30	1.622	2.802	1.842	3.243	2.932
1.4	0.658	0.930	0.870	1.317	1.112	40	1.713	2.985	1.934	3.426	3.113
1.5	0.679	0.966	0.892	1.358	1.148	50	1.784	3.127	2.005	3.568	3.254
1.6	0.698	1.000	0.912	1.396	1.181	60	1.842	3.243	2.063	3.684	3.370
1.7	0.716	1.033	0.931	1.433	1.213	70	1.891	3.341	2.112	3.783	3.467
1.8	0.734	1.064	0.949	1.467	1.244	80	1.934	3.426	2.154	3.868	3.552
1.9	0.750	1.094	0.966	1.500	1.273	90	1.971	3.501	2.192	3.943	3.627
2	0.766	1.122	0.982	1.532	1.300	100	2.005	3.568	2.225	4.010	3.693
2.2	0.795	1.176	1.012	1.590	1.353	200	2.225	4.010	2.446	4.451	4.134
2.4	0.822	1.226	1.039	1.644	1.401	300	2.355	4.268	2.575	4.709	4.391
2.5	0.835	1.250	1.052	1.669	1.424	400	2.446	4.451	2.667	4.892	4.574
3	0.892	1.358	1.110	1.783	1.527	500	2.517	4.593	2.738	5.034	4.717
3.5	0.940	1.450	1.159	1.880	1.616	600	2.575	4.709	2.796	5.150	4.833
4	0.982	1.532	1.201	1.964	1.694	700	2.624	4.807	2.845	5.248	4.931
4.5	1.019	1.604	1.239	2.038	1.763	800	2.667	4.892	2.887	5.333	5.015
5	1.052	1.669	1.272	2.105	1.826	900	2.704	4.967	2.925	5.408	5.092
6	1.110	1.783	1.330	2.220	1.935	10^3	2.738	5.034	2.958	5.476	5.158
7	1.159	1.880	1.379	2.318	2.028	10^4	3.471	6.500	3.691	6.941	6.623
8	1.201	1.964	1.422	2.403	2.110	10^5	4.204	7.966	4.424	8.407	8.089
9	1.239	2.038	1.459	2.477	2.182	10^6	4.937	9.432	5.157	9.874	9.555
10	1.272	2.105	1.493	2.544	2.246	∞	∞	∞	∞	∞	∞

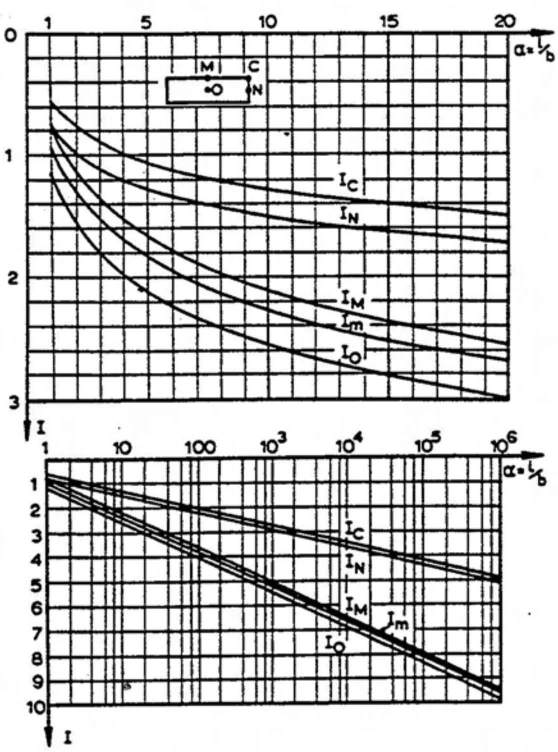


FIG.3.31 Influence factors for vertical surface displacement beneath rectangle. (Giroud, 1968).

For a point on the centre-line of the rectangle, distance x from the centre (Fig.3.32), Giroud (1969) gives the following expression for the horizontal surface displacement ρ_x:

$$\rho_x = \frac{(1+\nu)(1-2\nu)}{2\pi E} p\ell \left[\frac{b}{2\ell} \ln \frac{4(\ell-x)^2+b^2}{4x^2+b^2} + 2\left(1-\frac{x}{\ell}\right) \arctan \frac{b}{2(\ell-x)} - 2\frac{x}{\ell} \arctan \frac{b}{2x} \right] \quad \ldots (3.21)$$

From this expression, the solution for ρ_x beneath the corner of a rectangle of proportions $2\frac{\ell}{b}$ may be obtained by taking half the value of ρ_x obtained from equation (3.21) when $x=\ell$.

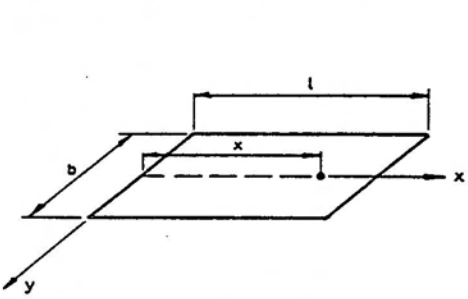


FIG.3.32

3.4.2 LINEARLY VARYING VERTICAL LOADING (Fig. 3.33)

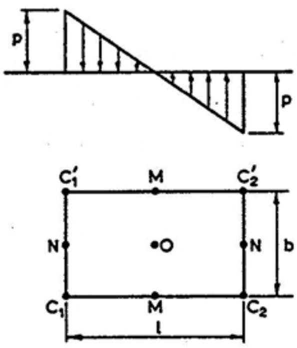


FIG. 3.33

The normal stresses are expressed as follows by Giroud (1970):

Under the corners:

$$\sigma_z = \varepsilon\, p\, M_0 \qquad \ldots (3.22a)$$

$$\sigma_x = \varepsilon\, p[\, M_2 - (1-2\nu)M_2^1] \qquad \ldots (3.22b)$$

$$\sigma_y = \varepsilon\, q[\, N_2 - (1-2\nu)N_2^1] \qquad \ldots (3.22c)$$

Under the centre:

$$\sigma_z = 0 \qquad \ldots (3.23a)$$

$$\sigma_x = 0 \qquad \ldots (3.23b)$$

$$\sigma_y = 0 \qquad \ldots (3.23c)$$

where $\varepsilon = +1$ at C_1 and C_1^1, and,
-1 at C_2 and C_2^1,
$M_0, M_2, M_2^1, N_2, N_2^1$ are influence factors which are given in Tables 3.21 to 3.25.

Influence factors for the vertical surface displacement beneath various points have been obtained by Giroud (1968), and are tabulated in Table 3.20. Explicit expressions for the displacements are given by Giroud.

At the corners

$$\rho_z = \frac{(1-\nu^2)p\ell}{E} I_c \quad \text{if } b \geq \ell \qquad \ldots (3.24a)$$

or

$$\frac{(1-\nu^2)pb}{E} I_c' \quad \text{if } \ell \geq b \qquad \ldots (3.24b)$$

where

$$I_c = \frac{\alpha}{\pi}\left[\alpha - \sqrt{1+\alpha^2} + \ell n\, \frac{1+\sqrt{1+\alpha^2}}{\alpha}\right]$$

$$\text{and } \alpha = b/\ell \qquad \ldots (3.24c)$$

$$I_c' = \frac{1}{\pi}\left[\frac{1}{\alpha} + \frac{\sqrt{1+\alpha^2}}{\alpha} + \ell n(\alpha + \sqrt{1+\alpha^2})\right]$$

$$\text{and } \alpha = \ell/b \qquad \ldots (3.24d)$$

At N,

$$\rho_z = \frac{(1-\nu^2)p\ell}{E} I_N \quad \text{if } b \geq \ell \qquad \ldots (3.24e)$$

or

$$\frac{(1-\nu^2)pb}{E} I_N' \quad \text{if } \ell \geq b \qquad \ldots (3.24f)$$

$$\rho_z = 0 \quad \text{at points } M \text{ and } O \qquad \ldots (3.24g)$$

TABLE 3.20

INFLUENCE FACTORS FOR VERTICAL SURFACE DISPLACEMENT DUE TO LINEARLY VARYING COMPRESSION TO TENSION LOADING

(Giroud, 1968)

$b \geq \ell$			$\ell \geq b$					
b/ℓ	I_C	I_N	ℓ/b	I_C'	I_N'	ℓ/b	I_C'	I_N'
1	0.149	0.263	1	0.149	0.263	15	0.785	0.995
1.1	0.150	0.269	1.1	0.162	0.282	20	0.872	1.084
1.2	0.151	0.274	1.2	0.174	0.300	25	0.940	1.154
1.3	0.152	0.279	1.3	0.187	0.317	30	0.995	1.211
1.4	0.153	0.282	1.4	0.198	0.334	40	1 084	1.301
1.5	0.154	0.286	1.5	0.210	0.349	50	1.154	1.371
1.6	0.154	0.289	1.6	0.221	0.364	60	1.211	1.429
1.7	0.155	0.291	1.7	0.232	0.379	70	1.259	1.478
1.8	0.155	0.294	1.8	0.243	0.392	80	1.301	1.520
1.9	0.156	0.296	1.9	0.253	0.406	90	1.338	1.557
2	0.156	0.297	2	0.263	0.418	100	1.371	1.590
2.2	0.157	0.300	2.2	0.282	0.442	200	1.590	1.810
2.4	0.157	0.303	2.4	0.300	0.465	300	1.719	1.939
2.5	0.157	0.304	2.5	0.309	0.475	400	1.810	2.031
3	0.158	0.308	3	0.349	0.524	500	1.881	2.101
3.5	0.158	0.310	3.5	0.386	0.566	600	1.939	2.159
4	0.158	0.312	4	0.418	0.603	700	1.988	2.208
4.5	0.159	0.313	4.5	0.448	0.636	800	2.031	2.251
5	0.159	0.314	5	0.475	0.666	900	2.068	2.288
6	0.159	0.315	6	0.524	0.719	10^3	2.101	2.322
7	0.159	0.316	7	0.566	0.765	10^4	2.834	3.055
8	0.159	0.317	8	0.603	0.804	10^5	3.567	3.788
10	0.159	0.317	9	0.636	0.840	10^6	4.300	4.521
∞	0.159	0.318	10	0.666	0.872	∞	∞	∞

TABLE 3.21

VALUES OF M_o

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.109	0.160	0.180	0.183	0.186	0.187	0.188	0.188	0.188	0.188	0.188	0.188	0.188	0.188
0.4	0.000	0.047	0.083	0.111	0.118	0.126	0.131	0.134	0.134	0.135	0.135	0.135	0.135	0.135	0.135
0.5	0.000	0.033	0.060	0.085	0.092	0.100	0.107	0.111	0.112	0.113	0.113	0.113	0.113	0.113	0.113
0.6	0.000	0.023	0.044	0.065	0.072	0.079	0.087	0.092	0.093	0.094	0.094	0.094	0.094	0.094	0.094
0.8	0.000	0.013	0.025	0.038	0.043	0.049	0.056	0.062	0.064	0.065	0.065	0.065	0.065	0.065	0.065
1	0.000	0.007	0.015	0.023	0.026	0.031	0.036	0.042	0.045	0.045	0.045	0.045	0.045	0.045	0.045
1.2	0.000	0.005	0.009	0.014	0.017	0.020	0.024	0.029	0.031	0.032	0.032	0.032	0.032	0.032	0.032
1.4	0.000	0.003	0.006	0.009	0.011	0.013	0.016	0.020	0.022	0.023	0.023	0.023	0.023	0.023	0.023
1.5	0.000	0.002	0.005	0.007	0.009	0.011	0.013	0.017	0.019	0.020	0.020	0.020	0.020	0.020	0.020
1.6	0.000	0.002	0.004	0.006	0.007	0.009	0.011	0.014	0.016	0.017	0.017	0.017	0.017	0.017	0.017
1.8	0.000	0.001	0.003	0.004	0.005	0.006	0.008	0.010	0.012	0.013	0.013	0.013	0.013	0.013	0.013
2	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.007	0.009	0.010	0.010	0.010	0.010	0.010	0.010
2.5	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.005	0.005	0.006	0.006	0.006	0.006
3	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.003
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.22

VALUES OF M_2

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.026	0.040	0.046	0.046	0.046	0.045	0.043	0.042	0.042	0.042	0.042	0.042	0.042	0.042
0.4	0.000	0.002	0.003	0.002	0.001	-0.000	-0.002	-0.005	-0.007	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
0.5	0.000	-0.001	-0.002	-0.005	-0.006	-0.008	-0.010	-0.013	-0.015	-0.016	-0.016	-0.016	-0.016	-0.016	-0.016
0.6	0.000	-0.002	-0.004	-0.007	-0.009	-0.010	-0.013	-0.016	-0.018	-0.019	-0.019	-0.020	-0.020	-0.020	-0.020
0.8	0.000	-0.002	-0.005	-0.007	-0.009	-0.010	-0.013	-0.016	-0.018	-0.019	-0.019	-0.019	-0.019	-0.019	-0.019
1	0.000	-0.002	-0.003	-0.006	-0.007	-0.008	-0.010	-0.013	-0.015	-0.015	-0.016	-0.016	-0.016	-0.016	-0.016
1.2	0.000	-0.001	-0.002	-0.004	-0.005	-0.006	-0.007	-0.009	-0.011	-0.012	-0.012	-0.012	-0.013	-0.013	-0.013
1.4	0.000	-0.001	-0.002	-0.003	-0.003	-0.004	-0.005	-0.007	-0.008	-0.009	-0.009	-0.009	-0.010	-0.010	-0.010
1.5	0.000	-0.001	-0.001	-0.002	-0.003	-0.003	-0.004	-0.006	-0.007	-0.008	-0.008	-0.008	-0.008	-0.009	-0.009
1.6	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.006	-0.007	-0.007	-0.007	-0.007	-0.008	-0.008
1.8	0.000	-0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.005	-0.005	-0.006	-0.006	-0.006	-0.006
2	0.000	-0.000	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.004	-0.004	-0.004	-0.005	-0.005	-0.00
2.5	0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.00	-0.001	-0.002	-0.002	-0.002	-0.002	-0.003	-0.003	-0.00
3	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

TABLE 3.23

VALUES OF M_2'

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.161	0.115	0.077	0.063	0.048	0.031	0.015	0.006	0.003	0.002	0.001	0.000	0.000	0.000
0.2	0.000	0.026	0.037	0.036	0.033	0.028	0.021	0.011	0.005	0.002	0.001	0.001	0.000	0.000	0.000
0.4	0.000	0.007	0.012	0.015	0.015	0.015	0.012	0.008	0.004	0.002	0.001	0.001	0.000	0.000	0.000
0.5	0.000	0.004	0.008	0.010	0.010	0.010	0.009	0.006	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.6	0.000	0.003	0.005	0.007	0.007	0.008	0.007	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.8	0.000	0.001	0.002	0.003	0.004	0.004	0.004	0.003	0.002	0.001	0.001	0.001	0.000	0.000	0.000
1	0.000	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
1.2	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000
1.4	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
1.5	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000
1.6	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000
1.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.24

VALUES OF N_2

(Giroud, 1972)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.2	0.000	0.008	0.034	0.067	0.078	0.090	0.101	0.110	0.114	0.114	0.115	0.115	0.115	0.115	0.115
0.4	0.000	0.001	0.006	0.018	0.024	0.033	0.044	0.056	0.061	0.062	0.063	0.063	0.063	0.063	0.063
0.5	0.000	0.000	0.003	0.010	0.014	0.020	0.029	0.040	0.045	0.047	0.048	0.048	0.048	0.048	0.048
0.6	0.000	0.000	0.002	0.006	0.008	0.013	0.019	0.028	0.034	0.036	0.036	0.037	0.037	0.037	0.037
0.8	0.000	0.000	0.000	0.002	0.003	0.005	0.009	0.015	0.019	0.021	0.022	0.022	0.023	0.023	0.023
1	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.008	0.011	0.013	0.014	0.014	0.015	0.015	0.015
1.2	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.007	0.008	0.009	0.009	0.010	0.010	0.010
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.005	0.006	0.006	0.007	0.007	0.007
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.004	0.005	0.005	0.006	0.006	0.006
1.6	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.004	0.004	0.004	0.005	0.005	0.005
1.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.003	0.003	0.004	0.004	0.004
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.003	0.003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.002
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.25

VALUES OF N_2'

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.039	0.135	0.173	0.187	0.202	0.219	0.235	0.244	0.247	0.248	0.249	0.250	0.250	0.250
0.2	0.000	0.022	0.042	0.062	0.070	0.079	0.091	0.103	0.110	0.112	0.113	0.114	0.115	0.115	0.115
0.4	0.000	0.009	0.018	0.028	0.033	0.038	0.045	0.054	0.059	0.061	0.062	0.063	0.063	0.063	0.063
0.5	0.000	0.006	0.013	0.020	0.023	0.027	0.033	0.040	0.044	0.046	0.047	0.047	0.048	0.048	0.048
0.6	0.000	0.005	0.009	0.014	0.017	0.020	0.024	0.029	0.033	0.035	0.036	0.036	0.037	0.037	0.037
0.8	0.000	0.002	0.005	0.008	0.009	0.011	0.013	0.017	0.020	0.021	0.022	0.022	0.023	0.023	0.023
1	0.000	0.001	0.003	0.004	0.005	0.006	0.008	0.010	0.012	0.013	0.014	0.014	0.015	0.015	0.015
1.2	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.006	0.008	0.009	0.009	0.009	0.010	0.010	0.010
1.4	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.004	0.005	0.006	0.006	0.006	0.007	0.007	0.007
1.5	0.000	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.004	0.005	0.005	0.005	0.006	0.006	0.006
1.6	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005
1.8	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.004
2	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.003	0.003
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000

For a point at a distance x from the edge $C_1^1NC_1$ on the axis NON (Fig.3.33), Giroud (1969) gives the following solution for the horizontal displacement ρ_x:

$$\rho_x = \frac{(1+\nu)(1-2\nu)}{2\pi E}\,\ell\left[\frac{b}{2\ell}\left(1-2\frac{x}{\ell}\right)\ln\frac{4(\ell-x)^2+b^2}{4x^2+b^2} - 2\frac{x}{\ell}\left(1-\frac{x}{\ell}\right)\left(\arctan\frac{b}{2x}+\arctan\frac{b}{2(\ell-x)}\right) + \frac{b^2}{2\ell^2}\left(\arctan\frac{2x}{b}+\arctan\frac{2(\ell-x)}{b}\right) - \frac{b}{\ell}\right] \qquad \text{... (3.25)}$$

As in Section 3.4.1, the solution for the corner of a rectangle of proportions $2\ell/b$ may be obtained by taking half the value of ρ_x for $x=\ell$.

The results in this section can be combined with those in Section 3.4.1 to give the results for a trapezoidal distribution of loading. For the particular case when the loading varies linearly across the rectangle to zero at one edge, expressions and graphs for the vertical stress are given by Gray (1943,1948), expressions for the horizontal and shear stresses by Ambraseys (1960) and graphs and expressions for vertical displacement by Stamatopoulos (1959).

3.4.3 VERTICAL EMBANKMENT LOADING (Fig.3.34)

Vertical surface displacements ρ_z have been evaluated for several points by Giroud (1968b).

Influence factors K_O, K_C etc for the seven points marked in Fig.3.34 are shown in Figs. 3.35 to 3.41.

In all cases

$$\rho_z = \frac{1-\nu^2}{E}\,p\,a\,K \qquad \text{... (3.26)}$$

Expressions for the influence factors K_O(*centre*) and K_C(*corner*) are as follows:

$$K_O = \frac{1}{2\pi}\left[\frac{1}{\beta}\ln\left(\alpha+\sqrt{1+\alpha^2}\right) + \frac{\alpha^2}{\beta}\ln\frac{1+\sqrt{1+\alpha^2}}{\alpha} - \frac{(1-2\beta)^2}{\beta}\ln\frac{\alpha-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{1-2\beta} - \frac{(\alpha-2\beta)^2}{\beta}\ln\frac{1-2\beta+\sqrt{(1-2\beta)^2+(\alpha-2\beta)^2}}{\alpha-2\beta}\right.$$

$$- \frac{(\alpha-1)^2}{\sqrt{2}\,\beta} \ln \frac{\sqrt{2(1+\alpha^2)}+\alpha+1}{\sqrt{2(1-2\beta)^2+2(\alpha-2\beta)^2}-4\beta+\alpha+1} \Bigg] \quad \text{... (3.27a)}$$

$$K_C = \frac{1}{2\pi}\Bigg[\frac{1}{\beta} \ln (\alpha+\sqrt{1+\alpha^2}) + \frac{\alpha^2}{\beta} \ln \frac{1+\sqrt{1+\alpha^2}}{\alpha}$$

$$+\beta \ln \frac{\beta-\alpha+\sqrt{\beta^2+(\alpha-\beta)^2}}{1-\beta+\sqrt{\beta^2+(1-\beta)^2}}$$

$$- \frac{1}{\sqrt{2}\beta} \ln \frac{\sqrt{2\beta^2+2(1-\beta)^2}+2\beta-1}{\sqrt{2}-1}$$

$$- \frac{\alpha^2}{\sqrt{2}\beta} \ln \frac{\sqrt{2\beta^2+2(\alpha-\beta)^2}+2\beta-\alpha}{\alpha(\sqrt{2}-1)}$$

$$- \frac{(\alpha-1)^2}{\sqrt{2}\beta} \ln \frac{\sqrt{2(1-\beta)^2+2(\alpha-\beta)^2}+2\beta-\alpha-1}{\sqrt{2(1+\alpha^2)}-(1+\alpha)}$$

$$- \frac{(1-\beta)^2}{\beta} \ln \frac{\sqrt{\beta^2+(1-\beta)^2}-\beta}{\sqrt{(1-\beta)^2+(\alpha-\beta)^2}+\beta-\alpha}$$

$$- \frac{(\alpha-\beta)^2}{\beta} \ln \frac{\sqrt{\beta^2+(\alpha-\beta)^2}-\beta}{\sqrt{(1-\beta)^2+(\alpha-\beta)^2}+\beta-1}$$

$$+ 2\beta \ln(1+\sqrt{2}) \Bigg] \quad \text{... (3.27b)}$$

For the point A,

$$K_A = \frac{1}{2\pi}\Bigg[\frac{1}{2\beta} \ln (2\alpha+\sqrt{1+4\alpha^2}) + \frac{2\alpha^2}{\beta} \ln \frac{1+\sqrt{1+4\alpha^2}}{2\alpha}$$

$$- \frac{1}{2\sqrt{2}\beta} \ln \frac{\sqrt{2(1-2\beta)^2+8\beta^2}+4\beta-1}{\sqrt{2}-1}$$

$$- \frac{(2\alpha-1)^2}{2\sqrt{2}\beta} \ln \frac{\sqrt{2(1-2\beta)^2+8(\alpha-\beta)^2}+4\beta-2\alpha-1}{\sqrt{2(1+4\alpha^2)}-(1+2\alpha)}$$

$$- \frac{(1-2\beta)^2}{2\beta} \ln \frac{\sqrt{4\beta^2+(1-2\beta)^2}-2\beta}{\sqrt{(1-2\beta)^2+4(\alpha-\beta)^2}+2(\beta-\alpha)}$$

$$- \frac{2(\alpha-\beta)^2}{\beta} \ln \frac{2(\alpha-\beta)}{\sqrt{(1-2\beta)^2+4(\alpha-\beta)^2}+2\beta-1}$$

$$- 2\beta \ln \frac{2\beta}{\sqrt{(1-2\beta)^2+4\beta^2}+2\beta-1} \Bigg]$$

$$\text{... (3.27c)}$$

In the above expressions,

$\alpha = b/a$

$\beta = c/a.$

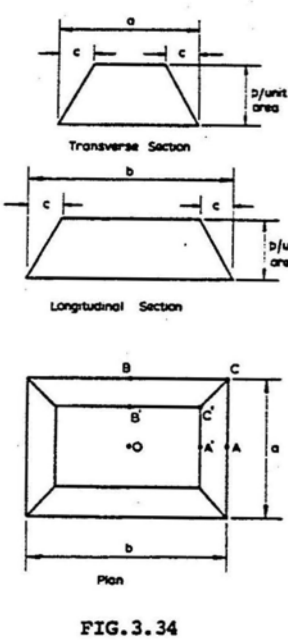


FIG.3.34

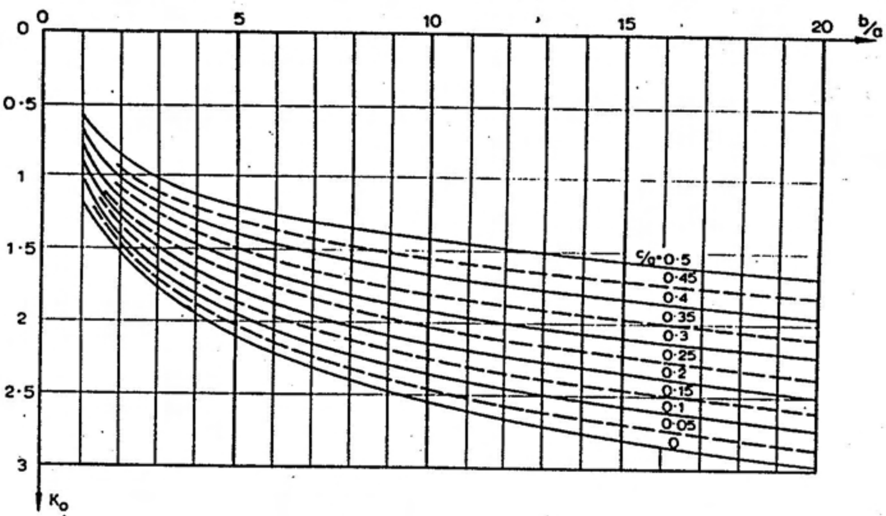


FIG.3.35 Displacement Influence Factors K_o. (Giroud, 1968).

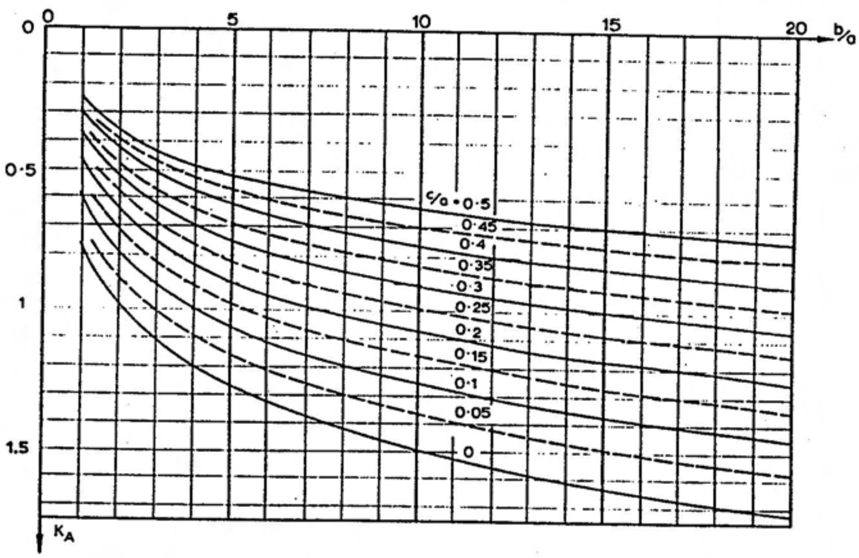


FIG.3.36 Displacement Influence Factors K_A. (Giroud, 1968).

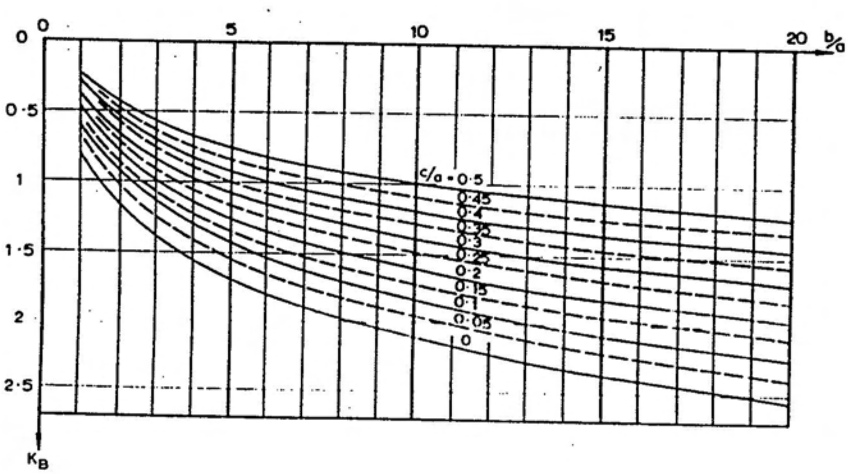


FIG.3.37 Displacement Influence Factors K_B. (Giroud, 1968).

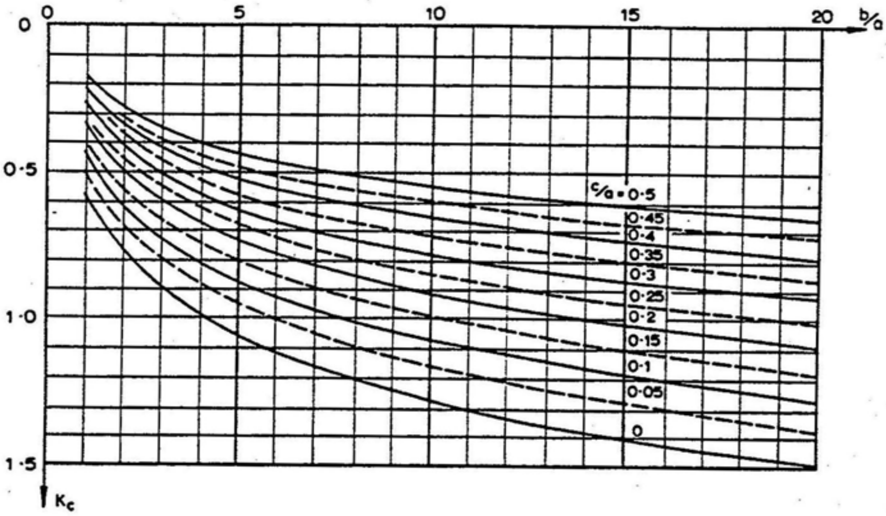


FIG.3.38 Displacement Influence Factors K_c. (Giroud, 1968).

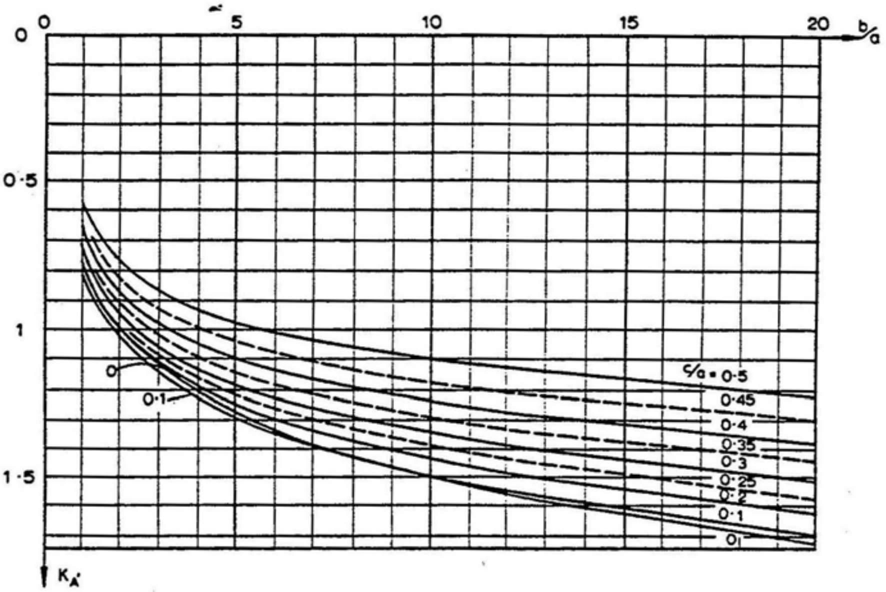


FIG.3.39 Displacement Influence Factors K_A'. (Giroud, 1968).

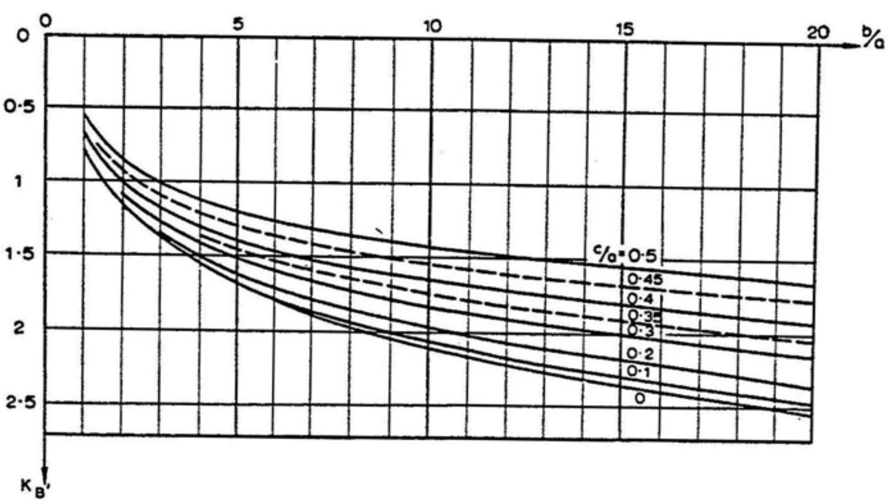


FIG.3.40 Displacement Influence Factors K_B'. (Giroud, 1968).

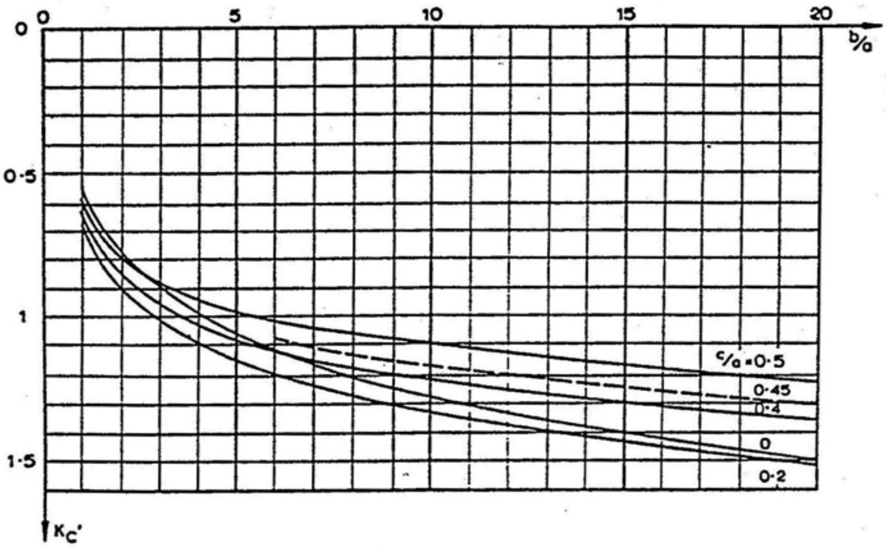


FIG.3.41 Displacement Influence Factors K_c'. (Giroud, 1968).

3.4.4 UNIFORM HORIZONTAL LOADING (Fig.3.42)

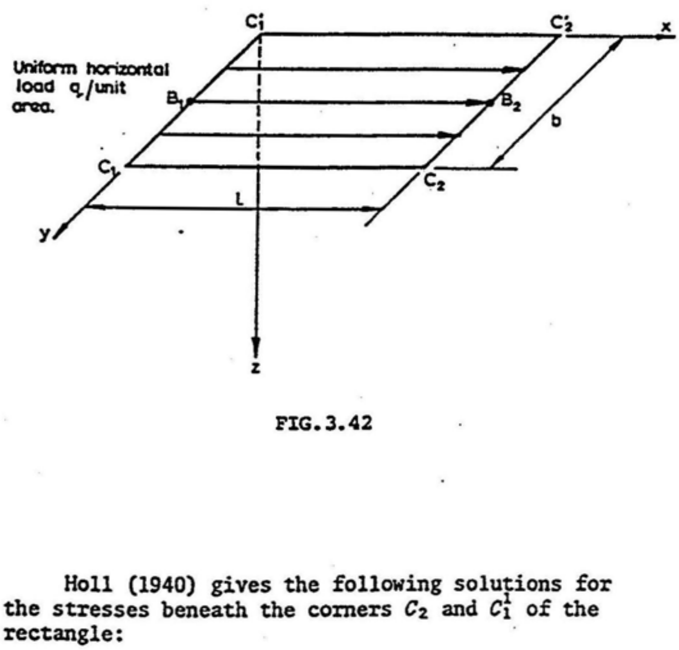


FIG.3.42

Holl (1940) gives the following solutions for the stresses beneath the corners C_2 and C_1^1 of the rectangle:

$$\sigma_z = \frac{q}{2\pi}\left[\frac{b}{R_2} - \frac{z^2 b}{R_1^2\, R_3}\right] \quad \ldots (3.28a)$$

$$\sigma_x = \frac{q}{\pi}\left[\ell n\,\frac{R_1(b+R_2)}{z(b+R_3)} - \frac{\ell^2 b}{2R_1^2\, R_3}\right] \quad \ldots (3.28b)$$

$$\sigma_y = \frac{q}{2\pi}\left[\ell n\,\frac{R_1(b+R_2)}{z(b+R_3)} - b\left(\frac{1}{R_2} - \frac{1}{R_3}\right)\right] \quad \ldots (3.28c)$$

$$\tau_{xz} = \frac{q}{2\pi}\left[\tan^{-1}\frac{\ell b}{zR_3} - \frac{\ell\, bz}{R_1^2\, R_3}\right] \quad \ldots (3.28d)$$

$$\tau_{yz} = \frac{q}{2\pi}\left[1 + \frac{z}{R_3} - z\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right] \quad \ldots (3.28e)$$

$$\tau_{xy} = \frac{q}{2\pi}\left[\ell n\,\frac{(R_1+\ell)(R_3-\ell)}{z\,R_2} + \ell\left(\frac{1}{R_3} - \frac{1}{R_1}\right)\right] \quad \ldots (3.28f)$$

where $R_1 = (\ell^2+z^2)^{\frac{1}{2}}$

$R_2 = (b^2+z^2)^{\frac{1}{2}}$

$R_3 = (\ell^2+b^2+z^2)^{\frac{1}{2}}$

It should be noted that the values of τ_{xz}, τ_{yz} and σ_z for uniform horizontal loading correspond to the values of σ_x, τ_{xy} and τ_{xz} for uniform vertical loading (from the reciprocal theorem).

The principle of superposition may be applied to determine the stresses at points not beneath the corner of the rectangle.

Influence factors for the normal stresses have been obtained by Giroud (1970). The stresses are expressed as follows:

Under the corners,

$$\sigma_z = \varepsilon\, q\, K_1 \quad \ldots (3.29a)$$

$$\sigma_x = \varepsilon\, q\, [K_3 - (1-2\nu)K_3^1] \quad \ldots (3.29b)$$

$$\sigma_y = \varepsilon\, q\, [K_5 - (1-2\nu)K_5^1] \quad \ldots (3.29c)$$

where $\varepsilon = +1$ at C_2 and C_2^1 (see Fig.3.42)
and -1 at C_1 and C_1^1.

K_1, K_3, K_3^1, K_5, K_5^1 are influence factors which are given in Tables 3.26 to 3.30.

Under the centre,

$\sigma_z = 0$

$\sigma_x = 0$

$\sigma_y = 0$.

TABLE 3.26

VALUES OF K_1 (Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159
0.2	0.000	0.071	0.111	0.135	0.140	0.145	0.149	0.152	0.153	0.153	0.153	0.153	0.153	0.153	0.153
0.4	0.000	0.037	0.067	0.095	0.105	0.115	0.125	0.133	0.136	0.137	0.137	0.137	0.137	0.137	0.137
0.5	0.000	0.028	0.054	0.079	0.089	0.100	0.111	0.121	0.125	0.127	0.127	0.127	0.127	0.127	0.127
0.6	0.000	0.023	0.043	0.066	0.075	0.085	0.097	0.109	0.115	0.116	0.117	0.117	0.117	0.117	0.117
0.8	0.000	0.015	0.029	0.046	0.053	0.062	0.073	0.086	0.093	0.095	0.096	0.097	0.097	0.097	0.097
1	0.000	0.010	0.020	0.032	0.037	0.045	0.054	0.067	0.075	0.077	0.079	0.079	0.079	0.080	0.080
1.2	0.000	0.007	0.014	0.023	0.027	0.033	0.040	0.051	0.059	0.062	0.064	0.064	0.065	0.065	0.065
1.4	0.000	0.005	0.010	0.017	0.020	0.024	0.030	0.040	0.047	0.050	0.052	0.053	0.054	0.054	0.054
1.5	0.000	0.004	0.009	0.014	0.017	0.021	0.026	0.035	0.042	0.045	0.047	0.048	0.049	0.049	0.049
1.6	0.000	0.004	0.008	0.013	0.015	0.018	0.023	0.031	0.038	0.041	0.043	0.044	0.045	0.045	0.045
1.8	0.000	0.003	0.006	0.010	0.011	0.014	0.018	0.024	0.030	0.034	0.035	0.036	0.037	0.038	0.038
2	0.000	0.002	0.004	0.007	0.009	0.011	0.014	0.019	0.025	0.028	0.029	0.030	0.032	0.032	0.032
2.5	0.000	0.001	0.003	0.004	0.005	0.006	0.008	0.011	0.015	0.018	0.019	0.020	0.022	0.022	0.022
3	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.007	0.010	0.012	0.013	0.014	0.015	0.016	0.016
4	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.006	0.007	0.007	0.009	0.009	0.009
5	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.003	0.004	0.004	0.005	0.006	0.006
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.27

VALUES OF K_3 (Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.107	0.189	0.259	0.282	0.307	0.332	0.353	0.362	0.364	0.365	0.365	0.365	0.366	0.366
0.4	0.000	0.037	0.069	0.104	0.117	0.133	0.150	0.167	0.175	0.177	0.177	0.178	0.178	0.178	0.178
0.5	0.000	0.023	0.045	0.069	0.079	0.091	0.104	0.118	0.125	0.127	0.128	0.129	0.129	0.129	0.129
0.6	0.000	0.016	0.030	0.047	0.054	0.063	0.074	0.085	0.091	0.093	0.094	0.094	0.094	0.095	0.095
0.8	0.000	0.007	0.014	0.023	0.026	0.031	0.038	0.045	0.050	0.052	0.052	0.052	0.053	0.053	0.053
1	0.000	0.004	0.007	0.012	0.014	0.016	0.020	0.025	0.028	0.030	0.030	0.030	0.031	0.031	0.031
1.2	0.000	0.002	0.004	0.006	0.007	0.009	0.011	0.014	0.017	0.018	0.018	0.018	0.019	0.019	0.019
1.4	0.000	0.001	0.002	0.004	0.004	0.005	0.006	0.009	0.010	0.011	0.011	0.012	0.012	0.012	0.012
1.5	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.007	0.008	0.009	0.009	0.009	0.010	0.010	0.010
1.6	0.000	0.001	0.001	0.002	0.003	0.003	0.004	0.005	0.006	0.007	0.007	0.008	0.008	0.008	0.008
1.8	0.000	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.004	0.005	0.005	0.005	0.005	0.005	0.005
2	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.004	0.004
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	0.000

TABLE 3.28

VALUES OF K_5^1

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.143	0.128	0.109	0.100	0.088	0.071	0.047	0.027	0.017	0.011	0.008	0.003	0.001	0.000
0.2	0.000	0.025	0.040	0.048	0.049	0.048	0.044	0.033	0.021	0.014	0.010	0.007	0.003	0.001	0.000
0.4	0.000	0.009	0.016	0.023	0.025	0.026	0.026	0.023	0.016	0.011	0.008	0.006	0.003	0.001	0.000
0.5	0.000	0.006	0.011	0.016	0.018	0.020	0.021	0.019	0.014	0.010	0.008	0.006	0.003	0.001	0.000
0.6	0.000	0.004	0.008	0.012	0.013	0.015	0.016	0.016	0.012	0.009	0.007	0.005	0.002	0.001	0.000
0.8	0.000	0.002	0.004	0.007	0.008	0.009	0.010	0.011	0.009	0.007	0.006	0.005	0.002	0.001	0.000
1	0.000	0.001	0.003	0.004	0.005	0.006	0.007	0.008	0.007	0.006	0.005	0.004	0.002	0.001	0.000
1.2	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.006	0.006	0.005	0.004	0.004	0.002	0.001	0.000
1.4	0.000	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.003	0.002	0.001	0.000
1.5	0.000	0.000	0.001	0.002	0.002	0.002	0.003	0.004	0.004	0.004	0.003	0.003	0.002	0.001	0.000
1.6	0.000	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.003	0.002	0.001	0.000
1.8	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.003	0.002	0.001	0.001	0.000
2	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000
2.5	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	-0.000	0.000	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.29

VALUES OF K_5

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.005	0.027	0.066	0.085	0.109	0.142	0.185	0.218	0.234	0.242	0.247	0.255	0.258	0.259
0.4	0.000	0.001	0.005	0.017	0.025	0.038	0.059	0.091	0.118	0.133	0.141	0.145	0.153	0.156	0.158
0.5	0.000	0.000	0.003	0.010	0.015	0.024	0.039	0.065	0.090	0.104	0.111	0.116	0.123	0.127	0.128
0.6	0.000	0.000	0.002	0.006	0.009	0.015	0.026	0.048	0.070	0.082	0.089	0.094	0.101	0.105	0.106
0.8	0.000	0.000	0.001	0.002	0.004	0.007	0.013	0.026	0.042	0.053	0.059	0.063	0.070	0.074	0.075
1	0.000	0.000	0.000	0.001	0.002	0.003	0.007	0.015	0.027	0.035	0.040	0.044	0.051	0.054	0.055
1.2	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.009	0.017	0.024	0.028	0.032	0.038	0.041	0.042
1.4	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.005	0.011	0.016	0.020	0.023	0.029	0.032	0.033
1.5	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.009	0.013	0.017	0.020	0.025	0.028	0.029
1.6	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.007	0.011	0.015	0.017	0.022	0.025	0.026
1.8	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.008	0.011	0.013	0.017	0.020	0.021
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.008	0.010	0.014	0.017	0.018
2.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.004	0.005	0.008	0.011	0.012
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.005	0.007	0.008
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.005
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.003
10	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
15	0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 3.30

VALUES OF K_5^1

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.036	0.069	0.105	0.120	0.139	0.164	0.197	0.223	0.236	0.244	0.248	0.255	0.258	0.259
0.4	0.000	0.016	0.031	0.049	0.058	0.069	0.085	0.107	0.127	0.137	0.143	0.147	0.153	0.157	0.158
0.5	0.000	0.011	0.023	0.036	0.043	0.051	0.064	0.083	0.100	0.109	0.115	0.118	0.124	0.127	0.128
0.6	0.000	0.009	0.017	0.028	0.032	0.039	0.050	0.065	0.080	0.088	0.093	0.096	0.102	0.105	0.106
0.8	0.000	0.005	0.010	0.017	0.020	0.024	0.031	0.042	0.053	0.059	0.063	0.066	0.071	0.074	0.075
1	0.000	0.003	0.007	0.011	0.013	0.016	0.020	0.028	0.036	0.041	0.045	0.047	0.052	0.054	0.055
1.2	0.000	0.002	0.004	0.007	0.009	0.011	0.014	0.019	0.025	0.030	0.033	0.035	0.039	0.041	0.042
1.4	0.000	0.002	0.003	0.005	0.006	0.007	0.010	0.014	0.018	0.022	0.024	0.026	0.030	0.032	0.033
1.5	0.000	0.001	0.003	0.004	0.005	0.006	0.008	0.012	0.016	0.019	0.021	0.023	0.026	0.028	0.029
1.6	0.000	0.001	0.002	0.004	0.004	0.005	0.007	0.010	0.014	0.016	0.019	0.020	0.023	0.025	0.026
1.8	0.000	0.001	0.002	0.003	0.003	0.004	0.005	0.008	0.010	0.013	0.014	0.016	0.019	0.020	0.021
2	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.006	0.008	0.010	0.011	0.012	0.015	0.017	0.018
2.5	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.006	0.007	0.007	0.009	0.011	0.012
3	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.004	0.004	0.005	0.006	0.008	0.008
4	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.004	0.005
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003	0.003
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Explicit expressions and influence factors for vertical displacement ρ_z beneath the points C_1, C_2, B_1 and B_2 of the rectangle have been evaluated by Giroud (1968) and are given in Table 3.31 (refer Fig. 3.42 for definition of ℓ and b).

At $\left.\begin{matrix} C_2 \\ C_1 \end{matrix}\right\}$ $$\rho_z = \frac{\pm(1+\nu)(1-2\nu)q\,b\,I_c}{E} \quad \text{if} \quad \ell \geq b \qquad \text{... (3.30a)}$$

$$\text{or} \quad \frac{\pm(1+\nu)(1-2\nu)q\,\ell\,I_c^1}{E} \quad \text{if} \quad b \geq \ell \qquad \text{... (3.30b)}$$

where $$I_c = \frac{1}{2\pi}\left[\arctan\alpha + \alpha\ell n\,\frac{\sqrt{1+\alpha^2}}{\alpha}\right] \qquad \text{... (3.30c)}$$

and $\alpha = b/\ell$

$$I_c^1 = \frac{\alpha}{2\pi}\left[\arctan\frac{1}{\alpha} + \frac{1}{\alpha}\,\ell n\,\sqrt{1+\alpha^2}\right] \qquad \text{... (3.30d)}$$

and $\alpha = \ell/b$.

At $\left.\begin{matrix} B_2 \\ B_1 \end{matrix}\right\}$ $$\rho_z = \frac{\pm(1+\nu)(1-2\nu)q\,b\,I_B}{E} \quad \text{if} \quad \ell \geq b \qquad \text{... (3.31a)}$$

$$\text{or} \quad \frac{\pm(1+\nu)(1-2\nu)q\,\ell\,I_B^1}{E} \quad b \geq \ell \qquad \text{... (3.31b)}$$

Giroud (1969a) gives the following expression for the horizontal displacement ρ_x of a point on the centre-line B_1B_2 (Fig.3.42), distance x from B_1:

$$\rho_x = \frac{(1+\nu)}{\pi E}\,q\ell\left[2(1-\nu)\left\{(1-\frac{x}{\ell})\ell n\,\frac{b+\sqrt{4(\ell-x)^2+b^2}}{2(\ell-x)} + \frac{x}{\ell}\,\ell n\,\frac{b+\sqrt{4x^2+b^2}}{2|x|}\right\} + \frac{b}{\ell}\,\ell n\,\frac{2(\ell-x)+\sqrt{4(\ell-x)^2+b^2}}{-2x+\sqrt{4x^2+b^2}}\right] \qquad \text{... (3.32)}$$

For the corners C_1 and C_2, Giroud (1969b) gives

$$\rho_x = \frac{(1+\nu)}{\pi E}\,q\ell\left[(1-\nu)\ell n\,\frac{b+\sqrt{\ell^2+b^2}}{\ell} + \frac{b}{\ell}\,\ell n\,\frac{\ell+\sqrt{\ell^2+b^2}}{b}\right] \qquad \text{... (3.33a)}$$

$$\rho_y = \frac{\pm\nu(1+\nu)}{\pi E}\,q(\ell+b-\sqrt{\ell^2+b^2}) \qquad \text{... (3.33b)}$$

(+ for C_1^1 and C_2
− for C_1 and C_2^1).

TABLE 3.31

INFLUENCE FACTORS FOR VERTICAL SURFACE DISPLACEMENT DUE TO UNIFORM HORIZONTAL LOADING

(Giroud, 1968)

$b \geq \ell$			$\ell \geq b$					
b/ℓ	I_C'	I_B'	ℓ/b	I_C	I_B	ℓ/b	I_C	I_B
1	0.180	0.276	1	0.180	0.276	15	0.590	0.701
1.1	0.185	0.288	1.1	0.192	0.290	20	0.636	0.746
1.2	0.190	0.299	1.2	0.204	0.303	25	0.671	0.782
1.3	0.194	0.309	1.3	0.214	0.315	30	0.701	0.811
1.4	0.197	0.318	1.4	0.225	0.326	40	0.746	0.857
1.5	0.200	0.327	1.5	0.234	0.337	50	0.782	0.892
1.6	0.203	0.335	1.6	0.243	0.347	60	0.811	0.921
1.7	0.206	0.342	1.7	0.252	0.356	70	0.835	0.946
1.8	0.208	0.348	1.8	0.260	0.365	80	0.857	0.967
1.9	0.210	0.355	1.9	0.268	0.373	90	0.875	0.986
2	0.212	0.360	2	0.276	0.381	100	0.892	1.002
2.2	9.215	0.371	2.2	0.290	0.396	200	1.002	1.113
2.5	0.219	0.384	2.4	0.303	0.410	300	1.067	1.177
3	0.224	0.401	2.5	0.309	0.416	400	1.113	1.223
3.5	0.228	0.413	3	0.337	0.445	500	1.148	1.259
4	0.230	0.423	3.5	0.361	0.469	600	1.177	1.288
4.5	0.232	0.431	4	0.381	0.491	700	1.202	1.312
5	0.234	0.438	4.5	0.400	0.509	800	1.223	1.333
7	0.239	0.455	5	0.416	0.526	900	1.242	1.352
10	0.242	0.468	6	0.445	0.555	10^3	1.259	1.369
15	0.245	0.479	7	0.469	0.579	10^4	1.625	1.735
20	0.246	0.484	8	0.491	0.601	10^5	1.991	2.102
40	0.248	0.492	9	0.509	0.619	10^6	2.358	2.469
∞	0.250	0.500	10	0.526	0.638	∞	∞	∞

3.4.5 LINEARLY VARYING HORIZONTAL LOADING (Fig.3.43)

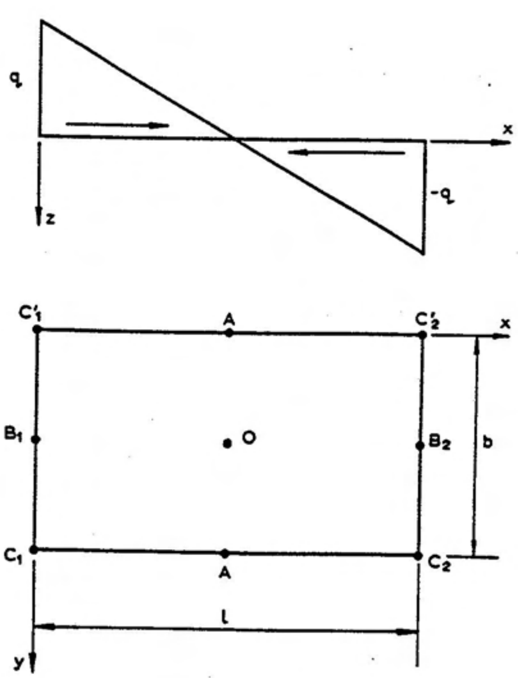


FIG.3.43

Influence factors for the normal stresses have been obtained by Giroud (1970). The stresses are expressed as follows:

Under the corners:

$$\sigma_z = -q\, M_1 \quad \text{... (3.34a)}$$

$$\sigma_x = -q\,[M_3 - (1-2\nu)M_3^1] \quad \text{... (3.34b)}$$

$$\sigma_y = -q\,[M_5 - (1-2\nu)M_5^1] \quad \text{... (3.34c)}$$

Under the centre,

$$\sigma_z = 2q(K_1 - M_1) \quad \text{... (3.35a)}$$

$$\sigma_x = 2q\,[K_3 - M_3 - (1-2\nu)(K_3^1 - M_3^1)] \quad \text{... (3.35b)}$$

$$\sigma_y = 2q\,[K_5 - M_5 - (1-2\nu)(K_5^1 - M_5^1)] \quad \text{... (3.35c)}$$

The influence factors $M_1, M_3, M_3^1, M_5, M_5^1$ are given in Tables 3.32 to 3.36. $K_1, K_3, K_3^1, K_5, K_5^1$ are given in Tables 3.26 to 3.30.

Explicit expressions and influence factors for vertical displacement ρ_z beneath the points $O, A, C_1, C_1^1, B_1, C_2, C_2^1, B_2$ of the rectangle have been evaluated by Giroud (1968) and are shown in Table 3.37. Influence factors for the mean settlement ρ_m are also given.

At C_2, C_2^1, C_1 & C_1^1

$$\rho_z = \frac{-(1+\nu)(1-2\nu)q\,\ell\,I_C}{E} \quad \text{if } b \geq \ell \quad \text{... (3.36a)}$$

or

$$\frac{-(1+\nu)(1-2\nu)q\,b\,I_C^1}{E} \quad \text{if } \ell \geq b \quad \text{... (3.36b)}$$

and similarly for points B_2 and B_1,

where

$$I_C = \frac{\alpha}{2\pi}\left[-1 + \alpha\arctan\frac{1}{\alpha} + \ln\frac{\sqrt{1+\alpha^2}}{\alpha}\right] \quad \text{... (3.36c)}$$

and $\alpha = b/\ell$

$$I_C^1 = \frac{1}{2\pi}\left[-1 + \frac{1}{\alpha}\arctan\alpha + \ln\sqrt{1+\alpha^2}\right] \quad \text{... (3.36d)}$$

and $\alpha = \ell/b$

At O,

$$\rho_z = \frac{(1+\nu)(1-2\nu)q\,\ell\,I_O}{E} \quad \text{if } b \geq \ell \quad \text{... (3.37a)}$$

or

$$\frac{(1+\nu)(1-2\nu)qb\,I_O^1}{E} \quad \text{if } \ell \geq b \quad \text{... (3.37b)}$$

and similarly for point A and the mean settlement ρ_m.

Giroud (1969b) gives the following expressions for the horizontal displacements at the corners of the rectangle:

$$\rho_x = \frac{\pm(1+\nu)}{\pi E} q \left[b \, \ln \frac{\ell + \sqrt{\ell^2 + b^2}}{b} - (1+\nu) \frac{b}{\ell} \left(\sqrt{\ell^2 + b^2} - b\right)\right] \quad \ldots (3.38a)$$

(+ for C_1 and C_1^1, - for C_2 and C_2^1)

$$\rho_y = \frac{\pm\nu(1+\nu)}{\pi E} q \left[b - \frac{b^2}{\ell} \ln \frac{\ell + \sqrt{\ell^2 + b^2}}{b}\right] \quad \ldots (3.38b)$$

(+ for C_1^1 and C_2^1, - for C_1 and C_2).

TABLE 3.32

VALUES OF M_1

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.159
0.2	0.000	0.043	0.065	0.075	0.076	0.078	0.078	0.078	0.078	0.078	0.078	0.078	0.078	0.078	0.078
0.4	0.000	0.012	0.021	0.027	0.029	0.030	0.031	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
0.5	0.000	0.006	0.011	0.015	0.016	0.017	0.017	0.016	0.015	0.015	0.015	0.015	0.015	0.015	0.015
0.6	0.000	0.003	0.005	0.007	0.007	0.008	0.007	0.006	0.005	0.005	0.005	0.005	0.005	0.004	0.004
0.8	0.000	-0.000	-0.000	-0.001	-0.001	-0.002	-0.003	-0.004	-0.006	-0.006	-0.007	-0.007	-0.007	-0.007	-0.007
1	0.000	-0.001	-0.002	-0.003	-0.004	-0.005	-0.006	-0.008	-0.010	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011
1.2	0.000	-0.001	-0.002	-0.004	-0.004	-0.005	-0.007	-0.009	-0.011	-0.011	-0.012	-0.012	-0.012	-0.012	-0.012
1.4	0.000	-0.001	-0.002	-0.003	-0.004	-0.005	-0.006	-0.008	-0.010	-0.011	-0.011	-0.012	-0.012	-0.012	-0.012
1.5	0.000	-0.001	-0.002	-0.003	-0.004	-0.004	-0.006	-0.008	-0.009	-0.010	-0.011	-0.011	-0.011	-0.011	-0.011
1.6	0.000	-0.001	-0.002	-0.003	-0.003	-0.004	-0.005	-0.007	-0.009	-0.010	-0.010	-0.011	-0.011	-0.011	-0.011
1.8	0.000	-0.001	-0.001	-0.002	-0.003	-0.003	-0.004	-0.006	-0.008	-0.009	-0.009	-0.009	-0.010	-0.010	-0.010
2	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.007	-0.007	-0.008	-0.008	-0.009	-0.009	-0.009
2.5	0.000	-0.000	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006
3	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.003	-0.004	-0.004	-0.004	-0.005	-0.005	-0.005
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

TABLE 3.33

VALUES OF M_3

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.013	0.017	0.012	0.008	0.002	-0.006	-0.015	-0.019	-0.020	-0.021	-0.021	-0.021	-0.021	-0.021
0.4	0.000	-0.006	-0.012	-0.020	-0.024	-0.028	-0.035	-0.042	-0.046	-0.047	-0.047	-0.048	-0.048	-0.048	-0.048
0.5	0.000	-0.006	-0.012	-0.019	-0.022	-0.026	-0.031	-0.037	-0.041	-0.042	-0.042	-0.043	-0.043	-0.043	-0.043
0.6	0.000	-0.005	-0.010	-0.015	-0.018	-0.021	-0.026	-0.031	-0.034	-0.035	-0.036	-0.036	-0.036	-0.036	-0.036
0.8	0.000	-0.003	-0.006	-0.009	-0.011	-0.013	-0.016	-0.020	-0.022	-0.023	-0.023	-0.023	-0.024	-0.024	-0.023
1	0.000	-0.002	-0.003	-0.005	-0.006	-0.008	-0.009	-0.012	-0.014	-0.014	-0.015	-0.015	-0.015	-0.015	-0.015
1.2	0.000	-0.001	-0.002	-0.003	-0.004	-0.004	-0.006	-0.007	-0.009	-0.009	-0.009	-0.010	-0.010	-0.010	-0.010
1.4	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.004	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006
1.5	0.000	-0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.004	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005
1.6	0.000	-0.000	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
1.8	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
2	0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
2.5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
3	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	0.000

TABLE 3.34

VALUES OF M_3^1

(Giroud, 1970)

b/ℓ z/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	0.080	0.043	0.017	0.009	0.001	-0.006	-0.009	-0.007	-0.005	-0.003	-0.003	-0.001	-0.000	0.000
0.2	0.000	0.004	0.005	0.001	-0.001	-0.003	-0.006	-0.007	-0.006	-0.004	-0.003	-0.002	-0.001	-0.000	0.000
0.4	0.000	-0.000	-0.001	-0.002	-0.003	-0.003	-0.004	-0.005	-0.004	-0.003	-0.003	-0.002	-0.001	-0.000	0.000
0.5	0.000	-0.001	-0.001	-0.002	-0.003	-0.003	-0.004	-0.004	-0.004	-0.003	-0.002	-0.002	-0.001	-0.000	0.000
0.6	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.004	-0.003	-0.003	-0.002	-0.002	-0.001	-0.000	0.000
0.8	0.000	-0.000	-0.001	-0.001	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002	-0.002	-0.001	-0.001	-0.000	0.000
1	0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.000	0.000
1.2	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.000	0.000
1.4	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	0.000
1.5	0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	0.000
1.6	0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	0.000
1.8	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	-0.000	0.000
2	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.000	-0.000	0.000
2.5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
3	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000

TABLE 3.35

VALUES OF M_5

(Giroud, 1970)

b/ℓ z/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.003	0.015	0.031	0.036	0.042	0.046	0.045	0.040	0.036	0.034	0.032	0.030	0.029	0.028
0.4	0.000	0.000	0.001	0.004	0.006	0.007	0.008	0.006	0.002	-0.002	-0.004	-0.005	-0.008	-0.009	-0.009
0.5	0.000	0.000	0.001	0.002	0.002	0.003	0.003	0.001	-0.004	-0.007	-0.009	-0.010	-0.013	-0.014	-0.014
0.6	0.000	0.000	0.000	0.001	0.001	0.001	0.000	-0.002	-0.006	-0.009	-0.011	-0.012	-0.014	-0.015	-0.016
0.8	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.003	-0.006	-0.009	-0.010	-0.012	-0.014	-0.015	-0.015
1	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.005	-0.007	-0.009	-0.010	-0.012	-0.013	-0.013
1.2	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.004	-0.005	-0.007	-0.008	-0.010	-0.011	-0.011
1.4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.003	-0.004	-0.005	-0.006	-0.008	-0.009	-0.009
1.5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.003	-0.004	-0.005	-0.007	-0.008	-0.008
1.6	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.003	-0.004	-0.005	-0.006	-0.007	-0.008
1.8	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.002	-0.003	-0.004	-0.005	-0.006	-0.006
2	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.005
2.5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.002	-0.003	-0.003	-0.004
3	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.002	-0.002	-0.003
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.002
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000

TABLE 3.36

VALUES OF M_S^1

(Giroud, 1970)

z/ℓ \ b/ℓ	0	0.1	0.2	1/3	0.4	0.5	2/3	1	1.5	2	2.5	3	5	10	∞
0	0.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.000	0.016	0.028	0.038	0.041	0.044	0.045	0.043	0.038	0.035	0.033	0.032	0.030	0.029	0.028
0.4	0.000	0.002	0.004	0.006	0.006	0.006	0.006	0.003	-0.000	-0.003	-0.005	-0.006	-0.008	-0.009	-0.009
0.5	0.000	0.001	0.001	0.001	0.001	0.001	0.000	-0.002	-0.006	-0.008	-0.010	-0.011	-0.013	-0.014	-0.014
0.6	0.000	-0.000	-0.000	-0.001	-0.001	-0.002	-0.003	-0.005	-0.008	-0.010	-0.012	-0.013	-0.014	-0.015	-0.016
0.8	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.006	-0.009	-0.010	-0.012	-0.012	-0.014	-0.015	-0.015
1	0.000	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.007	-0.009	-0.010	-0.011	-0.012	-0.013	-0.013
1.2	0.000	-0.000	-0.001	-0.002	-0.002	-0.002	-0.003	-0.004	-0.006	-0.007	-0.008	-0.009	-0.010	-0.011	-0.011
1.4	0.000	-0.000	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.005	-0.006	-0.006	-0.007	-0.008	-0.009	-0.009
1.5	0.000	-0.000	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.004	-0.005	-0.006	-0.006	-0.007	-0.008	-0.008
1.6	0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.003	-0.004	-0.004	-0.005	-0.006	-0.007	-0.007	-0.008
1.8	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.003	-0.004	-0.004	-0.005	-0.005	-0.006	-0.006
2	0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.004	-0.004	-0.005	-0.005
2.5	0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.003	-0.003	-0.004
3	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.003
4	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002
5	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001	-0.001
10	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
15	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
20	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
50	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

TABLE 3.37

INFLUENCE FACTORS FOR VERTICAL SURFACE DISPLACEMENTS DUE TO LINEARLY VARYING HORIZONTAL LOADING

(Giroud, 1968)

ℓ ≥ b				ℓ ≥ b						b ≥ ℓ					
ℓ/b	I_A'	I_O'	I_m'	ℓ/b	I_C'	I_B'	ℓ/b	I_C'	I_B'	b/ℓ	I_C	I_B	I_A	I_O	I_m
1	0.100	0.159	0.080	1	0.021	0.057	15	0.288	0.390	1	0.021	0.057	0.100	0.159	0.080
1.1	0.107	0.168	0.083	1.1	0.024	0.064	20	0.330	0.434	1.1	0.020	0.056	0.102	0.166	0.083
1.2	0.114	0.176	0.087	1.2	0.028	0.071	25	0.363	0.468	1.2	0.019	0.054	0.104	0.171	0.087
1.3	0.121	0.183	0.090	1.3	0.032	0.078	30	0.390	0.497	1.3	0.018	0.052	0.105	0.176	0.090
1.4	0.127	0.189	0.093	1.4	0.035	0.084	40	0.434	0.541	1.4	0.017	0.051	0.107	0.181	0.093
1.5	0.133	0.195	0.096	1.5	0.039	0.090	50	0.468	0.576	1.5	0.016	0.049	0.108	0.185	0.095
1.6	0.139	0.201	0.098	1.6	0.043	0.096	60	0.497	0.605	1.6	0.015	0.048	0.109	0.188	0.098
1.7	0.144	0.206	0.100	1.7	0.046	0.102	70	0.521	0.629	1.7	0.014	0.046	0.110	0.191	0.100
1.8	0.150	0.210	0.102	1.8	0.050	0.108	80	0.541	0.650	1.8	0.014	0.045	0.110	0.194	0.102
1.9	0.154	0.215	0.104	1.9	0.053	0.114	90	0.560	0.669	1.9	0.013	0.043	0.111	0.197	0.104
2	0.159	0.219	0.106	2	0.057	0.119	100	0.576	0.685	2	0.012	0.042	0.112	0.199	0.106
2.2	0.168	0.226	0.110	2.2	0.064	0.129	200	0.685	0.795	2.2	0.011	0.039	0.113	0.204	0.109
2.5	0.179	0.235	0.114	2.4	0.071	0.139	300	0.749	0.859	2.5	0.010	0.036	0.114	0.209	0.113
3	0.195	0.247	0.119	2.5	0.074	0.144	400	0.795	0.905	3	0.009	0.031	0.116	0.215	0.119
3.5	0.208	0.255	0.124	3	0.090	0.165	500	0.830	0.940	3.5	0.007	0.028	0.117	0.220	0.124
4	0.219	0.262	0.127	3.5	0.105	0.185	600	0.859	0.969	4	0.007	0.025	0.118	0.224	0.127
5	0.235	0.273	0.133	4	0.119	0.202	700	0.884	0.994	5	0.005	0.020	0.120	0.229	0.133
7	0.255	0.285	0.139	4.5	0.132	0.217	800	0.905	1.015	7	0.004	0.015	0.121	0.235	0.140
10	0.273	0.294	0.145	5	0.144	0.232	900	0.924	1.034	10	0.003	0.010	0.122	0.239	0.146
15	0.287	0.302	0.149	6	0.165	0.257	10^3	0.940	1.051	15	0.002	0.007	0.123	0.243	0.151
20	0.294	0.306	0.151	7	0.185	0.278	10^4	1.307	1.417	20	0.001	0.005	0.124	0.245	0.155
50	0.308	0.313	0.156	8	0.202	0.297	10^5	1.673	1.784	50	0.001	0.002	0.124	0.248	0.161
100	0.313	0.316	0.158	9	0.217	0.315	10^6	2.039	2.150	100	0.000	0.001	0.125	0.249	0.163
∞	0.318	0.318	0.159	10	0.232	0.330	∞	∞	∞	∞	0	0	0.125	0.250	0.167

3.5 Loading on an Elliptical Area

3.5.1 UNIFORM VERTICAL LOADING

Stresses and displacements at the surface and on the axis of the ellipse have been obtained by Deresiewicz (1960) (Fig.3.44).

Expressions are derived for the stresses and displacements on the axis within the mass, and on the surface.

The variation of maximum shear stress with depth for various e values is shown in Fig.3.45. Stress distributions along the axis for four values of e are given in Fig.3.46. In all cases, $\nu=0.3$, and e is defined as $e=(1-a^2/b^2)^{\frac{1}{2}}$.

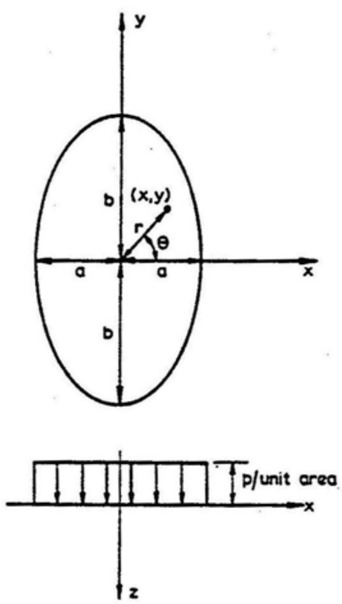


FIG.3.44

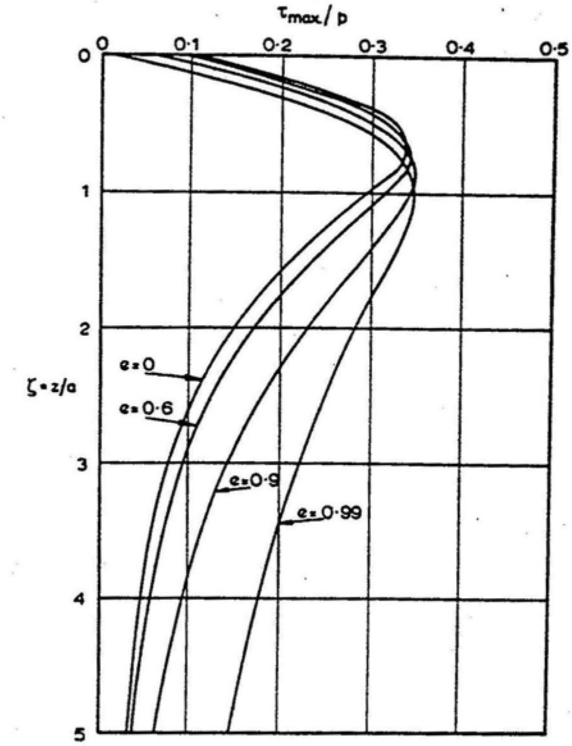


FIG.3.45 Maximum shear stress down axis of ellipse. (Deresiewicz, 1960).

Values of the horizontal stresses on the axis are tabulated in Table 3.38.

TABLE 3.38

HORIZONTAL STRESSES ON AXIS OF ELLIPSE ($\nu=0.3$) (Deresiewicz, 1960)

e	0		0.3		0.6		0.9		0.99	
$\xi=z/a$	σ_x/p	σ_y/p	σ_x/p	σ_y/p	σ_x/p	σ_y/p	σ_x/p	σ_y/p	σ_x/p	σ_y/p
0	0.8000	0.8000	0.8047	0.7953	0.8222	0.7778	0.8786	0.7214	0.9636	0.6364
0.05	0.7351	0.7351	0.7404	0.7330	0.7585	0.7244	0.8157	0.6875	0.8987	0.6158
0.1	0.6711	0.6711	0.6765	0.6716	0.6954	0.6716	0.7535	0.6538	0.8343	0.5954
0.2	0.5488	0.5488	0.5542	0.5542	0.5744	0.5694	0.6338	0.5875	0.7100	0.5551
0.4	0.3428	0.3428	0.3488	0.3531	0.3681	0.3894	0.4236	0.4647	0.5003	0.4849
0.7	0.1488	0.1488	0.1524	0.1599	0.1668	0.1999	0.2094	0.3137	0.2587	0.3839
1.0	0.0575	0.0575	0.0600	0.0648	0.0673	0.0949	0.0928	0.2060	0.1235	0.3105
1.5	0.0064	0.0064	0.0067	0.0090	0.0086	0.0229	0.0134	0.0997	0.0179	0.2270
2	-0.0050	-0.0050	-0.0051	-0.0037	-0.0055	0.0017	-0.0050	0.0457	-0.0277	0.1721
3	-0.0064	-0.0064	-0.0062	-0.0067	-0.0078	-0.0054	-0.0115	0.0080	-0.0604	0.1095
4	-0.0046	-0.0046	-0.0030	-0.0064	-0.0058	-0.0046	-0.0092	-0.0011	-0.0723	0.0761
5	-0.0033	-0.0033	-0.0019	-0.0050	-0.0038	-0.0040	-0.0069	-0.0030	-0.0783	0.0566
10	-0.0009	-0.0009	-0.0015	-0.0001	-0.0010	-0.0013	-0.0021	-0.0018	-0.0898	0.0252

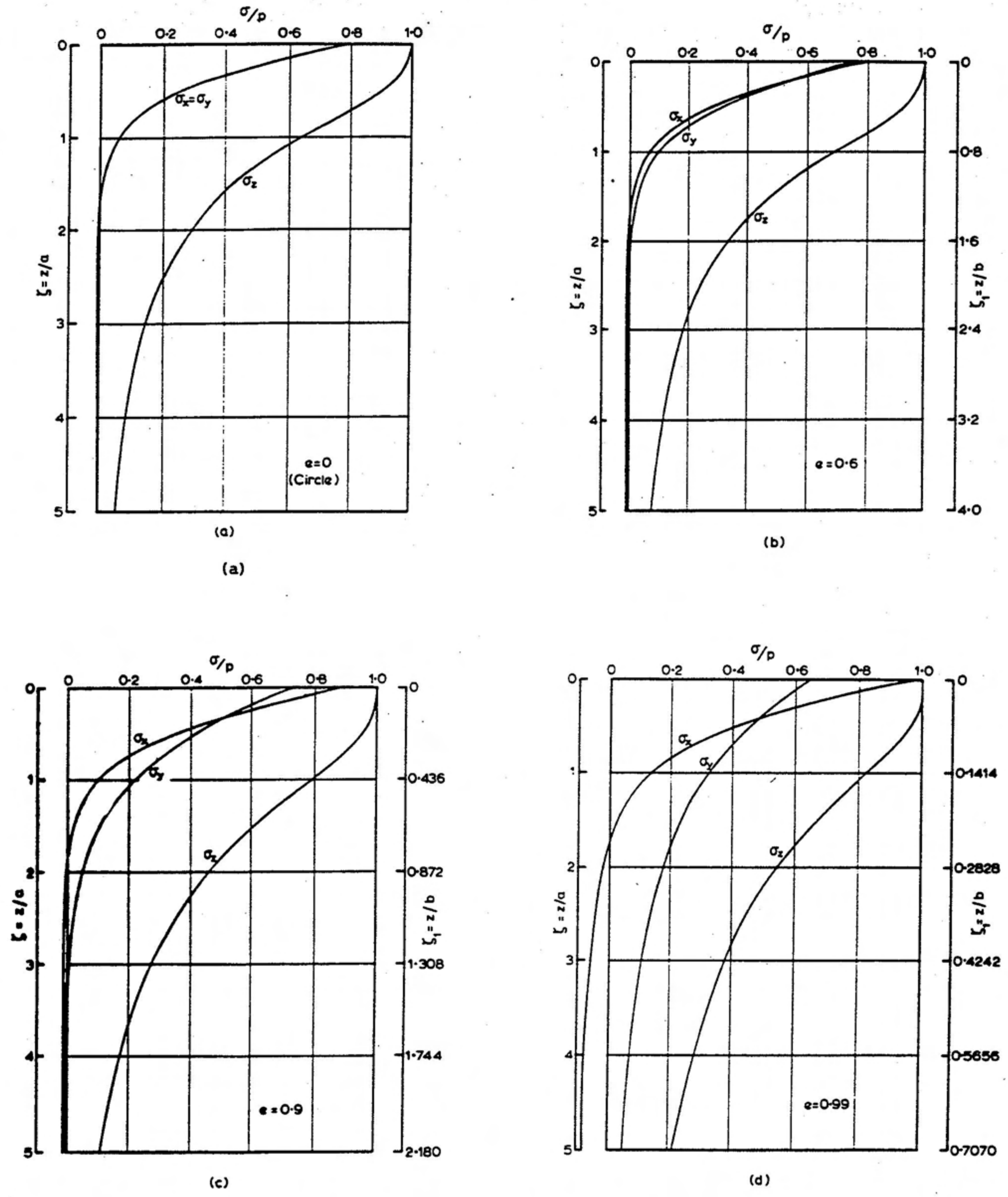


FIG.3.46 Variation of normal stresses with depth on axis of ellipse. ν=0.3. (Deresiewicz, 1960).

On the axis the displacements are given by

$$\rho_z = \frac{4(1-\nu^2)pa}{\pi E} K(e) \qquad \text{... (3.39a)}$$

$$\rho_x = \rho_y = 0 \qquad \text{... (3.39b)}$$

where $K(e)$ = complete elliptic integral of the first kind.

Relative vertical displacements ρ/ρ_o on the axis of the ellipse are shown in Fig.3.47 for $\nu=0.3$.

The variation of ρ/ρ_o along the boundary of the ellipse with position is given in Fig.3.48, while Fig.3.49 shows the variation of the displacements at the extremity of the major axis (ρ_M) and the minor axis (ρ_m). ρ_z is expressed in all cases as a ratio of the surface displacement ρ_o.

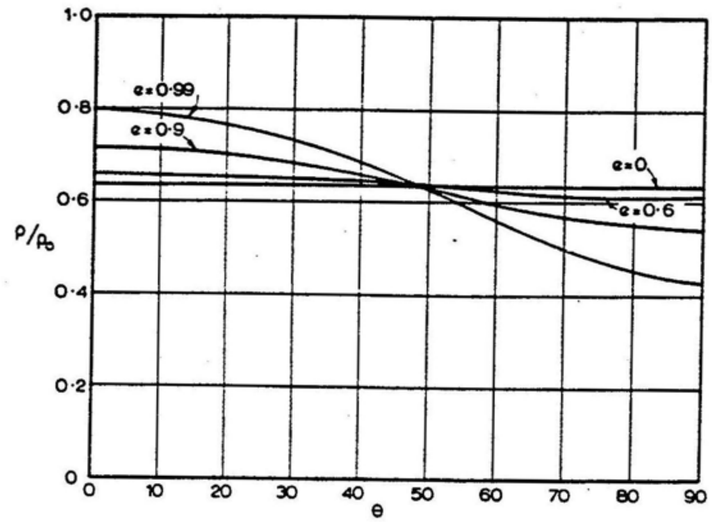


FIG.3.48 Variation of boundary surface displacement ρ to centre value ρ_o with position along boundary. (Deresiewicz, 1960).

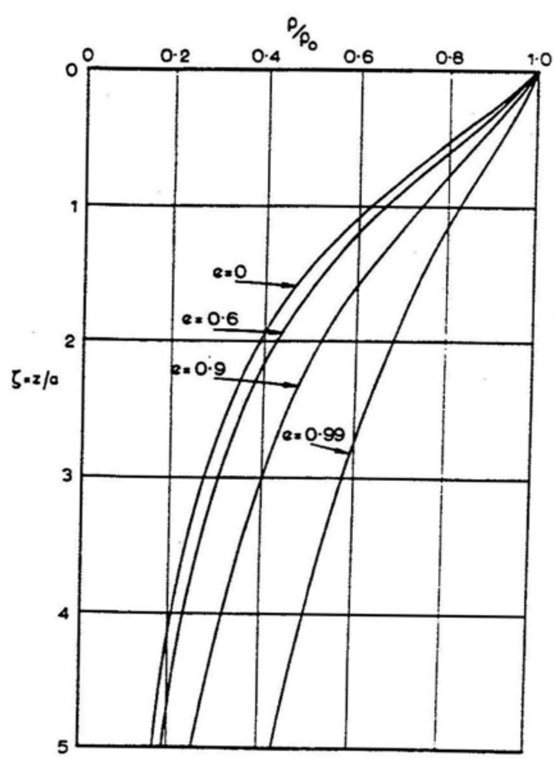


FIG.3.47 Vertical displacement on axis as ratio of surface value ρ_o. (Deresiewicz, 1960).

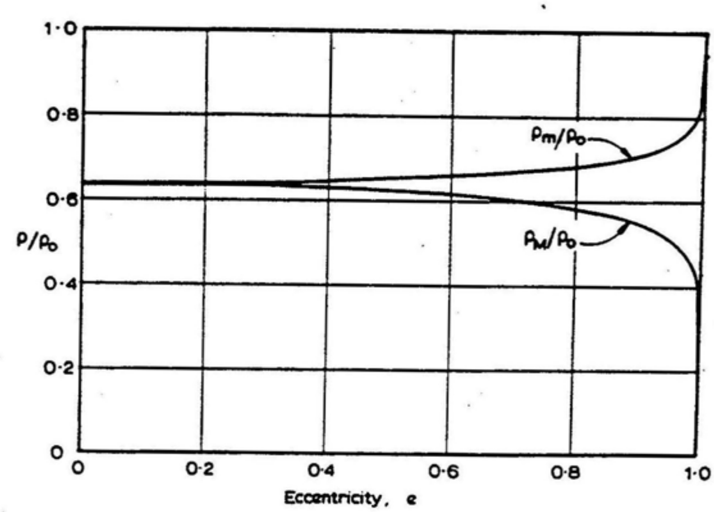


FIG.3.49 Variation of ratio of displacement at extremity of major axis, ρ_m, and minor axis, ρ_m, to that at centre, ρ_o. (Deresiewicz, 1960).

3.5.2 VERTICAL SEMI-ELLIPSOIDAL LOADING

This type of loading has been used to simulate wheel loading on road pavements. Vertical stresses and vertical displacements within the mass have been evaluated by Sanborn and Yoder (1967).

3.6 Loading over Any Area

3.6.1 "NEWMARK CHARTS"

The basis for, and use of, "Newmark Charts", is described in 1.7.2. Charts for vertical stress σ_z, horizontal stress, bulk stress Θ and shear stresses τ_{xz} and τ_{xy} (all as a function of the applied stress), originally presented by Newmark (1942), are reproduced in Figs. 3.50 to 3.54. Fig. 3.55 gives correction factors for τ_{xy} when Poisson's ratio is different from 0.5, while Fig. 3.56 gives part of the correction factor for σ_x. When $\nu \neq 0.5$, σ_x is given by the value of σ_x for $\nu = 0.5$, plus $(1-2\nu)/6$ times the value of Θ for $\nu = 0.5$ (Fig. 3.52) plus $(1-2\nu)$ times the quantity obtained from Fig. 3.56.

Similar charts for vertical displacement ρ_z on the surface and below the surface were obtained by Newmark (1947) and are shown in Figs.3.57 and 3.58. A chart for correcting the vertical subsurface displacements in Fig.3.58, which are for $\nu = 0.5$, for other values of ν, is given in Fig.3.59. Figs. 3.50 to 3.59 are for vertical surface loading.

Charts for the horizontal normal stress due to an applied surface horizontal shear loading have been prepared by Barber (1965) and are given in Figs.3.60 to 3.63. Stresses parallel to, and perpendicular to, the applied loading are considered for both $\nu = 0.5$ and $\nu = 0$. As pointed out by Barber (1966), the vertical stress due to shear loading is, by the reciprocal theorem, identical to the shear stress due to a vertical load and may thus be determined from Fig.3.53.

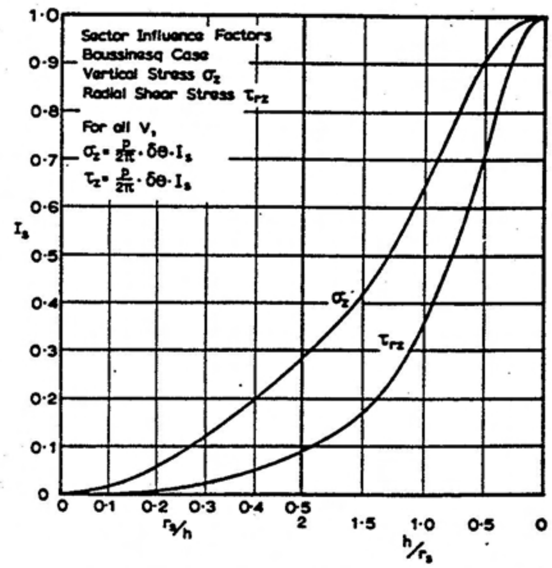


FIG.3.64 Sector Influence Values for σ_z and τ_{rz}.

3.6.2 SECTOR CURVES

The sector method and the use of sector curves have been described in 1.7.3. Sector curves for the normal and shear stresses due to vertical loading, obtained by Poulos (1967a), are shown in Figs.3.64 and 3.65. For the vertical and radial displacements ρ_z and ρ_r, plots of the curves are unnecessary, as the sector curves have the following simple explicit form:

$$\rho_z = \frac{p}{\pi}.\delta\theta.\frac{(1-\nu^2)}{E}\, r_s \qquad \text{... (3.39a)}$$

$$\rho_r = \frac{p}{2\pi}.\delta\theta.\frac{(1+\nu)(1-2\nu)}{E}\, r_s \qquad \text{... (3.39b)}$$

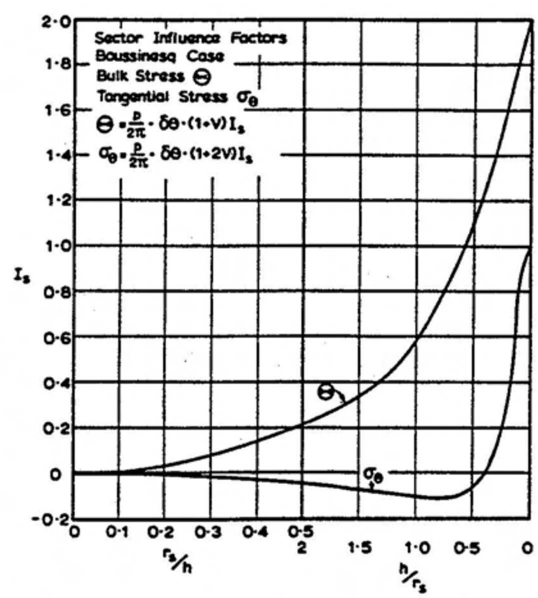


FIG.3.65 Sector Influence values for Θ and σ_θ.

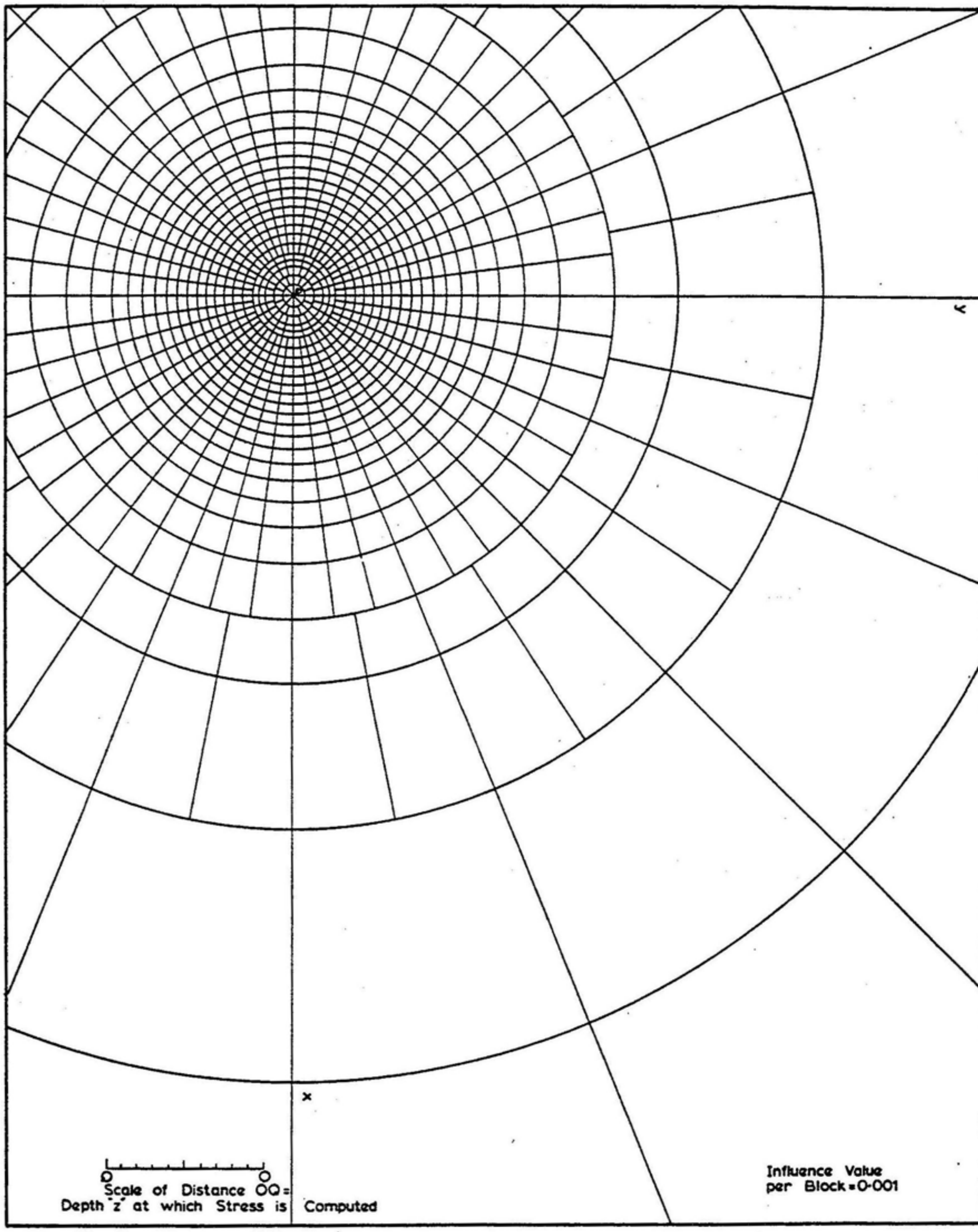


FIG.3.50 Influence chart for vertical stress σ_z (Newmark, 1942)
(All values of ν)

$\sigma_z = .001Np$
where N=no.of blocks.

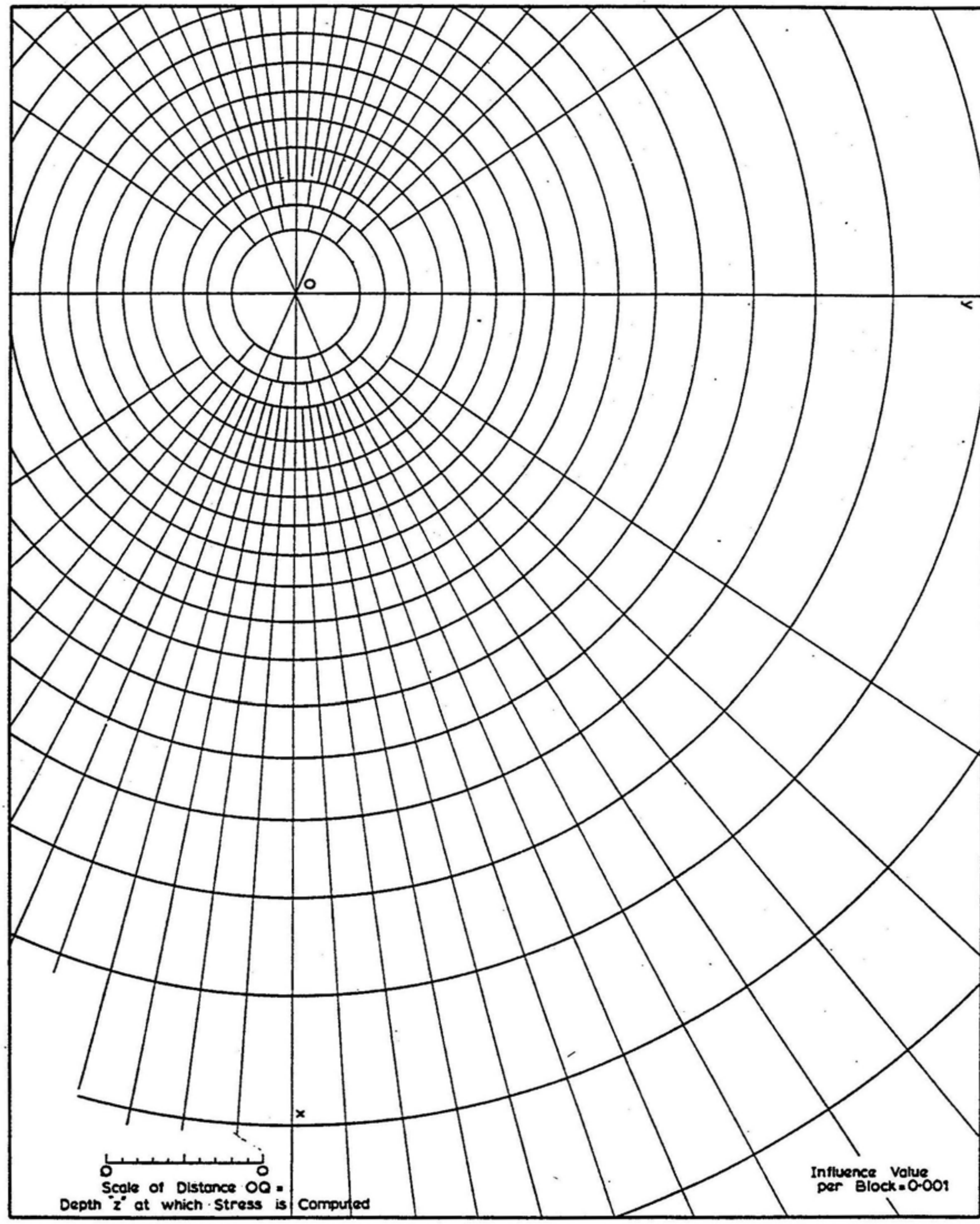


FIG.3.51 Influence chart for horizontal stress σ_x (Newmark, 1942).
$\nu = 0.5$
$\sigma_x =$ .001Np
where N=no. of blocks.

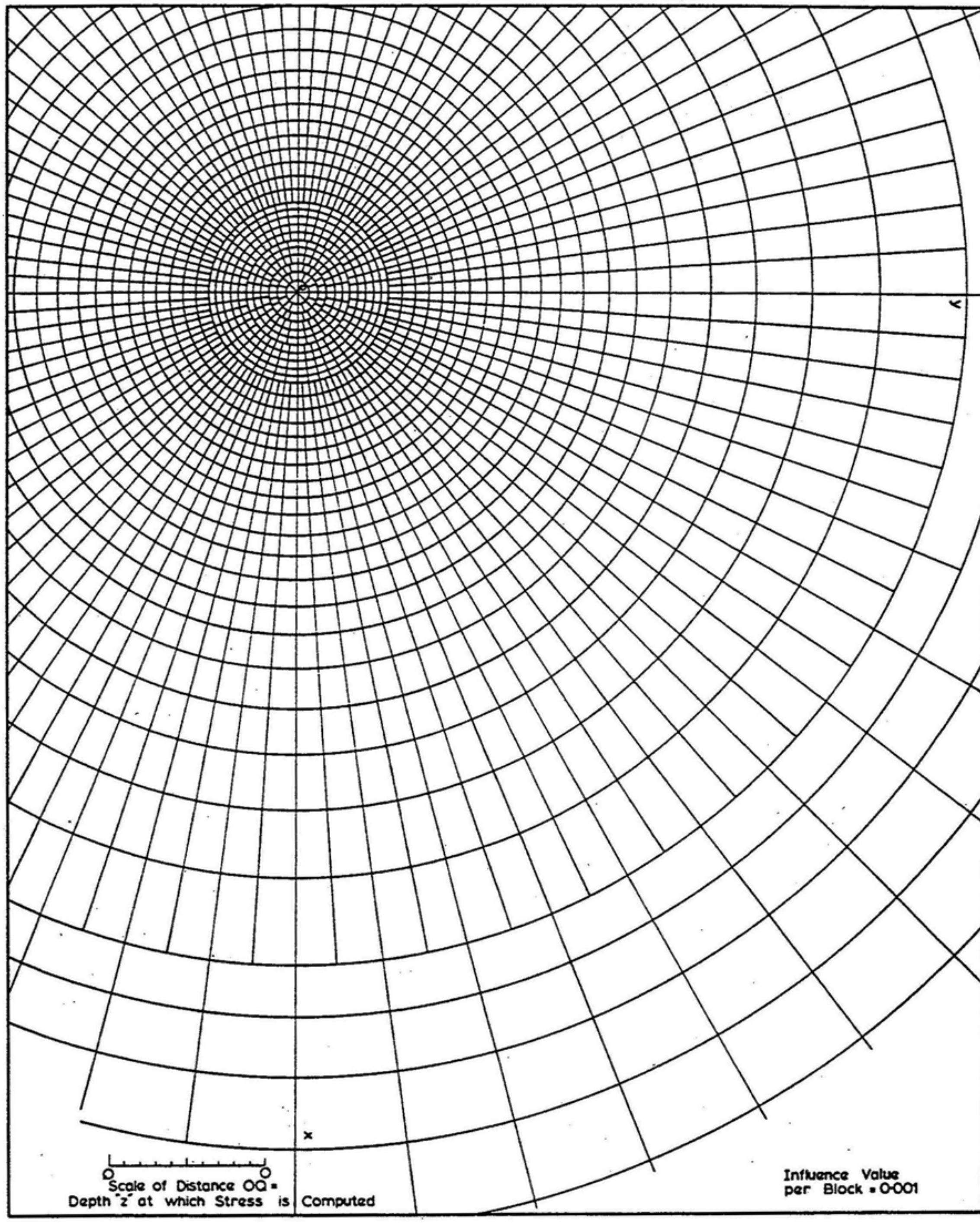


FIG.3.52 Influence chart for bulk stress θ (Newmark, 1942) (for all values of ν)

$$\theta = \frac{2(1+\nu)}{3} \ .001Np$$

where N=no.of blocks.

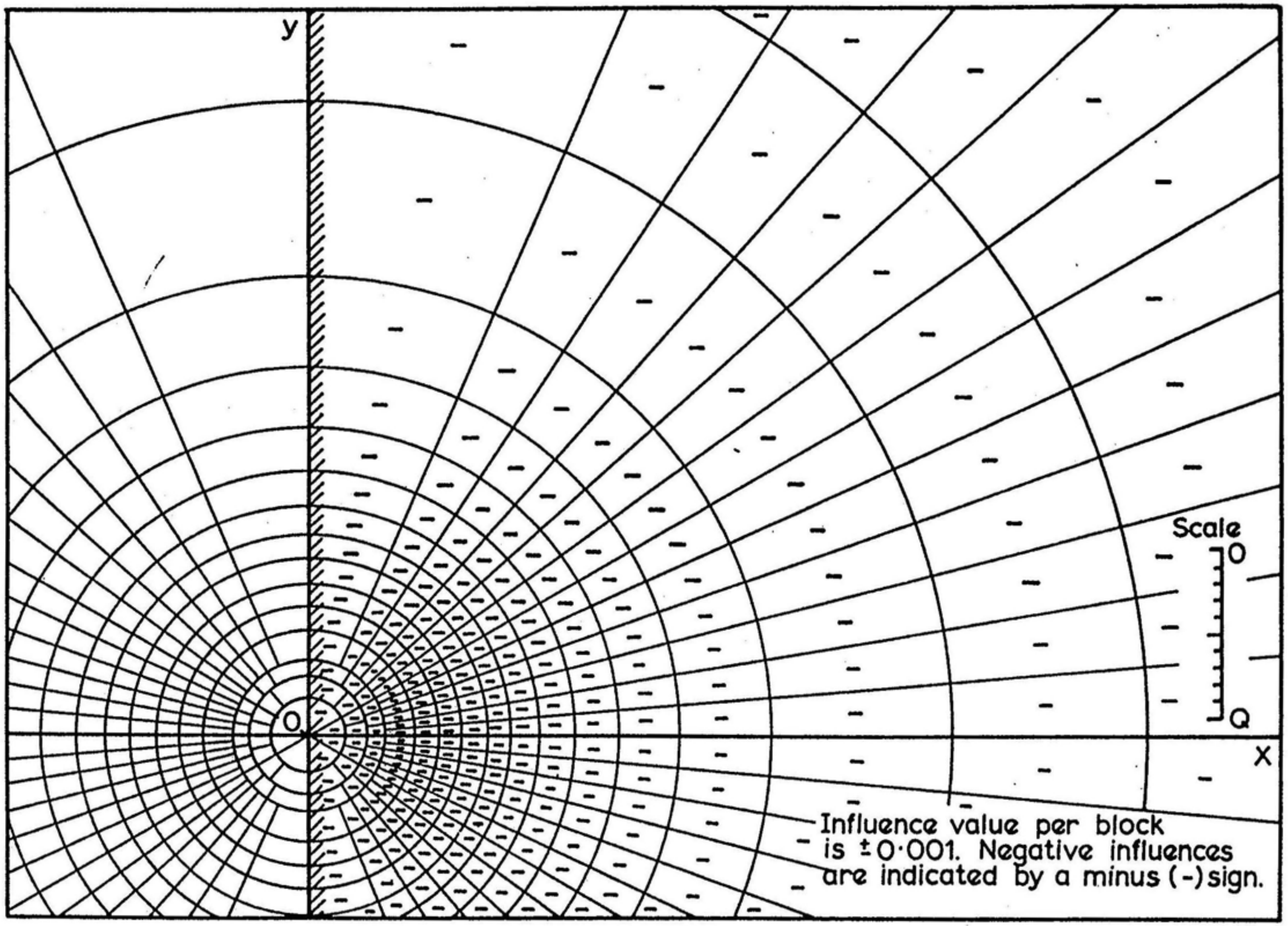


FIG.3.53 Influence chart for shear stress τ_{xz}. (Newmark, 1942).
(for all values of ν)
τ_{xz} = .001NP
where N=no.of blocks.

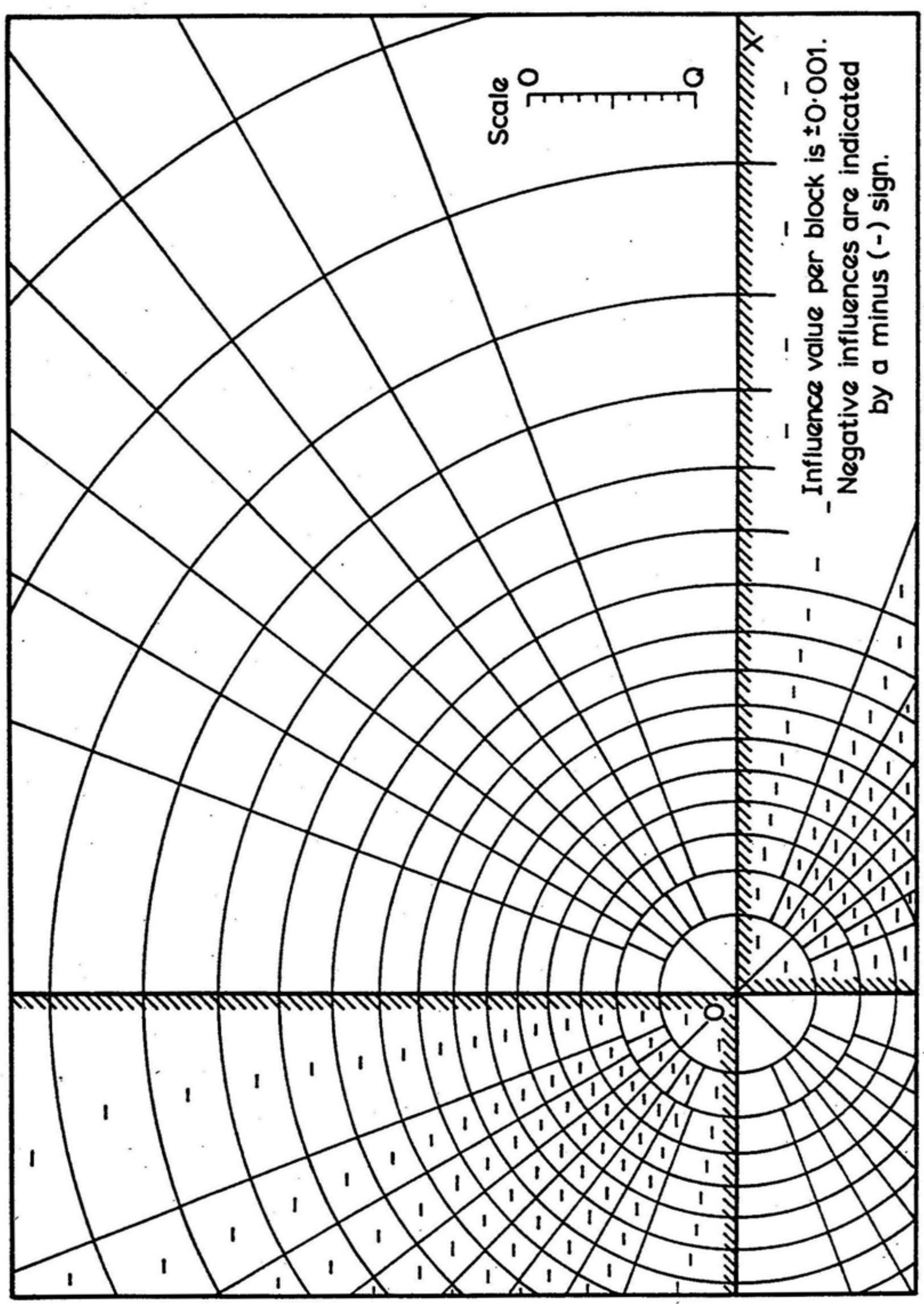


FIG.3.54 Influence chart for shear stress τ_{xy} for ν=0.5 (Newmark, 1942).
τ_{xy} = .001N p
where N=no.of blocks.

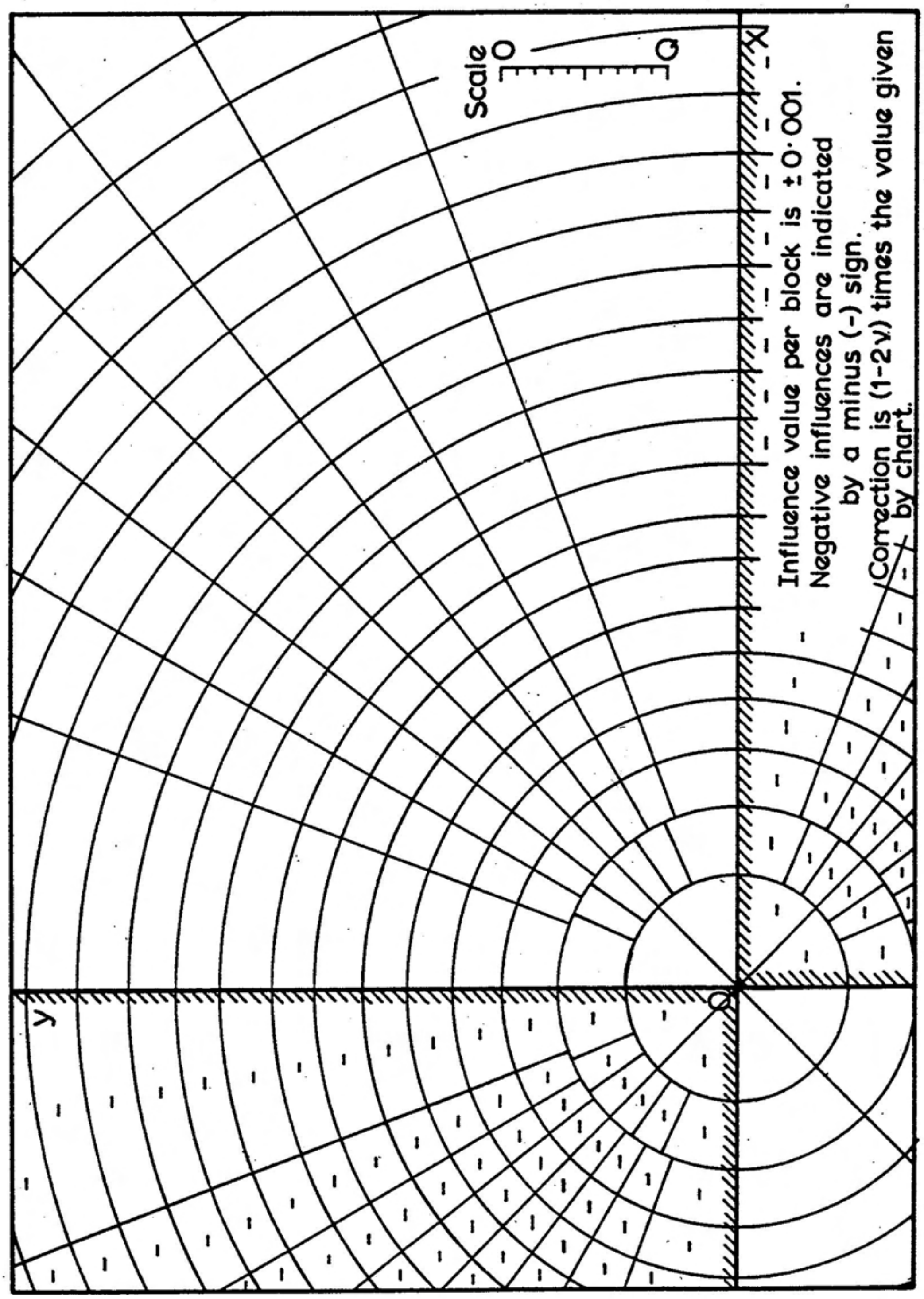


FIG.3.55 Influence chart for correction to τ_{xy} when ν=0.5. (Newmark, 1942).

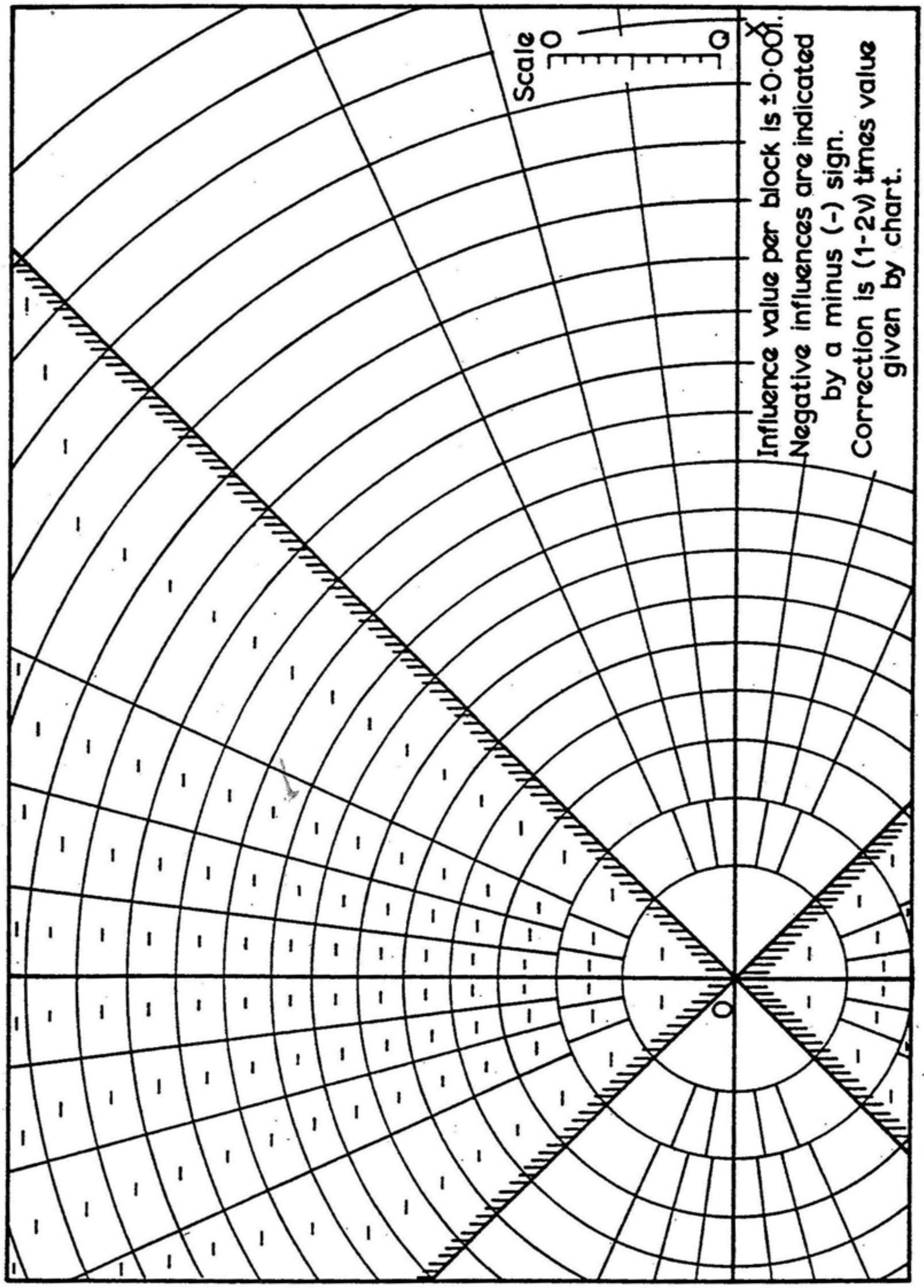


FIG.3.56 Influence chart for part of correction to σ_x for ν=0.5. (Newmark,1942).

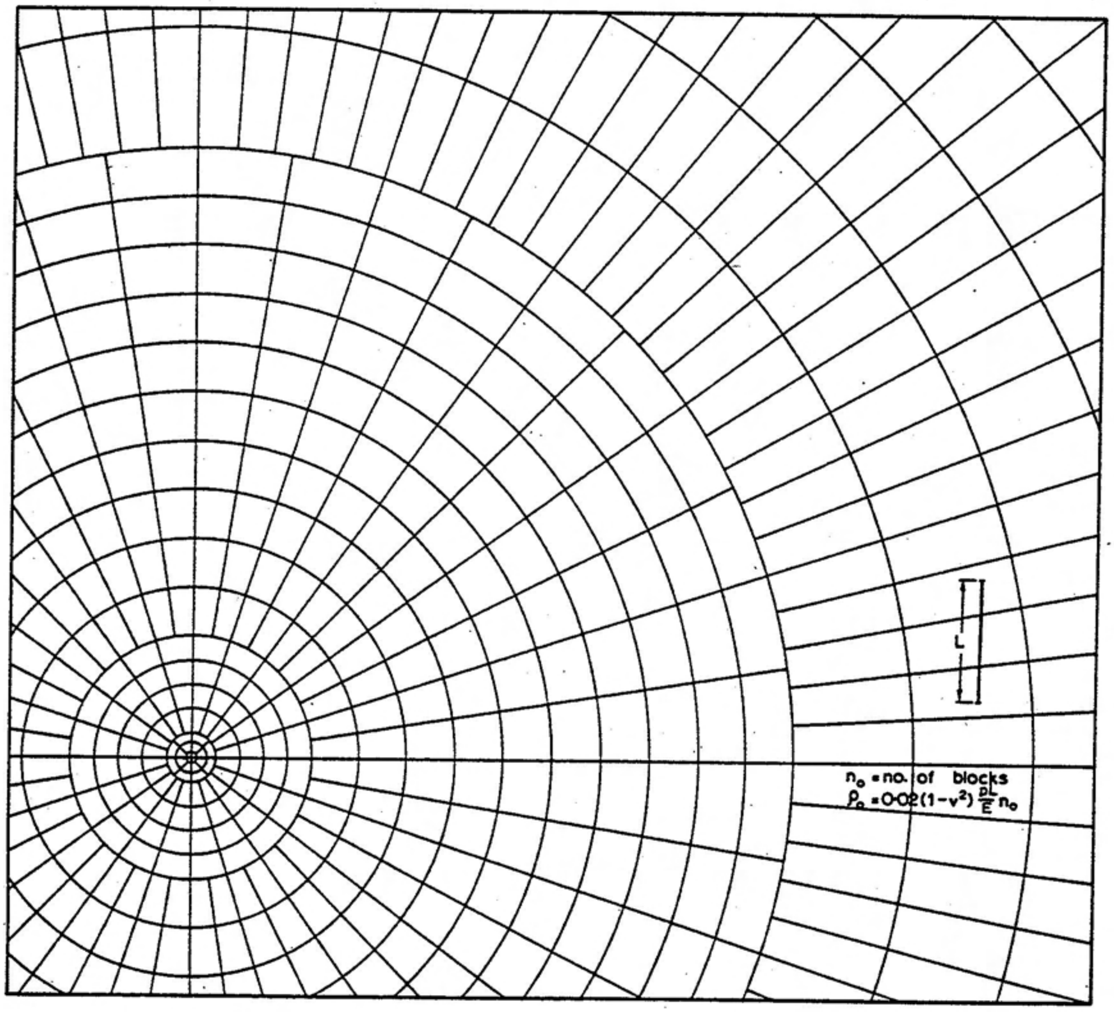


FIG.3.57 Influence chart for vertical displacement at surface. (Newmark, 1947).

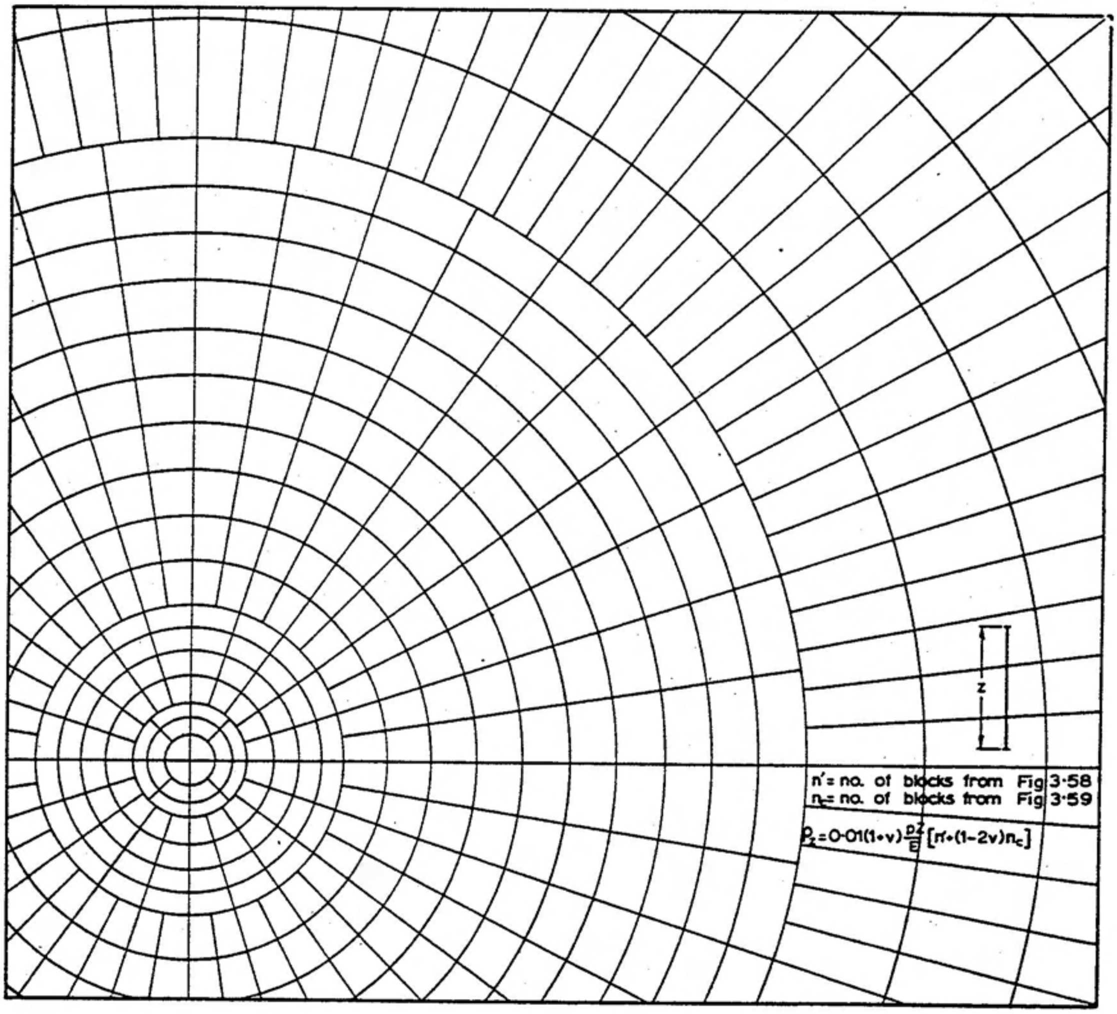


FIG.3.58 Influence chart for vertical displacement at depth z below surface. ν=0.5. (Newmark, 1947).

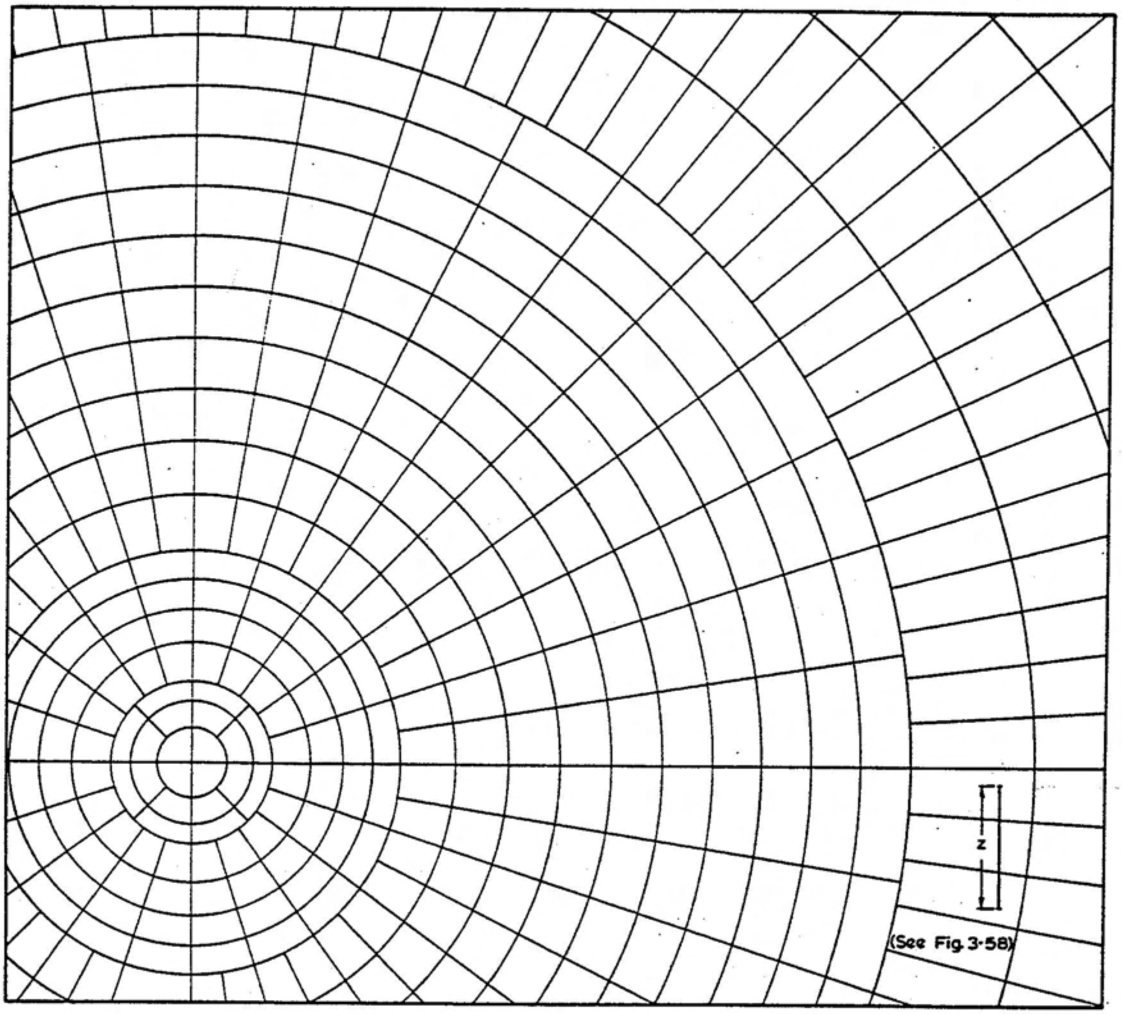


FIG. 3.59 Influence chart for Poisson's ratio correction for vertical displacement at depth z below surface. (Newmark, 1947).

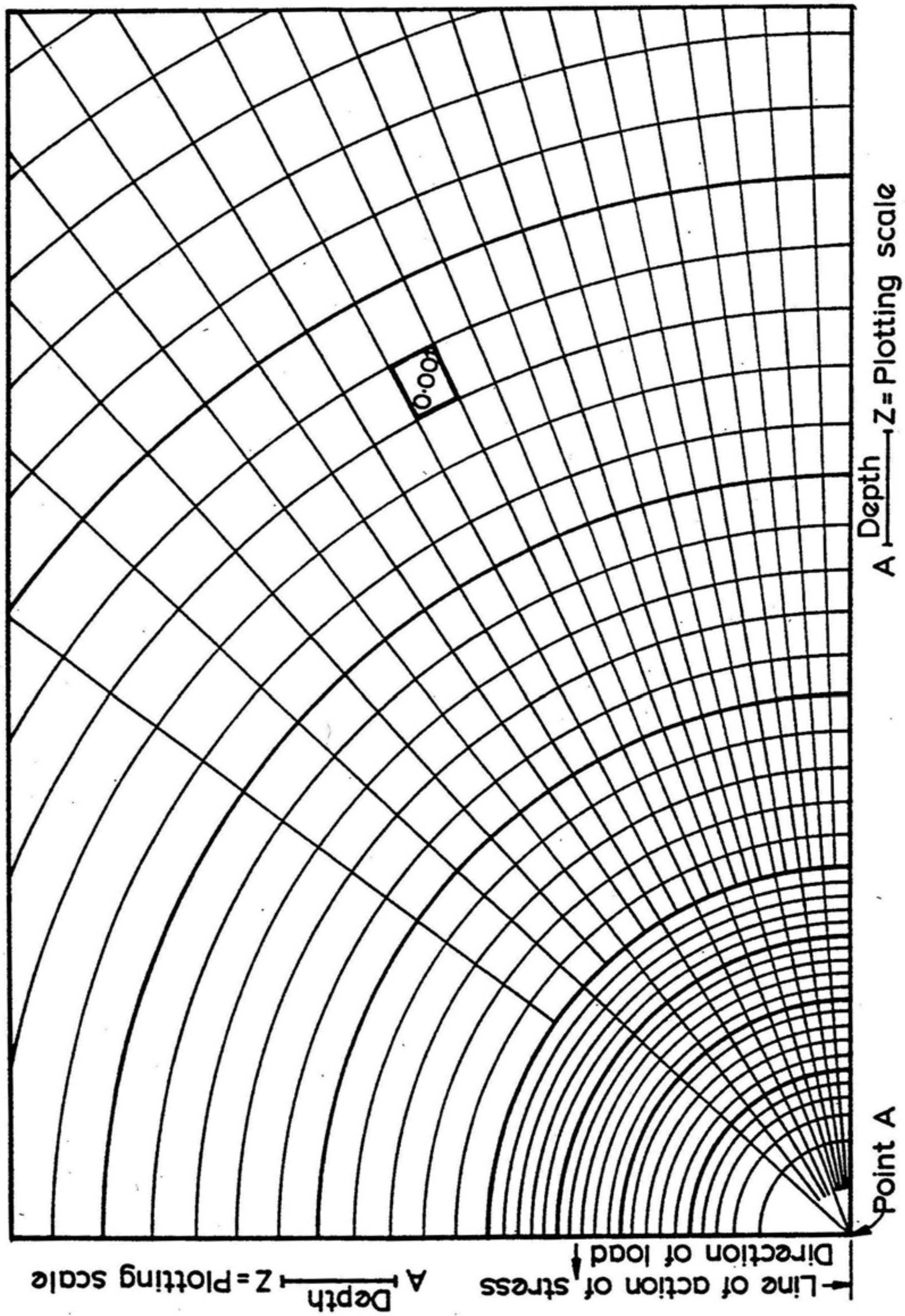


FIG.3.60 Influence chart for horizontal stress, parallel to an applied shear load. ν=0.5. (Barber, 1964).

σ_h = .001Nq
where N=no.of blocks.

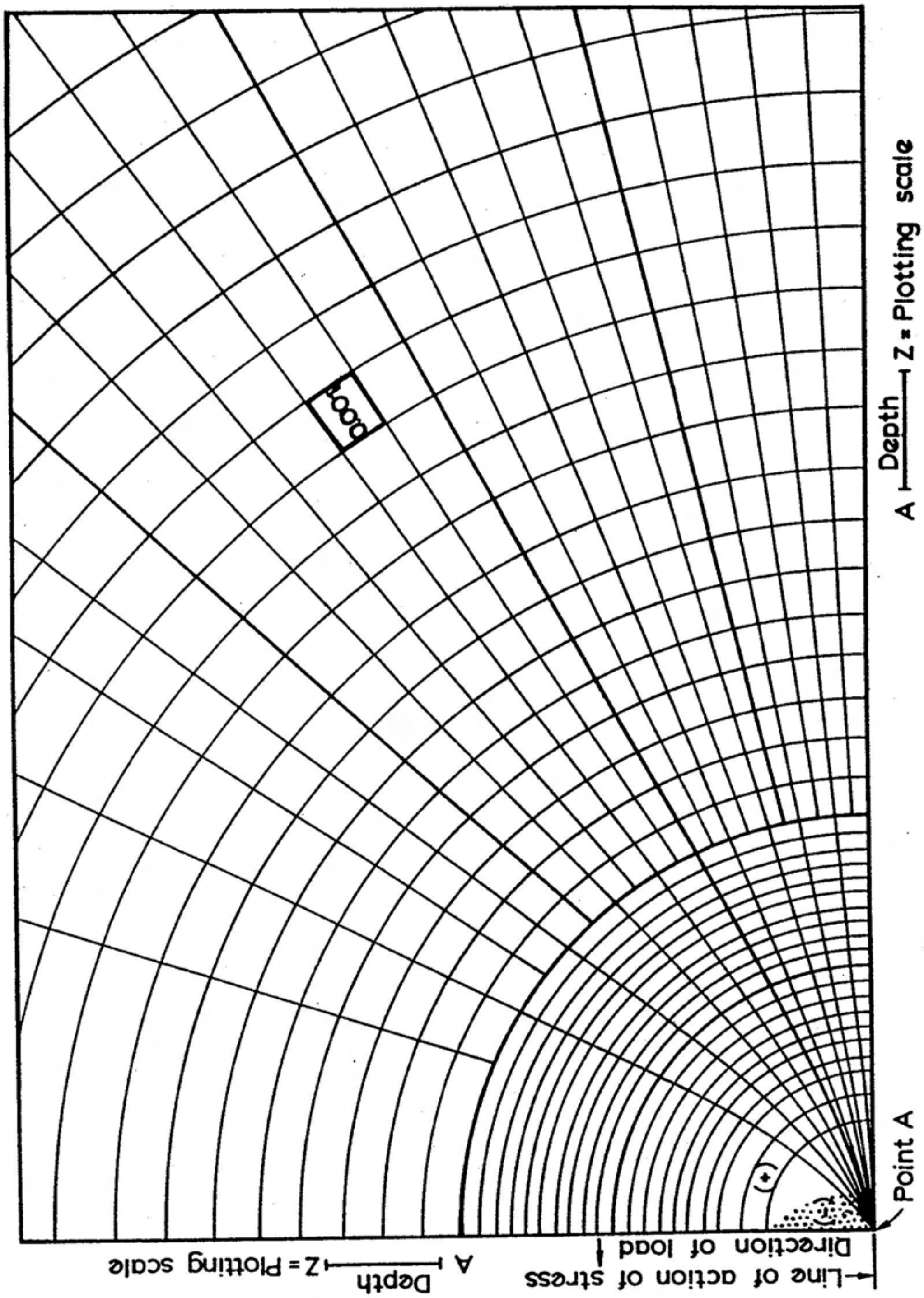


FIG.3.61 Influence chart for horizontal stress, parallel to an applied shear load. ν=0. (Barber, 1965).
σ_h = .001N q
where N=no.of blocks.

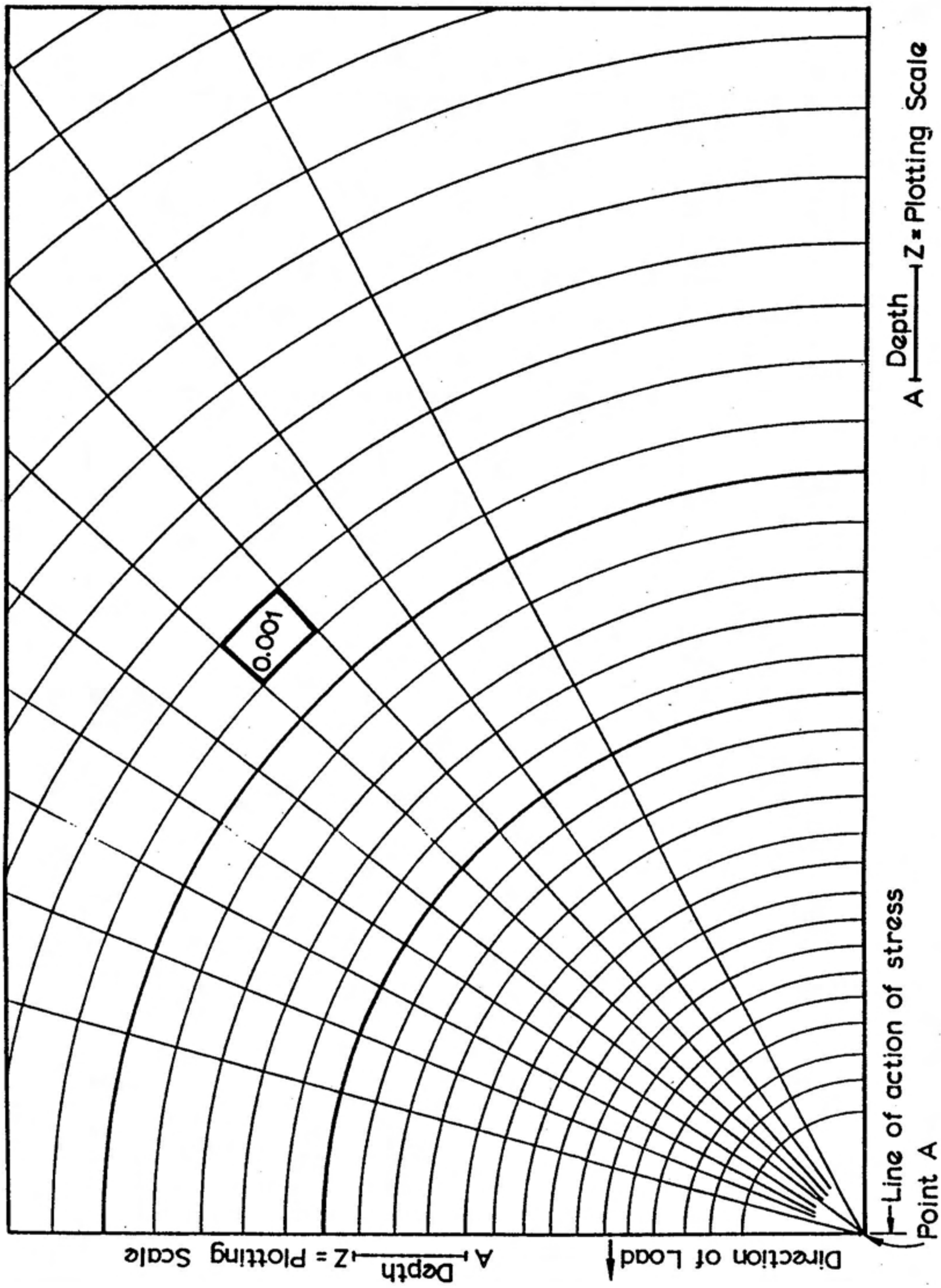


FIG.3.62 Influence chart for horizontal stress, perpendicular to an applied shear load. ν=0.5. (Barber, 1965).
σ_h = .001Nq
where N=no.of blocks.

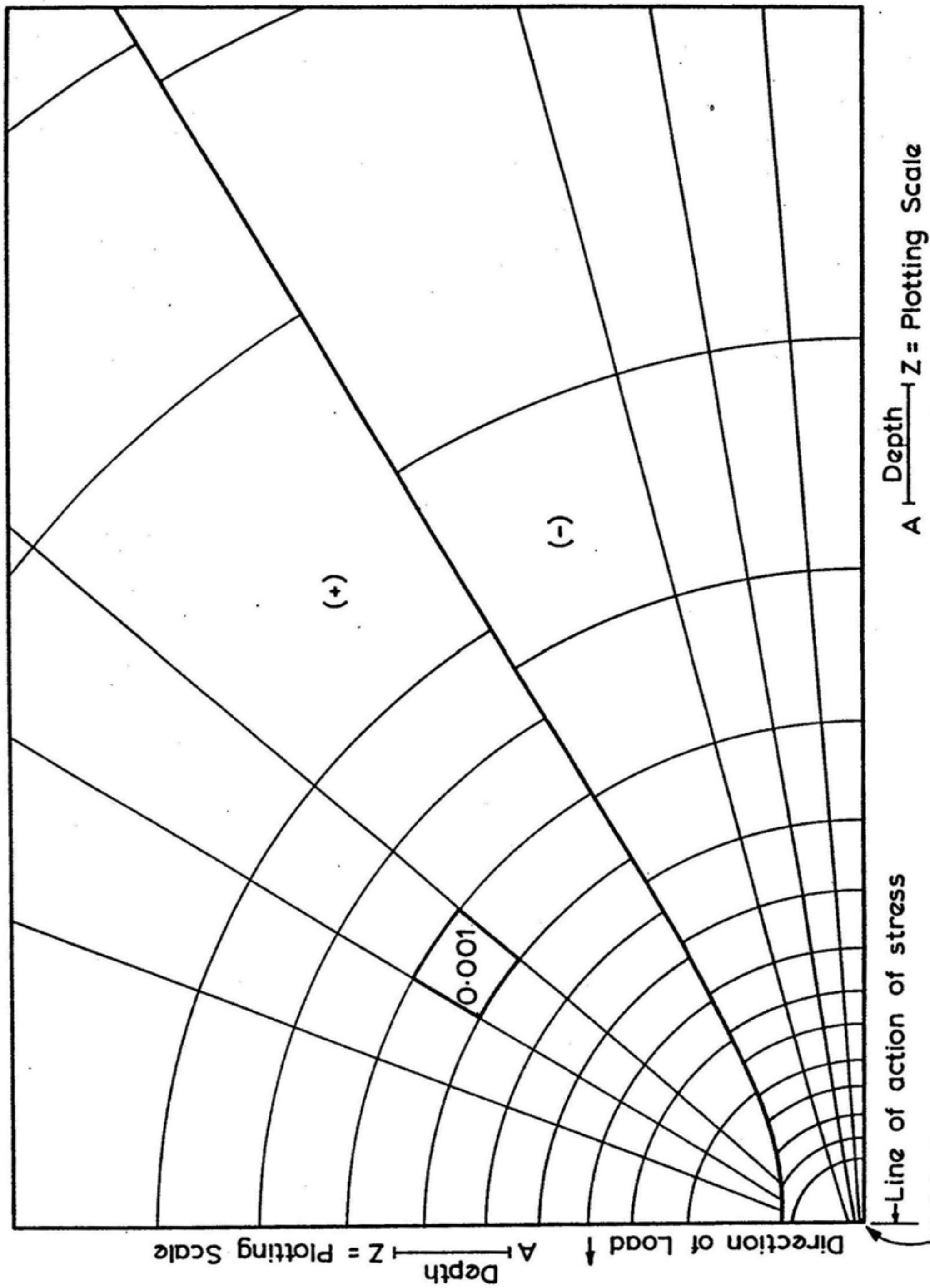


FIG.3.63 Influence chart for horizontal stress, perpendicular to an applied shear load. ν=O. (Barber, 1965).

σ_h = .OO1Nq
where N=no.of blocks.