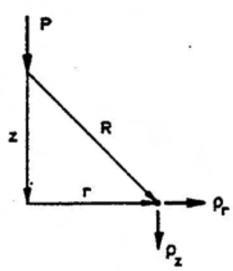
Chapter 2
BASIC SOLUTIONS FOR CONCENTRATED LOADING

2.1 Point Loading

2.1.1 KELVIN PROBLEM

Point load acting within an infinite elastic mass (Fig.2.1).

FIG.2.1



$$\sigma_z = \frac{P}{8\pi(1-\nu)}\left[\frac{3z^3}{R^5} + \frac{(1-2\nu)z}{R^3}\right] \quad \text{... (2.1a)}$$

$$\sigma_r = \frac{P}{8\pi(1-\nu)}\frac{z}{R^3}\left[\frac{3r^2}{R^2} - (1-2\nu)\right] \quad \text{... (2.1b)}$$

$$\sigma_\theta = -\frac{P(1-2\nu)}{8\pi(1-\nu)}\frac{z}{R^3} \quad \text{... (2.1c)}$$

$$\theta = \frac{P}{8\pi(1-\nu)} \cdot \frac{2(1+\nu)z}{R^3} \quad \text{... (2.1d)}$$

$$\tau_{rz} = \frac{P}{8\pi(1-\nu)}\frac{r}{R^3}\left[\frac{3z^2}{R^2} + (1-2\nu)\right] \quad \text{... (2.1e)}$$

$$\rho_z = \frac{P(1+\nu)}{8\pi(1-\nu)ER}\left[3 - 4\nu + \frac{z^2}{R^2}\right] \quad \text{... (2.1f)}$$

$$\rho_r = -\frac{P(1+\nu)}{8\pi(1-\nu)E} \cdot \frac{rz}{R^3} \quad \text{... (2.1g)}$$

2.1.2 BOUSSINESQ PROBLEM

Point load acting on the surface of a semi-infinite mass (Fig.2.2).

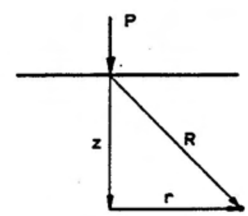


FIG.2.2

$$\sigma_z = \frac{3Pz^3}{2\pi R^5} \quad \text{... (2.2a)}$$

$$\sigma_r = -\frac{P}{2\pi R^2}\left[\frac{-3r^2 z}{R^3} + \frac{(1-2\nu)R}{R+z}\right] \quad \text{... (2.2b)}$$

$$\sigma_\theta = -\frac{(1-2\nu)P}{2\pi R^2}\left[\frac{z}{R} - \frac{R}{R+z}\right] \quad \text{... (2.2c)}$$

$$\theta = \frac{(1+\nu)\,Pz}{\pi R^3} \quad \text{... (2.2d)}$$

$$\tau_{rz} = \frac{3Prz^2}{2\pi R^5} \quad \text{... (2.2e)}$$

$$\rho_z = \frac{P(1+\nu)}{2\pi ER}\left[2(1-\nu) + \frac{z^2}{R^2}\right] \quad \text{... (2.2f)}$$

$$\rho_r = \frac{P(1+\nu)}{2\pi ER}\left[\frac{rz}{R^2} - \frac{(1-2\nu)r}{R+z}\right] \quad \text{... (2.2g)}$$

2.1.3 CERUTTI'S PROBLEM

Horizontal point load acting along the surface of a semi-infinite mass (Fig.2.3).

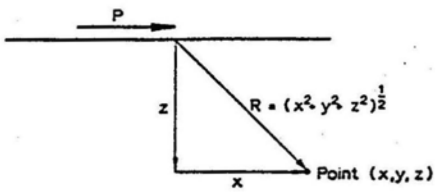


FIG. 2.3

$$\sigma_z = \frac{3Pxz^2}{2\pi R^5} \quad \ldots (2.3a)$$

$$\sigma_x = \frac{-Px}{2\pi R^3}\left[\frac{-3x^2}{R^2} + \frac{1-2\nu}{(R+z)^2}\left(R^2 - y^2 - \frac{2Ry^2}{R+z}\right)\right] \quad \ldots (2.3b)$$

$$\sigma_y = \frac{-Px}{2\pi R^3}\left[\frac{-3y^2}{R^2} + \frac{1-2\nu}{(R+z)^2}\left(3R^2 - x^2 - \frac{2Rx^2}{R+z}\right)\right] \quad \ldots (2.3c)$$

$$\theta = \frac{(1+\nu)Px}{\pi R^3} \quad \ldots (2.3d)$$

$$\tau_{xy} = \frac{-Py}{2\pi R^3}\left[-\frac{3x^2}{R^2} + \frac{(1-2\nu)}{(R+z)^2}\left(-R^2 + x^2 + \frac{2Rx^2}{R+z}\right)\right] \quad \ldots (2.3e)$$

$$\tau_{yz} = \frac{3Pxyz}{2\pi R^5} \quad \ldots (2.3f)$$

$$\tau_{zx} = \frac{3Px^2z}{2\pi R^5} \quad \ldots (2.3g)$$

$$\rho_z = \frac{P(1+\nu)}{2\pi ER}\left[\frac{xz}{R^2} + \frac{(1-2\nu)x}{R+z}\right] \quad \ldots (2.3h)$$

$$\rho_x = \frac{P(1+\nu)}{2\pi ER}\left[1 + \frac{x^2}{R^2} + (1-2\nu)\left(\frac{R}{R+z} - \frac{x^2}{(R+z)^2}\right)\right] \quad \ldots (2.3i)$$

$$\rho_y = \frac{P(1+\nu)}{2\pi ER}\left[\frac{xy}{R^2} - \frac{(1-2\nu)xy}{(R+z)^2}\right] \quad \ldots (2.3j)$$

2.1.4 MINDLIN'S PROBLEM NO.1

Vertical point load P acting beneath the surface of a semi-infinite mass. (Mindlin, 1936). (Fig.2.4).

$$\sigma_x = \frac{-P}{8\pi(1-\nu)}\left[\frac{(1-2\nu)(z-c)}{R_1^3} - \frac{3x^2(z-c)}{R_1^5} + \frac{(1-2\nu)[3(z-c)-4\nu(z+c)]}{R_2^3} - \frac{3(3-4\nu)x^2(z-c)-6c(z+c)[(1-2\nu)z-2\nu c]}{R_2^5} - \frac{30cx^2z(z+c)}{R_2^7} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)} \times \left(1 - \frac{x^2}{R_2(R_2+z+c)} - \frac{x^2}{R_2^2}\right)\right] \quad \ldots (2.4a)$$

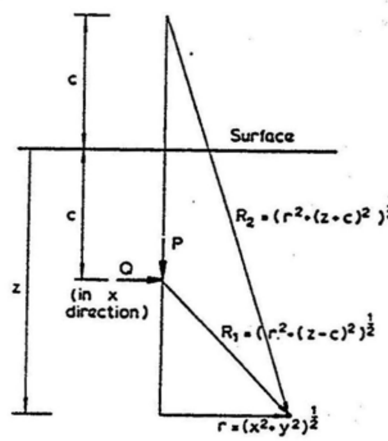


FIG. 2.4

$$\sigma_y = \frac{-P}{8\pi(1-\nu)}\left[\frac{(1-2\nu)(z-c)}{R_1^3} - \frac{3y^2(z-c)}{R_1^5} + \frac{(1-2\nu)[3(z-c)-4\nu(z+c)]}{R_2^3} - \frac{3(3-4\nu)y^2(z-c)-6c(z+c)[(1-2\nu)z-2\nu c]}{R_2^5} - \frac{30cy^2z(z+c)}{R_2^7} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)} \times \left(1 - \frac{y^2}{R_2(R_2+z+c)} - \frac{y^2}{R_2^2}\right)\right] \quad \ldots (2.4b)$$

$$\sigma_z = \frac{-P}{8\pi(1-\nu)}\left[-\frac{(1-2\nu)(z-c)}{R_1^3} + \frac{(1-2\nu)(z-c)}{R_2^3} - \frac{3(z-c)^3}{R_1^5} - \frac{3(3-4\nu)z(z+c)^2-3c(z+c)(5z-c)}{R_2^5} - \frac{30cz(z+c)^3}{R_2^7}\right] \quad \ldots (2.4c)$$

$$\tau_{yz} = \frac{-Py}{8\pi(1-\nu)} \left[-\frac{(1-2\nu)}{R_1^3} + \frac{1-2\nu}{R_2^3} - \frac{3(z-c)^2}{R_1^5} - \frac{3(3-4\nu)z(z+c)-3c(3z+c)}{R_2^5} - \frac{30cz(z+c)^2}{R_2^7} \right] \quad \text{... (2.4d)}$$

$$\tau_{zx} = \frac{-Px}{8\pi(1-\nu)} \left[-\frac{1-2\nu}{R_1^3} + \frac{1-2\nu}{R_2^3} - \frac{3(z-c)^2}{R_1^5} - \frac{3(3-4\nu)z(z+c)-3c(3z+c)}{R_2^5} - \frac{30cz(z+c)^2}{R_2^7} \right] \quad \text{... (2.4e)}$$

$$\tau_{xy} = \frac{-Pxy}{8\pi(1-\nu)} \left[-\frac{3(z-c)}{R_1^5} - \frac{3(3-4\nu)(z-c)}{R_2^5} + \frac{4(1-\nu)(1-2\nu)}{R_2^2(R_2+z+c)} \left(\frac{1}{R_2+z+c} + \frac{1}{R_2}\right) - \frac{30cz(z+c)}{R_2^7} \right] \quad \text{... (2.4f)}$$

$$\sigma_r = \frac{-P}{8\pi(1-\nu)} \left[\frac{(1-2\nu)(z-c)}{R_1^3} - \frac{(1-2\nu)(z+7c)}{R_2^3} + \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)} - \frac{3r^2(z-c)}{R_1^5} + \frac{6c(1-2\nu)(z+c)^2-6c^2(z+c)-3(3-4\nu)r^2(z-c)}{R_2^5} - \frac{30cr^2z(z+c)}{R_2^7} \right] \quad \text{... (2.4g)}$$

$$\sigma_\theta = \frac{-P(1-2\nu)}{8\pi(1-\nu)} \left[\frac{z-c}{R_1^3} + \frac{(3-4\nu)(z+c)-6c}{R_2^3} - \frac{4(1-\nu)}{R_2(R_2+z+c)} + \frac{6c(z+c)^2}{R_2^5} - \frac{6c^2(z+c)}{(1-2\nu)R_2^5} \right] \quad \text{...(2.4h)}$$

$$\tau_{rz} = \frac{-Pr}{8\pi(1-\nu)} \left[-\frac{1-2\nu}{R_1^3} + \frac{1-2\nu}{R_2^3} - \frac{3(z-c)^2}{R_1^5} - \frac{3(3-4\nu)z(z+c)-3c(3z+c)}{R_2^5} - \frac{30cz(z+c)^2}{R_2^7} \right] \quad \text{... (2.4i)}$$

$$\rho_r = \frac{Pr}{16\pi G(1-\nu)} \left[\frac{z-c}{R_1^3} + \frac{(3-4\nu)(z-c)}{R_2^3} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)} + \frac{6cz(z+c)}{R_2^5} \right] \quad \text{... (2.4j)}$$

$$\rho_z = \frac{P}{16\pi G(1-\nu)} \left[\frac{3-4\nu}{R_1} + \frac{8(1-\nu)^2-(3-4\nu)}{R_2} + \frac{(z-c)^2}{R_1^3} + \frac{(3-4\nu)(z+c)^2-2cz}{R_2^3} + \frac{6cz(z+c)^2}{R_2^5} \right] \quad \text{... (2.4k)}$$

Influence factors for σ_z, and σ_r and σ_θ on the axis have been tabulated by Geddes (1966).

2.1.5 MINDLIN'S PROBLEM NO.2.

Horizontal point load Q acting beneath the surface of a semi-infinite mass. (Mindlin, 1936). (Fig.2.4).

$$\sigma_x = \frac{-Qx}{8\pi(1-\nu)} \left[-\frac{(1-2\nu)}{R_1^3} + \frac{(1-2\nu)(5-4\nu)}{R_2^3} - \frac{3x^2}{R_1^5} - \frac{3(3-4\nu)x^2}{R_2^5} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)^2} \times \left(3 - \frac{x^2(3R_2+z+c)}{R_2^2(R_2+z+c)}\right) + \frac{6c}{R_2^5}\left(3c - (3-2\nu)(z+c) + \frac{5x^2z}{R_2^2}\right) \right] \quad \text{... (2.5a)}$$

$$\sigma_y = \frac{-Qx}{8\pi(1-\nu)} \left[\frac{(1-2\nu)}{R_1^3} + \frac{(1-2\nu)(3-4\nu)}{R_2^3} - \frac{3y^2}{R_1^5} - \frac{3(3-4\nu)y^2}{R_2^5} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)^2} \times \left(1 - \frac{y^2(3R_2+z+c)}{R_2^2(R_2+z+c)}\right) \right.$$

(Continued)

$$+\frac{6c}{R_2^5}\left(c-(1-2\nu)(z+c)+\frac{5y^2z}{R_2^2}\right)\Bigg] \qquad \ldots (2.5b)$$

$$\sigma_z = \frac{-Qx}{8\pi(1-\nu)}\Bigg[\frac{(1-2\nu)}{R_1^3}-\frac{(1-2\nu)}{R_2^3}-\frac{3(z-c)^2}{R_1^5}-\frac{3(3-4\nu)(z+c)^2}{R_2^5}+\frac{6c}{R_2^5}\left(c+(1-2\nu)(z+c)+\frac{5z(z+c)^2}{R_2^2}\right)\Bigg] \qquad \ldots (2.5c)$$

$$\tau_{yz} = \frac{-Qxy}{8\pi(1-\nu)}\Bigg[-\frac{3(z-c)}{R_1^5}-\frac{3(3-4\nu)(z+c)}{R_2^5}+\frac{6c}{R_2^5}\left(1-2\nu+\frac{5z(z+c)}{R_2^2}\right)\Bigg] \qquad \ldots (2.5d)$$

$$\tau_{zx} = \frac{-Q}{8\pi(1-\nu)}\Bigg[-\frac{(1-2\nu)(z-c)}{R_1^3}+\frac{(1-2\nu)(z-c)}{R_2^3}-\frac{3x^2(z-c)}{R_1^5}-\frac{3(3-4\nu)x^2(z+c)}{R_2^5}-\frac{6c}{R_2^5}\left(z(z+c)-(1-2\nu)x^2-\frac{5x^2z(z+c)}{R_2^2}\right)\Bigg] \qquad \ldots (2.5e)$$

$$\tau_{xy} = \frac{-Qy}{8\pi(1-\nu)}\Bigg[-\frac{(1-2\nu)}{R_1^3}+\frac{(1-2\nu)}{R_2^3}-\frac{3x^2}{R_1^5}-\frac{3(3-4\nu)x^2}{R_2^5}-\frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)^2}\times\left(1-\frac{x^2(3R_2+z+c)}{R_2^2(R_2+z+c)}\right)-\frac{6cz}{R_2^5}\left(1-\frac{5x^2}{R_2^2}\right)\Bigg] \qquad \ldots (2.5f)$$

$$\rho_x = \frac{Q}{16\pi G(1-\nu)}\Bigg[\frac{(3-4\nu)}{R_1}+\frac{1}{R_2}+\frac{x^2}{R_1^3}+\frac{(3-4\nu)x^2}{R_2^3}+\frac{2cz}{R_2^3}\left(1-\frac{3x^2}{R_2^2}\right)+\frac{4(1-\nu)(1-2\nu)}{R_2+z+c}\times\left(1-\frac{x^2}{R_2(R_2+z+c)}\right)\Bigg] \qquad \ldots (2.5g)$$

$$\rho_y = \frac{Qxy}{16\pi G(1-\nu)}\Bigg[\frac{1}{R_1^3}+\frac{(3-4\nu)}{R_2^3}-\frac{6cz}{R_2^5}-\frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)^2}\Bigg] \qquad \ldots (2.5h)$$

$$\rho_z = \frac{Qx}{16\pi G(1-\nu)}\Bigg[\frac{z-c}{R_1^3}+\frac{(3-4\nu)(z-c)}{R_2^3}-\frac{6cz(z+c)}{R_2^5}+\frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+c)}\Bigg] \qquad \ldots (2.5i)$$

2.1.6 POINT LOAD ON FINITE LAYER

Vertical point load acting at the surface of a layer underlain by a rough rigid base (Fig.2.5). This problem has been studied in detail by Burmister (1943,1945).

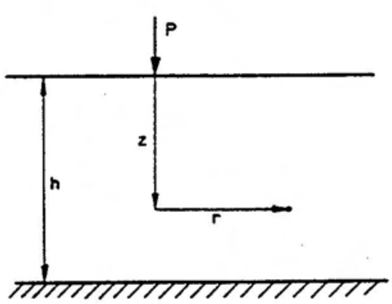


FIG. 2.5

Numerical values for the stresses and displacements in this problem have been tabulated by Poulos (1967b), and are given in TABLES 2.1 to 2.7.

2.1.7 OTHER SOLUTIONS

(a) *Point Loads Inside an Infinite Two Layer System* - Solutions for stresses and displacements have been given by Plevako (1969) for both vertical and horizontal point loading.

(b) *Point Load Within an Elastic Layer* - A formal solution for stresses and displacements has been given by Shekter and Prikhodchenko (1964), but no numerical values are evaluated.

TABLE 2.1

INFLUENCE FACTORS I_{σ_z} FOR VERTICAL STRESS σ_z

POINT LOAD

$$\sigma_z = I_{\sigma_z} P/2\pi h^2$$

r/h \ z/h	I_{σ_z} for Poisson's Ratio ν = 0.0									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.422	4.993	5.779	7.005	9.017	12.50	19.10	33.55	75.11	300.0
0.1	4.308	4.844	5.571	6.680	8.448	11.37	16.45	25.82	43.04	53.07
0.2	3.989	4.431	5.005	5.826	7.022	8.738	11.05	13.49	13.36	5.400
0.3	3.523	3.842	4.225	4.713	5.316	5.970	6.426	6.067	4.028	0.980
0.4	2.982	3.178	3.386	3.601	3.782	3.827	3.554	2.741	1.419	0.280
0.5	2.432	2.528	2.606	2.642	2.595	2.397	1.977	1.323	0.594	0.112
0.6	1.923	1.947	1.943	1.886	1.747	1.499	1.132	0.690	0.288	0.057
0.7	1.481	1.460	1.412	1.319	1.165	0.944	0.669	0.384	0.155	0.035
0.8	1.114	1.070	1.006	0.910	0.770	0.597	0.404	0.223	0.089	0.023
0.9	0.821	0.768	0.702	0.615	0.504	0.376	0.245	0.131	0.052	0.015
1.0	0.593	0.541	0.481	0.408	0.324	0.233	0.146	0.075	0.029	0.010
1.25	0.238	0.201	0.164	0.127	0.090	0.056	0.028	0.010	0.001	-0.001
1.5	0.074	0.055	0.036	0.020	0.006	-0.006	-0.012	-0.013	-0.010	-0.007
1.75	0.007	-0.003	-0.011	-0.018	-0.021	-0.023	-0.022	-0.018	-0.014	-0.010
2.0	-0.017	-0.020	-0.023	-0.025	-0.025	-0.023	-0.020	-0.017	-0.013	-0.010
2.5	-0.019	-0.018	-0.018	-0.016	-0.015	-0.013	-0.011	-0.009	-0.007	-0.006
3.0	-0.010	-0.009	-0.008	-0.007	-0.006	-0.005	-0.004	-0.003	-0.002	-0.001
3.5	-0.004	-0.003	-0.002	-0.002	-0.001	-0.000	0.000	0.001	0.001	0.001
4.0	-0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.002
6.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

r/h \ z/h	I_{σ_z} for Poisson's Ratio ν = 0.2									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.358	5.028	5.853	7.089	9.098	12.57	19.15	33.59	75.13	300.0
0.1	4.243	4.874	5.640	6.761	8.525	11.44	16.51	25.86	43.06	53.08
0.2	3.919	4.448	5.060	5.894	7.089	8.797	11.10	13.52	13.38	5.415
0.3	3.447	3.840	4.259	4.763	5.369	6.018	6.466	6.098	4.050	0.994
0.4	2.902	3.157	3.399	3.630	3.817	3.862	3.585	2.766	1.437	0.293
0.5	2.352	2.490	2.597	2.651	2.613	2.418	1.998	1.341	0.069	0.124
0.6	1.844	1.898	1.918	1.878	1.750	1.507	1.143	0.701	0.298	0.067
0.7	1.407	1.405	1.376	1.298	1.155	0.941	0.671	0.389	0.161	0.041
0.8	1.048	1.015	0.964	0.879	0.752	0.586	0.399	0.222	0.091	0.026
0.9	0.764	0.716	0.659	0.581	0.480	0.360	0.235	0.126	0.051	0.016
1.0	0.545	0.494	0.439	0.374	0.297	0.214	0.133	0.068	0.025	0.008
1.25	0.213	0.172	0.135	0.099	0.066	0.036	0.013	-0.001	-0.007	-0.006
1.5	0.067	0.042	0.021	0.004	-0.010	-0.018	-0.022	-0.021	-0.017	-0.012
1.75	0.010	-0.003	-0.014	-0.022	-0.026	-0.028	-0.026	-0.022	-0.017	-0.012
2.0	-0.008	-0.014	-0.019	-0.022	-0.024	-0.023	-0.020	-0.017	-0.013	-0.010
2.5	-0.008	-0.009	-0.009	-0.010	-0.009	-0.009	-0.007	-0.006	-0.005	-0.004
3.0	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.000	0.000	0.001
3.5	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002
4.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
6.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

r/h \ z/h	I_{σ_z} for Poisson's Ratio ν = 0.4									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.454	5.259	6.113	7.330	9.297	12.73	19.25	33.64	75.15	300.0
0.1	4.329	5.091	5.887	6.990	8.717	11.58	16.60	25.92	43.08	53.07
0.2	3.980	4.626	5.270	6.093	7.257	8.926	11.18	13.57	13.40	5.404
0.3	3.472	3.963	4.416	4.917	5.502	6.121	6.536	6.135	4.058	0.981
0.4	2.890	3.221	3.496	3.734	3.910	3.936	3.634	2.791	1.440	0.279
0.5	2.306	2.500	2.639	2.705	2.665	2.461	2.026	1.353	0.605	0.108
0.6	1.775	1.865	1.912	1.888	1.767	1.523	1.153	0.702	0.290	0.051
0.7	1.323	1.343	1.336	1.276	1.143	0.935	0.666	0.381	0.150	0.027
0.8	0.959	0.936	0.902	0.833	0.720	0.563	0.382	0.209	0.079	0.014
0.9	0.671	0.632	0.587	0.523	0.436	0.327	0.212	0.109	0.039	0.007
1.0	0.466	0.412	0.365	0.312	0.248	0.177	0.107	0.050	0.015	0.002
1.25	0.163	0.116	0.080	0.050	0.025	0.007	-0.007	-0.012	-0.010	-0.003
1.5	0.045	0.016	-0.007	-0.022	-0.031	-0.004	-0.031	-0.023	-0.013	-0.004
1.75	0.009	-0.007	-0.020	-0.028	-0.031	-0.030	-0.025	-0.017	-0.009	-0.002
2.0	0.003	-0.006	-0.013	-0.017	-0.018	-0.017	-0.014	-0.009	-0.005	0.001
2.5	0.007	0.005	0.003	0.001	0.001	-0.000	0.000	0.001	0.001	0.002
3.0	0.008	0.007	0.006	0.005	0.005	0.004	0.003	0.003	0.003	0.002
3.5	0.006	0.006	0.005	0.004	0.004	0.003	0.003	0.002	0.002	0.002
4.0	0.004	0.003	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001
6.0	0.000	0.000	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

r/h \ z/h	I_{σ_z} for Poisson's Ratio ν = 0.5									
	0.1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.695	5.603	6.457	7.631	9.542	12.91	19.39	33.75	75.26	300.0
0.1	4.556	5.417	6.215	7.279	8.951	11.76	16.73	26.01	43.17	53.17
0.2	4.171	4.905	5.554	6.345	7.463	9.080	11.29	13.64	13.44	5.438
0.3	3.612	4.175	4.638	5.118	5.666	6.241	6.610	6.167	4.053	0.947
0.4	2.974	3.359	3.649	3.876	4.027	4.019	3.679	2.795	1.403	0.205
0.5	2.341	2.571	2.726	2.791	2.738	2.512	2.049	1.344	0.561	0.032
0.6	1.770	1.882	1.945	1.927	1.803	1.549	1.162	0.692	0.257	-0.001
0.7	1.291	1.321	1.329	1.279	1.152	0.943	0.669	0.378	0.139	0.010
0.8	0.911	0.891	0.869	0.813	0.710	0.561	0.385	0.215	0.089	0.029
0.9	0.623	0.576	0.540	0.490	0.415	0.319	0.216	0.124	0.064	0.042
1.0	0.412	0.355	0.314	0.272	0.223	0.166	0.111	0.069	0.047	0.046
1.25	0.123	0.072	0.037	0.015	0.000	-0.007	-0.007	0.000	0.014	0.029
1.5	0.023	-0.011	-0.034	-0.048	-0.052	-0.048	-0.039	-0.025	-0.010	0.003
1.75	-0.003	-0.023	-0.037	-0.046	-0.048	-0.046	-0.039	-0.030	-0.021	-0.013
2.0	-0.005	-0.016	-0.024	-0.030	-0.032	-0.033	-0.030	-0.026	-0.021	-0.018
2.5	0.004	-0.012	-0.002	-0.005	-0.007	-0.008	-0.009	-0.010	-0.010	-0.011
3.0	0.008	0.007	0.005	-0.003	0.002	0.001	-0.001	-0.002	-0.003	-0.003
3.5	0.008	0.007	0.006	0.005	0.004	0.003	0.002	0.001	0.001	0.000
4.0	0.006	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002
6.0	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

TABLE 2.2

INFLUENCE FACTORS I_{σ_r} FOR RADIAL STRESS σ_r

POINT LOAD

$$\theta_r = I_{\sigma_r} P/2\pi h^2$$

r/h \ z/h	I_{σ_r} for Poisson's Ratio $\nu = 0.0$: 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	0	-0.444	-0.795	-1.150	-1.598	-2.273	-3.460	-5.960	-12.99	-50.61
0.1	0	-0.431	-0.718	-1.014	-1.350	-1.784	-2.308	-2.686	-0.311	23.14
0.2	0	-0.301	-0.516	-0.673	-0.767	-0.729	0.273	1.325	5.468	7.069
0.3	0	-0.167	-0.263	-0.275	-0.161	0.173	0.937	2.277	3.474	0.401
0.4	0	-0.038	-0.030	0.056	0.264	0.636	1.212	1.776	1.509	-1.197
0.5	0	0.062	0.140	0.269	0.475	0.750	1.043	1.099	0.438	-1.473
0.6	0	0.125	0.238	0.370	0.529	0.683	0.765	0.589	-0.077	-1.395
0.7	0	0.155	0.278	0.391	0.494	0.553	0.514	0.261	-0.307	-1.228
0.8	0	0.159	0.276	0.364	0.421	0.418	0.321	0.065	-0.394	-1.050
0.9	0	0.148	0.250	0.315	0.339	0.302	0.185	-0.048	-0.409	-0.885
1.0	0	0.128	0.213	0.258	0.261	0.209	0.091	-0.108	-0.390	-0.740
1.25	0	0.075	0.119	0.133	0.115	0.064	-0.024	-0.146	-0.296	-0.463
1.5	0	0.036	0.054	0.055	0.036	0.001	-0.054	-0.124	-0.204	-0.285
1.75	0	0.015	0.019	0.041	-0.001	-0.023	-0.055	-0.093	-0.135	-0.175
2.0	0	0.004	0.003	-0.003	-0.015	-0.028	-0.046	-0.066	-0.086	-0.104
2.5	0	-0.003	-0.006	-0.010	-0.015	-0.021	-0.026	-0.032	-0.036	-0.039
3.0	0	-0.003	-0.005	-0.007	-0.010	-0.012	-0.014	-0.015	-0.015	-0.015
3.5	0	-0.002	-0.003	-0.004	-0.005	-0.006	-0.007	-0.007	-0.007	-0.006
4.0	0	-0.001	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
6.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

r/h \ z/h	I_{σ_r} for Poisson's Ratio $\nu = 0.2$: 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	1.090	0.407	-0.065	-0.452	-0.845	-1.347	-2.143	-3.725	-8.033	-30.71
0.1	1.061	0.431	0.002	-0.327	-0.613	-0.878	-1.054	-0.627	3.865	34.75
0.2	0.980	0.488	0.172	-0.156	-0.677	0.112	0.843	3.011	8.351	12.50
0.3	0.863	0.552	0.376	0.338	0.484	0.926	1.884	3.586	5.412	3.347
0.4	0.726	0.595	0.547	0.615	0.845	1.296	2.000	2.785	2.855	0.614
0.5	0.589	0.604	0.649	0.768	0.990	1.320	1.695	1.885	1.413	-0.259
0.6	0.462	0.576	0.678	0.808	0.980	1.173	1.306	1.213	0.657	-0.528
0.7	0.352	0.522	0.650	0.769	0.885	0.972	0.966	0.766	0.264	-0.578
0.8	0.263	0.452	0.586	0.686	0.756	0.775	0.701	0.479	0.063	-0.544
0.9	0.192	0.378	0.504	0.586	0.624	0.605	0.505	0.297	-0.036	-0.479
1.0	0.137	0.307	0.420	0.485	0.503	0.466	0.363	0.182	-0.080	-0.292
1.25	0.054	0.282	0.266	0.291	0.263	0.226	0.176	0.087	-0.089	-0.181
1.5	0.017	0.080	0.121	0.140	0.136	0.112	0.067	0.008	-0.062	-0.134
1.75	0.008	0.053	0.067	0.074	0.069	0.051	0.031	-0.002	-0.038	-0.069
2.0	0.021	0.011	0.021	0.025	0.025	0.019	0.023	-0.002	-0.018	-0.031
2.5	-0.002	0.000	0.002	0.003	0.003	0.002	-0.002	-0.001	-0.003	-0.003
3.0	-0.001	-0.001	-0.000	-0.000	-0.000	-0.000	0.000	0.001	0.001	0.002
3.5	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.001	0.001	0.002
4.0	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
6.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

r/h \ z/h	I_{σ_r} for Poisson's Ratio $\nu = 0.4$: 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	2.969	1.785	1.024	0.492	0.073	-0.318	-0.775	-1.487	-3.121	-10.92
0.1	2.836	1.777	1.071	0.601	0.288	0.125	0.257	1.442	8.002	46.27
0.2	2.653	1.753	1.192	0.875	0.792	1.049	2.014	4.709	11.20	17.83
0.3	2.315	1.695	1.320	1.172	1.283	1.774	2.888	4.912	7.322	6.214
0.4	1.926	1.590	1.397	1.377	1.573	2.048	2.845	3.814	4.180	2.359
0.5	1.537	1.441	1.411	1.449	1.641	1.980	2.405	2.696	2.377	0.905
0.6	1.183	1.261	1.314	1.404	1.554	1.747	1.906	1.866	1.390	0.306
0.7	0.882	1.066	1.182	1.282	1.384	1.469	1.475	1.303	0.844	0.056
0.8	0.639	0.876	1.022	1.121	1.186	1.204	1.136	0.929	0.537	-0.037
0.9	0.451	0.701	0.856	0.949	0.990	0.973	0.878	0.679	0.362	-0.058
1.0	0.311	0.549	0.700	0.784	0.812	0.781	0.684	0.511	0.262	-0.045
1.25	0.108	0.278	0.391	0.453	0.470	0.445	0.381	0.282	0.158	0.023
1.5	0.030	0.131	0.204	0.247	0.263	0.254	0.224	0.179	0.125	0.073
1.75	0.006	0.061	0.104	0.133	0.147	0.148	0.140	0.124	0.107	0.092
2.0	0.002	0.031	0.055	0.073	0.085	0.091	0.092	0.091	0.089	0.090
2.5	0.005	0.013	0.021	0.029	0.036	0.042	0.047	0.052	0.057	0.062
3.0	0.006	0.009	0.013	0.016	0.020	0.024	0.027	0.031	0.034	0.037
3.5	0.004	0.006	0.008	0.010	0.008	0.014	0.016	0.018	0.020	0.021
4.0	0.003	0.004	0.005	0.007	0.008	0.009	0.010	0.011	0.011	0.012
6.0	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

r/h \ z/h	I_{σ_r} for Poisson's Ratio $\nu = 0.5$: 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.695	2.981	1.902	1.184	0.672	0.274	-0.067	-0.393	-0.733	-1.101
0.1	4.556	2.947	1.937	1.288	0.884	0.711	0.947	2.464	10.02	51.95
0.2	4.171	2.846	2.018	1.537	1.366	1.606	2.641	5.556	12.58	20.42
0.3	3.612	2.672	2.081	1.793	1.824	2.289	3.437	5.581	8.242	7.561
0.4	2.974	2.429	2.077	1.946	2.071	2.515	3.321	4.344	4.817	3.146
0.5	2.341	2.133	1.983	1.957	2.092	2.397	2.815	3.122	2.841	1.412
0.6	1.770	1.812	1.801	1.847	1.953	2.115	2.259	2.216	1.737	0.743
0.7	1.291	1.492	1.589	1.660	1.732	1.791	1.780	1.597	1.134	0.547
0.8	0.911	1.195	1.349	1.437	1.485	1.484	1.400	1.182	0.785	0.412
0.9	0.623	0.934	1.113	1.210	1.244	1.216	1.110	0.903	0.585	0.320
1.0	0.412	0.715	0.899	0.997	1.026	0.991	0.888	0.712	0.467	0.256
1.25	0.123	0.344	0.492	0.578	0.608	0.593	0.535	0.444	0.331	0.173
1.5	0.023	0.156	0.258	0.313	0.356	0.355	0.342	0.310	0.269	0.135
1.75	-0.003	0.104	0.139	0.203	0.227	0.234	0.240	0.238	0.216	0.102
2.0	-0.005	0.039	0.078	0.110	0.134	0.152	0.163	0.171	0.176	0.082
2.5	0.004	0.019	0.035	0.049	0.063	0.076	0.087	0.098	0.107	0.041
3.0	0.008	0.015	0.023	0.037	0.038	0.045	0.053	0.059	0.065	0.019
3.5	0.008	0.012	0.016	0.021	0.025	0.029	0.034	0.037	0.040	0.008
4.0	0.006	0.009	0.012	0.015	0.017	0.020	0.022	0.024	0.026	0.028
6.0	0.001	0.002	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.005
8.0	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 2.3

INFLUENCE FACTORS I_{σ_θ} FOR TANGENTIAL STRESS σ_θ

POINT LOAD

$$\sigma_\theta = I_{\sigma_\theta} \; P/2\pi h^2$$

r/h \ z/h	I_{σ_θ} for Poisson's Ratio $\nu = 0.0$									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	0	-0.444	-0.795	-1.150	-1.598	-2.273	-3.460	-5.960	-12.99	-50.61
0.1	0	-0.431	-0.769	-1.104	-1.513	-2.101	-3.054	-4.758	-7.822	-6.674
0.2	0	-0.394	-0.697	-0.970	-1.293	-1.681	-2.161	-2.600	-2.002	4.276
0.3	0	-0.341	-0.595	-0.810	-1.014	-1.200	-1.299	-1.064	0.206	3.849
0.4	0	-0.281	-0.482	-0.633	-0.743	-0.788	-0.691	-0.279	0.757	2.738
0.5	0	-0.222	-0.373	-0.472	-0.517	-0.486	-0.324	0.062	0.788	1.913
0.6	0	-0.169	-0.279	-0.339	-0.346	-0.283	-0.119	0.190	0.681	1.353
0.7	0	-0.124	-0.201	-0.235	-0.223	-0.154	-0.011	0.221	0.554	0.972
0.8	0	-0.089	-0.141	-0.158	-0.137	-0.073	0.042	0.213	0.440	0.708
0.9	0	-0.062	-0.096	-0.103	-0.080	-0.025	0.065	0.190	0.346	0.522
1.0	0	-0.042	-0.064	-0.064	-0.042	0.003	0.072	0.162	0.271	0.388
1.25	0	-0.014	-0.018	-0.013	0.003	0.028	0.062	0.103	0.148	0.192
1.5	0	-0.002	-0.001	0.004	0.014	0.027	0.044	0.063	0.081	0.099
1.75	0	0.001	0.004	0.008	0.014	0.021	0.029	0.038	0.046	0.052
2.0	0	0.002	0.005	0.008	0.011	0.015	0.019	0.023	0.026	0.028
2.5	0	0.001	0.003	0.004	0.005	0.007	0.008	0.008	0.009	0.009
3.0	0	0.001	0.001	0.002	0.002	0.003	0.003	0.003	0.003	0.003
3.5	0	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4.0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

r/h \ z/h	I_{σ_θ} for Poisson's Ratio $\nu = 0.2$									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	1.089	0.407	-0.065	-0.452	-0.845	-1.347	-2.143	-3.725	-8.033	-30.71
0.1	1.060	0.400	-0.057	-0.429	-0.797	-1.245	-1.900	-3.003	-4.929	-4.342
0.2	0.979	0.378	-0.039	-0.368	-0.672	-0.996	-1.365	-1.705	-1.430	2.237
0.3	0.861	0.344	-0.014	-0.287	-0.516	-0.712	-0.847	-0.779	-0.097	1.996
0.4	0.725	0.302	0.009	-0.207	-0.367	-0.471	-0.482	-0.303	0.246	1.349
0.5	0.587	0.257	0.025	-0.138	-0.247	-0.296	-0.261	-0.090	0.279	0.877
0.6	0.460	0.211	0.035	-0.086	-0.159	-0.181	-0.136	-0.006	0.231	0.566
0.7	0.351	0.169	0.038	-0.050	-0.100	-0.108	-0.070	0.023	0.171	0.363
0.8	0.261	0.131	0.036	-0.027	-0.061	-0.064	-0.035	0.027	0.119	0.231
0.9	0.190	0.099	0.031	-0.014	-0.037	-0.039	-0.018	0.023	0.079	0.144
1.0	0.136	0.073	0.025	-0.007	-0.023	-0.024	-0.010	0.016	0.050	0.087
1.25	0.053	0.030	0.011	-0.002	-0.008	-0.010	-0.006	0.002	0.011	0.018
1.5	0.016	0.009	0.002	-0.003	-0.005	-0.006	-0.005	-0.004	-0.002	-0.002
1.75	0.002	0.002	-0.002	-0.003	-0.004	-0.005	-0.005	-0.005	-0.005	-0.006
2.0	-0.002	-0.002	-0.003	-0.003	-0.003	-0.004	-0.004	-0.004	-0.004	-0.005
2.5	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
3.0	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.001
3.5	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
6.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

r/h \ z/h	I_{σ_θ} for Poisson's Ratio $\nu = 0.4$									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	2.970	1.785	1.024	0.492	0.073	-0.318	-0.775	-1.487	-3.121	-10.91
0.1	2.886	1.742	1.004	0.487	0.082	-0.287	-0.695	-1.245	-2.082	-2.115
0.2	2.653	1.621	0.945	0.470	0.103	-0.214	-0.518	-0.807	-0.903	0.099
0.3	2.315	1.440	0.855	0.440	0.123	-0.134	-0.348	-0.489	-0.439	0.052
0.4	1.926	1.227	0.745	0.397	0.134	-0.072	-0.228	-0.319	-0.298	-0.120
0.5	1.538	1.006	0.625	0.345	0.131	-0.033	-0.156	-0.234	-0.256	-0.226
0.6	1.183	0.796	0.507	0.288	0.118	-0.014	-0.116	-0.189	-0.238	-0.273
0.7	0.882	0.610	0.398	0.231	0.099	-0.007	-0.093	-0.162	-0.221	-0.283
0.8	0.640	0.454	0.303	0.179	0.077	-0.007	-0.079	-0.142	-0.202	-0.269
0.9	0.452	0.329	0.223	0.133	0.056	-0.010	-0.069	-0.124	-0.180	-0.244
1.0	0.311	0.232	0.160	0.096	0.038	-0.014	-0.061	-0.107	-0.156	-0.212
1.25	0.108	0.086	0.061	0.035	0.009	-0.017	-0.043	-0.070	-0.099	-0.132
1.5	0.030	0.026	0.018	0.009	-0.002	-0.014	-0.026	-0.039	-0.054	-0.070
1.75	0.006	0.006	0.005	0.002	-0.002	-0.007	-0.012	-0.018	-0.024	-0.031
2.0	0.002	0.003	0.003	0.002	0.001	-0.001	-0.003	-0.005	-0.007	-0.009
2.5	0.005	0.005	0.006	0.006	0.006	0.006	0.006	0.006	0.007	0.007
3.0	0.006	0.006	0.006	0.006	0.007	0.007	0.007	0.007	0.008	0.008
3.5	0.004	0.004	0.005	0.005	0.005	0.005	0.006	0.006	0.006	0.006
4.0	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004	0.004	0.004
6.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

r/h \ z/h	I_{σ_θ} for Poisson's Ratio $\nu = 0.5$									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.695	2.981	1.902	1.184	0.672	0.274	-0.067	-0.393	-0.733	-1.101
0.1	4.557	2.905	1.859	1.159	0.658	0.267	-0.069	-0.391	-0.729	-1.100
0.2	4.171	2.690	1.736	1.088	0.618	0.248	-0.074	-0.385	-0.716	-1.089
0.3	3.613	2.373	1.552	0.980	0.557	0.218	-0.080	-0.373	-0.688	-1.056
0.4	2.975	2.001	1.330	0.849	0.482	0.183	-0.086	-0.353	-0.644	-0.991
0.5	2.341	1.618	1.096	0.707	0.401	0.145	-0.090	-0.326	-0.586	-0.896
0.6	1.770	1.259	0.871	0.568	0.321	0.108	-0.092	-0.294	-0.517	-0.780
0.7	1.291	0.946	0.668	0.440	0.247	0.074	-0.091	-0.259	-0.443	-0.658
0.8	0.912	0.688	0.496	0.330	0.182	0.045	-0.087	-0.223	-0.371	-0.540
0.9	0.623	0.485	0.356	0.238	0.128	0.023	-0.082	-0.189	-0.305	-0.434
1.0	0.413	0.331	0.247	0.165	0.085	0.005	-0.075	-0.158	-0.247	-0.343
1.25	0.124	0.107	0.083	0.053	0.019	-0.017	-0.056	-0.096	-0.138	-0.181
1.5	0.023	0.023	0.017	0.008	-0.005	-0.020	-0.037	-0.054	-0.072	-0.089
1.75	-0.003	-0.001	-0.002	-0.005	-0.009	-0.014	-0.021	-0.027	-0.034	-0.040
2.0	-0.004	-0.003	-0.003	-0.004	-0.005	-0.007	-0.009	-0.011	-0.012	-0.013
2.5	0.005	0.005	0.004	0.004	0.004	0.005	0.005	0.005	0.006	0.007
3.0	0.009	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.010	0.011
3.5	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.010
4.0	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.008	0.008
6.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
8.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 2.4

INFLUENCE FACTORS I_θ FOR BULK STRESS Θ

POINT LOAD

$$\Theta = I_\theta P/2\pi h^2$$

I_θ for Poisson's Ratio $\nu = 0.0$

r/h \ z/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	4.422	4.105	4.188	4.705	5.821	7.958	12.18	21.63	49.12	198.8
0.1	4.308	4.008	4.084	4.563	5.586	7.495	11.09	18.38	34.90	69.54
0.2	3.989	3.736	3.793	4.174	4.963	6.337	8.614	12.21	16.82	16.74
0.3	3.523	3.333	3.368	3.628	4.142	4.951	6.063	7.280	7.708	5.229
0.4	2.982	2.859	2.874	3.024	3.303	3.683	4.075	4.238	3.685	1.822
0.5	2.432	2.368	2.372	2.440	2.552	2.667	2.696	2.484	1.820	0.552
0.6	1.923	1.903	1.902	1.917	1.930	1.904	1.778	1.469	0.891	0.015
0.7	1.481	1.490	1.489	1.475	1.436	1.347	1.171	0.867	0.402	-0.221
0.8	1.114	1.140	1.140	1.140	1.054	0.945	0.767	0.500	0.135	-0.319
0.9	0.821	0.854	0.856	0.827	0.762	0.655	0.494	0.273	-0.011	-0.348
1.0	0.593	0.627	0.630	0.602	0.542	0.446	0.309	0.130	-0.090	-0.342
1.25	0.238	0.262	0.266	0.247	0.208	0.147	0.066	-0.033	-0.147	-0.271
1.5	0.074	0.088	0.089	0.079	0.056	0.022	-0.022	-0.074	-0.132	-0.192
1.75	0.007	0.013	0.013	0.006	-0.006	-0.024	-0.047	-0.073	-0.101	-0.130
2.0	-0.017	-0.015	-0.016	-0.020	-0.027	-0.036	-0.047	-0.060	-0.073	-0.086
2.5	-0.018	-0.019	-0.021	-0.022	-0.025	-0.027	-0.029	-0.032	-0.034	-0.036
3.0	-0.010	-0.011	-0.011	-0.012	-0.013	-0.014	-0.014	-0.014	-0.014	-0.014
3.5	-0.003	-0.004	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.004
4.0	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
6.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

I_θ for Poisson's Ratio $\nu = 0.2$

r/h \ z/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	6.537	5.834	5.724	6.184	7.405	9.875	14.86	26.13	59.06	238.6
0.1	6.364	5.705	5.584	6.005	7.115	9.315	13.55	22.23	42.00	83.49
0.2	5.879	5.314	5.193	5.511	6.350	7.914	10.58	14.83	20.30	20.15
0.3	5.171	4.736	4.621	4.814	5.337	6.231	7.502	8.905	9.365	6.337
0.4	4.353	4.055	3.954	4.039	4.295	4.687	5.102	5.249	4.538	2.256
0.5	3.527	3.351	3.271	3.281	3.356	3.442	3.342	3.136	2.300	0.742
0.6	2.766	2.685	2.630	2.599	2.570	2.500	2.313	1.909	1.185	0.105
0.7	2.110	2.096	2.064	2.017	1.940	1.805	1.567	1.177	0.597	-0.174
0.8	1.572	1.598	1.585	1.537	1.447	1.297	1.065	0.728	0.274	-0.287
0.9	1.146	1.193	1.194	1.153	1.067	0.927	0.723	0.445	0.094	0.320
1.0	0.818	0.874	0.884	0.852	0.777	0.656	0.487	0.265	-0.005	-0.197
1.25	0.319	0.483	0.412	0.388	0.320	0.253	0.184	0.188	-0.085	-0.169
1.5	0.100	0.131	0.144	0.141	0.122	0.087	0.040	-0.017	-0.081	-0.148
1.75	0.020	0.050	0.051	0.049	0.038	0.018	0.001	-0.029	-0.059	-0.087
2.0	-0.011	-0.005	-0.001	-0.000	-0.002	-0.007	-0.001	-0.022	-0.035	-0.046
2.5	-0.011	-0.011	-0.009	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
3.0	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.000	0.001	0.002	0.003
3.5	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005
4.0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.004
6.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

I_θ for Poisson's Ratio $\nu = 0.4$

r/h \ z/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	10.39	8.826	8.158	8.309	9.438	12.08	17.70	30.66	68.90	278.2
0.1	10.10	8.610	7.961	8.077	9.086	11.42	16.17	26.11	49.00	97.23
0.2	9.286	8.000	7.406	7.438	8.153	9.761	12.68	17.47	23.69	23.34
0.3	8.102	7.098	6.592	6.529	6.909	7.761	9.075	10.56	10.94	7.248
0.4	6.742	6.038	5.638	5.507	5.617	5.911	6.251	6.286	5.321	2.518
0.5	5.381	4.947	4.675	4.499	4.438	4.407	4.275	3.816	2.726	0.788
0.6	4.142	3.921	3.733	3.580	3.438	3.256	2.943	2.379	1.442	0.084
0.7	3.088	3.019	2.916	2.788	2.626	2.396	2.048	1.522	0.773	-0.200
0.8	2.238	2.266	2.226	2.133	1.983	1.760	1.439	0.995	0.414	-0.292
0.9	1.580	1.662	1.166	1.605	1.482	1.290	1.021	0.665	0.221	-0.295
1.0	1.087	1.194	1.225	1.191	1.098	0.945	0.730	0.454	0.121	-0.255
1.25	0.379	0.479	0.531	0.538	0.504	0.434	0.330	0.120	0.049	-0.112
1.5	0.106	0.173	0.215	0.234	0.230	0.207	0.170	0.116	0.058	-0.001
1.75	0.022	0.061	0.089	0.106	0.113	0.112	0.103	0.089	0.073	0.059
2.0	0.007	0.028	0.045	0.059	0.068	0.073	0.076	0.077	0.078	0.081
2.5	0.017	0.023	0.030	0.036	0.042	0.048	0.053	0.059	0.065	0.071
3.0	0.019	0.022	0.025	0.028	0.031	0.034	0.038	0.041	0.044	0.047
3.5	0.015	0.016	0.018	0.020	0.021	0.023	0.025	0.026	0.028	0.029
4.0	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.017
6.0	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002
8.0	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000	0.000
10.0	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

I_θ for Poisson's Ratio $\nu = 0.5$

r/h \ z/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	14.08	11.56	10.26	9.999	10.88	13.46	19.26	32.97	73.79	298.0
0.1	13.67	11.27	10.01	9.726	10.49	12.74	17.61	28.09	53.45	104.0
0.2	12.52	10.44	9.308	8.970	9.447	10.93	13.86	18.81	25.30	24.77
0.3	10.84	9.219	8.271	7.891	8.047	8.748	9.968	11.38	11.61	7.452
0.4	8.923	7.789	7.056	6.670	6.581	6.716	6.914	6.786	5.576	2.360
0.5	7.022	6.322	5.805	5.455	5.231	5.053	4.773	4.140	2.816	0.548
0.6	5.310	4.953	4.627	4.342	4.077	3.771	3.329	2.614	1.487	-0.038
0.7	3.873	3.759	3.587	3.379	3.131	2.808	2.358	1.716	0.829	-0.101
0.8	2.734	2.774	2.714	2.580	2.376	2.091	1.698	1.173	0.503	-0.099
0.9	1.869	1.995	2.010	1.937	1.787	1.558	1.243	0.837	0.343	-0.072
1.0	1.237	1.401	1.460	1.434	1.333	1.163	0.925	0.623	0.267	-0.040
1.25	0.370	0.523	0.612	0.645	0.627	0.568	0.476	0.348	0.208	0.021
1.5	0.068	0.169	0.217	0.273	0.305	0.287	0.267	0.231	0.187	0.048
1.75	-0.010	0.072	0.108	0.153	0.170	0.174	0.182	0.180	0.162	0.049
2.0	-0.013	0.021	0.051	0.076	0.097	0.113	0.125	0.135	0.143	0.051
2.5	0.013	0.025	0.037	0.049	0.061	0.072	0.083	0.093	0.103	0.038
3.0	0.024	0.030	0.036	0.042	0.048	0.054	0.061	0.067	0.073	0.026
3.5	0.024	0.027	0.030	0.034	0.037	0.041	0.044	0.047	0.050	0.018
4.0	0.019	0.021	0.023	0.025	0.028	0.030	0.032	0.033	0.035	0.012
6.0	0.005	0.006	0.006	0.007	0.007	0.007	0.008	0.008	0.008	0.008
8.0	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002
10.0	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 2.5

INFLUENCE FACTORS $I_{\tau_{rz}}$ FOR SHEAR STRESS τ_{rz}

POINT LOAD

$$\tau_{rz} = I_{\tau_{rz}} \; P/2\pi h^2$$

z/h → r/h ↓	$I_{\tau_{rz}}$ for Poisson's Ratio $\nu = 0.0$									
r/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0	0	0	0	0	0	0	0	0	0	0
0.1	0.258	0.312	0.462	0.731	1.204	2.093	3.957	8.480	21.42	53.01
0.2	0.473	0.564	0.814	1.246	1.957	3.154	5.230	8.750	13.18	10.69
0.3	0.616	0.719	0.999	1.456	2.138	3.116	4.416	5.735	5.791	2.781
0.4	0.678	0.774	1.025	1.408	1.916	2.517	3.080	3.262	2.540	0.928
0.5	0.671	0.747	0.939	1.211	1.527	1.820	1.966	1.785	1.164	0.347
0.6	0.614	0.668	0.796	0.963	1.128	1.232	1.199	0.962	0.541	0.124
0.7	0.529	0.564	0.638	0.725	0.791	0.797	0.706	0.505	0.294	0.027
0.8	0.435	0.455	0.490	0.523	0.532	0.495	0.398	0.249	0.086	-0.016
0.9	0.344	0.353	0.363	0.365	0.345	0.294	0.209	0.104	0.008	-0.035
1.0	0.262	0.265	0.261	0.246	0.214	0.163	0.095	0.023	-0.031	-0.041
1.25	0.112	0.111	0.098	0.075	0.045	0.011	-0.022	-0.046	-0.052	-0.032
1.5	0.032	0.032	0.023	0.009	-0.009	-0.026	-0.038	-0.044	-0.036	-0.014
1.75	-0.004	-0.002	-0.006	-0.012	-0.020	-0.026	-0.029	-0.026	-0.017	0.001
2.0	-0.016	-0.013	-0.013	-0.014	-0.016	-0.017	-0.015	-0.010	-0.002	0.011
2.5	-0.014	-0.011	-0.008	-0.006	-0.003	-0.000	0.004	0.009	0.015	0.021
3.0	-0.007	-0.004	-0.001	0.002	0.005	0.008	0.012	0.015	0.019	0.023
3.5	-0.002	0.001	0.003	0.006	0.008	0.011	0.013	0.017	0.019	0.022
4.0	0.001	0.003	0.005	0.007	0.009	0.011	0.014	0.016	0.018	0.020
6.0	0.001	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010	0.011
8.0	0.001	0.001	0.002	0.002	0.003	0.004	0.004	0.005	0.005	0.006
10.0	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003

z/h → r/h ↓	$I_{\tau_{rz}}$ for Poisson's Ratio $\nu = 0.2$									
r/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0	0	0	0	0	0	0	0	0	0	0
0.1	0.330	0.342	0.472	0.731	1.199	2.086	3.949	8.472	21.42	53.00
0.2	0.607	0.623	0.835	1.248	1.949	3.141	5.215	8.734	13.16	10.68
0.3	0.797	0.804	1.032	1.462	2.129	3.099	4.395	5.713	5.771	2.765
0.4	0.889	0.880	1.071	1.420	1.909	2.499	3.056	3.237	2.516	0.909
0.5	0.896	0.868	0.996	1.231	1.524	1.804	1.943	1.758	1.138	0.326
0.6	0.839	0.798	0.864	0.992	1.131	1.220	1.178	0.937	0.515	0.102
0.7	0.746	0.697	0.713	0.761	0.801	0.790	0.689	0.482	0.215	0.005
0.8	0.636	0.586	0.570	0.566	0.550	0.495	0.386	0.229	0.064	-0.038
0.9	0.526	0.480	0.446	0.413	0.368	0.299	0.202	0.088	-0.012	-0.056
1.0	0.424	0.384	0.343	0.297	0.242	0.173	0.093	0.012	-0.048	-0.061
1.25	0.229	0.206	0.170	0.127	0.079	0.031	-0.015	-0.049	-0.063	-0.049
1.5	0.114	0.103	0.082	0.054	0.024	-0.005	-0.029	-0.043	-0.045	-0.031
1.75	0.054	0.050	0.039	0.024	0.007	-0.008	-0.020	-0.027	-0.026	-0.017
2.0	0.025	0.024	0.019	0.012	0.004	-0.004	-0.010	-0.013	-0.012	-0.008
2.5	0.007	0.007	0.007	0.006	0.004	0.003	0.001	0.000	-0.000	-0.000
3.0	0.004	0.005	0.005	0.005	0.005	0.004	0.004	0.003	0.002	0.001
3.5	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.002	0.001
4.0	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.002	0.002	0.001
6.0	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
8.0	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

z/h → r/h ↓	$I_{\tau_{rz}}$ for Poisson's Ratio $\nu = 0.4$									
r/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	0	0	0	0	0	0	0	0	0	0
0.1	0.436	0.375	0.470	0.713	1.175	2.060	3.923	8.448	21.40	52.99
0.2	0.806	0.687	0.834	1.216	1.904	3.091	5.165	8.689	13.12	10.65
0.3	1.064	0.896	1.037	1.423	2.069	3.032	4.327	5.649	5.717	2.727
0.4	1.195	0.995	1.085	1.381	1.843	2.421	2.976	3.161	2.450	0.862
0.5	1.214	1.001	1.024	1.198	1.459	1.723	1.856	1.675	1.065	0.273
0.6	1.149	0.941	0.904	0.969	1.073	1.142	1.091	0.851	0.439	0.045
0.7	1.033	0.843	0.765	0.752	0.753	0.720	0.607	0.391	0.139	-0.052
0.8	0.893	0.730	0.631	0.570	0.514	0.435	0.313	0.152	-0.008	-0.094
0.9	0.750	0.616	0.512	0.428	0.346	0.252	0.140	0.020	-0.078	-0.109
1.0	0.616	0.510	0.412	0.322	0.232	0.139	0.042	-0.047	-0.107	-0.110
1.25	0.352	0.301	0.235	0.164	0.091	0.022	-0.038	-0.082	-0.102	-0.088
1.5	0.191	0.170	0.135	0.091	0.045	0.002	-0.034	-0.059	-0.068	-0.060
1.75	0.102	0.096	0.079	0.055	0.030	0.004	-0.017	-0.032	-0.040	-0.040
2.0	0.056	0.055	0.048	0.036	0.022	0.008	-0.005	-0.016	-0.023	-0.028
2.5	0.020	0.020	0.019	0.016	0.012	0.006	0.001	-0.006	-0.012	-0.019
3.0	0.009	0.008	0.007	0.006	0.004	0.001	-0.003	-0.007	-0.011	-0.016
3.5	0.003	0.002	0.001	0.000	-0.001	-0.003	-0.006	-0.008	-0.011	-0.015
4.0	-0.000	-0.001	-0.002	-0.003	-0.004	-0.006	-0.007	-0.009	-0.011	-0.013
6.0	-0.004	-0.004	-0.005	-0.005	-0.005	-0.006	-0.006	-0.007	-0.007	-0.008
8.0	-0.003	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.005	-0.005
10.0	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003

z/h → r/h ↓	$I_{\tau_{rz}}$ for Poisson's Ratio $\nu = 0.5$									
r/h	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	0	0	0	0	0	0	0	0	0	0
0.1	0.530	0.395	0.459	0.690	1.148	2.035	3.905	8.440	21.40	53.01
0.2	0.980	0.728	0.815	1.173	1.853	3.043	5.128	8.668	13.13	10.67
0.3	1.294	0.957	1.017	1.367	2.001	2.965	4.270	5.610	5.702	2.739
0.4	1.456	1.074	1.071	1.322	1.766	2.340	2.901	3.101	2.412	0.851
0.5	1.481	1.094	1.020	1.144	1.380	1.636	1.771	1.599	1.007	0.241
0.6	1.406	1.044	0.915	0.927	1.002	1.057	1.004	0.769	0.372	0.007
0.7	1.268	0.952	0.791	0.726	0.696	0.646	0.527	0.322	0.075	-0.087
0.8	1.102	0.841	0.671	0.562	0.476	0.378	0.249	0.089	-0.059	-0.118
0.9	0.932	0.725	0.565	0.438	0.328	0.216	0.096	-0.023	-0.110	-0.119
1.0	0.773	0.616	0.474	0.348	0.234	0.124	0.020	-0.068	-0.119	-0.105
1.25	0.463	0.394	0.309	0.219	0.132	0.053	-0.013	-0.058	-0.075	-0.057
1.5	0.274	0.251	0.209	0.158	0.105	0.056	0.015	-0.012	-0.024	-0.020
1.75	0.169	0.164	0.146	0.120	0.091	0.063	0.038	0.019	0.006	0.001
2.0	0.111	0.113	0.106	0.094	0.078	0.062	0.046	0.032	0.020	0.009
2.5	0.059	0.061	0.060	0.057	0.053	0.046	0.039	0.030	0.021	0.011
3.0	0.037	0.038	0.037	0.036	0.033	0.030	0.026	0.021	0.015	0.008
3.5	0.025	0.025	0.024	0.023	0.021	0.019	0.016	0.013	0.009	0.005
4.0	0.017	0.017	0.016	0.015	0.014	0.012	0.010	0.008	0.006	0.003
6.0	0.004	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.001	0.001
8.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000
10.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 2.6
INFLUENCE VALUES FOR SURFACE DISPLACEMENTS
POINT LOAD
$\rho = I_\rho P/2\pi hE$

VERTICAL DISPLACEMENT ρ_z

(Taylor, 1962)

r/h \ ν	0	0.2	0.4	0.5
0.05	37.580	35.921	31.052	27.351
0.1	17.586	16.728	14.260	13.360
0.2	7.624	7.162	5.897	4.914
0.3	4.327	4.016	3.154	2.480
0.4	2.720	2.478	1.827	1.320
0.5	1.792	1.599	1.092	0.699
0.6	1.212	1.048	0.635	0.290
0.7	0.823	0.690	0.352	0.051
0.8	0.560	0.450	0.168	-0.079
0.9	0.373	0.286	0.053	-0.160
1.0	0.250	0.182	-0.011	-0.183
1.25	0.080	0.031	-0.085	-0.194
1.5	0.013	-0.002	-0.077	-0.156
1.75	-0.007	-0.007	-0.048	-0.123
2.0	-0.012	-0.011	-0.039	-0.083
2.5	-0.004	-0.017	-0.025	-0.036
3.0	-0.003	0.001	-0.008	-0.025
3.5	-0.003	0.000	-0.004	-0.018
4.0	-0.001	0.000	-0.003	-0.012
6.0	-0.000	-0.000	-0.001	-0.002
8.0	-0.000	-0.000	-0.000	-0.000
10.0	-0.000	-0.000	-0.000	-0.000

TABLE 2.7
INFLUENCE VALUES FOR SURFACE DISPLACEMENTS
POINT LOAD
$\rho = I_\rho P/2\pi hE$

RADIAL DISPLACEMENT ρ_r

(Taylor, 1962)

r/h \ ν	0	0.2	0.4	0.5
0.05	19.959	14.362	5.559	-0.041
0.1	9.948	7.124	2.723	-0.078
0.2	4.896	3.455	1.250	-0.156
0.3	3.183	2.184	0.716	-0.225
0.4	2.308	1.523	0.426	-0.288
0.5	1.773	1.064	0.232	-0.326
0.6	1.277	0.824	0.102	-0.376
0.7	1.000	0.620	0.008	-0.405
0.8	0.789	0.465	-0.063	-0.420
0.9	0.627	0.349	-0.111	-0.421
1.0	0.499	0.259	-0.141	-0.417
1.25	0.292	0.150	-0.175	-0.380
1.5	0.167	0.048	-0.163	-0.315
1.75	0.097	0.012	-0.134	-0.250
2.0	0.060	0.002	-0.109	-0.195
2.5	0.027	0.003	-0.070	-0.118
3.0	0.010	-0.002	-0.038	-0.072
3.5	0.003	-0.008	-0.022	-0.046
4.0	0.002	-0.000	-0.014	-0.029
6.0	0.000	-0.000	-0.002	-0.002
8.0	0.000	-0.000	-0.000	-0.001
10.0	0.000	-0.000	-0.000	-0.000

2.2 Line Loading

In sub-sections 2.2.1 to 2.2.7, only solutions for stress are presented. Displacements due to line loading on or in a semi-infinite mass are only meaningful if evaluated as the displacement of one point relative to another point, both points being located neither at the origin of loading nor at infinity.

2.2.1 INFINITE LINE LOAD ACTING WITHIN AN INFINITE SOLID (Integrated Kelvin problem).(Fig.2.6).

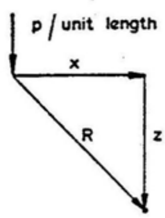


FIG. 2.6

$$\sigma_z = \frac{p}{2\pi(1-\nu)} \frac{z}{R^2} \left[\frac{(3-2\nu)}{2} - \frac{x^2}{R^2} \right] \quad \text{... (2.6a)}$$

$$\sigma_x = \frac{p}{2\pi(1-\nu)} \frac{z}{R^2} \left[-\frac{(1-2\nu)}{2} + \frac{x^2}{R^2} \right] \quad \text{... (2.6b)}$$

$$\sigma_y = \frac{p}{2\pi} \frac{\nu}{(1-\nu)} \frac{z}{R^2} \quad \text{... (2.6c)}$$

$$\tau_{xz} = \frac{p}{2\pi(1-\nu)} \frac{x}{R^2} \left[\frac{(1-2\nu)}{2} + \frac{z^2}{R^2} \right] \quad \text{... (2.6d)}$$

2.2.2 INFINITE VERTICAL LINE LOAD ON THE SURFACE OF A SEMI-INFINITE MASS (Integrated Boussinesq problem)(Fig.2.7).

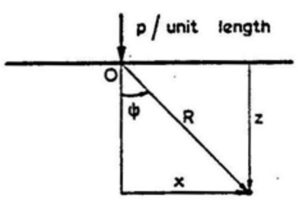


FIG. 2.7

$$\sigma_x = \frac{2p}{\pi} \frac{x^2 z}{R^4} \quad \text{... (2.7a)}$$

$$\sigma_z = \frac{2p}{\pi} \frac{z^3}{R^4} \quad \text{... (2.7b)}$$

$$\sigma_y = \frac{2p\nu}{\pi} \frac{z}{R^2} \quad \text{... (2.7c)}$$

$$\tau_{xz} = \frac{2p}{\pi} \frac{xz^2}{R^4} \quad \text{... (2.7d)}$$

Principal Stresses:

$$\sigma_1 = \sigma_R = \frac{2p}{\pi} \frac{z}{R^2} \quad \text{... (2.8a)}$$

$$\sigma_2 = \sigma_\psi = 0 \quad \text{... (2.8b)}$$

$$\tau_{max} = \frac{p}{\pi} \frac{z}{R^2} \quad \text{... (2.8c)}$$

Loci of constant σ_1, σ_2 and τ_{max} are circles tangent to x-axis at 0.

Trajectories of σ_1 are radial lines through 0.

Trajectories of σ_2 are a family of semi-circles, centres at 0.

Trajectories of τ_{max} are two orthogonal families of equi-angular spirals intersecting the radial lines at ±45°.

For the case of a vertical line load of finite length, influence values for σ_z are tabulated by Lysmer and Duncan (1969).

2.2.3 HORIZONTAL LINE LOAD ACTING ON SURFACE OF SEMI-INFINITE MASS (Integrated Cerruti Problem)(Fig.2.8).

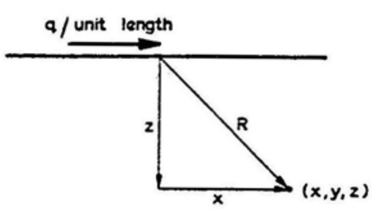


FIG. 2.8

$$\sigma_z = \frac{2qxz^2}{\pi R^4} \quad \ldots (2.9a)$$

$$\sigma_x = \frac{2qx^3}{\pi R^4} \quad \ldots (2.9b)$$

$$\sigma_y = \frac{2qxz^2}{\pi R^4} \quad \ldots (2.9c)$$

$$\tau_{xz} = \frac{2qx^2z}{\pi R^4} \quad \ldots (2.9d)$$

2.2.4 MELAN'S PROBLEM I

Vertical line loading p/unit length beneath the surface of a semi-infinite mass (Fig.2.9).

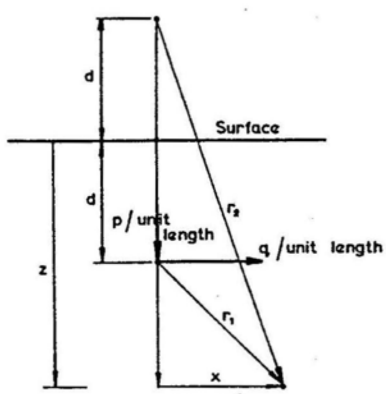


FIG. 2.9

$$\sigma_z = \frac{p}{\pi}\left[\frac{m+1}{2m}\left\{\frac{(z-d)^3}{r_1^4} + \frac{(z+d)\,[(z+d)^2 + 2dz]}{r_2^4} - \frac{8dz(d+z)x^2}{r_2^6}\right\} + \frac{m-1}{4m}\left(\frac{z-d}{r_1^2} + \frac{3z+d}{r_2^2} - \frac{4zx^2}{r_2^4}\right)\right] \quad \ldots (2.10a)$$

$$\sigma_x = \frac{p}{\pi}\left[\frac{m+1}{2m}\left\{\frac{(z-d)x^2}{r_1^4} + \frac{(z+d)(x^2+2d^2) - 2dx^2}{r_2^4} + \frac{8dz(d+z)x^2}{r_2^6}\right\} + \frac{m-1}{4m}\left(-\frac{z-d}{r_1^2} + \frac{z+3d}{r_2^2} + \frac{4zx^2}{r_2^4}\right)\right] \quad \ldots (2.10b)$$

$$\tau_{xz} = \frac{px}{\pi}\left[\frac{m+1}{2m}\left\{\frac{(z-d)^2}{r_1^4} + \frac{z^2-2dz-d^2}{r_2^4} + \frac{8dz(d+z)^2}{r_2^6}\right\} + \frac{m-1}{4m}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2} + \frac{4z(d+z)}{r_2^4}\right)\right] \quad \ldots (2.10c)$$

where $m = \frac{1-\nu}{\nu}$.

2.2.5 MELAN'S PROBLEM II

Horizontal line load q/unit length acting beneath the surface of a semi-infinite mass (Fig.2.9).

$$\sigma_z = \frac{qx}{\pi}\left[\frac{m+1}{2m}\left\{\frac{(z-d)^2}{r_1^4} - \frac{d^2-z^2+6dz}{r_2^4} + \frac{8dzx^2}{r_2^6}\right\} - \frac{m-1}{4m}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2} - \frac{4z(d+z)}{r_2^4}\right\}\right] \quad \ldots (2.11a)$$

$$\sigma_x = \frac{qx}{\pi}\left[\frac{m+1}{2m}\left\{\frac{x^2}{r_1^4} + \frac{x^2-4dz-2d^2}{r_2^4} + \frac{8dz(d+z)^2}{r_2^6}\right\} + \frac{m-1}{4m}\left(\frac{1}{r_1^2} + \frac{3}{r_2^2} - \frac{4z(d+z)}{r_2^4}\right)\right] \quad \ldots (2.11b)$$

$$\tau_{xz} = \frac{q}{\pi}\left[\frac{m+1}{2m}\left\{\frac{(z-d)x^2}{r_1^4} + \frac{(2dz+x^2)(d+z)}{r_2^4} - \frac{8dz(d+z)x^2}{r_2^6}\right\} + \frac{m-1}{4m}\left(\frac{z-d}{r_1^2} + \frac{3z+d}{r_2^2} - \frac{4z(d+z)^2}{r_2^4}\right)\right] \quad \ldots (2.11c)$$

2.2.6 VERTICAL LINE LOADING ON QUARTER-SPACE (Figure 2.10).

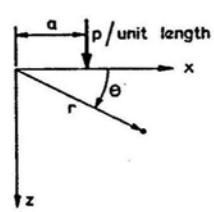


FIG. 2.10

Solutions for the stresses within the quarter-space have been given by Shepherd (1935). Hetenyi (1960) has obtained solutions for the stresses on the boundaries due to both a vertical and a horizontal line load.

Values of stresses obtained by Shepherd are given in Table 2.8. Polar coordinates are used as the problem originally considered was that of an infinite sector.

TABLE 2.8

STRESSES IN A QUARTER-SPACE
(After Shepherd, 1935)

Stress	r/a / $\theta°$	0.358	0.504	0.710	1.0	1.409	1.985	2.796
$\frac{2\pi a}{p}\sigma_\theta$	0	0	0	0	∞	0	0	0
	15	0.02	0.29	2.63	14.83	1.30	0.09	-0.06
	30	0.13	1.12	4.46	6.59	2.30	0.27	0.01
	45	0.29	1.36	3.03	3.36	1.52	0.34	0.04
	60	0.19	0.84	1.56	1.53	0.80	0.21	0.01
	75	0.10	0.23	0.41	0.36	0.22	0.05	-0.06
	90	0	0	0	0	0	0	0
$\frac{2\pi a}{p}\sigma_r$	0	-0.88	-1.26	-1.61	∞	-1.96	-1.90	-1.70
	15	0.04	0.75	1.37	-1.06	3.03	0.63	-0.31
	30	0.22	0.31	-0.42	-0.32	1.93	1.46	0.69
	45	0.04	0.03	-0.24	0.48	1.74	1.85	1.25
	60	0.02	0.09	0.46	1.16	2.09	2.18	1.89
	75	-0.16	0.35	0.45	1.56	2.55	2.79	2.47
	90	-0.64	-0.48	0.23	1.45	2.60	3.13	2.99
$\frac{2\pi a}{p}\tau_{r\theta}$	0	0	0	0	0	0	0	0
	15	-0.09	0.26	2.33	-2.49	-2.95	-0.77	-0.35
	30	0.05	0.48	0.70	-2.88	-3.31	-1.45	-0.69
	45	0.02	0.01	-0.90	-2.46	-2.74	-1.59	-0.84
	60	0.01	-0.38	-1.20	-1.80	-2.01	-1.35	-0.75
	75	-0.01	-0.28	-0.79	-1.19	-1.15	-0.81	-0.43
	90	0	0	0	0	0	0	0

2.2.7 LINE LOADING AT THE APEX OF AN INFINITE WEDGE (Fig.2.11)

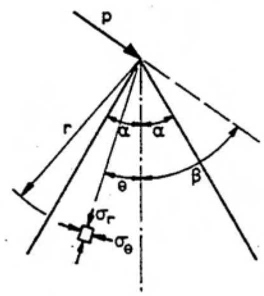


FIG. 2.11

Michell (1900) gives the following solutions:

$$\sigma_r = \frac{2p}{r}\left[\frac{\cos\beta\cos\theta}{2\alpha + \sin 2\alpha} + \frac{\sin\beta\sin\theta}{2\alpha - \sin 2\alpha}\right] \quad \text{... (2.12a)}$$

$$\sigma_\theta = 0 \quad \text{... (2.12b)}$$

$$\tau_{r\theta} = 0 \quad \text{... (2.12c)}$$

2.2.8 VERTICAL LINE LOAD ON FINITE LAYER (Fig.2.12)

Values of stresses and displacements are given in TABLES 2.9 to 2.13 (Poulos, 1966).

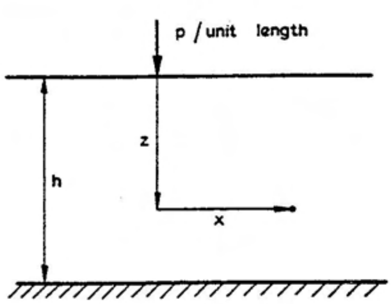


FIG. 2.12

TABLE 2.9

INFLUENCE VALUES I_{σ_z} FOR VERTICAL STRESS σ_z

LINE LOAD

$$\sigma_z = \frac{P}{\pi h} I_{\sigma_z}$$

$\nu=0$	$\nu=0.2$
$\nu=0.4$	$\nu=0.5$

z/h x/h	1.0		0.9		0.8		0.7		0.6		0.4		0.2	
0	2.634	2.566	2.787	2.759	2.980	2.975	3.249	3.256	3.641	3.653	5.157	5.127	9.891	9.899
	2.539	2.580	2.787	2.865	3.025	3.113	3.312	3.391	3.704	3.772	5.201	5.234	9.905	9.911
0.1	2.573	2.503	2.713	2.682	2.885	2.877	3.118	3.122	3.443	3.452	4.516	4.505	5.946	5.952
	2.471	2.508	2.702	2.774	2.920	3.001	3.171	3.243	3.498	3.562	4.585	4.586	5.957	5.960
0.2	2.400	2.331	2.505	2.471	2.627	2.614	2.774	2.773	2.948	2.954	3.251	3.217	2.341	2.347
	2.291	2.312	2.477	2.528	2.642	2.703	2.810	2.867	2.990	3.040	3.283	3.305	2.351	2.343
0.3	2.144	2.075	2.203	2.162	2.261	2.240	2.311	2.302	2.335	2.333	2.099	2.081	0.918	0.922
	2.021	2.023	2.145	2.170	2.245	2.279	2.318	2.349	2.352	2.383	2.117	2.128	0.923	0.907
0.4	1.840	1.774	1.855	1.810	1.857	1.828	1.830	1.812	1.751	1.741	1.301	1.257	0.407	0.409
	1.711	1.689	1.773	1.766	1.810	1.815	1.808	1.815	1.744	1.755	1.307	1.307	0.409	0.389
0.5	1.525	1.462	1.504	1.455	1.465	1.429	1.391	1.365	1.265	1.247	0.803	0.774	0.205	0.204
	1.389	1.350	1.397	1.365	1.387	1.365	1.338	1.320	1.231	1.223	0.792	0.786	0.201	0.187
0.6	1.223	1.168	1.179	1.130	1.117	1.077	1.024	0.993	0.889	0.867	0.497	0.443	0.110	0.107
	1.092	1.039	1.061	1.008	1.021	0.978	0.951	0.916	0.837	0.814	0.476	0.466	0.105	0.099
0.7	0.954	0.906	0.898	0.850	0.827	0.785	0.733	0.698	0.611	0.584	0.308	0.270	0.062	0.056
	0.830	0.771	0.774	0.711	0.718	0.661	0.644	0.594	0.543	0.509	0.276	0.264	0.053	0.058
0.8	0.721	0.683	0.661	0.619	0.592	0.553	0.508	0.473	0.408	0.379	0.185	0.124	0.032	0.024
	0.615	0.553	0.547	0.478	0.485	0.421	0.416	0.360	0.335	0.294	0.150	0.137	0.023	0.035
0.9	0.536	0.507	0.479	0.443	0.417	0.380	0.345	0.311	0.267	0.237	0.108	0.066	0.015	0.006
	0.447	0.390	0.375	0.309	0.314	0.251	0.254	0.195	0.192	0.149	0.070	0.057	0.005	0.020
1.0	0.357	0.338	0.306	0.279	0.254	0.224	0.199	0.169	0.144	0.116	0.045	-0.020	-0.000	-0.011
	0.294	0.241	0.224	0.162	0.167	0.106	0.117	0.062	0.075	0.032	0.010	-0.005	-0.007	0.005
1.25	0.121	0.123	0.091	0.082	0.062	0.047	0.035	0.018	0.013	-0.005	-0.016	-0.057	-0.014	-0.024
	0.111	0.079	0.059	0.020	0.020	-0.023	-0.010	-0.055	-0.028	-0.064	-0.037	-0.057	-0.015	-0.021
1.5	0.010	0.027	-0.005	0.003	-0.018	-0.016	-0.028	-0.031	-0.033	-0.040	-0.035	-0.097	-0.019	-0.026
	0.043	0.026	0.011	-0.013	-0.014	-0.042	-0.031	-0.064	-0.040	-0.070	-0.035	-0.061	-0.011	-0.034
1.75	-0.030	-0.006	-0.036	-0.019	-0.040	-0.029	-0.042	-0.036	-0.039	-0.038	-0.032	-0.064	-0.017	-0.019
	0.024	0.018	0.005	-0.006	-0.010	-0.027	-0.021	-0.046	-0.026	-0.049	-0.021	-0.047	-0.005	-0.034
2.0	-0.042	-0.014	-0.042	-0.019	-0.041	-0.022	-0.039	-0.026	-0.035	-0.026	-0.025	-0.131	-0.013	-0.011
	0.025	0.024	0.015	0.010	0.006	-0.003	-0.001	-0.014	-0.005	-0.020	-0.005	-0.026	0.002	-0.025
2.5	-0.025	-0.003	-0.022	-0.004	-0.019	-0.004	-0.016	-0.004	-0.013	-0.004	-0.006	-0.042	0.000	0.002
	0.027	0.034	0.022	0.027	0.018	0.020	0.014	0.001	0.011	0.009	0.007	-0.000	0.006	-0.006
3.0	-0.010	0.004	-0.007	0.004	-0.005	0.004	-0.003	0.004	-0.000	0.004	0.003	0.005	0.006	0.006
	0.022	0.032	0.019	0.028	0.017	0.023	0.014	0.015	0.012	0.015	0.008	0.008	0.005	0.003
4.0	0.006	0.005	0.006	0.005	0.007	0.004	0.007	0.005	0.008	0.005	0.009	0.005	0.009	0.004
	0.008	0.019	0.006	0.017	0.005	0.015	0.004	0.013	0.003	0.011	0.001	0.008	0.000	0.007
6.0	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000
	0.000	0.006	-0.000	0.005	-0.000	0.004	-0.001	0.004	-0.001	0.004	-0.001	0.003	-0.001	0.002
8.0	0.002	-0.000	0.002	-0.000	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001
	-0.001	0.001	-0.001	0.000	-0.001	0.000	-0.001	0.000	-0.001	0.000	-0.001	-0.000	-0.001	-0.001

TABLE 2.10

INFLUENCE VALUES I_θ FOR BULK STRESS θ

LINE LOAD

$$\theta = \frac{P}{\pi h} I_\theta$$

ν=0	ν=0.2
ν=0.4	ν=0.5

x/h \ z/h	1.0		0.9		0.8		0.7		0.6		0.4		0.2	
0	2.634	3.789	2.546	3.567	2.569	3.508	2.706	3.611	2.986	3.902	4.292	5.419	8.885	10.92
	5.923	7.739	5.419	6.933	5.145	6.433	5.120	6.217	5.322	6.295	6.846	7.709	13.02	14.12
0.1	2.573	3.707	2.492	3.499	2.512	3.441	2.637	3.530	2.887	3.786	3.975	5.042	6.647	8.230
	5.766	7.524	5.294	6.761	5.012	6.291	4.997	6.075	5.162	6.121	6.390	7.217	9.880	10.76
0.2	2.400	3.436	2.338	3.267	2.352	3.214	2.444	3.272	2.620	3.445	3.253	4.165	3.759	4.764
	5.346	6.936	4.960	6.318	4.726	5.903	4.673	5.683	4.755	5.651	5.371	6.116	5.848	6.432
0.3	2.144	3.063	2.107	2.947	2.115	2.900	2.166	2.920	2.256	2.997	2.458	3.206	1.961	2.605
	4.716	6.068	4.447	5.632	4.216	5.315	4.188	5.106	4.176	4.997	4.226	4.875	3.326	3.737
0.4	1.840	2.559	1.829	2.538	1.832	2.504	1.847	2.492	1.862	2.490	1.770	2.363	0.970	1.413
	3.992	5.068	3.847	4.840	3.718	4.620	3.635	4.434	3.553	4.272	3.239	3.803	1.952	2.275
0.5	1.525	2.142	1.536	2.129	1.536	2.107	1.524	2.075	1.489	2.021	1.239	1.716	0.423	0.754
	3.240	4.050	3.206	3.992	3.112	3.882	3.057	3.739	2.937	3.565	2.452	2.947	1.183	1.476
0.6	1.223	1.685	1.249	1.709	1.249	1.701	1.221	1.662	1.157	1.583	0.844	1.220	0.107	0.370
	2.548	3.117	2.599	3.203	2.568	3.175	2.517	3.079	2.390	2.919	1.868	2.308	0.754	1.045
0.7	0.954	1.302	0.989	1.349	0.989	1.352	0.955	1.313	0.879	1.228	0.559	0.867	-0.070	0.153
	1.937	2.314	2.045	2.484	2.037	2.533	2.026	2.486	1.910	2.362	1.423	1.820	0.499	0.808
0.8	0.721	0.949	0.759	1.008	0.760	1.019	0.724	0.987	0.647	0.909	0.350	0.590	-0.175	0.020
	1.436	1.661	1.575	1.890	1.612	1.979	1.608	1.973	1.514	1.887	1.100	1.462	0.360	0.679
0.9	0.537	0.696	0.574	0.756	0.575	0.773	0.541	0.747	0.470	0.679	0.207	0.409	-0.224	-0.044
	1.044	1.171	1.195	1.409	1.230	1.533	1.263	1.560	1.194	1.514	0.858	1.191	0.279	0.610
1.0	0.357	0.394	0.390	0.451	0.392	0.472	0.362	0.457	0.300	0.409	0.080	0.213	-0.258	-0.098
	0.686	0.724	0.837	0.972	0.901	1.105	0.936	1.160	0.895	1.150	0.651	0.956	0.234	0.565
1.25	0.121	0.079	0.142	0.112	0.142	0.128	0.122	0.127	0.082	0.109	-0.052	0.029	-0.242	-0.090
	0.258	0.238	0.373	0.417	0.411	0.547	0.484	0.619	0.483	0.666	0.389	0.634	0.212	0.514
1.5	0.010	-0.014	0.020	0.002	0.018	0.011	0.004	0.013	-0.020	0.009	-0.097	-0.017	-0.200	-0.056
	0.101	0.079	0.176	0.229	0.214	0.299	0.268	0.353	0.285	0.405	0.272	0.475	0.222	0.465
1.75	-0.030	-0.021	-0.027	-0.015	-0.030	-0.011	-0.039	-0.009	-0.053	-0.009	-0.095	-0.014	-0.148	-0.020
	0.056	0.055	0.102	0.141	0.142	0.200	0.170	0.252	0.191	0.299	0.211	0.376	0.217	0.419
2.0	-0.042	-0.022	-0.043	-0.018	-0.047	-0.015	-0.052	-0.013	-0.060	-0.012	-0.081	-0.013	-0.105	-0.013
	0.058	0.072	0.084	0.118	0.110	0.163	0.130	0.205	0.149	0.243	0.179	0.311	0.206	0.368
2.5	-0.025	-0.005	-0.027	-0.004	-0.030	-0.003	-0.033	-0.001	-0.036	-0.000	-0.041	0.002	-0.045	0.005
	0.062	0.102	0.073	0.124	0.086	0.147	0.097	0.170	0.108	0.193	0.131	0.238	0.152	0.281
3.0	-0.010	0.005	-0.012	0.005	-0.013	0.005	-0.015	0.006	-0.016	0.007	-0.017	0.009	-0.019	0.013
	0.052	0.097	0.058	0.111	0.064	0.125	0.073	0.139	0.079	0.154	0.093	0.182	0.105	0.208
4.0	0.006	0.007	0.005	0.007	0.005	0.007	0.004	0.007	0.004	0.007	0.004	0.007	-0.000	0.008
	0.018	0.058	0.021	0.064	0.026	0.070	0.027	0.075	0.029	0.081	0.034	0.091	0.037	0.101
6.0	0.005	0.001	0.005	0.001	0.005	0.001	0.005	0.001	0.005	0.001	0.005	0.001	0.003	0.000
	0.000	0.017	0.001	0.018	0.001	0.020	0.002	0.021	0.003	0.022	0.004	0.025	0.004	0.027
8.0	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001
	-0.002	0.002	-0.002	0.002	-0.002	0.003	-0.001	0.003	-0.001	0.003	-0.001	0.004	-0.001	0.004

TABLE 2.11

INFLUENCE VALUES $I_{\tau_{xz}}$ FOR HORIZONTAL SHEAR STRESS τ_{xz}

LINE LOAD

$$\tau_{xz} = \frac{P}{\pi h} I_{\tau_{xz}}$$

ν=0	ν=0.2
ν=0.4	ν=0.5

z/h / x/h	1.0		0.9		0.8		0.7		0.6		0.4		0.2	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.143	0.195	0.164	0.193	0.219	0.234	0.309	0.315	0.447	0.448	1.031	1.026	2.914	2.906
	0.266	0.330	0.223	0.250	0.242	0.248	0.310	0.307	0.436	0.427	1.008	0.999	2.888	2.888
0.2	0.265	0.364	0.300	0.357	0.391	0.422	0.536	0.550	0.744	0.747	1.441	1.432	2.230	2.215
	0.496	0.617	0.416	0.468	0.439	0.453	0.542	0.537	0.725	0.710	1.396	1.380	2.181	2.178
0.3	0.348	0.485	0.388	0.470	0.490	0.536	0.643	0.665	0.843	0.849	1.322	1.309	1.224	1.203
	0.666	0.830	0.554	0.630	0.563	0.588	0.657	0.655	0.820	0.803	1.260	1.238	1.155	1.147
0.4	0.391	0.554	0.427	0.529	0.517	0.578	0.645	0.676	0.792	0.802	1.008	0.994	0.638	0.612
	0.766	0.960	0.635	0.732	0.616	0.655	0.671	0.677	0.771	0.757	0.936	0.911	0.553	0.541
0.5	0.391	0.572	0.421	0.539	0.489	0.561	0.576	0.615	0.660	0.676	0.694	0.680	0.325	0.295
	0.799	1.007	0.660	0.775	0.610	0.666	0.617	0.634	0.646	0.640	0.617	0.594	0.227	0.215
0.6	0.367	0.552	0.385	0.514	0.427	0.510	0.476	0.524	0.510	0.532	0.448	0.435	0.155	0.122
	0.780	0.993	0.644	0.774	0.571	0.643	0.534	0.567	0.508	0.516	0.372	0.358	0.050	0.045
0.7	0.319	0.506	0.332	0.465	0.353	0.442	0.370	0.424	0.370	0.397	0.270	0.258	0.059	0.025
	0.724	0.933	0.600	0.740	0.511	0.599	0.444	0.494	0.380	0.404	0.198	0.198	0.048	-0.042
0.8	0.268	0.446	0.271	0.407	0.273	0.372	0.275	0.335	0.256	0.290	0.150	0.141	0.008	-0.027
	0.649	0.849	0.540	0.688	0.447	0.550	0.363	0.432	0.280	0.324	0.086	0.105	-0.100	-0.077
0.9	0.212	0.383	0.215	0.348	0.210	0.307	0.197	0.261	0.169	0.205	0.072	0.066	-0.018	-0.054
	0.567	0.757	0.475	0.627	0.385	0.500	0.295	0.380	0.205	0.267	0.018	0.055	-0.124	-0.084
1.0	0.152	0.303	0.148	0.274	0.139	0.234	0.119	0.185	0.090	0.129	0.014	0.010	-0.030	-0.066
	0.460	0.636	0.392	0.544	0.312	0.438	0.226	0.329	0.139	0.222	-0.028	0.034	-0.120	-0.070
1.25	0.055	0.177	0.054	0.161	0.046	0.131	0.030	0.092	0.010	0.049	-0.026	-0.028	-0.020	-0.059
	0.285	0.440	0.252	0.401	0.199	0.338	0.136	0.263	0.069	0.186	-0.050	0.049	-0.112	-0.022
1.5	0.012	0.098	0.004	0.090	0.003	0.073	-0.004	0.048	-0.012	0.021	-0.020	-0.025	0.006	-0.038
	0.170	0.311	0.157	0.300	0.127	0.268	0.087	0.223	0.043	0.174	-0.038	0.080	-0.085	0.020
1.75	-0.014	0.053	-0.015	0.051	-0.011	0.042	-0.010	0.028	-0.009	0.013	-0.000	-0.013	0.031	-0.019
	0.099	0.228	0.094	0.228	0.079	0.213	0.055	0.189	0.028	0.159	-0.025	0.095	-0.064	0.041
2.0	0.011	0.029	-0.018	0.029	-0.010	0.026	-0.004	0.020	0.003	0.013	0.021	0.000	0.052	-0.005
	0.054	0.172	0.053	0.175	0.045	0.169	0.033	0.156	0.017	0.139	-0.018	0.096	-0.051	0.050
2.5	0.002	0.017	-0.005	0.018	0.005	0.018	0.015	0.016	0.025	0.014	0.048	0.010	0.073	0.005
	0.016	0.116	0.015	0.118	0.011	0.114	0.005	0.109	-0.003	0.100	-0.023	0.075	-0.048	0.042
3.0	-0.004	0.015	0.005	0.015	0.015	0.015	0.026	0.015	0.036	0.014	0.057	0.011	0.079	0.007
	-0.000	0.087	-0.003	0.087	-0.006	0.084	-0.011	0.079	-0.016	0.073	-0.031	0.056	-0.051	0.031
4.0	0.007	0.012	0.015	0.012	0.023	0.011	0.030	0.010	0.038	0.009	0.054	0.007	0.070	0.004
	-0.020	0.045	-0.022	0.043	-0.025	0.040	-0.028	0.037	-0.021	0.034	-0.039	0.025	-0.049	0.014
6.0	0.007	0.007	0.012	0.006	0.017	0.005	0.022	0.005	0.027	0.004	0.037	0.003	0.047	0.001
	-0.026	0.015	-0.028	0.014	-0.029	0.013	-0.030	0.012	-0.032	0.010	-0.035	0.007	-0.038	0.004
8.0	0.004	0.004	0.007	0.003	0.011	0.003	0.014	0.003	0.017	0.002	0.024	0.001	0.030	0.001
	-0.028	0.006	-0.024	0.005	-0.025	0.005	-0.026	0.004	-0.026	0.004	-0.028	0.003	-0.030	0.001

TABLE 2.12

INFLUENCE VALUES $I_{\rho x}$ FOR SURFACE DISPLACEMENTS

LINE LOAD

$\rho_x = \frac{p}{\pi E} I_{\rho x}$

HORIZONTAL DISPLACEMENT ρ_x

x/h \ ν	0	0.2	0.4	0.5
0.1	1.366	0.952	0.316	-0.094
0.2	1.246	0.834	0.217	-0.184
0.3	1.130	0.727	0.127	-0.269
0.4	1.019	0.613	0.042	-0.342
0.5	0.872	0.518	-0.033	-0.407
0.6	0.749	0.436	-0.097	-0.460
0.7	0.646	0.353	-0.151	-0.499
0.8	0.556	0.288	-0.194	-0.524
0.9	0.476	0.227	-0.223	-0.538
1.0	0.390	0.180	-0.247	-0.537
1.25	0.257	0.084	-0.261	-0.509
1.5	0.167	0.026	-0.241	-0.448
1.75	0.111	0.005	-0.211	-0.384
2.0	0.074	0.001	-0.175	-0.308
2.5	0.034	-0.002	-0.114	-0.210
3.0	0.012	-0.015	-0.067	-0.135
4.0	0.002	-0.002	-0.026	-0.053
6.0	0.000	0.000	-0.005	-0.007
8.0	0.000	0.000	-0.000	-0.000

TABLE 2.13

INFLUENCE VALUES $I_{\rho z}$ FOR SURFACE DISPLACEMENTS

LINE LOAD

$\rho_z = \frac{p}{\pi E} I_{\rho z}$

VERTICAL DISPLACEMENT ρ_z

x/h \ ν	0	0.2	0.4	0.5
0.1	3.756	3.466	2.633	1.926
0.2	2.461	2.222	1.558	0.973
0.3	1.730	1.533	0.964	0.458
0.4	1.244	1.069	0.583	0.132
0.5	0.896	0.749	0.324	-0.079
0.6	0.643	0.511	0.145	-0.217
0.7	0.453	0.347	0.045	-0.299
0.8	0.313	0.218	-0.057	-0.344
0.9	0.212	0.141	-0.101	-0.358
1.0	0.126	0.059	-0.139	-0.359
1.25	0.023	-0.006	-0.146	-0.315
1.5	-0.012	-0.024	-0.112	-0.254
1.75	-0.023	-0.023	-0.086	-0.198
2.0	-0.017	-0.015	-0.071	-0.134
2.5	-0.010	-0.009	-0.034	-0.085
3.0	-0.008	-0.001	-0.014	-0.057
4.0	-0.002	-0.000	-0.007	-0.025
6.0	-0.000	-0.000	-0.002	-0.004
8.0	-0.000	-0.000	-0.000	-0.000

2.3 Line Loading—Axial Symmetry

2.3.1 UNIFORM VERTICAL RING LOADING ON SURFACE OF SEMI-INFINITE MASS (Fig. 2.13)

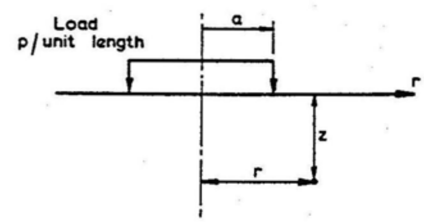


FIG. 2.13

On the axis (r=0),

$$\sigma_z = \frac{3pz^3 a}{(a^2+z^2)^{5/2}} \quad \text{... (2.13a)}$$

$$\sigma_r = \sigma_\theta = \frac{pza}{2(a^2+z^2)^{5/2}}\left[2(1+\nu)\,(a^2+z^2) - 3z^2\right] \quad \text{... (2.13b)}$$

$$\Theta = \frac{2p(1+\nu)za}{(a^2+z^2)^{3/2}} \quad \text{... (2.13c)}$$

$$\tau_{rz} = 0 \quad \text{... (2.13d)}$$

$$\rho_z = \frac{p(1+\nu)a}{E(a^2+z^2)^{\frac{1}{2}}}\left[2(1-\nu) + \frac{z^2}{(a^2+z^2)}\right] \quad \text{... (2.13e)}$$

$$\rho_r = 0 \quad \text{... (2.13f)}$$

2.3.2 UNIFORM VERTICAL SUBSURFACE LINE LOAD (Fig. 2.14)

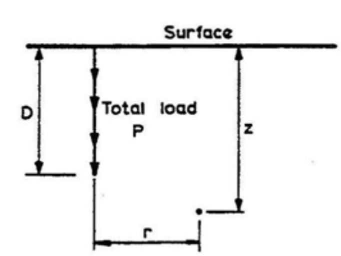


FIG. 2.14

Geddes (1966) evaluated the following expressions for the stresses from Mindlin's equations:

$$K_{zz} = \sigma_z \frac{D^2}{P} = \frac{-1}{8\pi(1-\nu)} \Bigg[-\frac{2(2-\nu)}{A} + \frac{2(2-\nu) + 2(1-2\nu)\frac{m}{n}\left(\frac{m}{n}+\frac{1}{n}\right)}{B} - \frac{(1-2\nu)2\left(\frac{m}{n}\right)^2}{F} + \frac{n^2}{A^3} + \frac{4m^2-4(1+\nu)\left(\frac{m}{n}\right)^2 m^2}{F^3} + \frac{4m(1+\nu)(m+1)\left(\frac{m}{n}+\frac{1}{n}\right)^2 - (4m^2+n^2)}{B^3} + \frac{6m^2\left(\frac{m^4-n^4}{n^2}\right)}{F^5} + \frac{6m\left(mn^2 - \frac{1}{n^2}[m+1]^5\right)}{B^5} \Bigg] \quad \ldots (2.14a)$$

$$K_{rr} = \sigma_r \frac{D^2}{P} = \frac{-1}{8\pi(1-\nu)} \Bigg[\frac{(1-2\nu)}{A} + \frac{(7-2\nu)-12(1-\nu)\frac{m}{n}\left(\frac{m}{n}+\frac{1}{n}\right)}{B} - \left\{ \frac{4(2-\nu)-12(1-\nu)\left(\frac{m}{n}\right)^2}{F} \right\} - \frac{n^2}{A^3} + \frac{4n^2-2m^2+2(1+2\nu)\left(\frac{m}{n}\right)^2 m^2}{F^3} - \left\{ \frac{3n^2-2m^2+2(1+2\nu)\frac{m}{n}(m+1)^2\left(\frac{m}{n}+\frac{1}{n}\right)}{B^3} \right\} + \frac{6\left[n^2m^2-m^4\left(\frac{m}{n}\right)^2\right]}{F^5} + \frac{6\left[\frac{m}{n}(m+1)^4\left(\frac{m}{n}+\frac{1}{n}\right) - m^2n^2\right]}{B^5} + 4(1-\nu)(1-2\nu)\left\{ \frac{1}{F+m} - \frac{1}{B+m+1} \right\} \Bigg] \quad \ldots (2.14b)$$

$$K_{\theta\theta} = \sigma_\theta \frac{D^2}{P} = \frac{-1}{8\pi(1-\nu)} \Bigg[\frac{(1-2\nu)}{A} + \frac{6-(1-2\nu)(3-4\nu)+6(1-2\nu)\frac{m}{n}\left(\frac{m}{n}+\frac{1}{n}\right)}{B} + \frac{2(1-2\nu)^2-6(1-2\nu)\left(\frac{m}{n}\right)^2 - 6}{F} + \frac{2m^2-4\nu n^2+2(1+2\nu)\frac{m}{n}(m+1)^2\left(\frac{m}{n}+\frac{1}{n}\right)}{B^3} + \frac{4\nu n^2-2m^2-2(1+2\nu)m^2\left(\frac{m}{n}\right)^2}{F^3} - 4(1-\nu)(1-2\nu)\left\{ \frac{1}{F+m} - \frac{1}{B+m+1} \right\} \Bigg] \quad \ldots (2.14c)$$

$$K_{rz} = \tau_{rz} \frac{D^2}{P} = \frac{-n}{8\pi(1-\nu)} \Bigg[\frac{(1-2\nu)\frac{1}{n}\left(\frac{m}{n}-\frac{1}{n}\right)}{A} + \frac{(m-1)\left(\frac{m}{n}-\frac{1}{n}\right)^2}{A^3} + \frac{(1-2\nu)\frac{1}{n}\left(\frac{m}{n}+\frac{1}{n}\right) - \frac{6}{n}\left(\frac{m}{n}\right)^2\left(\frac{m}{n}+\frac{1}{n}\right)}{B} + \frac{12m-4\nu m+(m+1)^3\left[\left(\frac{1}{n}\right)^2 + 12\frac{m^2}{n^4}\right]}{B^3} - \frac{6\left(\frac{m}{n}\right)^2(m+1)^3\left(\frac{m}{n}+\frac{1}{n}\right)^2 + 6mn^2}{B^5} + \frac{6\left(\frac{m}{n}\right)^3\left(\frac{1}{n}\right) - 2(1-2\nu)\frac{m}{n^2}}{F} + \frac{4\nu m-12m-m^3\left[\frac{2}{n^2} + 12\frac{m^2}{n^4}\right]}{F^3} + \frac{6mn^2+6m^3\left(\frac{m}{n}\right)^4}{F^5} \Bigg] \quad \ldots (2.14d)$$

On the axis with $n=0$ and $m>1.0$,

$$K_{zz} = \frac{-1}{8\pi(1-\nu)} \left[-\frac{4(1-\nu)}{m} - \frac{2(2-\nu)}{(m-1)} + \frac{2(2-\nu)}{(m+1)} + \frac{4m(2-\nu)}{(m+1)^2} - \frac{4m^2}{(m+1)^3} \right] \quad \ldots (2.14e)$$

$$K_{rr} = K_{\theta\theta} = \frac{-1}{8\pi(1-\nu)} \left[-\frac{2+2\nu(1-2\nu)}{m} + \frac{(1-2\nu)}{(m-1)} + \frac{6-(1-2\nu)^2}{(m+1)} - \frac{6m}{(m+1)^2} + \frac{2m^2}{(m+1)^3} \right] \quad \ldots (2.14f)$$

$$K_{rz} = 0 \quad \ldots (2.14g)$$

where $n = r/D$, $m = z/D$

$F^2 = n^2 + m^2$

$A^2 = [n^2 + (m-1)^2]$

$B^2 = [n^2 + (m+1)^2]$.

K_{zz} throughout the mass, and K_{rr} and $K_{\theta\theta}$ on the axis, are tabulated by Geddes (1966).

2.3.3 LINEARLY VARYING SUBSURFACE LINE LOAD (Fig. 2.15)

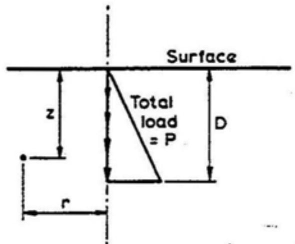


FIG. 2.15

The following expressions for stress have been obtained by Geddes (1966):

$$K_{zz} = \sigma_z \frac{D^2}{P} = \frac{-1}{4\pi(1-\nu)} \Bigg[\frac{-2(2-\nu)}{A} + \frac{2(2-\nu)(4m+1) - 2(1-2\nu)\left(\frac{m}{n}\right)^2 (m+1)}{B} + \frac{2(1-2\nu)\frac{m^3}{n^2} - 8(2-\nu)m}{F} + \frac{mn^2 + (m-1)^3}{A^3} + \frac{4\nu n^2 m + 4m^3 - 15n^2 m - 2(5+2\nu)\left(\frac{m}{n}\right)^2 (m+1)^3 + (m+1)^3}{B^3} + \frac{2(7-2\nu)mn^2 - 6m^3 + 2(5+2\nu)\left(\frac{m}{n}\right)^2 m^3}{F^3} + \frac{6mn^2(n^2-m^2) + 12\left(\frac{m}{n}\right)^2 (m+1)^5}{B^5} - \frac{12\left(\frac{m}{n}\right)^2 m^5 + 6mn^2(n^2-m^2)}{F^5} - 2(2-\nu)\log_e\left(\frac{A+m-1}{F+m} \cdot \frac{B+m+1}{F+m}\right) \Bigg] \quad \ldots (2.15a)$$

$$K_{rr} = \sigma_r \frac{D^2}{P} = \frac{-1}{4\pi(1-\nu)} \Bigg[\frac{(1-2\nu)}{A} + \frac{(7-2\nu) - 12m + 12(1-\nu)\left(\frac{m}{n}\right)^2 (m+1)}{B} + \frac{12m - 12(1-\nu)\frac{m^3}{n^2}}{F} - \frac{(m-1)^3 + mn^2}{A^3} + \frac{3(m+1)^3 - 2m^3 + (21-4\nu)mn^2 + 2(5+2\nu)\left(\frac{m}{n}\right)^2 (m+1)^3}{B^3} - \frac{2(5+2\nu)\frac{m^5}{n^2} + 4(5-\nu)mn^2}{F^3} + \frac{6mn^2(m^2-n^2) - 12\left(\frac{m}{n}\right)^2 (m+1)^5}{B^5} - \frac{6mn^2(m^2-n^2) - 12\frac{m^7}{n^2}}{F^5} + (1-2\nu)\log_e\frac{A+m-1}{F+m} + \{(1-2\nu)^2 - 6\}\log\left(\frac{B+m+1}{F+m}\right) + 2(1-\nu)(1-2\nu)\left\{\frac{m-1}{B+m+1} - \frac{m}{F+m}\right\} \Bigg] \quad \ldots (2.15b)$$

$$K_{\theta\theta} = \sigma_\theta \frac{D^2}{P} = \frac{-1}{4\pi(1-\nu)} \Bigg[\frac{(1-2\nu)}{A} - \frac{(1-2\nu)(3-4\nu) + 6(1-2\nu)\left(\frac{m}{n}\right)^2 (m+1) + 6(2m-1)}{B} + \frac{6(1-2\nu)\frac{m^3}{n^2} + 12m}{F} - (1-2\nu)\left\{\frac{2(m+1)^3 + 4mn^2 - 2\left(\frac{m}{n}\right)^2 (m+1)^3}{B^3}\right\} + \frac{2(m+1)^3 + 6mn^2 - 2m^3 - 6\left(\frac{m}{n}\right)^2 (m+1)^3}{B^3} + \frac{\left(2m^3 + 4mn^2 - 2\frac{m^5}{n^2}\right)(1-2\nu)}{F^3} - \frac{6mn^2 - 6\frac{m^5}{n^2}}{F^3} + (1-2\nu)\log_e\left(\frac{A+m-1}{F+m}\right) + \{(1-2\nu)^2 - 6\} \cdot \log_e\left(\frac{B+m+1}{F+m}\right) - 2(1-\nu)(1-2\nu)\left\{\frac{m-1}{B+m+1} - \frac{m}{F+m}\right\} \Bigg] \quad \ldots (2.15c)$$

and,

$$K_{rz} = \tau_{rz} \frac{D^2}{P} = \frac{-n}{4\pi(1-\nu)} \Bigg[\frac{2(2-\nu) + (1-2\nu)\frac{m}{n^2}(m-1)}{A} - \frac{6\left(\frac{m}{n}\right)^4}{F} - \frac{2(2-\nu) + (1-2\nu)\frac{m}{n^2}(m+1) - 6\frac{m^3}{n^4}(m+1)}{B} + \frac{\frac{m}{n^2}(m-1)^3 - n^2}{A^3}$$

$$- \frac{(3-4\nu)\,[\frac{m}{n^2}(m+1)^3+m^2]+17m^2-n^2+12\,\frac{m^3}{n^4}[m+1]^3}{B^3}$$

$$+ \frac{2(1-2\nu)\,\frac{m^4}{n^2}+4(5-\nu)m^2+12\,\frac{m^6}{n^4}}{F^3}$$

$$+ \frac{6\,\frac{m^3}{n^4}\,(m+1)^5-6\,\frac{m}{n^2}(m+1)^5+12m^2n^2}{B^5}$$

$$- \frac{6\,\frac{m^8}{n^4}-6\,\frac{m^6}{n^2}+12m^2n^2}{F^5} \Bigg]$$

... (2.15d)

On the loading axis with $m>1.0$

$$K_{zz} = \frac{-1}{4\pi(1-\nu)}\left[2 - \frac{2(2-\nu)m}{(m-1)} + \frac{6(2-\nu)m}{(m+1)} - \frac{2(7-2\nu)m^2}{(m+1)^2}\right.$$

$$\left. + \frac{4m^3}{(m+1)^3} - 2(2-\nu)\log_e\{\frac{m^2-1}{m^2}\}\right]$$

... (2.15e)

$$K_{rr} = K_{\theta\theta} = \frac{1}{4\pi(1-\nu)}\Bigg[11-2(1-2\nu)(1-\nu)+(1-2\nu)$$

$$.\ \log_e\{\frac{m-1}{m}\}+(1-2\nu)^2\log_e\{\frac{m+1}{m}\}-6\log_e\{\frac{m+1}{m}\}$$

$$+ (1-2\nu)\,\frac{m}{(m-1)}+[(1-2\nu)^2-18]\,\frac{m}{(m+1)}$$

$$+ \frac{9m^2}{(m+1)^2} - \frac{2m^3}{(m+1)^3}\Bigg]$$

... (2.15f)

$$K_{rz} = 0$$

... (2.15g)

where $n = r/D, \quad m = z/D$

$F^2 = n^2 + m^2$

$A^2 = [n^2 + (m-1)^2]$

$B^2 = [n^2 + (m+1)^2]$

Values of K_{zz} throughout the mass, and K_{rr} and $K_{\theta\theta}$ on the axis, are tabulated by Geddes (1966).