Lista de exercícios - Técnicas de Integração

Método da Substituição:

7. $\int (2x + 1)(x^2 + x + 1)^3 \, dx$

8. $\int x^3(1 - x^4)^5 \, dx$

9. $\int \sqrt{x - 1} \, dx$

10. $\int \sqrt[3]{1 - x} \, dx$

11. $\int x^3 \sqrt{2 + x^4} \, dx$

12. $\int x(x^2 + 1)^{3/2} \, dx$

13. $\int \frac{2}{(t + 1)^6} \, dt$

14. $\int \frac{1}{(1 - 3t)^4} \, dt$

15. $\int (1 - 2y)^{1,3} \, dy$

16. $\int \sqrt[5]{3 - 5y} \, dy$

17. $\int \cos 2\theta \, d\theta$

18. $\int \sec^2 3\theta \, d\theta$

19. $\int \frac{3x - 1}{(3x^2 - 2x + 1)^4} \, dx$

20. $\int \frac{x}{\sqrt{x^2 + 1}} \, dx$

36-43 Calcule a integral definida, se existir.

36. $\int_0^1 \cos \pi t \, dt$

37. $\int_0^{\pi/4} \text{sen } 4t \, dt$

38. $\int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} \, dx$

39. $\int_0^3 \frac{dx}{2x + 3}$

40. $\int_0^1 (2x - 1)^{100} \, dx$

41. $\int_0^{-4} \sqrt{1 - 2x} \, dx$

42. $\int_0^1 (x^4 + x)^5(4x^3 + 1) \, dx$

43. $\int_2^3 \frac{3x^2 - 1}{(x^3 - x)^2} \, dx$

Respostas:

7. $\frac{1}{4}\left(x^2+x+1\right)^4+C$ **8.** $-\frac{1}{24}\left(1-x^4\right)^6+C$

9. $\frac{2}{3}(x-1)^{3/2}+C$ **10.** $-\frac{3}{4}(1-x)^{4/3}+C$

11. $\frac{1}{6}\left(2+x^4\right)^{3/2}+C$ **12.** $\frac{1}{5}\left(x^2+1\right)^{5/2}+C$

13. $-\dfrac{2}{5(t+1)^5}+C$ **14.** $\dfrac{1}{9(1-3t)^3}+C$

15. $-\dfrac{(1-2y)^{2,3}}{4,6}+C$ **16.** $-\frac{1}{6}(3-5y)^{6/5}+C$

17. $\frac{1}{2}\operatorname{sen}2\theta+C$ **18.** $\frac{1}{3}\operatorname{tg}3\theta+C$

19. $-\dfrac{1}{6(3x^2-2x+1)^3}+C$ **20.** $\sqrt{x^2+1}+C$

36. 0

37. $\frac{1}{2}$

38. $\frac{4\sqrt{2}}{3}-\frac{5\sqrt{5}}{12}$

39. $\frac{1}{2}\ln 3$

40. $\frac{1}{101}$

41. $-\frac{26}{3}$

42. $\frac{32}{3}$

43. $\frac{1}{8}$

Integrais Trigonométricas:

1-36 Calcule a integral.

1. $\int \operatorname{sen}^3 x\,dx$ **2.** $\int \operatorname{sen}^3 x\cos^4 x\,dx$

3. $\int \operatorname{sen}^4 x\cos^3 x\,dx$ **4.** $\int \cos^5 x\operatorname{sen}^5 x\,dx$

5. $\int_0^{\pi/2} \operatorname{sen}^2 3x\,dx$ **6.** $\int_0^{\pi/2} \cos^2 x\,dx$

7. $\int \cos^4 t\,dt$ **8.** $\int \operatorname{sen}^6 \pi x\,dx$

9. $\int (1-\operatorname{sen}2x)^2\,dx$ **10.** $\int \operatorname{sen}\left(\theta+\dfrac{\pi}{6}\right)\cos\theta\,d\theta$

23. $\int_0^{\pi/4} \text{tg}^4 t \sec^2 t \, dt$

24. $\int_0^{\pi/4} \text{tg}^2 x \sec^4 x \, dx$

25. $\int \text{tg}^3 x \sec^3 x \, dx$

26. $\int_0^{\pi/3} \text{tg}^5 x \sec x \, dx$

27. $\int \frac{\sec^2 x}{\text{cotg}\, x} dx$

28. $\int \text{cotg}^2 w \, \text{cossec}^4 w \, dw$

29. $\int \text{cotg}^3 x \, \text{cossec}^4 x \, dx$

30. $\int e^x \cos^7(e^x) \, dx$

31. $\int \frac{\cos^2 x}{\text{sen}\, x} dx$

32. $\int \frac{dx}{\text{sen}^4 x}$

33. $\int \cos 3x \cos 4x \, dx$

34. $\int \text{sen}\, 3x \, \text{sen}\, 6x \, dx$

Respostas:

1. $-\cos x + \frac{1}{3}\cos^3 x + C$

2. $\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$

3. $\frac{1}{5}\text{sen}^5 x - \frac{1}{7}\text{sen}^7 x + C$

4. $\frac{1}{10}\text{sen}^{10} x - \frac{1}{4}\text{sen}^8 x + \frac{1}{6}\text{sen}^6 x + C$

5. $\frac{\pi}{4}$ **6.** $\frac{\pi}{4}$

7. $\frac{3}{8}t + \frac{1}{4}\text{sen}\, 2t + \frac{1}{32}\text{sen}\, 4t + C$

8. $\frac{5}{16}x - \frac{1}{4\pi}\text{sen}\, 2\pi x + \frac{3}{64\pi}\text{sen}\, 4\pi x + \frac{1}{48\pi}\text{sen}^3 2\pi x + C$

9. $\frac{3}{2}x + \cos 2x - \frac{1}{8}\text{sen}\, 4x + C$

10. $-\frac{\sqrt{3}}{8}\cos 2\theta + \frac{1}{4}\theta + \frac{1}{8}\text{sen}\, 2\theta + C$

22. $\frac{28}{15}$ **23.** $\frac{1}{5}$ **24.** $\frac{8}{15}$

25. $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

26. $\frac{38}{15}$

27. $\frac{1}{2}\text{tg}^2 x + C$

28. $-\frac{1}{3}\text{cotg}^3 w - \frac{1}{5}\text{cotg}^5 w + C$

29. $-\frac{1}{6}\text{cotg}^6 x - \frac{1}{4}\text{cotg}^4 x + C$

30. $\text{sen}\, e^x - \text{sen}^3 e^x + \frac{3}{5}\text{sen}^5 e^x - \frac{1}{7}\text{sen}^7 e^x + C$

31. $\ln|\text{cossec}\, x - \text{cotg}\, x| + \cos x + C$

32. $-\frac{1}{3}\text{cotg}^3 x - \text{cotg}\, x + C$

33. $\frac{1}{2}\text{sen}\, x + \frac{1}{14}\text{sen}\, 7x + C$

34. $\frac{1}{6}\text{sen}\, 3x - \frac{1}{18}\text{sen}\, 9x + C$

Integração por Partes:

1-15 Calcule a integral.

1. $\int xe^{2x}\,dx$
2. $\int x\cos x\,dx$
3. $\int x\,\text{sen}\,4x\,dx$
4. $\int x^2\cos 3x\,dx$
5. $\int x^2\,\text{sen}\,ax\,dx$
6. $\int \theta\,\text{sen}\,\theta\,\cos\theta\,d\theta$
7. $\int t^2\ln t\,dt$
8. $\int e^{-\theta}\cos 3\theta\,d\theta$
9. $\int_0^1 te^{-t}\,dt$
10. $\int_1^4 \ln\sqrt{x}\,dx$
11. $\int_0^{\pi/2} x\cos 2x\,dx$
12. $\int_0^1 x^2e^{-x}\,dx$
13. $\int x^3e^{x^2}\,dx$
14. $\int \text{sen}(\ln x)\,dx$
15. $\int x\,\text{tg}^{-1}x\,dx$

Respostas:

1. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
2. $x\,\text{sen}\,x + \cos x + C$
3. $-\frac{1}{4}x\cos 4x + \frac{1}{16}\text{sen}\,4x + C$
4. $\frac{1}{3}x^2\,\text{sen}\,3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\,\text{sen}\,3x + C$
5. $-\frac{x^2}{a}\cos ax + \frac{2x}{a^2}\text{sen}\,ax + \frac{2}{a^3}\cos ax + C$
6. $\frac{1}{8}(\text{sen}\,2\theta - 2\theta\cos 2\theta) + C$
7. $\frac{1}{9}t^3(3\ln t - 1) + C$
8. $\frac{1}{10}e^{-\theta}(3\,\text{sen}\,3\theta - \cos 3\theta) + C$
9. $1 - 2/e$
10. $2\ln 4 - \frac{3}{2}$
11. $-\frac{1}{2}$
12. $2 - 5/e$
13. $\frac{1}{2}e^{x^2}(x^2 - 1) + C$
14. $\frac{1}{2}x\,[\text{sen}(\ln x) - \cos(\ln x)] + C$
15. $\frac{1}{2}\left(x^2\,\text{tg}^{-1}x + \text{tg}^{-1}x - x\right) + C$

Substituição Trigonométrica:

1-21 Calcule a integral.

1. $\int_{1/2}^{\sqrt{3}/2} \frac{1}{x^2\sqrt{1-x^2}}\,dx$

2. $\int_0^2 x^3\sqrt{4-x^2}\,dx$

3. $\int \frac{x}{\sqrt{1-x^2}}\,dx$

4. $\int x\sqrt{4-x^2}\,dx$

5. $\int_0^2 \frac{x^3}{\sqrt{x^2+4}}\,dx$

6. $\int_0^3 \frac{dx}{\sqrt{9+x^2}}$

7. $\int \frac{dx}{x^3\sqrt{x^2-16}}$

8. $\int \frac{x}{(x^2+4)^{5/2}}\,dx$

9. $\int \frac{dx}{x\sqrt{x^2+3}}$

10. $\int x\sqrt{25+x^2}\,dx$

11. $\int \frac{\sqrt{9x^2-4}}{x}\,dx$

12. $\int_0^3 x\sqrt{9-x^2}\,dx$

Respostas:

1. $\frac{2}{\sqrt{3}}$

2. $\frac{64}{15}$

3. $-\sqrt{1-x^2}+C$

4. $-\frac{1}{3}\left(4-x^2\right)^{3/2}+C$

5. $\frac{8}{3}\left(2-\sqrt{2}\right)$

6. $\ln\left(\sqrt{2}+1\right)$

7. $\frac{1}{128}\left(\sec^{-1}\frac{x}{4}+\frac{4\sqrt{x^2-16}}{x^2}\right)+C$

8. $-\frac{1}{3}\left(x^2+4\right)^{-3/2}+C$

9. $\frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{x^2+3}-\sqrt{3}}{x}\right|+C$

10. $\frac{1}{3}\left(x^2+25\right)^{3/2}+C$

11. $\sqrt{9x^2-4}-2\sec^{-1}\left(\frac{3x}{2}\right)+C$

12. 9

Integração por Frações Parciais:

31. $\int \frac{dx}{(x-1)(x-2)(x-3)}$

32. $\int \frac{dx}{x(x+1)(2x+3)}$

33. $\int \frac{3x^2-6x+2}{2x^3-3x^2+x}\,dx$

34. $\int_0^1 \frac{x^3}{x^2+1}\,dx$

35. $\int_1^2 \frac{x^2+3}{x^3+2x}\,dx$

36. $\int \frac{3x^2-4x+5}{(x-1)(x^2+1)}\,dx$

37. $\int \frac{2t^3-t^2+3t-1}{(t^2+1)(t^2+2)}\,dt$

38. $\int \frac{x^4}{x^4-1}\,dx$

39. $\int \frac{2x^3-x}{x^4-x^2+1}\,dx$

40. $\int_0^1 \frac{x-1}{x^2+2x+2}\,dx$

41. $\int_0^1 \frac{x}{x^2+x+1}\,dx$

42. $\int_{-1/2}^{1/2} \frac{4x^2+5x+7}{4x^2+4x+5}\,dx$

43. $\int \frac{3x^3-x^2+6x-4}{(x^2+1)(x^2+2)}\,dx$

44. $\int \frac{(2\operatorname{sen}x-3)\cos x}{\operatorname{sen}^2 x-3\operatorname{sen}x+2}\,dx$

Respostas:

31. $\frac{1}{2}\ln\frac{|(x-1)(x-3)|}{(x-2)^2}+C$

32. $\frac{1}{3}\ln|x|-\ln|x+1|+\frac{2}{3}\ln|2x+3|+C$

33. $2\ln|x|-\ln|x-1|+\frac{1}{2}\ln|2x-1|+C$

34. $\frac{1}{2}(1-\ln 2)$

35. $\frac{5}{4}\ln 2$

36. $(x-1)^2+\ln\sqrt{x^2+1}-3\,\mathrm{tg}^{-1}x+C$

37. $\frac{1}{2}\ln(t^2+1)+\frac{1}{2}\ln(t^2+2)-\frac{1}{\sqrt{2}}\,\mathrm{tg}^{-1}\left(\frac{1}{\sqrt{2}}t\right)+C$

38. $x+\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\,\mathrm{tg}^{-1}x+C$

39. $\frac{1}{2}\ln(x^4-x^2+1)+C$

40. $\frac{1}{2}\ln\frac{5}{2}-2\,\mathrm{tg}^{-1}2+\frac{\pi}{2}$

41. $\ln\sqrt{3}-\frac{\pi}{6\sqrt{3}}$

42. $1+\frac{1}{8}\ln 2+\frac{3\pi}{32}$

43. $\frac{3}{2}\ln(x^2+1)-3\,\mathrm{tg}^{-1}x+\sqrt{2}\,\mathrm{tg}^{-1}\left(\frac{1}{\sqrt{2}}x\right)+C$

44. $\ln\left|\mathrm{sen}^2x-3\,\mathrm{sen}\,x+2\right|+C$

Rodovia Itajubá-Lorena, Km 74,5 - Caixa Postal 116
CEP 12600-970 - Lorena - SP
Fax (12) 3153-3133
Tel. (Direto) (12) 3159-5007/3153-3209/3159-5093
www.faenquil.br
Pólo Urbo-Industrial AI-6 - Caixa Postal 116
CEP 12600-970 - Lorena - SP
Fax (12) 3153-3006
Tel. (PABX) (12) 3159-9900