ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO
Departamento de Engenharia Mecatrônica
Sistemas Dinâmicos II para Mecatrônica
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Table of Continuous-time Pulsation Fourier Transform Pairs

	$x(t) = \mathcal{F}_\omega^{-1}\{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t}d\omega$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega) = \mathcal{F}_\omega\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$	
transform	$x(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$	
time reversal	$x(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(-\omega)$	frequency reversal
complex conjugation	$x^*(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(-\omega)$	reversed conjugation
reversed conjugation	$x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(\omega)$	complex conjugation
	$x(t)$ is purely real	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega) = X^*(-\omega)$	even/symmetry
	$x(t)$ is purely imaginary	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega) = -X^*(-\omega)$	odd/antisymmetry
even/symmetry	$x(t) = x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely real	
odd/antisymmetry	$x(t) = -x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely imaginary	
time shifting	$x(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)e^{-j\omega t_0}$	
	$x(t)e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega-\omega_0)$	frequency shifting
time scaling	$x(af)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	
	$\frac{1}{\lvert a\rvert}x\left(\frac{f}{a}\right)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(a\omega)$	frequency scaling
linearity	$ax_1(t)+bx_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$aX_1(\omega)+bX_2(\omega)$	
time multiplication	$x_1(t)x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	frequency convolution
frequency convolution	$x_1(t)*x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X_1(\omega)X_2(\omega)$	frequency multiplication
delta function	$\delta(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$e^{-j\omega t_0}$	
	1	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega)$	delta function
	$e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega-\omega_0)$	shifted delta function
two-sided exponential decay	$e^{-a\lvert t\rvert}\quad a>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2a}{a^2+\omega^2}$	
exponential decay	$e^{-at}u(t)\quad \Re\{a\}>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a+j\omega}$	
reversed exponential decay	$e^{-at}u(-t)\quad \Re\{a\}>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a-j\omega}$	
	$e^{\frac{t^2}{2\sigma^2}}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$	
sine	$\sin(\omega_0 t+\phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j\pi\left[e^{-j\phi}\delta(\omega+\omega_0)-e^{j\phi}\delta(\omega-\omega_0)\right]$	
cosine	$\cos(\omega_0 t+\phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\left[e^{-j\phi}\delta(\omega+\omega_0)+e^{j\phi}\delta(\omega-\omega_0)\right]$	
sine modulation	$x(t)\sin(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{j}{2}\left[X(\omega+\omega_0)-X(\omega-\omega_0)\right]$	
cosine modulation	$x(t)\cos(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2}\left[X(\omega+\omega_0)+X(\omega-\omega_0)\right]$	
squared sine	$\sin^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2\left[2\delta(\omega)-\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$	
squared cosine	$\cos^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2\left[2\delta(\omega)+\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$	
rectangular	$\mathrm{rect}\left(\frac{t}{T}\right)=\begin{cases}1 & \lvert t\rvert\leqslant\frac{T}{2}\\0 & \lvert t\rvert>\frac{T}{2}\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$	
triangular	$\mathrm{triang}\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T} & \lvert t\rvert\leqslant T\\0 & \lvert t\rvert>T\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\,\mathrm{sinc}^2\left(\frac{\omega T}{2}\right)$	
step	$u(t)=1_{[0,+\infty]}(t)=\begin{cases}1 & t\geqslant 0\\0 & t<0\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
signum	$\mathrm{sgn}(t)=\begin{cases}1 & t\geqslant 0\\-1 & t<0\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2}{j\omega}$	
sinc	$\mathrm{sinc}(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\,\mathrm{rect}\left(\frac{\omega}{2\pi T}\right)=\frac{1}{T}1_{[-\pi T,+\pi T]}(f)$	
squared sinc	$\mathrm{sinc}^2(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\,\mathrm{triang}\left(\frac{\omega}{2\pi T}\right)$	
integration	$\int_{-\infty}^{t}x(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$	
n-th time derivative	$\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$(j\omega)^n X(\omega)$	
n-th frequency derivative	$t^n f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j^n\frac{d^n}{df^n}X(\omega)$	
time inverse	$\frac{1}{t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$-j\pi\,\mathrm{sgn}(\omega)$	

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