TRANSFORMADA DE FOURIER

- CONSIDERE A EXPANSÃO DE FOURIER:

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t}} \quad (1)$$

- ONDE:

$$\boxed{c_n = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} f(t)\, e^{-in\omega_0 t}\, dt} \quad (2)$$

INTERVALO:

$\boxed{-\tau/2 < t < \tau/2}$

$\boxed{\text{PERÍODO} = \tau}$

SENDO:

$$\boxed{\tau = \frac{2\pi}{\omega_0}} \qquad \boxed{\omega_0 = \frac{2\pi}{\tau}}$$

O ÍNDICE u FOI USADO PARA NÃO CAUSAR CONFUSÃO COM t, QUE APARECE NA EQ. 1

ASSIM:

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty}\left[\frac{\omega_0}{2\pi}\int_{-\tau/2}^{\tau/2} f(u)\, e^{-in\omega_0 u}\, du\right] e^{in\omega_0 t}} \quad (3)$$

- QUAL SERIA A FORMA DA EXPANSÃO EM SÉRIE DE FOURIER DE UMA FUNÇÃO NÃO-PERIÓDICA? NESTE CASO, $\tau \to \infty$ NA EQ. 2. ASSIM, $\omega_0 \to 0$. REESCREVENDO A EQUAÇÃO (3), SUBSTITUINDO $\boxed{\omega_0 \to \Delta\omega}$:

$$f(t) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\Delta\omega \int_{-\tau/2}^{\tau/2} f(u)\, e^{-in\Delta\omega u}\, e^{in\Delta\omega t}\, du$$

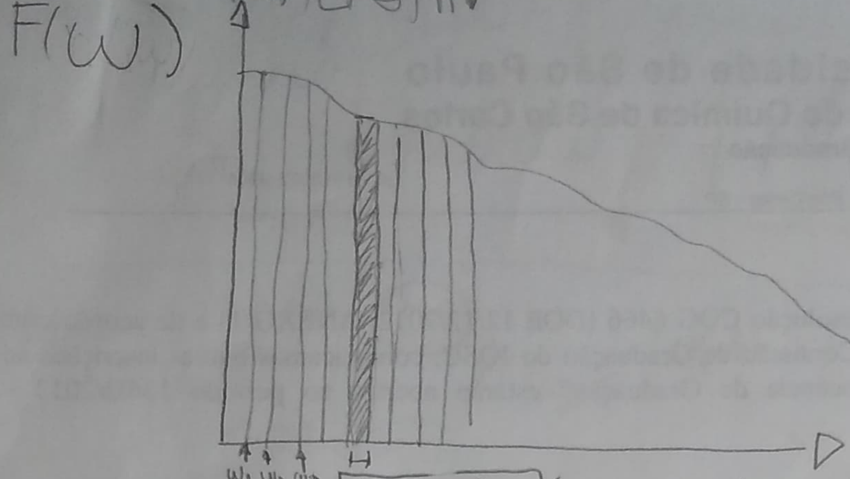
- COMO $\tau \to \infty$:

$$\boxed{f(t)} = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\Delta\omega \underbrace{\int_{-\infty}^{\infty} f(u)\, e^{in\Delta\omega(t-u)}\, du}_{F(n\Delta\omega)\ (\text{PARA } t \text{ CTE})} = \boxed{\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\Delta\omega\, F(n\Delta\omega)} \quad (4)$$

$$\boxed{\int_a^b f(x)\, dx = n^{\underline{o}}}$$

QUE NÃO DEPENDE DE x

• VAMOS ANALISAR:



$\boxed{\omega_n = n\Delta\omega}$ $(\Delta\omega = CTE)$

$\boxed{n = -\infty, \dots, -1, 0, 1, 2, 3, 4 \dots, +\infty}$

$$\int_{-\infty}^{\infty} F(\omega)\,d\omega \approx \sum_{n=-\infty}^{\infty} F(\omega_n)\cdot\Delta\omega_n = \sum_{n=-\infty}^{\infty} F(n\Delta\omega)\,\Delta\omega$$

$\Delta\omega \to 0$

$\Delta\omega = CTE$

• QUANDO $\Delta\omega \to 0$, $n\Delta\omega = n\,d\omega = \omega$. ASSIM

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, d\omega$$

$$f(t) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}\left[\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} f(u)\, e^{-i\omega u}\, du\right] e^{i\omega t}\, d\omega$$

$$\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{i\omega(t-u)}\, du\, d\omega} \quad (5)$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{-i\omega u}\, e^{i\omega t}\, du\, d\omega$$

• DESTA FORMA, DEFININDO

$$\boxed{\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \hat{F}(\omega)} \quad (6)$$

COMPARE, VOLTANDO A USAR t NO LUGAR DE u:

• A EQUAÇÃO (5) SE TORNA:

$$\boxed{f(t) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} \hat{F}(\omega)\, e^{i\omega t}\, d\omega} \quad (7)$$

$\hat{F}(\omega)$: TRANSFORMADA DE FOURIER DE $f(t)$

ALGUMAS TRANSFORMADAS DE FOURIER

EX1 $\boxed{f(t) = e^{-\alpha|t|}}$ $\boxed{-\infty < t < \infty}$

$$\hat{F}(\omega) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} e^{-\alpha|t|}\, e^{-i\omega t}\, dt$$

MAS $e^{-i\omega t} = \cos(\omega t) - i\,\mathrm{sen}(\omega t)$

$$\hat{F}(\omega) = \frac{1}{(2\pi)^{1/2}}\left[\int_{-\infty}^{\infty} \cos(\omega t)\, e^{-\alpha|t|}\, dt - i\, \cancel{\int_{-\infty}^{\infty} \operatorname{sen}(\omega t)\, e^{-\alpha|t|}\, dt}\right] \to 0$$

$\boxed{e^{-\alpha|t|}}$ ⇒ FUNÇÃO PAR $\boxed{f(t) = f(-t)}$

$$\boxed{\hat{F}(\omega)} = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \cos(\omega t)\, e^{-\alpha|t|}\, dt$$

MAS:

$$\boxed{\int_0^{\infty} e^{-\alpha t} \cos(\omega t)\, dt = \frac{\alpha}{\alpha^2+\omega^2}} \quad \boxed{\text{PARA } \alpha > 0}$$

PARA FUNÇÕES PARES, A INTEGRAL É SIMÉTRICA!!!

$$\boxed{\hat{F}(\omega)} = \frac{\boxed{2}}{(2\pi)^{1/2}} \cdot \frac{\alpha}{\alpha^2+\omega^2} = \boxed{\left(\frac{2}{\pi}\right)^{1/2} \frac{\alpha}{\alpha^2+\omega^2}}$$

- PARA REENCONTRAR $f(t)$ A PARTIR DE $\hat{F}(\omega)$, USAMOS A EQ. 7:

$$f(t) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \hat{F}(\omega)\, e^{i\omega t}\, d\omega = \frac{1}{(2\pi)^{1/2}} \left(\frac{2}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} \frac{\alpha}{\alpha^2+\omega^2}\, e^{i\omega t}\, d\omega$$

$$f(t) = \frac{\alpha}{\pi} \int_{-\infty}^{\infty} \frac{1}{\alpha^2+\omega^2}\, e^{i\omega t}\, d\omega = \frac{\alpha}{\pi} \int_{-\infty}^{\infty} \left[\frac{1}{\alpha^2+\omega^2}\left(\cos(\omega t) + i \operatorname{sen}(\omega t)\right)\right] d\omega$$

$$f(t) = \frac{\alpha}{\pi}\left[\int_{-\infty}^{\infty} \frac{1}{\alpha^2+\omega^2} \cos(\omega t)\, d\omega + i\, \cancel{\int_{-\infty}^{\infty} \frac{1}{\alpha^2+\omega^2} \operatorname{sen}(\omega t)\, d\omega}\right] \to 0$$

$\boxed{\frac{1}{\alpha^2+\omega^2}}$ ⇒ FUNÇÃO PAR ($f(\omega) = f(-\omega)$)

$$\boxed{f(t)} = \frac{\alpha}{\pi}\int_{-\infty}^{\infty}\frac{\cos(\omega t)}{\alpha^2+\omega^2}d\omega = \boxed{e^{-\alpha|t|}}$$

WOLFRAM

ALGUMAS TRANSFORMADAS DE FOURIER:

$f(t)$	$\hat{F}(\omega)$
$e^{-a\|t\|}$	$\left(\frac{2}{\pi}\right)^{1/2}\frac{a}{\omega^2+a^2}$
$e^{-a^2t^2}$	$\frac{1}{(2a^2)^{1/2}}e^{-\omega^2/4a^2}$
$\frac{1}{t^2+a^2}$	$\left(\frac{\pi}{2a^2}\right)^{1/2}e^{-a\|\omega\|}$

HÁ UMA RELAÇÃO ENTRE t E ω DADA PELA TRANSFORMADA