EX: MOSTRE QUE $\boxed{z^{-1} = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}}$ VII

$$z^{-1} = \frac{1}{z} = \frac{1}{x+iy}\left(\frac{x-iy}{x-iy}\right) = \frac{x-iy}{x^2-(i^2)y} = \frac{x-iy}{x^2+y^2}$$

$\boxed{i^2=-1}$

$$\boxed{z^{-1} = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}}$$

EX: DETERMINE A CURVA NO PLANO COMPLEXO DADA POR $|z-1|=2$

$$|z-1| = |x+iy-1| = 2 = |\underbrace{(x-1)}_{\text{PARTE REAL}} + i\underbrace{y}_{\text{PARTE IMAGINÁRIA}}|$$

$$|z-1| = |x'+iy|$$

$\boxed{\text{ONDE } x' = x-1}$

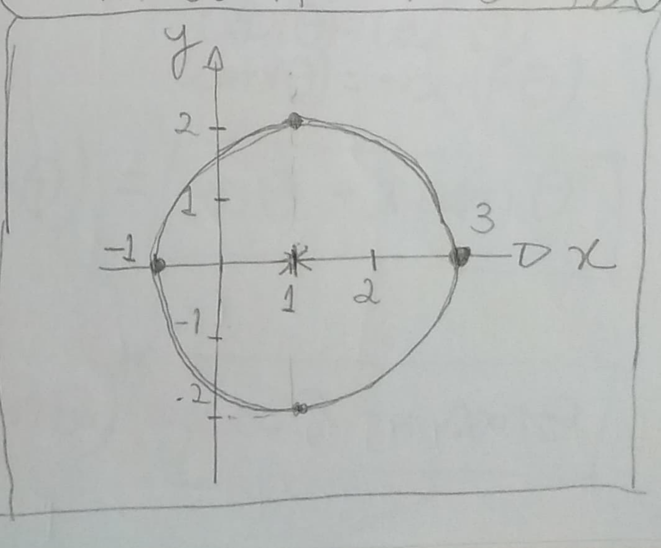
$$|z-1| = ((x'-iy)(x'+iy))^{1/2} = (x'^2+y^2)^{1/2} = 2$$

(ELEVANDO AO QUADRADO)

- LEMBRANDO QUE $x' = (x-1)$:

$$\boxed{x'^2+y^2=4}$$

EQ. DE UM CÍRCULO



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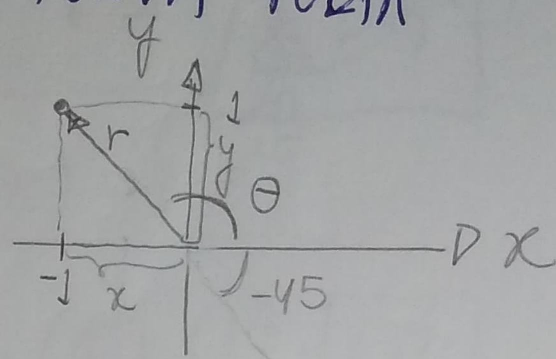
EX: EXPRESSE $z = -1 + i$ NA FORMA POLAR

$z = x + iy$ $\quad \boxed{x = -1} \boxed{y = 1}$



$z = r(\cos\theta + i\,\text{sen}\,\theta)$

$\boxed{r} = |z| = (z^* z)^{1/2} = (x^2 + y^2)^{1/2} = (1+1)^{1/2} = \boxed{\sqrt{2}}$

$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$ $\quad \boxed{\theta = -45°}$ OU $\boxed{\theta = 135°}$

A FUNÇÃO $\tan^{-1}$ É MULTIVALORADA

$$\boxed{\theta = 135° = \frac{3\pi}{4}}$$

$$\boxed{R: z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\,\text{sen}\,\frac{3\pi}{4}\right)}$$

EX: MOSTRE QUE $\underbrace{e^{-i\theta}}_{\text{EULER}} = \underbrace{\cos\theta - i\,\text{sen}\,\theta}_{\text{POLAR}}$

SABEMOS QUE:

$e^{i\theta} = \cos\theta + i\,\text{sen}\,\theta$

ASSIM

$\cos(\theta) = \cos(-\theta)$
$\text{sen}(\theta) = -\text{sen}(-\theta)$

$\boxed{e^{-i\theta}} = \boxed{e^{i(-\theta)}} = \cos(-\theta) + i\,\text{sen}(-\theta) = \boxed{\cos\theta - i\,\text{sen}\,\theta}$

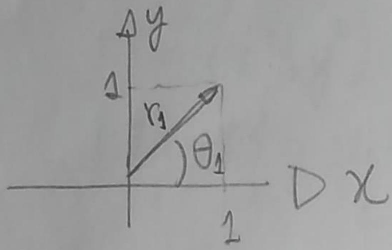
OU AINDA

$\boxed{e^{-i\theta}} = (e^{i\theta})^* = (\cos\theta + i\,\text{sen}\,\theta)^* = \boxed{\cos\theta - i\,\text{sen}\,\theta}$

EX: EXPRESSE $z_1 = 1 + i$ E $z_2 = -1 - i$ EM TERMOS DA FORMULA DE EULER $\boxed{\frac{z_2}{z_1} = ?}$

$$\boxed{z = x + iy = r(\cos\theta + i\,\mathrm{sen}\,\theta) = re^{i\theta}}$$

$$\boxed{r_1} = (z_1^* z_1)^{1/2} = ((1-i)(1+i))^{1/2} = (1+1)^{1/2} = \boxed{\sqrt{2}}$$



$$\boxed{\begin{matrix} x_1 = 1 \\ y_1 = 1 \end{matrix}}$$

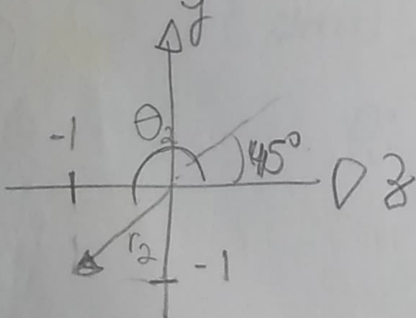
$$\tan\theta_1 = \frac{1}{1} = 1 \qquad \boxed{\theta_1 = \frac{\pi}{4} = 45^\circ}$$

OU

$$\boxed{\theta_1 = \frac{\pi}{4} + \pi = \frac{5\pi}{4} = 225^\circ}$$

R_1: $\boxed{z_1 = \sqrt{2}\, e^{i\frac{\pi}{4}}}$

$$\boxed{r_2} = (z_2^* z_2)^{1/2} = ((-1+i)(-1-i))^{1/2} = (1+1)^{1/2} = \boxed{\sqrt{2}}$$



$$\boxed{\begin{matrix} x_2 = -1 \\ y_2 = -1 \end{matrix}}$$

$$\boxed{\theta_2 = \frac{\pi}{4} = 45^\circ}$$

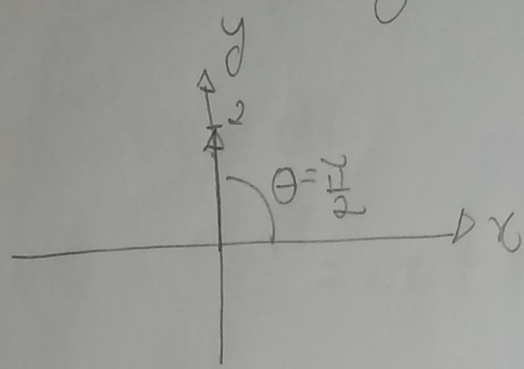
OU

$$\tan\theta_2 = \frac{1}{1} = 1 \qquad \boxed{\theta_2} = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}} = \boxed{225^\circ}$$

R_2: $\boxed{z_2 = \sqrt{2}\, e^{i\frac{5\pi}{4}}}$

EX:

$$z = 2e^{\frac{i\pi}{2}} \rightarrow z = x + iy$$



$$\begin{cases} r = (x^2+y^2)^{1/2} = 2 \\ \tan\frac{\pi}{2} = \frac{y}{x} = \infty \rightarrow x = 0 \end{cases}$$

$$(0+y^2)^{1/2} = 2$$

$$\boxed{y = \pm 2} \qquad \boxed{R: y = 0 + 2i}$$

OU AINDA:

$$\begin{cases} x = r\cos\theta = 2\cos(\pi/2) = 0 \\ y = r\,\mathrm{sen}\,\theta = 2\,\mathrm{sen}(\pi/2) = 2 \end{cases}$$

$$R: \boxed{y = 0 + 2i}$$

EX: ENCONTRE O RESULTADO DAS INTEGRAIS

$$\left[\begin{array}{l} I_1 = \int_0^\infty e^{-\alpha t} \operatorname{sen} t \, dt \\ I_2 = \int_0^\infty e^{-\alpha t} \cos t \, dt \end{array}\right.$$

SABENDO QUE: $\boxed{\alpha > 0}$

SABEMOS QUE $e^{it} = \cos t + i \operatorname{sen} t$

- ASSIM:

$$e^{-\alpha t} \cdot e^{it} = e^{-(\alpha - i)t} = e^{-\alpha t}(\cos t + i \operatorname{sen} t)$$

- ENTÃO:

$$\int_0^\infty e^{-(\alpha-i)t} dt = \underbrace{\int_0^\infty e^{-\alpha t} \cos t \, dt}_{\text{PARTE REAL (Re)}} + i \underbrace{\int_0^\infty e^{-\alpha t} \operatorname{sen} t \, dt}_{\text{PARTE IMAGINÁRIA (Im)}}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$I_2 \qquad\qquad\qquad\qquad I_1$$

- OU SEJA:

$$I_2 = \operatorname{Re}\left[\int_0^\infty e^{-(\alpha-i)t} dt\right] \qquad I_1 = \operatorname{Im}\left[\int_0^\infty e^{-(\alpha-i)t} dt\right]$$

- MAS

$$\int_0^\infty e^{-(\alpha-i)t} dt = \left.\frac{e^{-(\alpha-i)t}}{-(\alpha-i)}\right]_0^\infty = -\frac{1}{\alpha-i}\left[e^{-(\alpha-i)t}\right]_0^\infty = -\frac{1}{\alpha-i}\left[e^{-\alpha t} \cdot e^{it}\right]_0^\infty$$

$$\int_0^\infty e^{-(\alpha-i)t} dt = -\frac{1}{\alpha-i}\left[e^{-\alpha t} \cdot (\cos t + i \operatorname{sen} t)\right]_0^\infty = -\frac{1}{\alpha-i}\left[\underset{\boxed{\text{POIS } \alpha > 0}}{\underset{\uparrow}{0}} - 1 \cdot (1+0)\right] = \frac{1}{\alpha-i}$$

5

$$a = (\alpha - i)$$

$$\lim_{t \to \infty} e^{-at} = \lim_{t \to \infty} \left(e^{-\alpha t} \cdot e^{it}\right) = \lim_{t \to \infty} \left(e^{-\alpha t}(\cos t + i \operatorname{sen} t)\right)$$

$$\alpha > 0$$

$$\lim_{t \to \infty} e^{-\alpha t} = 0$$

$\cos t \Rightarrow$ VALOR ENTRE -1 E 1

$\operatorname{sen} t \Rightarrow$ VALOR ENTRE -1 E 1

$$\boxed{\lim_{t \to \infty} e^{-at} = 0}$$

$$\frac{1}{\alpha - i} = \frac{1}{\alpha - i} \cdot \frac{\alpha + i}{\alpha + i} = \frac{\alpha + i}{\alpha^2 + 1} = \underbrace{\frac{\alpha}{\alpha^2+1}}_{Re} + i \underbrace{\frac{1}{\alpha^2+1}}_{Im}$$

ASSIM:

$$\boxed{I_2 = \frac{\alpha}{\alpha^2+1}} \qquad \boxed{I_1 = \frac{1}{\alpha^2+1}}$$