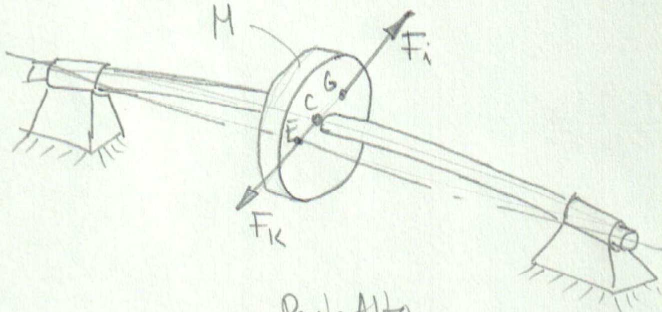
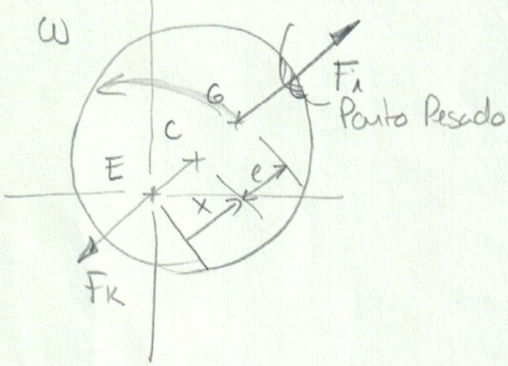
③ Balanceamento de Rotores Flexíveis e Velocidades Críticas:

Roteiro:

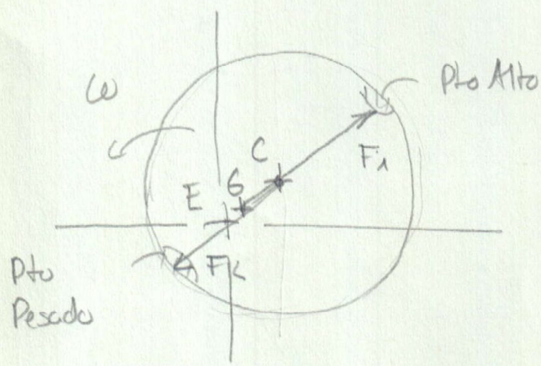
- velocidades críticas (revisão)
- Balanceamento de Rotores Flexíveis (ISO 11342: 1998)
  - Balanceamento Modal
  - Método dos Coeficientes de Influência para n planos de correção;
- Qualidade de Funcionamento (ISO 10816)
- experimento
  - velocidade crítica {analítica / experimental
  - balanceamento do rotor flexível em um plano de correção com medição de fase;
  - avaliação da qualidade de funcionamento.

Velocidade Crítica de Rotores (Vibração Transversal)





$\omega \ll \omega_{cr}$



$\omega \gg \omega_{cr}$

equilíbrio: $F_i = F_k$

$$M(x+e)\omega^2 = kx \Rightarrow kx - M\omega^2 x = Me\omega^2$$

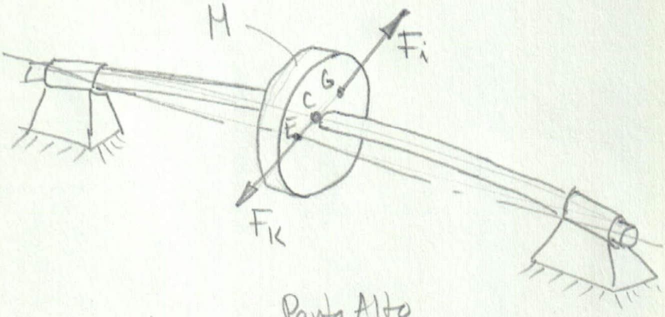
$$\boxed{x = \frac{Me\omega^2}{k - \omega^2 M}} \quad k - \omega^2 M = 0 \Rightarrow \boxed{\omega_{cr} = \sqrt{\frac{k}{M}}}$$

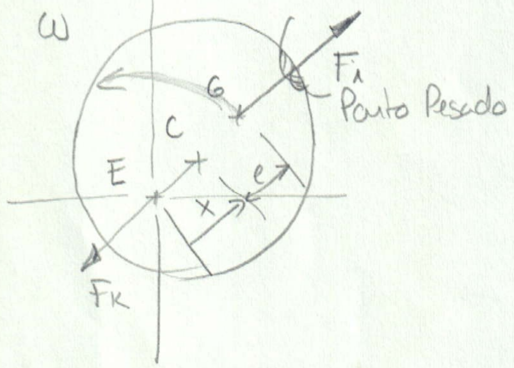
Velocidade Crítica

$$\div M \Rightarrow \quad x = \frac{e\,\omega^2}{\omega_{cr}^2 - \omega^2} = \frac{e}{(\omega_{cr}/\omega)^2 - 1}$$

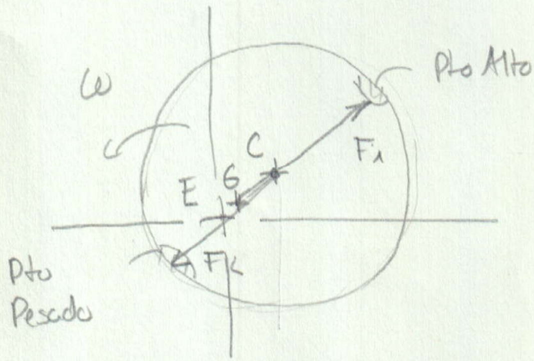
$$\div k \Rightarrow \quad x = \frac{e\,(\omega/\omega_{cr})^2}{1 - (\omega/\omega_{cr})^2}$$

Velocidade Crítica de Rotores (Vibração Transversal)





$\omega \ll \omega_{CR}$



$\omega \gg \omega_{CR}$

equilíbrio: $F_i = F_k$

$$M(x+e)\omega^2 = kx \Rightarrow kx - M\omega^2 x = Me\omega^2$$

$$\boxed{x = \frac{Me\omega^2}{k - \omega^2 M}} \qquad k - \omega^2 M = 0 \Rightarrow \boxed{\omega_{CR} = \sqrt{\frac{k}{M}}}$$

Velocidade Crítica

$$\div M \Rightarrow x = \frac{e\,\omega^2}{\omega_{CR}^2 - \omega^2} = \frac{e}{(\omega_{CR}/\omega)^2 - 1}$$

$$\div k \Rightarrow x = \frac{e\,(\omega/\omega_{CR})^2}{1 - (\omega/\omega_{CR})^2}$$

$\omega = 0 \Rightarrow x = 0$

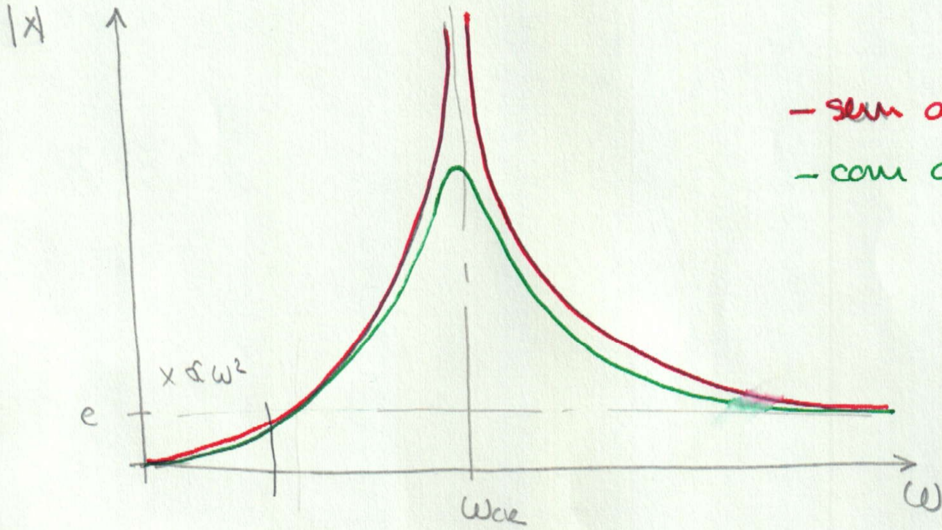
$\omega \ll \omega_{cr} \Rightarrow \frac{\omega}{\omega_{cr}} \ll 1 \Rightarrow x = \frac{e}{\omega_{cr}^2}\omega^2$

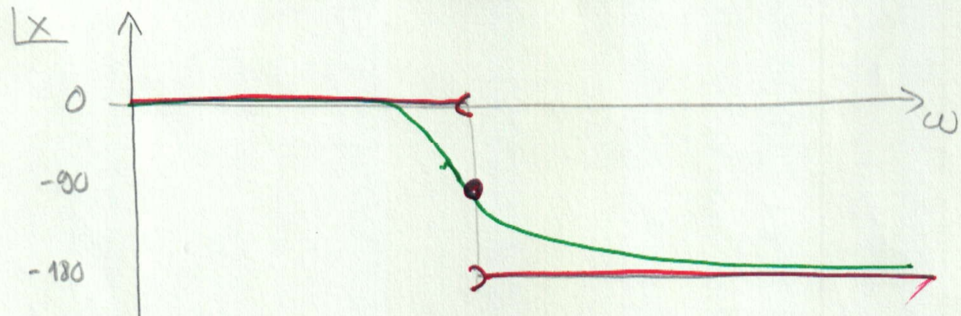
$\omega < \omega_{cr} \Rightarrow x > 0$
pto alto e pto pesado em fase

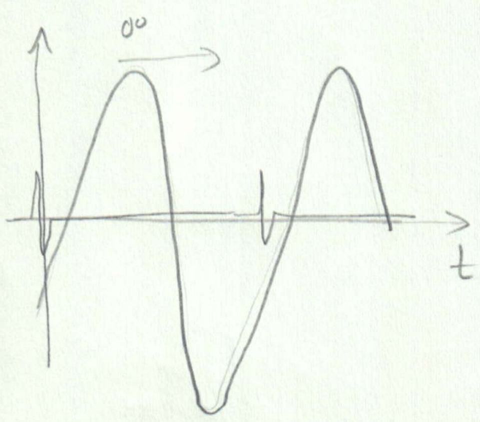
$\omega = \omega_{cr} \Rightarrow x \to \infty$

$\omega \gg \omega_{cr} \Rightarrow \frac{\omega_{cr}}{\omega} \ll 1 \Rightarrow x = -e$

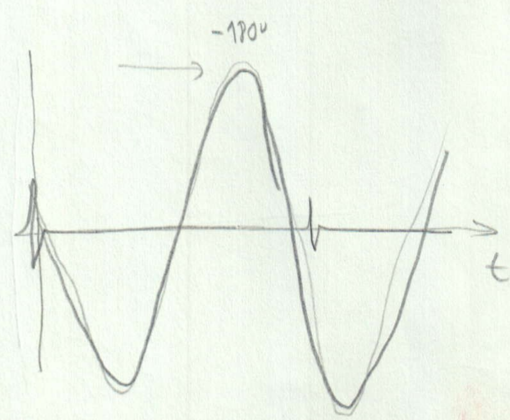
$\omega > \omega_{cr} \Rightarrow x < 0$
pto alto e pto pesado em contra-fase.







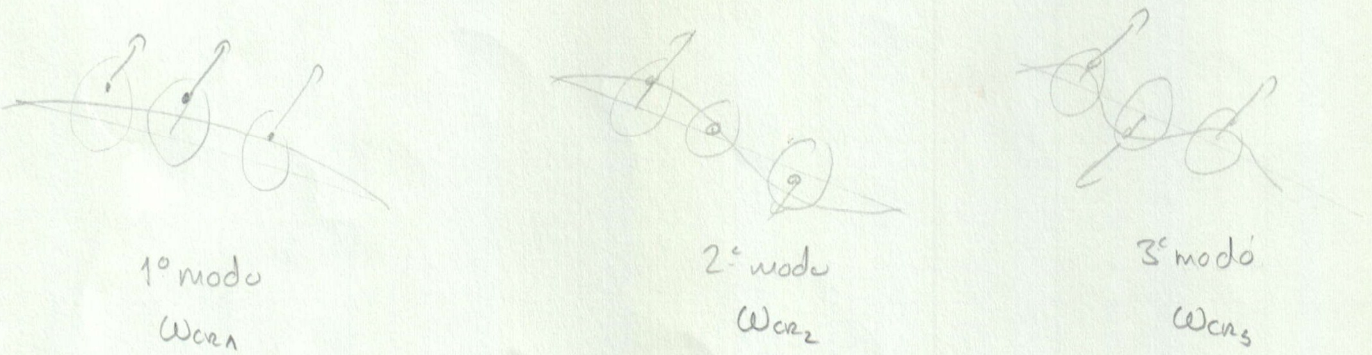
$\omega < \omega_{cr}$

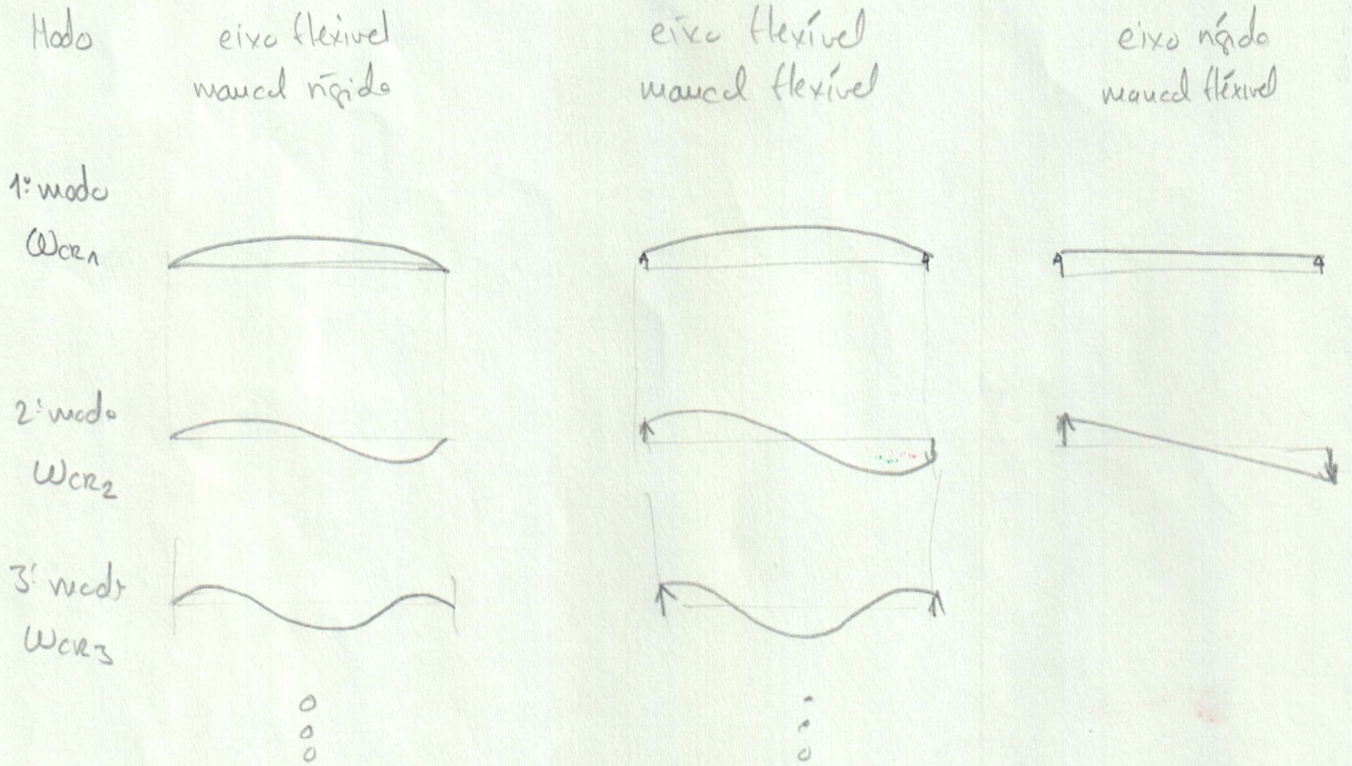


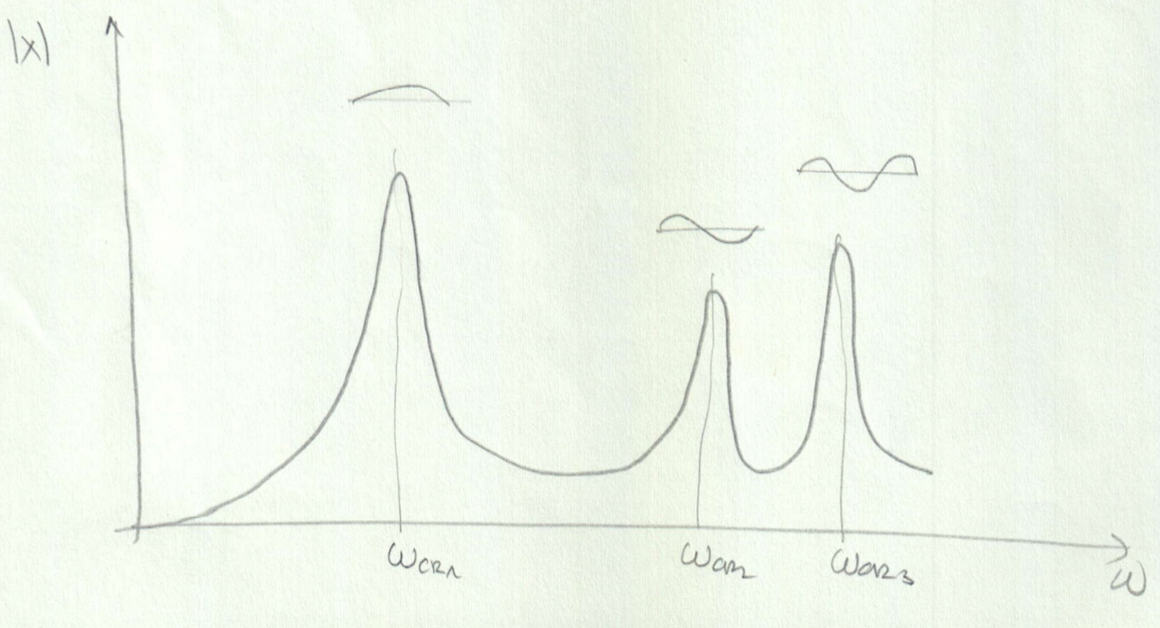
$\omega > \omega_{cr}$

Balanceamento de Rotores Flexíveis:

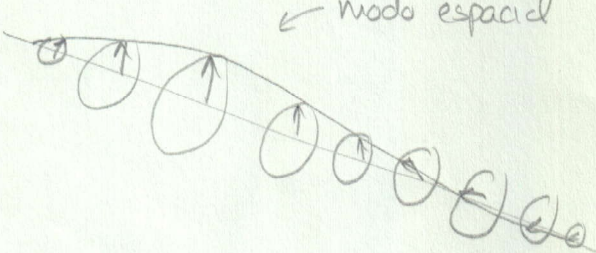
- modos críticos

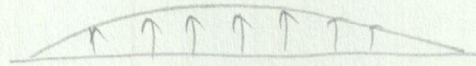






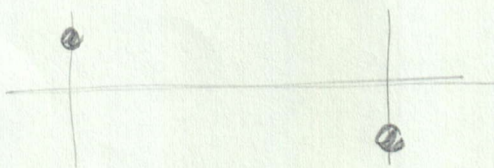
Balanceamento Modal:





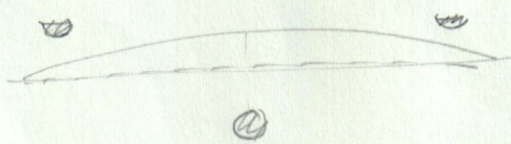
nº de planos de correção:

$\omega \ll \omega_{cr_1}$
rotor rígido



$n = 2$

1º modo
$\omega \simeq \omega_{cr_1}$

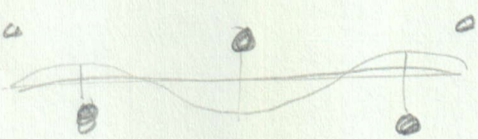


$n = 3$

2º modo
$\omega \simeq \omega_{cr_2}$

$n = 4$

3º modo
$\omega \simeq \omega_{cr_3}$



$n = 5$

Balanceamento pelo
Método dos Coeficientes de Influência p/ n planos de correção:

$$\begin{bmatrix} \alpha_{AI} & \alpha_{AII} & \alpha_{AIII} & \cdots & \alpha_{An} \\ \alpha_{BI} & \alpha_{BII} & \alpha_{BIII} & \cdots & \alpha_{Bn} \end{bmatrix} \begin{Bmatrix} m_I \vec{r}_I \\ m_{II} \vec{r}_{II} \\ m_{III} \vec{r}_{III} \\ \vdots \\ m_n \vec{r}_n \end{Bmatrix} = \begin{Bmatrix} \vec{\delta}_A \\ \vec{\delta}_B \end{Bmatrix}$$

$(2 \times n)$ $(n \times 1)$ (2×1)

nº de incógnitas n > nº de equações 2

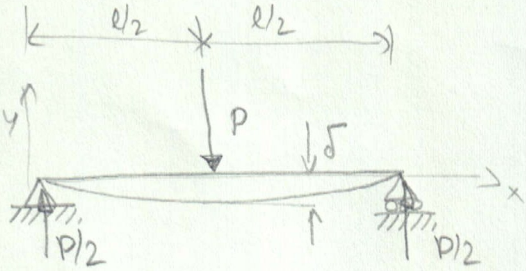
$$\begin{bmatrix} \alpha_{AI_1} & \alpha_{AII_1} & \cdots & \alpha_{An_1} \\ \alpha_{AI_2} & \alpha_{AII_2} & \cdots & \alpha_{An_2} \\ \vdots & & & \\ \alpha_{BI_1} & \alpha_{BII_1} & \cdots & \alpha_{Bn_1} \\ \alpha_{BI_2} & \alpha_{BII_2} & \cdots & \alpha_{Bn_2} \end{bmatrix} \begin{Bmatrix} m_I \vec{r}_I \\ m_{II} \vec{r}_{II} \\ \vdots \\ m_n \vec{r}_n \end{Bmatrix} = \begin{Bmatrix} \vec{\delta}_A(\omega_1) \\ \vec{\delta}_A(\omega_2) \\ \vdots \\ \vec{\delta}_B(\omega_1) \\ \vec{\delta}_B(\omega_2) \\ \vdots \end{Bmatrix}$$

matriz dos coef. de influência $(n \times n)$ — desbalanceamento $(n \times 1)$ — resposta $(n \times 1)$

$$A \vec{u} = \vec{\delta} \Rightarrow \boxed{\vec{u} = A^{-1} \vec{\delta}}$$

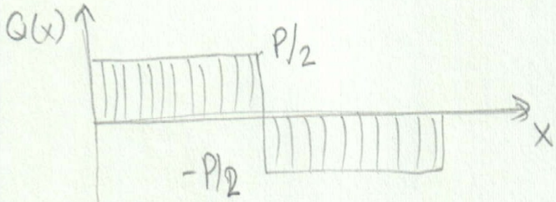
não há garantia que $\exists A^{-1}$

é necessário escolher os pontos de medição e as velocidades para balanceamento. (balanceamento modal).



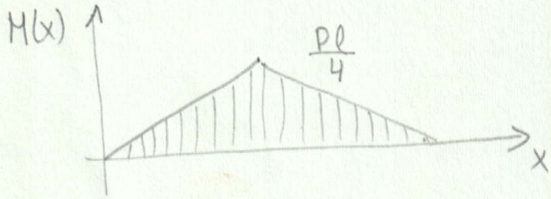
$P = k\delta \Rightarrow k = \dfrac{P}{\delta}$

↑ rigidez



$0 \leq x \leq l/2$

$$M(x) = \frac{Px}{2}$$



$l/2 \leq x \leq l$

$$M(x) = \frac{Pl}{2} - \frac{Px}{2}$$

linha elástica: $\dfrac{d^2y}{dx^2} = \dfrac{-M(x)}{EI}$

p/ $0 \leq x \leq l/2$

$$\frac{d^2y}{dx^2} = -\frac{Px}{2EI} \Rightarrow \varphi(x) = \frac{dy}{dx} = \int -\frac{Px}{2EI}\,dx + C_1$$

$$\varphi(x) = -\frac{Px^2}{4EI} + C_1$$

$$y(x) = \int -\frac{Px^2}{4EI}\,dx + \int C_1\,dx + C_2$$

$$y(x) = -\frac{Px^3}{12EI} + C_1 x + C_2$$

Condições de Contorno (C.C.) $y(0) = 0 \qquad y(l) = 0$

simplesmente apoiado

por simetria: $\varphi(l/2) = 0$

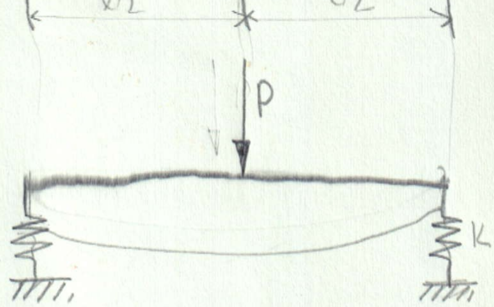
$$y(0) = 0 + 0 + C_2 = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\varphi(l/2) = -\frac{Pl^2}{16EI} + C_1 = 0 \Rightarrow \boxed{C_1 = \frac{Pl^2}{16EI}}$$

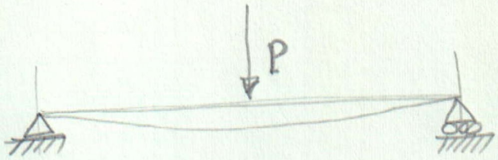
$$\boxed{k = \frac{P}{\delta} = \frac{48EI}{l^3}}$$

⇑

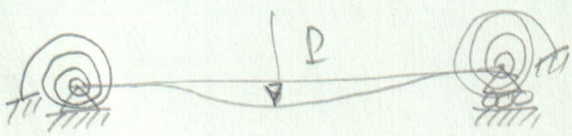
$$y(x) = \frac{Pl^3}{4EI}\left(\frac{1}{4}\frac{x}{l} - \frac{1}{3}\frac{x^3}{l^3}\right) \Rightarrow y(l/2) = \frac{Pl^3}{4EI}\left(\frac{1}{8} + \frac{1}{8 \cdot 3}\right) \Rightarrow \delta = y(l/2) = \frac{Pl^3}{48EI}$$



$$k = \frac{2k\,k'}{k' + 2k} \qquad k' = 48\,\frac{EI}{l^3}$$

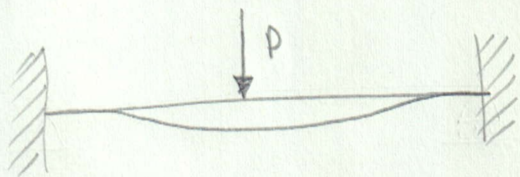


$$k = 48\,\frac{EI}{l^3}$$



$$k = \alpha\,\frac{EI}{l^3}$$

$48 < \alpha < 192$

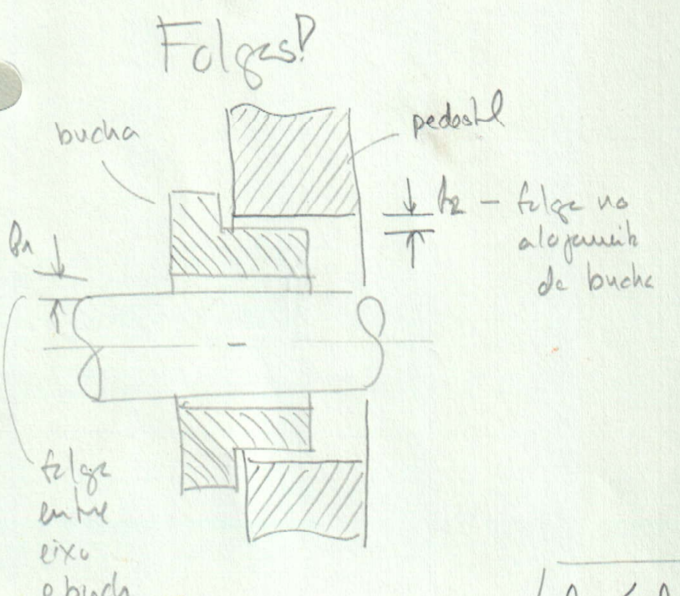


$$k = 192\,\frac{EI}{l^3}$$

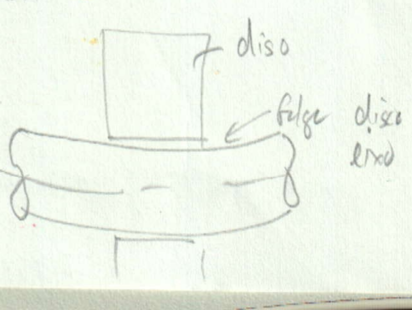
$$P = k\,\delta \;\Rightarrow\; \boxed{k = \frac{P}{\delta}}$$

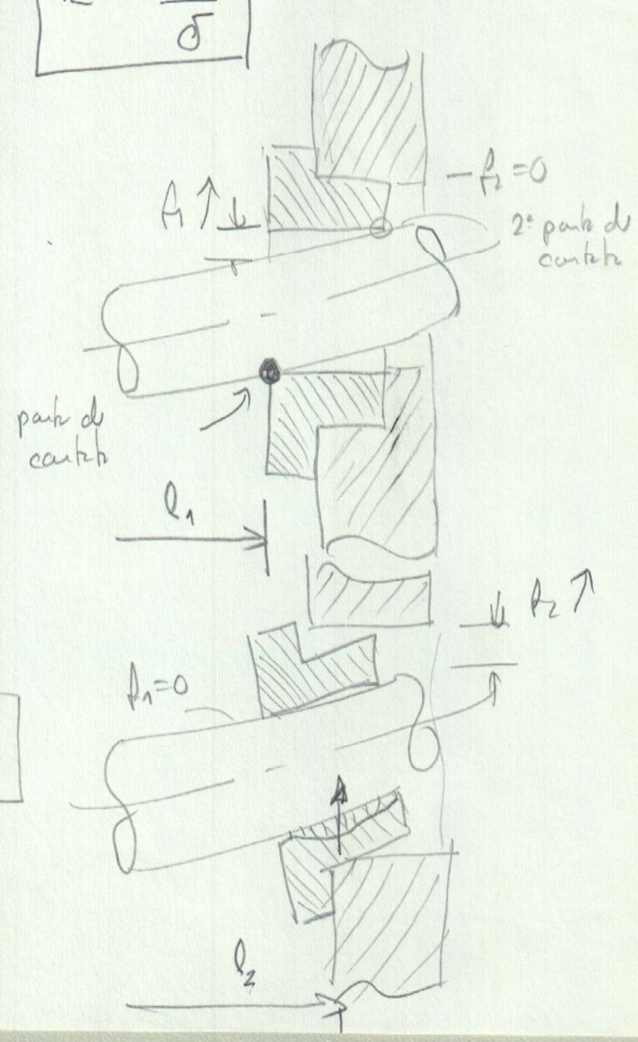
Comprimento efetivo do eixo:

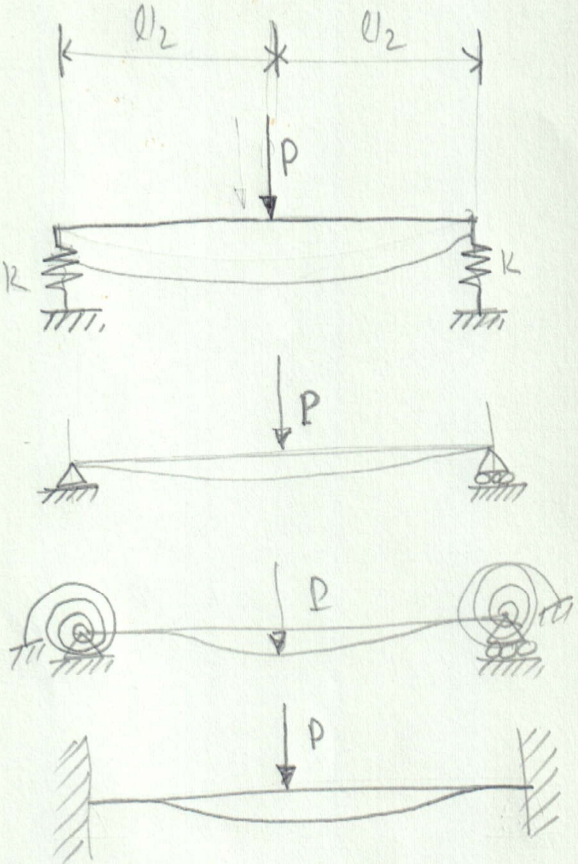
Folgas!



$$\boxed{l_1 < l < l_2}$$







$$k = \frac{2K\,k'}{k' + 2K} \qquad k' = 48\,\frac{EI}{l^3}$$

$$k = 48\,\frac{EI}{l^3}$$

$$k = \alpha\,\frac{EI}{l^3}$$

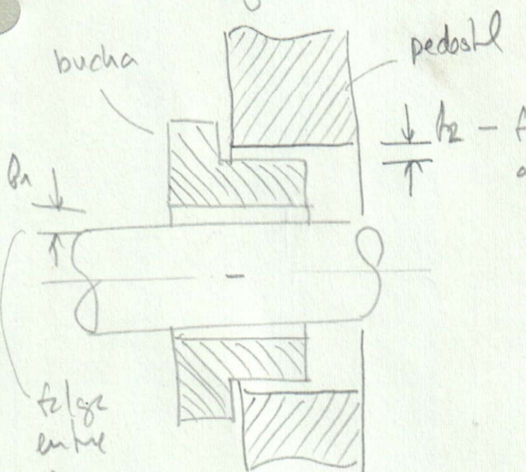
$$48 < \alpha < 192$$

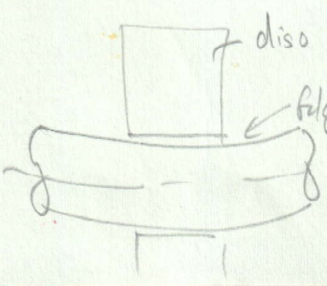
$$k = 192\,\frac{EI}{l^3}$$

$$P = k\,\delta \Rightarrow \boxed{k = \frac{P}{\delta}}$$

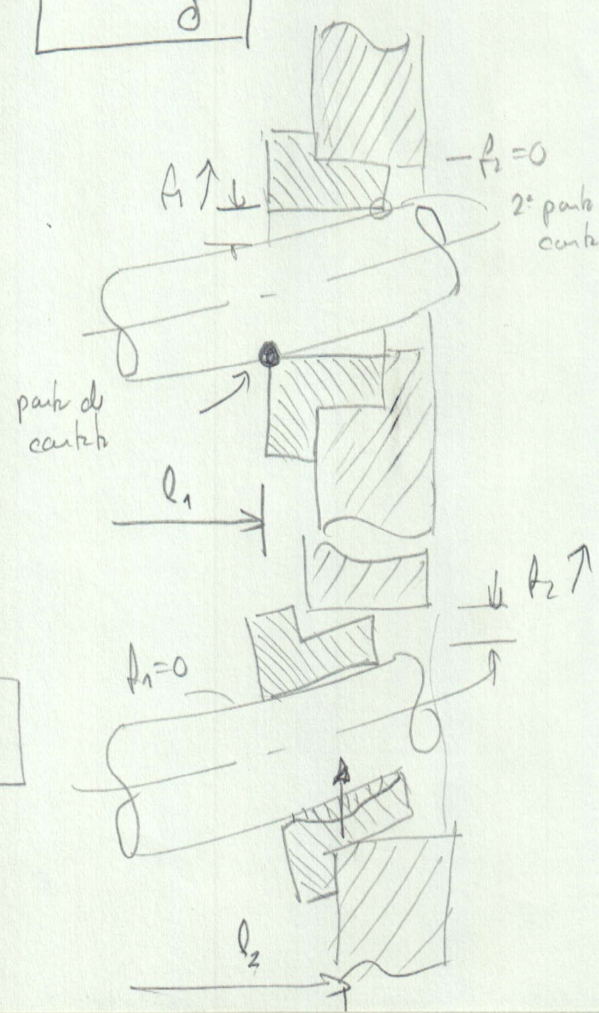
Comprimento efetivo do eixo:

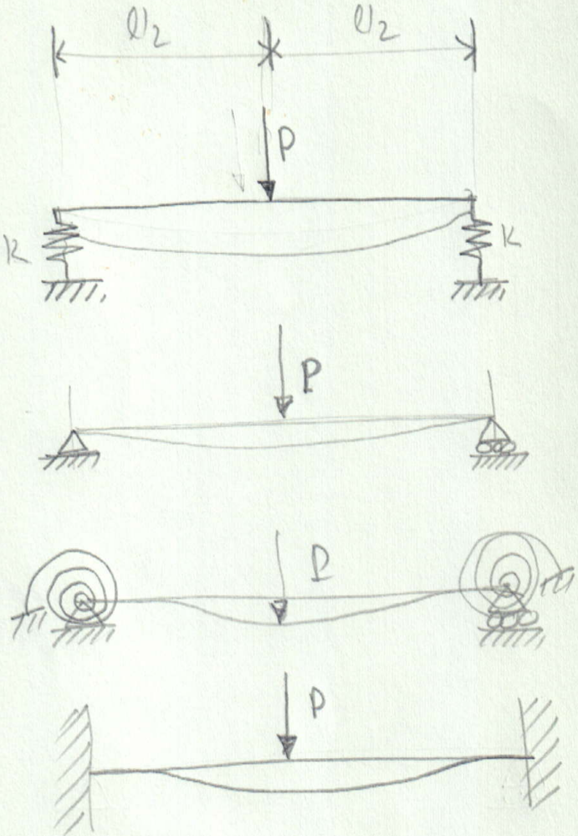
Folgas!





$$\boxed{l_1 < l < l_2}$$





$$k = \frac{2K\,k'}{k' + 2K} \qquad k' = 48\,\frac{EI}{l^3}$$

$$k = 48\,\frac{EI}{l^3}$$

$$k = \alpha\,\frac{EI}{l^3}$$

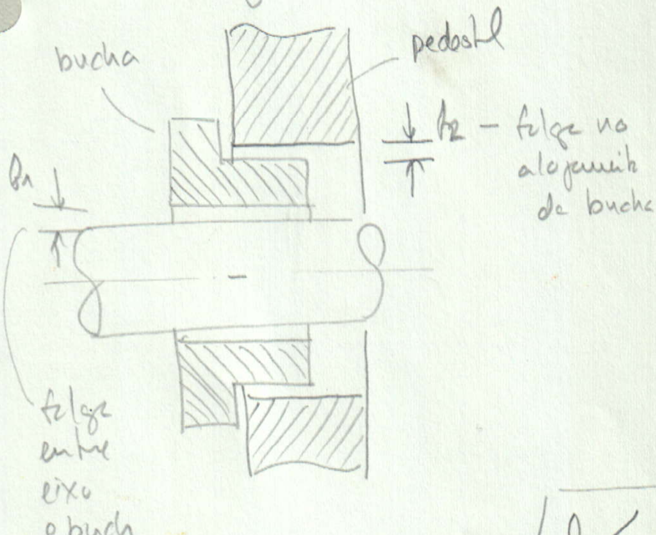
$48 < \alpha < 192$

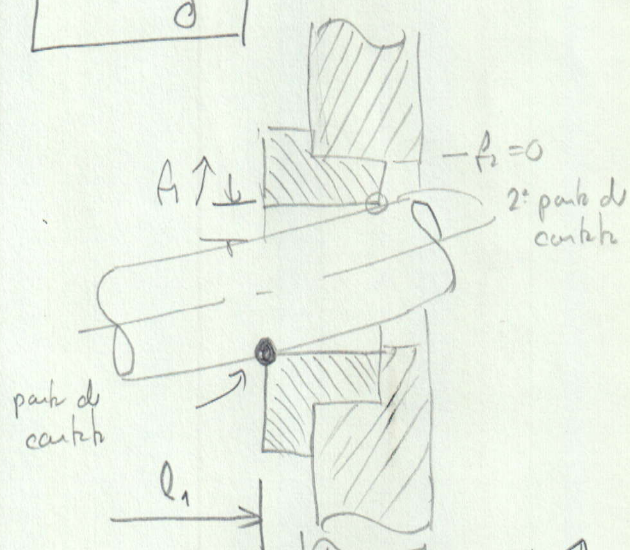
$$k = 192\,\frac{EI}{l^3}$$

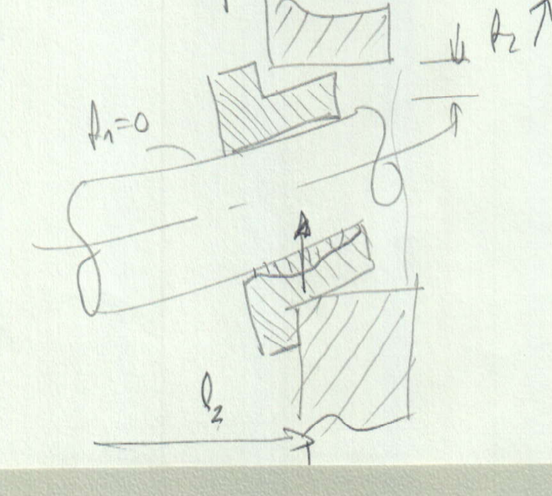
$$P = k\,\delta \Rightarrow \boxed{k = \frac{P}{\delta}}$$

Comprimento efetivo do eixo:

Folgas!



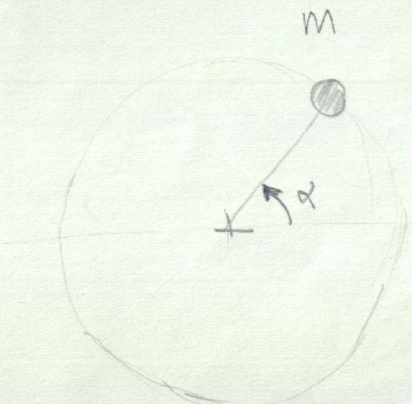




$$\boxed{l_1 < l < l_2}$$

Plano de Correção

Vibração de Resposta ao Desbalanceamento

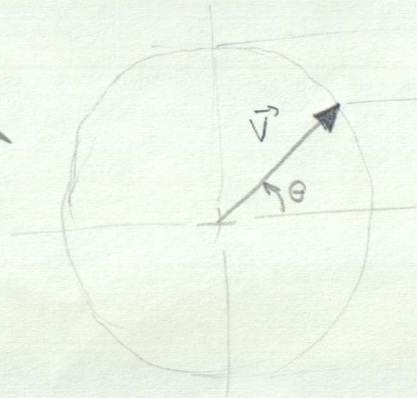


Função de Transferência

$V \propto m$

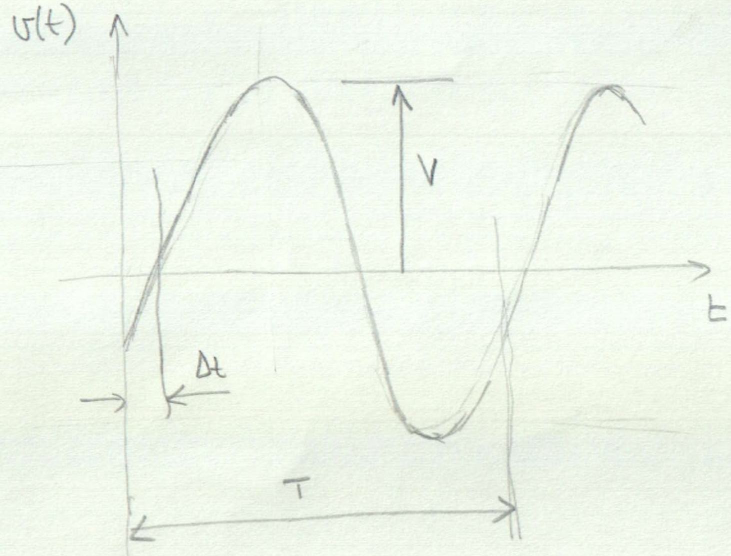
$\theta = \alpha + \Delta\theta$

$\omega = cte$



$\vec{v} = V \underline{|\theta}$

$\vec{v} = V \sin\theta \, \vec{i} + V \cos\theta \, \vec{j}$

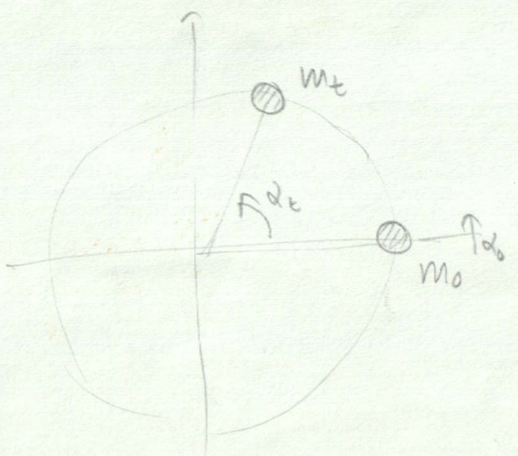


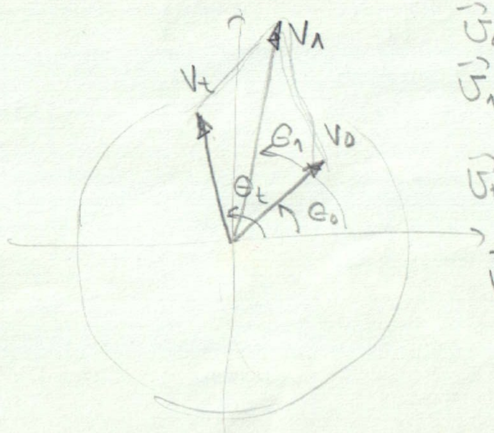
$v(t) = V \sin(\omega t - \theta)$

$$\boxed{\theta = 2\pi \frac{\Delta t}{T}}$$

2π — T

θ — Δt





resposta ao

$\vec{v}_0$ - desbal. inicial

$\vec{v}_1$ - desbal. inicial + massa de teste

$\vec{v}_t$ - massa de teste

$\vec{v}_1 = \vec{v}_0 + \vec{v}_t$

$\vec{v}_t = \vec{v}_1 - \vec{v}_0$

$\begin{cases} m_t \text{ — } V_t \\ m_0 \text{ — } V_0 \end{cases}$

$$\boxed{m_0 = m_t \, V_t / V_0}$$

$\theta_0 = \alpha_0 + \Delta\theta$

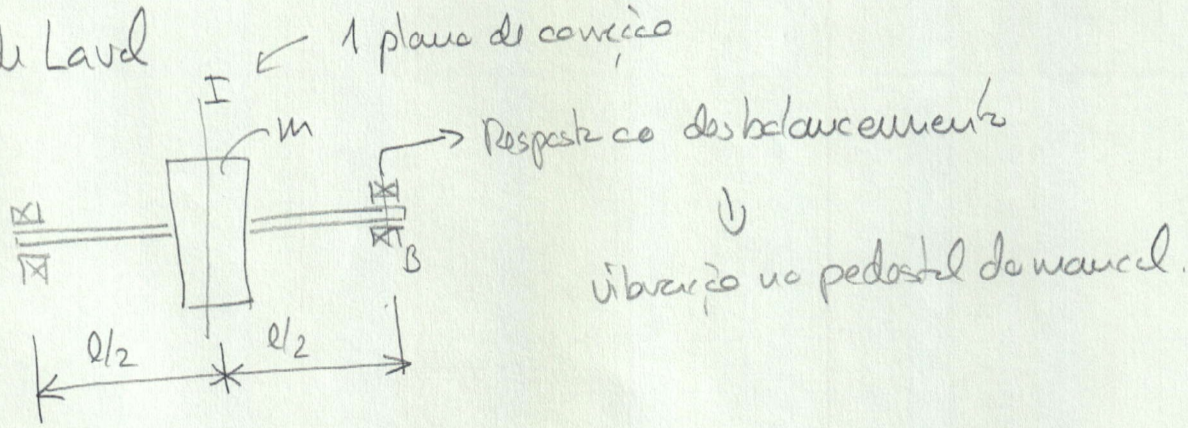
$\theta_t = \alpha_t + \Delta\theta$ (-)

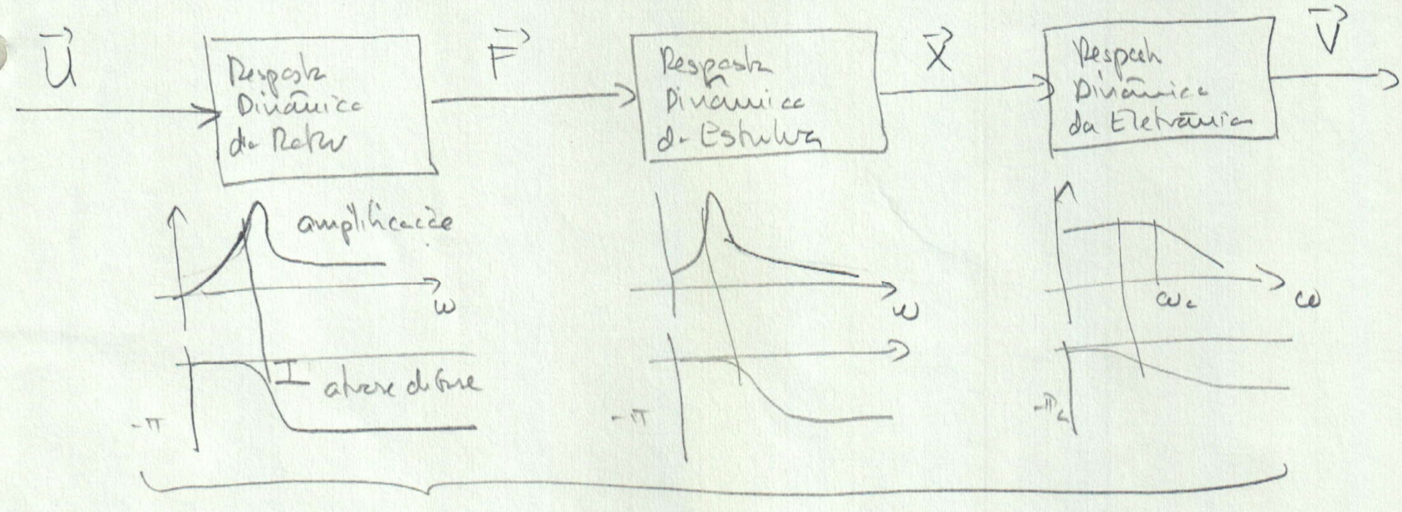
$$\boxed{\theta_0 = \theta_t + (\alpha_0 - \alpha_t)}$$

Metodo dos Coeficientes de Influência

p/ um plano de correção

Rotor de Laval





atraso $\Delta\theta$

$$\hat{V} = V \angle \theta = V e^{i\theta}$$

$$\hat{U} = m \angle \alpha = m e^{i\alpha}$$

$$\hat{\alpha} = A \angle \Delta\theta = A e^{i\Delta\theta}$$

$$\boxed{\hat{\alpha}\,\hat{U} = \hat{V}}$$

- $\hat{V}$ → resposta
- $\hat{U}$ → desbalanceamento
- $\hat{\alpha}$ → coeficiente de influência (complexo)

assim:

$$A e^{i\Delta\theta} \cdot m e^{i\alpha} = V e^{i\theta} \Rightarrow A \cdot m \; e^{i(\alpha + \Delta\theta)} = V e^{i\theta}$$

logo:

$$A \cdot m = V \Rightarrow A = \frac{V}{m} \left[\frac{mm/s}{g}\right] \rightarrow V \propto m \Rightarrow \begin{cases} V \longrightarrow m \\ V' \longrightarrow m' \end{cases}$$

//

$$\alpha + \Delta\theta = \theta$$

$$(-)\quad \alpha' + \Delta\theta = \theta'$$

$$\alpha - \alpha' = \theta - \theta' \Rightarrow \alpha = \alpha' + (\theta - \theta') \; //$$

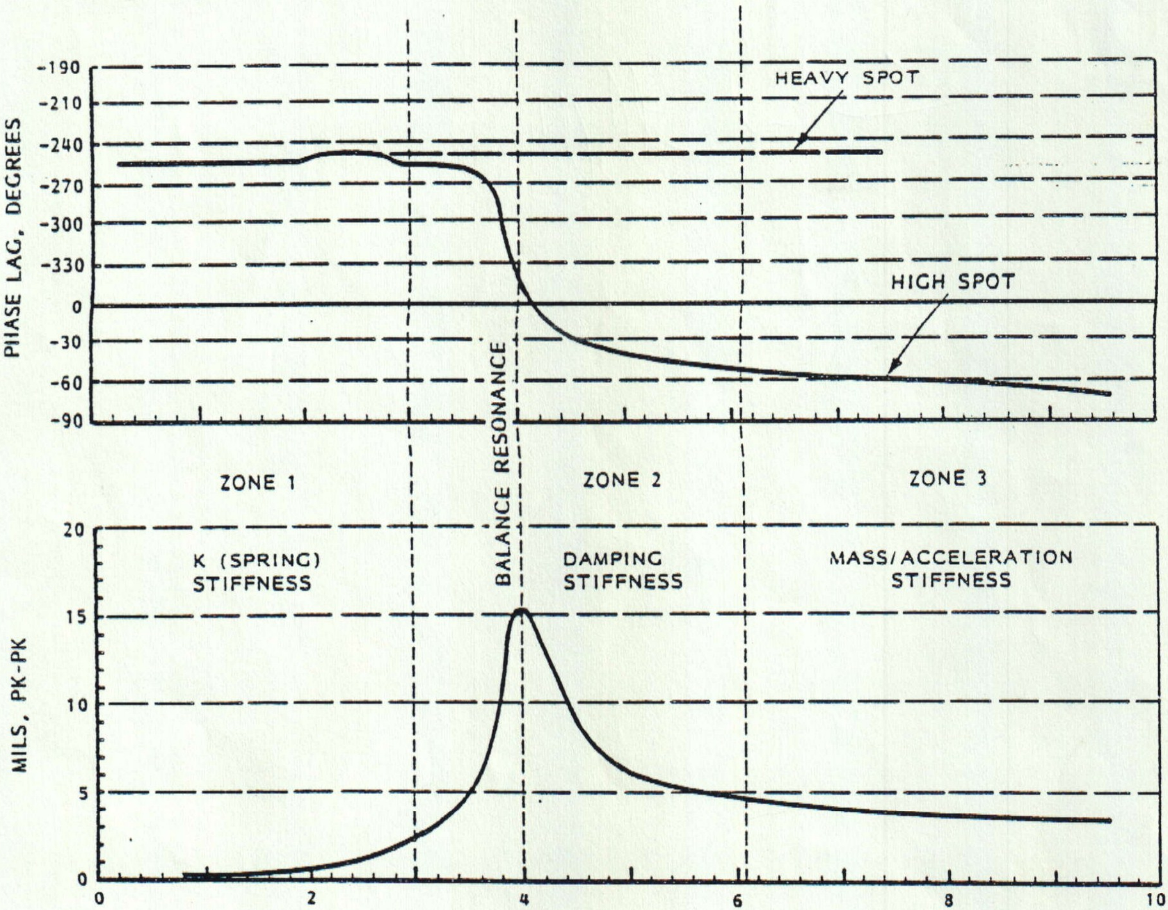


Figure 5. Typical Dynamic Motion Response

INTERNATIONAL STANDARD

ISO 11342

Second edition
1998-04-15

Mechanical vibration — Methods and criteria for the mechanical balancing of flexible rotors

Vibrations mécaniques — Méthodes et critères pour l'équilibrage mécanique des rotors flexibles

Table 1 — Flexible rotors

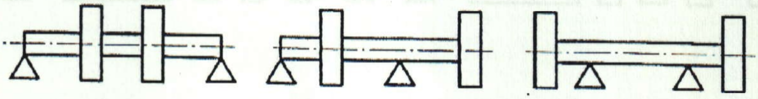
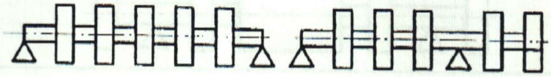
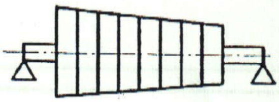
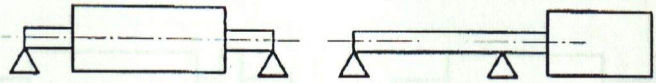
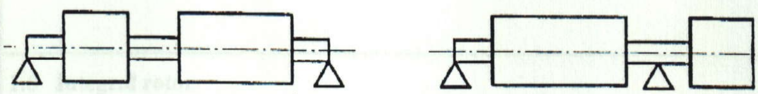
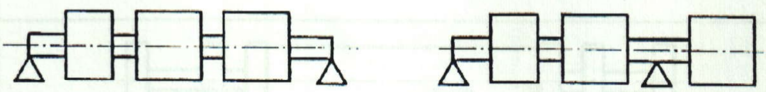
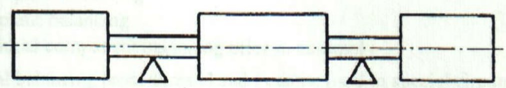
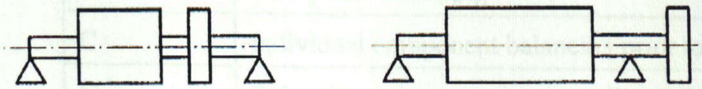
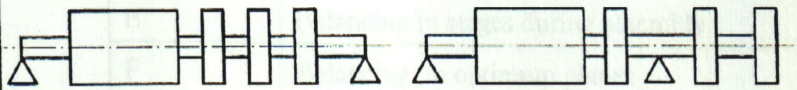
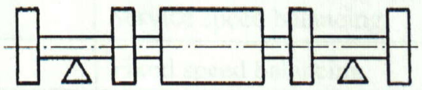
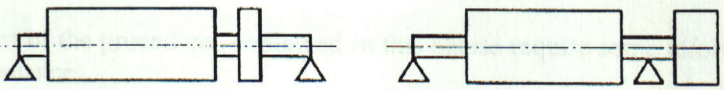
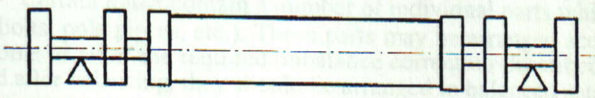
Configuration	Rotor characteristics	Recommended balancing procedure
1.1 Discs	Elastic shaft without unbalance, rigid disc(s)	(see table 2) (see next page for key to A-G)
	**Single disc** - perpendicular to shaft axis - with axial runout	 A; C B; C
	**Two discs** - perpendicular to shaft axis - with axial runout • at least one removable • integral	 B; C B + C, E G
 	**More than two discs** - all (but one) removable - integral	 B + C, D, E G
1.2 Rigid sections	Elastic shafts without unbalances, rigid sections	
	**Single rigid section** - removable - integral	 B; C; E B
	**Two rigid sections** - at least one removable - integral	 B + C; E G
 	**More (than two) rigid section** - all (but one) removable - integral	 B + C; E G

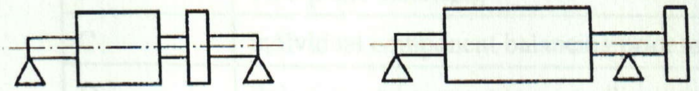
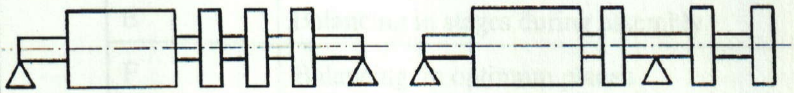
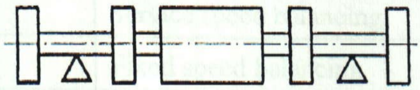
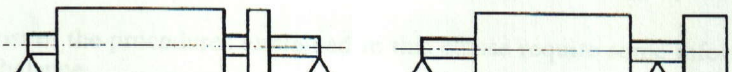
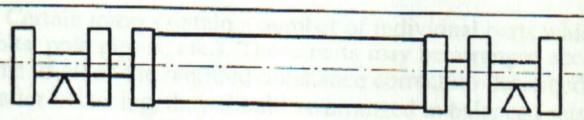
Table 1 — Flexible rotors (*concluded*)

Configuration	Rotor characteristics	Recommended balancing procedure
1.3 Discs and rigid sections	Elastic shaft without unbalance, rigid discs and sections	(see table 2) 1)
	**One each** - at least one part removable - integral	 B + C; E G
 	**More parts** - all (but one) removable - integral	 B + C; E G
1.4 Rolls	Mass, elasticity and unbalance distribution along the rotor	
	- under special conditions - in general	F G
1.5 Rolls and discs/rigid sections	Flexible roll, rigid discs, rigid sections	
	- discs/rigid sections/removable - under special conditions - in general - integral	 C + F; E + F G G
1.6 Integral rotor	Mass, elasticity and unbalance distribution along the rotor	
	Main parts with unbalances not detachable	G

1) A = Single-plane balancing
B = Two-plane balancing
C = Individual component balancing prior to assembly
E = Balancing in stages during assembly
F = Balancing in optimum planes
G = Multiple speed balancing

Two additional balancing procedures H and I can be used in special circumstances, see 7.4 and 7.5.

Table 1 — Flexible rotors (*concluded*)

Configuration	Rotor characteristics	Recommended balancing procedure (see table 2) 1)
1.3 Discs and rigid sections	Elastic shaft without unbalance, rigid discs and sections	
	**One each**	
	- at least one part removable	B + C; E
	- integral	G
	**More parts**	
	- all (but one) removable	B + C; E
	- integral	G
1.4 Rolls	Mass, elasticity and unbalance distribution along the rotor	
	- under special conditions	F
	- in general	G
1.5 Rolls and discs/rigid sections	Flexible roll, rigid discs, rigid sections	
	- discs/rigid sections/removable	
	- under special conditions	C + F; E + F
	- in general	G
	- integral	G
1.6 Integral rotor	Mass, elasticity and unbalance distribution along the rotor	
	Main parts with unbalances not detachable	G

1) A = Single-plane balancing
B = Two-plane balancing
C = Individual component balancing prior to assembly
E = Balancing in stages during assembly
F = Balancing in optimum planes
G = Multiple speed balancing

Two additional balancing procedures H and I can be used in special circumstances, see 7.4 and 7.5.

Table 2 — Balancing procedures

Procedure	Description	Subclause
	Low-speed balancing	
A	Single-plane balancing	6.5.1
B	Two-plane balancing	6.5.2
C	Individual component balancing prior to assembly	6.5.3
D	Balancing subsequent to controlling initial unbalance	6.5.4
E	Balancing in stages during assembly	6.5.5
F	Balancing in optimum planes	6.5.6
	High-speed balancing	
G	Multiple speed balancing	7.3
H	Service speed balancing	7.4
I	Fixed speed balancing	7.5

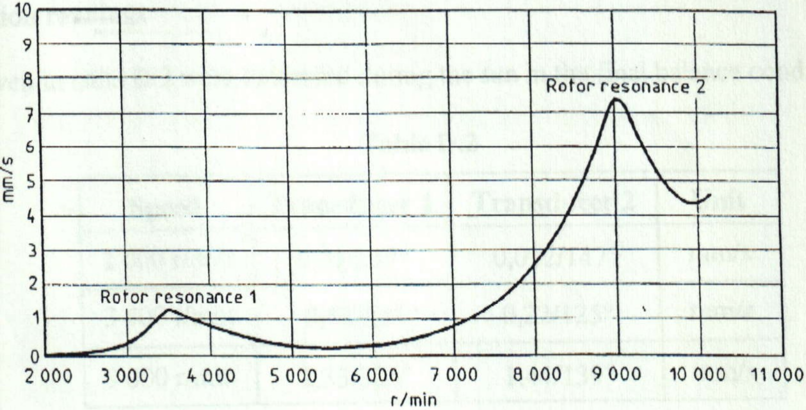


Figure D.2 — Run-up curve — Before balancing

Annex B
(informative)

Interim broad-band vibration criteria for specific machine groups

This part of ISO 10816 is a basic document which sets out general guidelines for the measurement and evaluation of mechanical vibration of machines, as measured on non-rotating parts. It is intended that evaluation criteria for specific machine types will be provided in additional parts of ISO 10816 for different machine types. However, as a short-term expedient only, limited evaluation criteria are provided in table B.1 until the relevant parts become available. The values given are for the upper limits of zones A to C, respectively (see 5.3.1), for the machine classes defined below. It is important, therefore, prior to using these values, to check that they have not been superseded by an additional part of ISO 10816. This annex will be deleted when all of the relevant parts have been published.

The machine classifications are as follows.

Class I: Individual parts of engines and machines, integrally connected to the complete machine in its normal operating condition. (Production electrical motors of up to 15 kW are typical examples of machines in this category.)

Class II: Medium-sized machines (typically electrical motors with 15 kW to 75 kW output) without special foundations, rigidly mounted engines or machines (up to 300 kW) on special foundations.

Class III: Large prime-movers and other large machines with rotating masses mounted on rigid and heavy foundations which are relatively stiff in the direction of vibration measurements.

Class IV: Large prime-movers and other large machines with rotating masses mounted on foundations which are relatively soft in the direction of vibration measurements (for example, turbogenerator sets and gas turbines with outputs greater than 10 MW).

Table B.1 — Typical zone boundary limits

<table>
<tr><th>R.m.s. vibration velocity
mm/s</th><th>Class I</th><th>Class II</th><th>Class III</th><th>Class IV</th></tr>
<tr><td>0,28</td><td rowspan="3">A</td><td rowspan="4">A</td><td rowspan="5">A</td><td rowspan="6">A</td></tr>
<tr><td>0,45</td></tr>
<tr><td>0,71</td></tr>
<tr><td>1,12</td><td rowspan="2">B</td></tr>
<tr><td>1,8</td><td rowspan="2">B</td></tr>
<tr><td>2,8</td><td rowspan="2">C</td><td rowspan="2">B</td></tr>
<tr><td>4,5</td><td rowspan="2">C</td><td rowspan="2">B</td></tr>
<tr><td>7,1</td><td rowspan="6">D</td><td rowspan="2">C</td></tr>
<tr><td>11,2</td><td rowspan="4">D</td><td rowspan="2">C</td></tr>
<tr><td>18</td><td rowspan="3">D</td></tr>
<tr><td>28</td><td rowspan="2">D</td></tr>
<tr><td>45</td></tr>
</table>