Eyesight and the solar Wien peak

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It is sometimes said that humans see best at yellow–green wavelengths because they have evolved under a Sun whose blackbody spectrum has a Wien peak in the green part of the spectrum. However, as a function of frequency, the solar blackbody spectrum peaks in the infrared. Why did human vision not evolve toward a peak sensitivity in this range, if the eye is an efficient quantum detector of photons? The puzzle is resolved if we assume that natural selection acted in such a way as to maximize the amount of energy that can be detected by the retina across a range of wavelengths (whose upper and lower limits are fixed by biological constraints). It is then found that our eyes are indeed perfectly adapted to life under a class G2 star. Extending this reasoning allows educated guesses to be made about the kind of eyesight that might have evolved in extrasolar planetary systems such as that of the red dwarf Gliese 876.
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I. INTRODUCTION

Astronomy textbooks teach us that human eyesight is most sensitive in the optical range (peaking at 500–560 nm) because it has evolved over time to take advantage of the Sun's blackbody spectrum. When evaluated as a function of wavelength, the latter has a Wien peak at 502 nm, so this argument appears logical at first sight. However, the real story must be more complicated than this, because the solar blackbody spectrum peaks at 3.39×10^{14} Hz when evaluated as a function of frequency. This frequency corresponds to a wavelength of 884 nm. If the eye is an efficient quantum photon detector, should it not have evolved toward this latter peak, where it will collect the greatest number of photons? Why, then, do we not see in the infrared?

This question has been raised by Brecher,[1] who draws the conclusion that other factors (such as the availability of suitable pigments) must have played a more important role than the shape of the solar blackbody spectrum in determining the peak sensitivity of the human eye. Although many complex biochemical and other factors must certainly have been involved, I would like to interpret these facts another way.

Suppose that nature did once experiment with life-forms whose eyesight was fine-tuned for maximum sensitivity at precisely the solar Wien peak. Such creatures would not have lasted long against competitors whose visual acuity was not so sharply peaked, but who were able to function over a broader range of lighting conditions. In a complex and changing environment, in other words, natural selection would have operated to bring about the best possible balance between the wavelength of peak sensitivity, λ_p, and *range of wavelengths*, $\Delta\lambda$, to which the retina is capable of responding. In the case of the human eye this range extends roughly from 400 nm to 700 nm, so that $\Delta\lambda\approx300$ nm.

The best possible balance will be influenced by many factors, but insofar as the Sun is concerned, we may presume that the relevant quantity is the *total energy* available to the eyes. This quantity is not proportional to the blackbody spectrum (as a function of either λ or ν), but to its integral over either quantity. If we assume a roughly symmetrical spectral sensitivity, the integral may be taken from $\lambda_p-\Delta\lambda/2$ to $\lambda_p+\Delta\lambda/2$. Following this reasoning, we will find that that human eyesight evolving under the light of a G2 star such as the Sun should have a peak sensitivity near $\lambda_p\approx560$ nm—exactly what is observed.

II. THE SOLAR BLACKBODY FUNCTION

The Planck function describing the spectrum of radiation emitted by a blackbody at temperature T can be written as a function of either λ or ν,[2]

$$B_\lambda(T)=\frac{2hc^2/\lambda^5}{\exp(hc/kT\lambda)-1}, \tag{1a}$$

$$B_\nu(T)=\frac{2hc^2\nu^3/c^2}{\exp(h\nu/kT)-1}. \tag{1b}$$

These expressions have dimensions of intensity (energy per unit time, per unit area) per unit wavelength in the first case and per unit frequency in the second. The substitution of $\nu=c/\lambda$ into $B_\nu(T)$ does *not* give $B_\lambda(T)$. It is the integral of either function (over λ or ν, as appropriate) which must give the same intensity.

To plot the two functions $B_\lambda(T)$ and $B_\nu(T)$ for sunlight, we need to estimate the latter's blackbody temperature. The Sun is usually modelled as a blackbody with an effective temperature of 5800 K. (This is actually the temperature of a blackbody whose total radiated power is the same as that of the Sun.) However, the spectrum of sunlight that we receive on the Earth's surface is modified by the Earth's atmosphere. The most important effects are Rayleigh scattering and continuum absorption by ozone, both of which shift the spectrum toward longer wavelengths. Absorption by molecular oxygen, water vapor, and other gases produces a host of narrow line features that further modify the shape of the spectrum, especially on the long-wavelength side of the solar blackbody peak. As measured in the direction of the Sun, these processes combine to reduce the effective solar blackbody temperature to about 5200 K under a variety of weather conditions.[3] Ambient daylight (that is, sunlight in other directions) has a higher effective temperature, because Rayleigh scattering preferentially scatters short-wavelength light. (This is why the sky is blue.) So the average spectrum of light that we receive on Earth can be modeled in a very

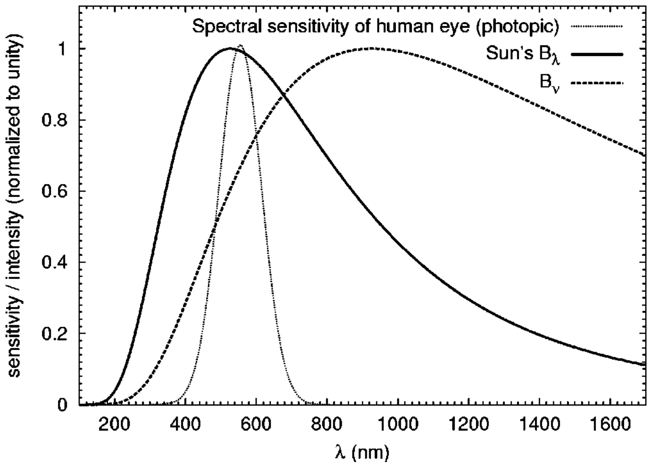

Fig. 1. The Planck blackbody spectrum, evaluated as a function of wavelength λ (solid line) and as a function of frequency ν, where $\lambda = c/\nu$ (dashed line). The spectral sensitivity of the human eye (dotted line) is shown for comparison.

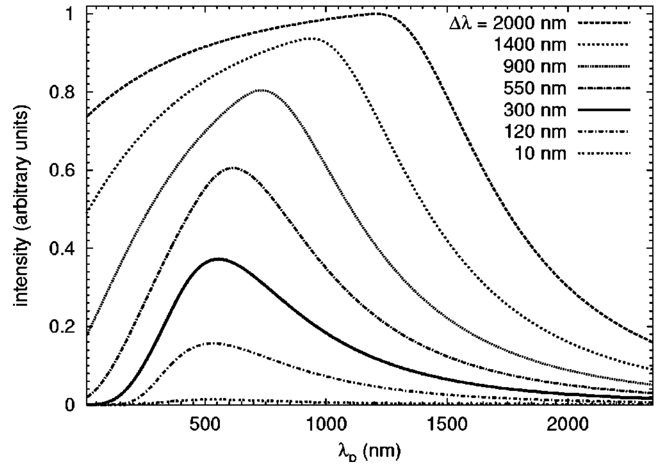

Fig. 2. The intensity of solar radiation emitted between wavelengths $\lambda_p - \Delta\lambda/2$ and $\lambda_p + \Delta\lambda/2$, plotted as a function of λ_p for seven values of $\Delta\lambda$.

rough way as that of a blackbody with a temperature somewhere between 5200 K and 5800 K. Let us write this as $T_\odot = 5500 \pm 300$ K. The two functions $B_\lambda(T_\odot)$ and $B_\nu(T_\odot)$ are plotted as functions of wavelength in Fig. 1.

The approximate spectral sensitivity of the human eye is also shown in Fig. 1 for comparison (dotted line). For photopic or light-adapted vision, this curve is roughly Gaussian in shape and is centered at 555 nm. I have taken $\Delta\lambda/2$ as a 3σ standard deviation, so that $\sigma \approx 50$ nm. This picture is of course simplified. The details are more complicated and far more interesting. Photopic vision in primates arises from three different kinds of cone cells whose individual sensitivities peak at about 430 nm (violet–blue), 535 nm (green), and 562 nm (yellow).[4] This particular configuration may have evolved to aid in the detection of fruits[5] or young leaves[6] against a predominantly green background. Red-sensitive cells would in principle allow for better color discrimination, but may not have arisen in practice because less energy is carried by photons at these wavelengths.[7] (This is a telling point in the context of the present article.) Scotopic or dark-adapted vision relies on rod cells whose sensitivity peaks near 500 nm. These appear to have evolved after the cone cells.[8] Very few photons of wavelength shorter than 400 nm reach the retina. Those with $315\ \text{nm} < \lambda < 400$ nm are absorbed by the lens (where they are one cause of cataracts), while those with $\lambda < 315$ nm do not get farther than the cornea. Similar constraints must be operative for any life-form, providing the biological basis for the quantity $\Delta\lambda$.

Let us now compare the wavelength of peak sensitivity in the human eye (that is, 555 nm for photopic vision) with the locations of the two blackbody peaks in Fig. 1. We can obtain analytical approximations to these peaks in the following way. We differentiate either of the expressions in Eq. (1) and set the result equal to zero (for a maximum at $\lambda = c/\nu = \lambda_B$) to obtain an equation of the following form:

$$1 - hc/mkT\lambda_B = \exp(-hc/kT\lambda_B), \tag{2}$$

where $m = 5$ for $B_\lambda(T)$ and 3 for $B_\nu(T)$. Both sides of Eq. (2) are small. If we relabel the left-hand side as ϵ, then the right-hand side becomes $\exp[-m(1-\epsilon)]$ and Eq. (2) can be written as

$$\epsilon \exp(m) = \exp(\epsilon m). \tag{3}$$

If we use the series approximation for $\exp(\epsilon)$, we obtain the quadratic equation

$$m^2\epsilon^2/2 - (e^m - m)\epsilon + 1 = 0. \tag{4}$$

We solve Eq. (4) for ϵ and find that the peak of the blackbody spectrum occurs at

$$\lambda_B = \frac{hc}{m(1-\epsilon)kT} = \frac{hc}{kT} \Big/ \left[\sqrt{(e^m/m-1)^2 - 2} - (e^m/m - 1) + m\right]. \tag{5}$$

For $m = 5$ Eq. (5) reduces to the familiar Wien formula $\lambda_{B_\lambda} = 2.90$ mm K/T; for $m = 3$ the peak is found at $\lambda_{B_\nu} = 5.10$ mm K/T. For sunlight as received on the Earth's surface ($T_\odot = 5500 \pm 300$ K) these expressions yield wavelengths 530 ± 30 nm and 930 ± 30 nm, respectively, in agreement with the locations of the peaks in Fig. 1.

These values of the peaks are close enough to 555 nm to give us some assurance that the shape of the solar blackbody function had something to do with the evolution of human vision. But was it the dominant factor? And if so, then which (if either) of the two blackbody peaks in Fig. 1 is the relevant one?

III. INTENSITY AND THE OPTIMAL EYE

In fact, neither of the blackbody peaks in Fig. 1 is particularly relevant in itself. It seems reasonable to assume that the eye has evolved over time so as to avail itself of the greatest possible *energy* from the Sun, subject to the biological constraint of a finite (and nonzero) window of wavelengths $\Delta\lambda$ centered on λ_p. In terms of $B_\lambda(T)$ this assumption means maximizing the quantity:

$$I_B(\Delta\lambda, \lambda_p) = \int_{\lambda_p - \Delta\lambda/2}^{\lambda_p + \Delta\lambda/2} B_\lambda(T)\, d\lambda. \tag{6}$$

The integral has dimensions of energy per unit time per unit area, or *intensity*. The results are plotted in Fig. 2 as a function of λ_p over the range $10\ \text{nm} < \lambda_p < 2200$ nm for seven values of $\Delta\lambda$. [The integration of $B_\nu(T)$ over the correspond-

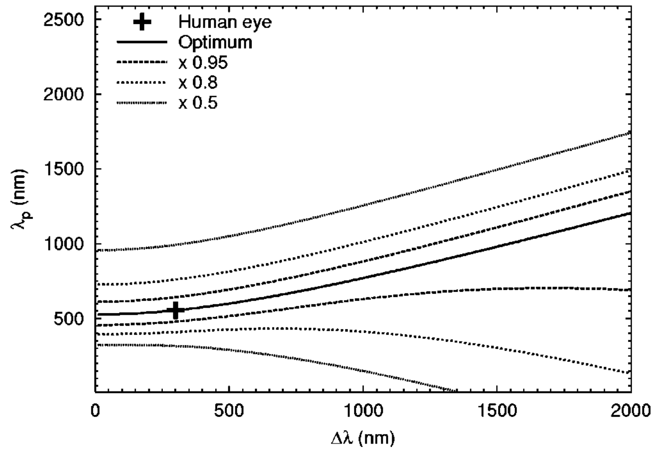

Fig. 3. Contours of equal solar intensity in the phase space defined by the parameters $\Delta\lambda$ and λ_p. We would expect the human eye to evolve to the solid line, where it is sensitive to the greatest amount of energy from the Sun. This does in fact seem to have occurred (location of cross).

ing frequency interval $\Delta\nu = c/\Delta\lambda$ would produce exactly the same figure.]

Suppose that biology had dictated that the eye be sensitive to a very slender range of wavelengths, $\Delta\lambda = 10$ nm. Such a situation would be represented in Fig. 2 by the lowest-intensity (double-dotted) line, which peaks at $\lambda_p = 530$ nm—the solar Wien peak, as measured on Earth. (In fact the Wien peak is just the $\Delta\lambda \to 0$ limit of the optimum value of λ_p, as we have defined the latter quantity.) On the other hand, had we been so fortunate as to possess retinas sensitive to a wavelength range as wide as $\Delta\lambda = 2000$ nm, Fig. 2 shows that natural selection would have encouraged us to center this range deep in the infrared, near $\lambda_p = 1210$ nm (long-dashed line).

As it happens, human spectral sensitivity is characterized by $\Delta\lambda \approx 300$ nm. The corresponding curve in Fig. 2 (solid line) peaks at $\lambda_p \approx 560$ nm. Thus it is no surprise to find that human photopic (or light-adapted) vision is most sensitive at wavelengths near 555 nm. The puzzle of which blackbody peak to use is thus resolved: the appropriate solar referent is not the blackbody function itself, but its integral. The latter is the same regardless of whether we integrate $B_\lambda(T)$ over λ or $B_\nu(T)$ over ν. The lesson is one that can be generalized to many other situations in physics, where one has a distribution or density function that is essentially differential in nature (for example, the intensity B_λ per unit wavelength; other examples in astronomy are the mass per unit radial distance of a spherical shell inside a star and the absorption cross-section per unit mass in the interstellar medium). It is a general rule that integrated densities (for example, the mass of the star and the fractional absorption along the line of sight) are physically more meaningful than the densities themselves.

The results we have obtained so far can be presented in a way that is both more compact and more suggestive. Let us solve numerically for the maximum value of $I_B(\Delta\lambda, \lambda_p)$ as a function of $\Delta\lambda$ and λ_p, and plot the results as contours in the phase space defined by these two variables. This procedure leads (in the case of sunlight on Earth, with $T_\odot \approx 5500$ K) to the diagram shown in Fig. 3. The solid line corresponds to values of $(\Delta\lambda, \lambda_p)$ for which I_B reaches its maximum value, $I_{B,\max}$. The other contours (labeled lines) correspond to combinations of $(\Delta\lambda, \lambda_p)$ for which I_B is given by $0.95 I_{B,\max}$, $0.8 I_{B,\max}$, and $0.5 I_{B,\max}$ respectively. The heavy cross marks the location in phase space of the human eye (300 nm, 555 nm), and we can see immediately that the latter is indeed perfectly adapted to life around a G2 star.

Other factors, of course, must also play a role. The spectrum of sunlight as filtered through the Earth's atmosphere is not truly blackbody in shape.[3] The absorption and scattering characteristics of the atmosphere may have varied significantly over the course of human evolution. Survival requires visual acuity under moonlit, starlit, and artificially lit as well as sunlit conditions. The reflectivities of individual surfaces in our environment produce spectra quite different from that of sunlight. And the sensitivity of the retina itself depends on wavelength in a nontrivial way. Considerations of this kind could be accommodated by multiplying $B_\lambda(\lambda)$ under the integral in Eq. (6) by other functions of λ to weight various wavelengths as appropriate. However, there seems to be no reason to think that such effects are more important than the shape of the solar blackbody spectrum itself. Indeed, it is remarkable that we can come so close to predicting the actual characteristics of the eye without taking refinements such as these into account.

IV. EYESIGHT AROUND OTHER STARS?

As an application of our reasoning, let us now ask how eyesight might have evolved under the light of a different star. More than one hundred extrasolar planets have now been discovered, orbiting around home stars with a wide range of properties. One of the most interesting such systems is Gliese 876, a class M4 red dwarf located about 15 light years away in the constellation Aquarius. Like the Sun, this star lies on the main sequence, but it is much smaller, fainter, and cooler than our home star. The mass, diameter, and luminosity of Gliese 876 are estimated at about 1/3, 1/5, and 1/800 those of the Sun, respectively. Two teams independently discovered a planet orbiting this star in 1998,[9,10] and a second was detected three years later.[11] Both of these planets are gas giants, with masses about 1.9 and 0.6 times that of Jupiter. They orbit their home star in just 61 and 30 days, respectively. Although life as we know it would not likely have evolved on either of these planets, the right conditions could conceivably arise on a terrestrial-type planet very close to the star, or perhaps on a moon of one of the gas giants. Figure 4 shows a view of the Gliese 876 system as it might appear from one of these moons.

From the differences in its V- (optical), I-, and K-band (infrared) colors, the effective surface temperature of Gliese 876 is inferred to lie between 3100 K and 3250 K.[10] If we assume for argument's sake that other life-bearing planets have atmospheres similar to our own, then the effective blackbody temperature of starlight from Gliese 876 on the surface of such a world would be roughly $T_{\mathrm{G876}} = 2900 \pm 300$ K. The peaks of the functions $B_\lambda(T_{\mathrm{G876}})$ and $B_\nu(T_{\mathrm{G876}})$ are given by Eq. (5) as $\lambda_{B_\lambda} = 1000 \pm 120$ nm and $\lambda_{B_\nu} = 1760 \pm 200$ nm. Replacing $T_\odot$ by T_{G876} and repeating the optimization procedure described in Sec. III, we arrive at the diagram in Fig. 5. As before, this is a phase plot in the space defined by the visual characteristics $\Delta\lambda$ and λ_p. The position of the human eye is again marked by a heavy cross. It is clear from this diagram that humans would be very

Fig. 4. A view of Gliese 876 and its two gas-giant planets, as seen from a rocky moon. Figure © Lynette Cook, reproduced by permission. Contact the artist at lynette@spaceart.org to license and obtain high-resolution color files depicting this and other extrasolar systems. To see more of the artist's work, go to ⟨http://extrasolar.spaceart.org⟩.

poorly adapted to life around Gliese 876. Were we suddenly transported to this system, we would find that we were sensitive to less than 50% of the stellar energy that was potentially available.

Let us suppose that some other form of life did evolve on one of the planets or moons of this system, and let us further guess that biological constraints there were comparable to those on Earth so that retinal sensitivity developed over approximately the same range of wavelengths, $\Delta\lambda \approx 300$ nm. By tracing vertically up from the location of the heavy cross in Fig. 5, we see that natural selection would have encouraged creatures of this kind to evolve to a peak sensitivity in the neighborhood of $\lambda_p \approx 1000$ nm. Had we evolved around Gliese 876, in other words, we would likely see in the infrared.

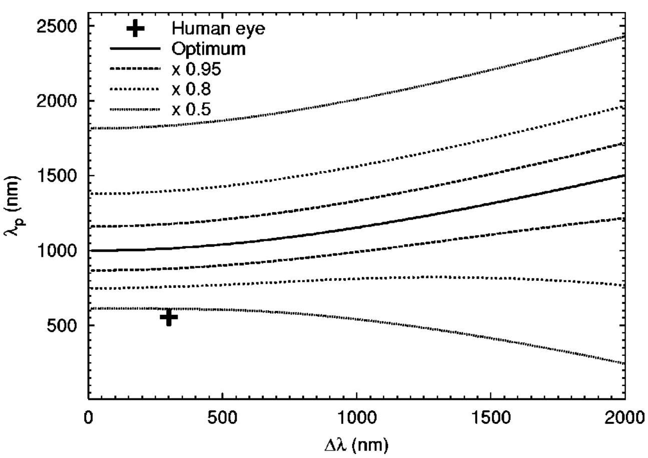

Fig. 5. The same diagram as in Fig. 3, but now assuming a class M4 star like Gliese 876 in place of the Sun. The human eye would be poorly adapted to life around this star (location of cross). Based on this diagram, we would expect life-forms whose range of vision was comparable to ours ($\Delta\lambda \approx 300$ nm) to evolve to a peak sensitivity of $\lambda_p \approx 1000$ nm.

V. CONCLUSIONS

The human eye has evolved to take advantage of the shape of the solar blackbody curve. However, it is not the curve itself that matters (as either a function of wavelength or frequency) but its *integral*, which is proportional to the intensity of light from the Sun. When we take this fact into account, together with the observation that human vision is characterized by a finite range of wavelengths $\Delta\lambda$ as well as a peak sensitivity at λ_p, then we can readily explain why we see best in the yellow part of the spectrum. Around other stars, the same line of reasoning leads to quite different conclusions, and we have shown as an example that life-forms in the extrasolar planetary system around Gliese 876 would likely evolve toward a peak sensitivity in the infrared.

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