

Ondas I

Física II – Módulo II – Fenômenos Ondulatórios

Ondas estão por toda parte...

... e elas carregam energia, momento linear, momento angular, som, luz — e tudo isso sem transportar matéria!

La Ola



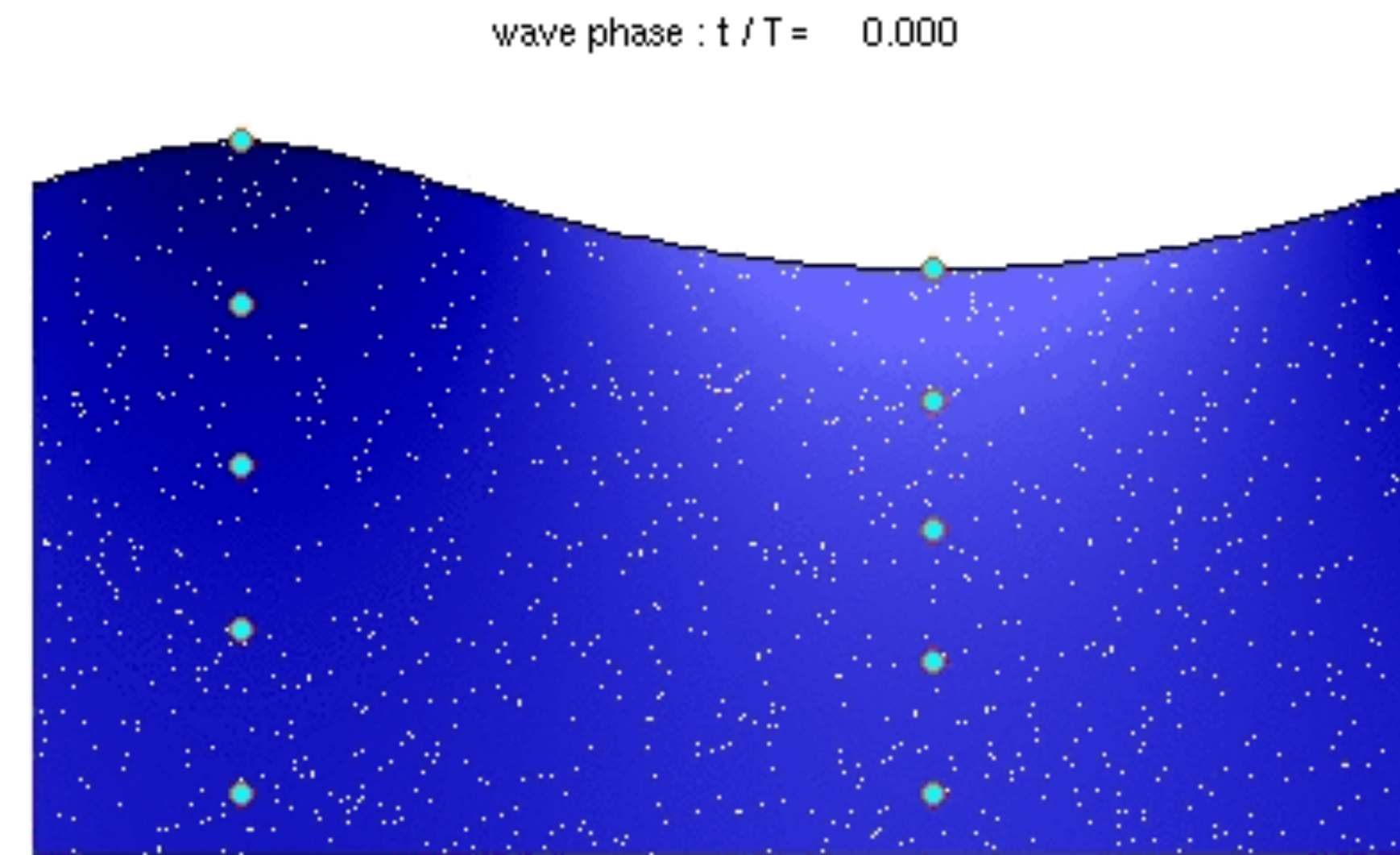
Ondas Na Água



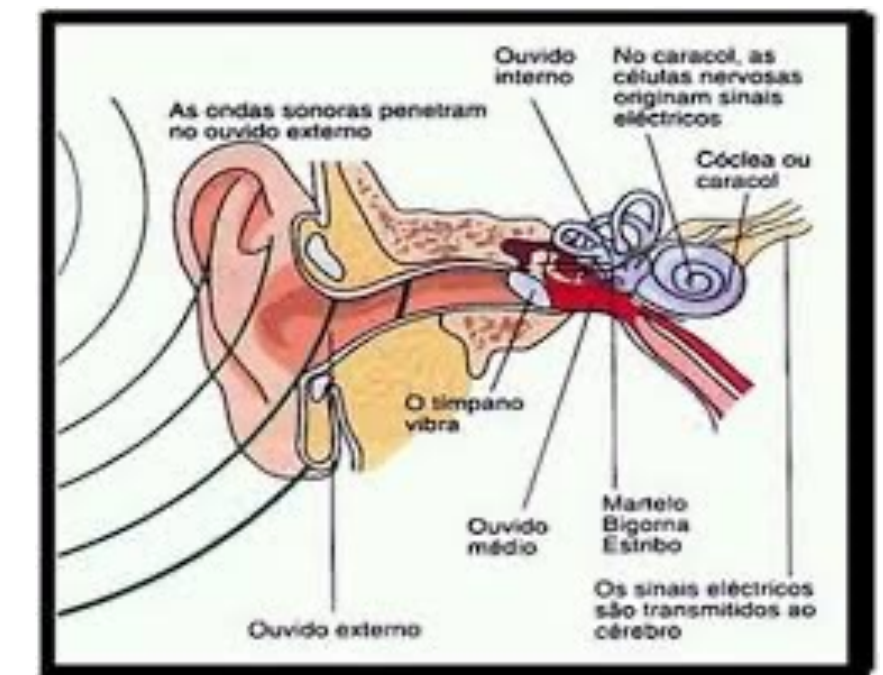
Ondas Na Água



O vento empurra uma pilha de água, e essa pilha cria uma onda, que eventualmente vai parar na costa. Mesmo depois que o vento já terminou, as ondas viajam centenas ou milhares de km, transmitindo a energia do vento que originou aquelas ondas — mas as moléculas de água só se moveram alguns centímetros !



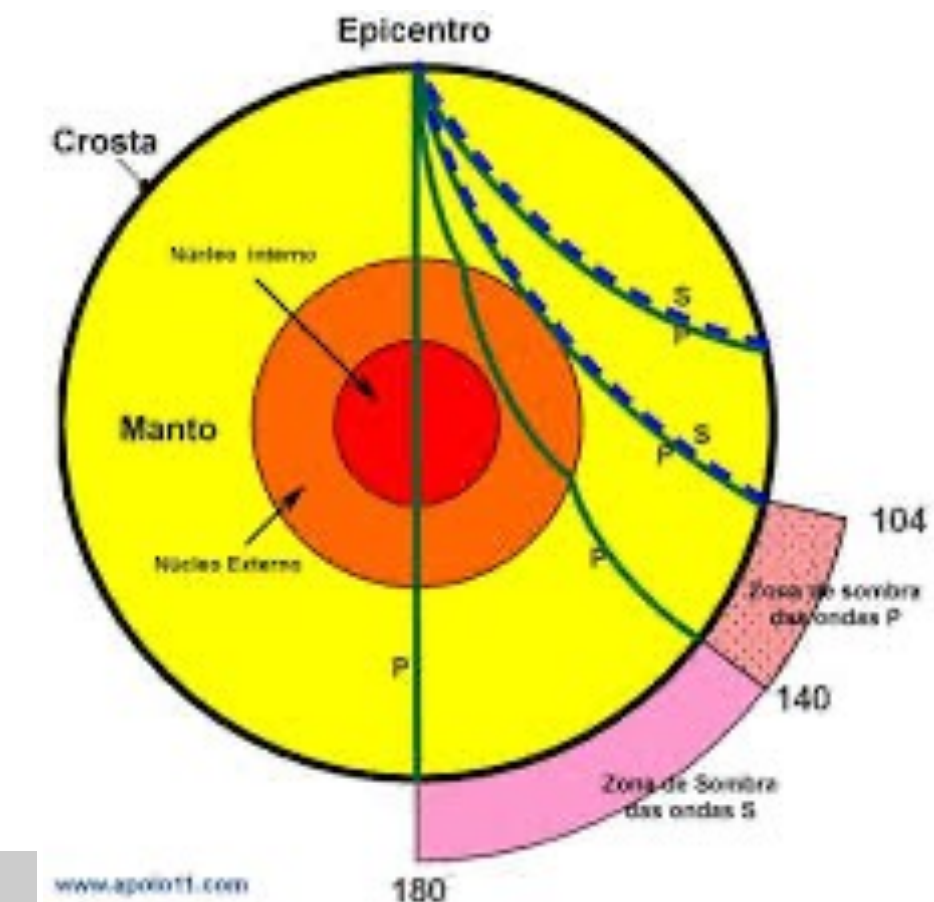
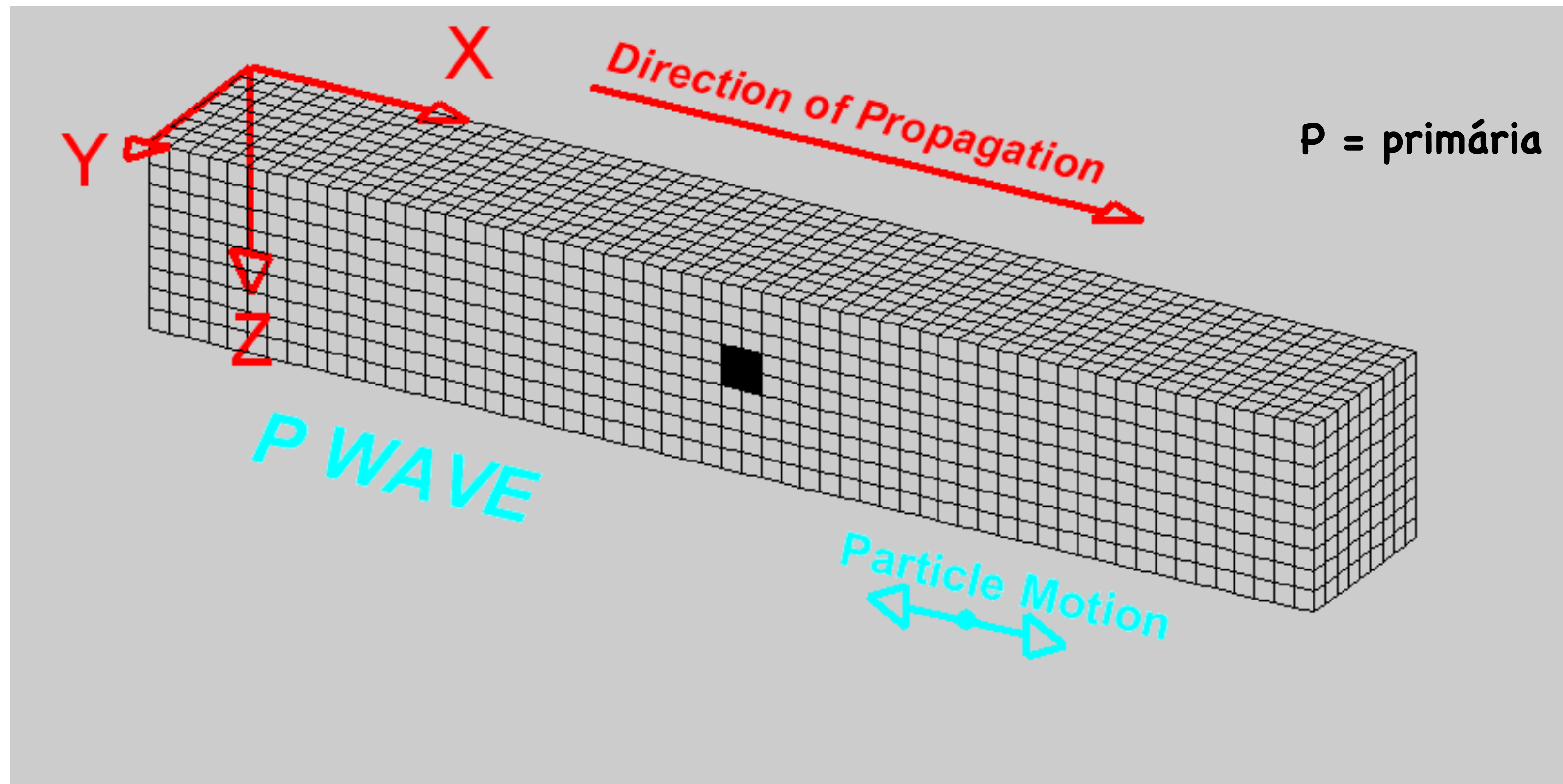
Ondas Sonoras



Ondas de som/sísmicas

São ondas sonoras se propagando na matéria

onda de corpo
longitudinal

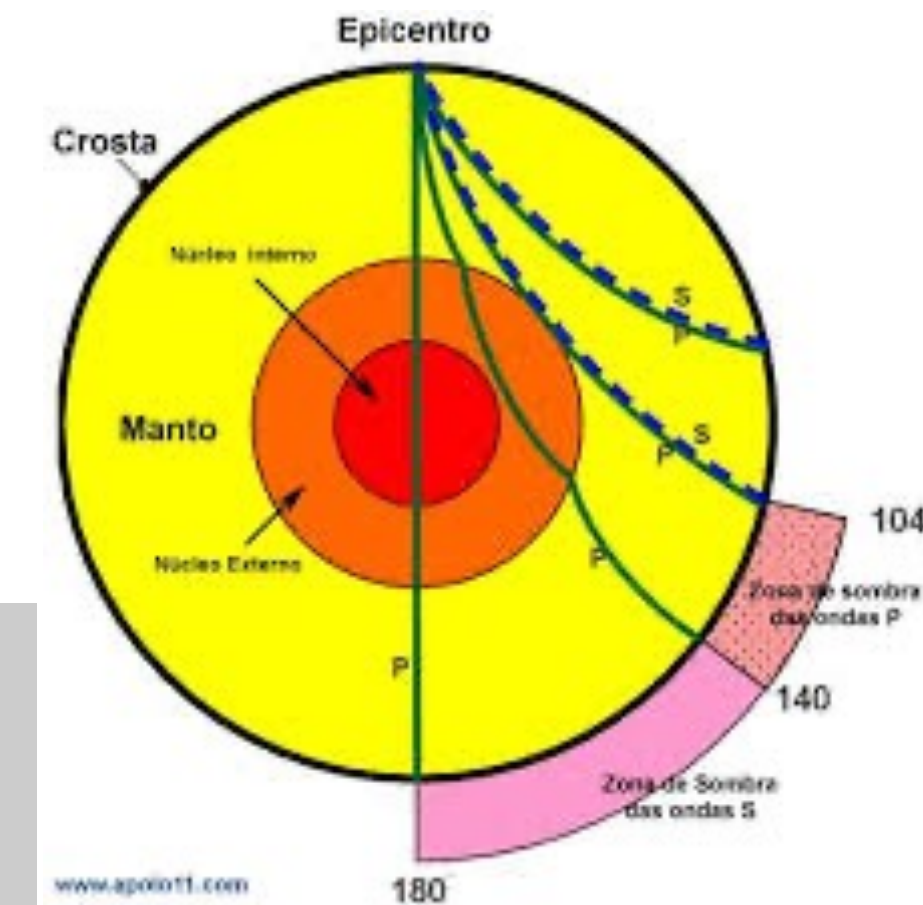
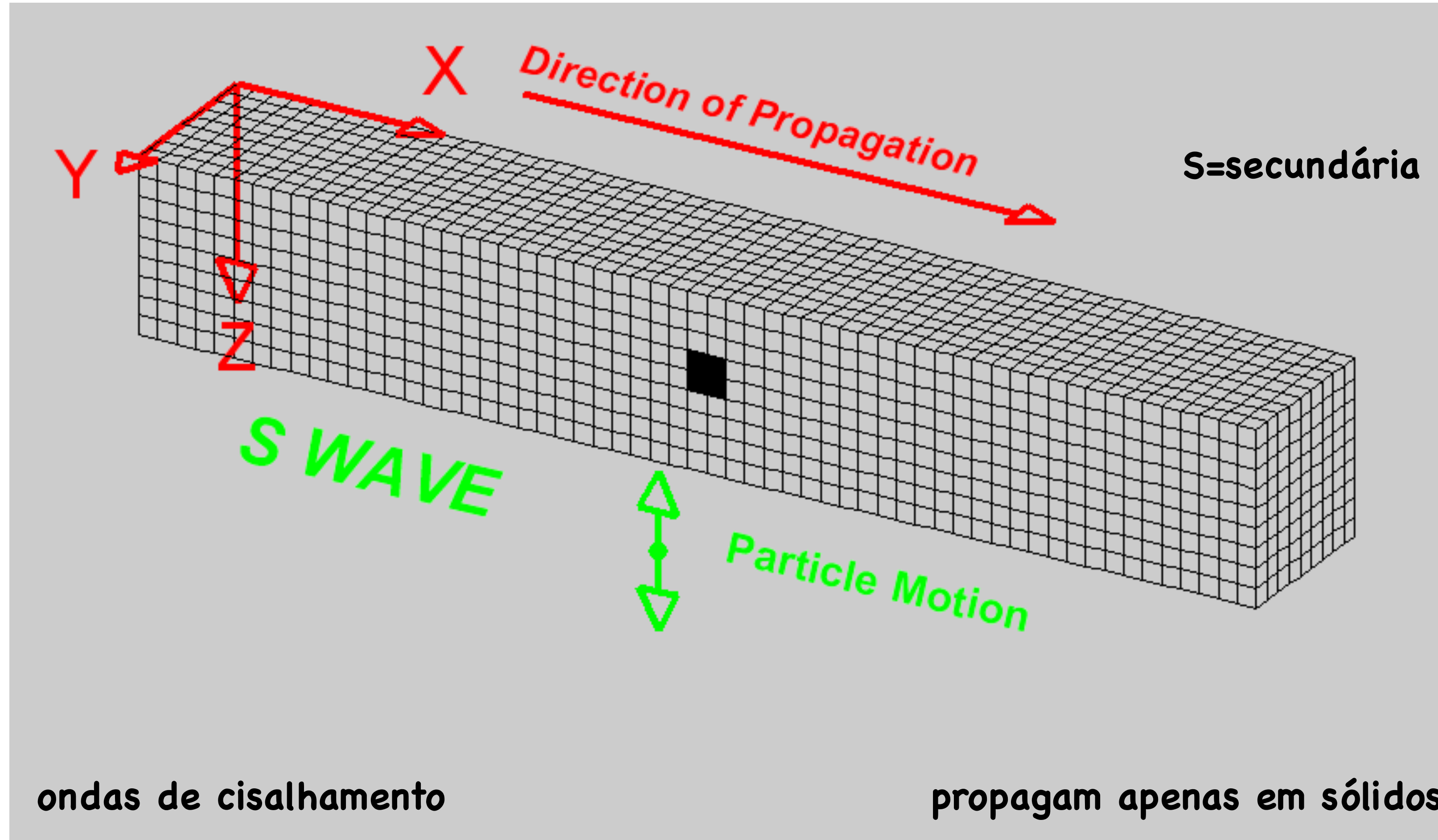


ar 330 m/s
água 1450 m/s
crosta terrestre: 5-7 km/s
manto/núcleo: 8 km/s

Ondas Sísmicas

ondas de corpo
transversal

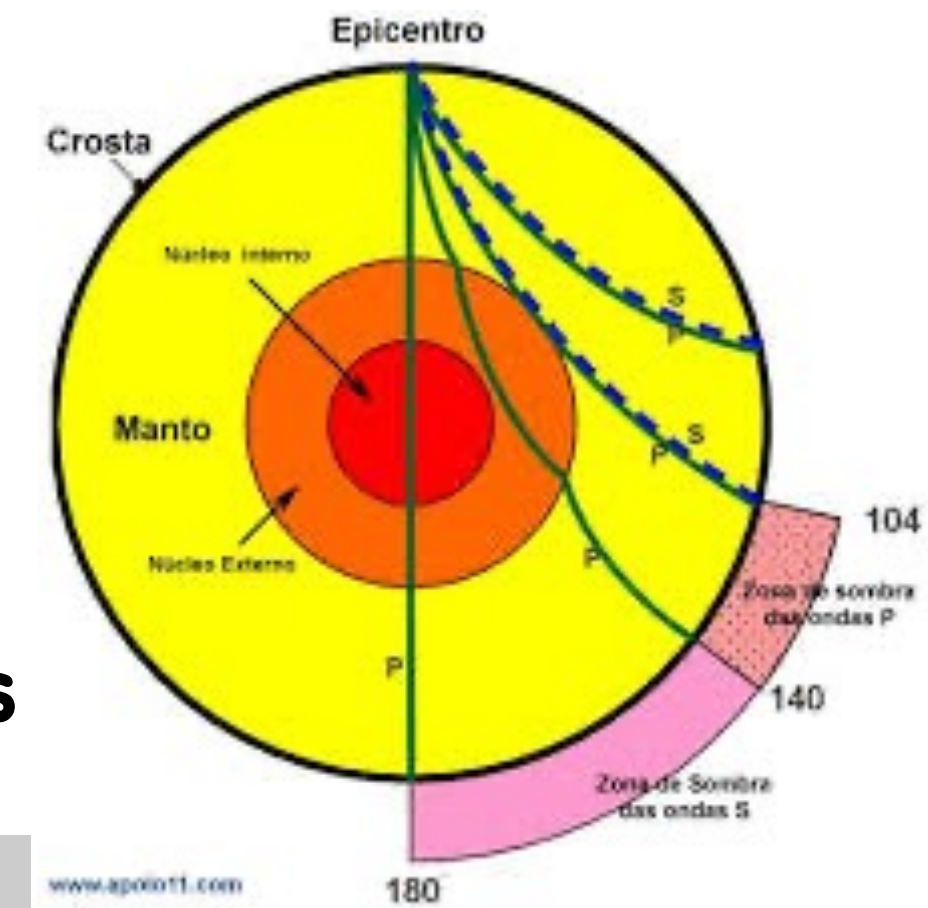
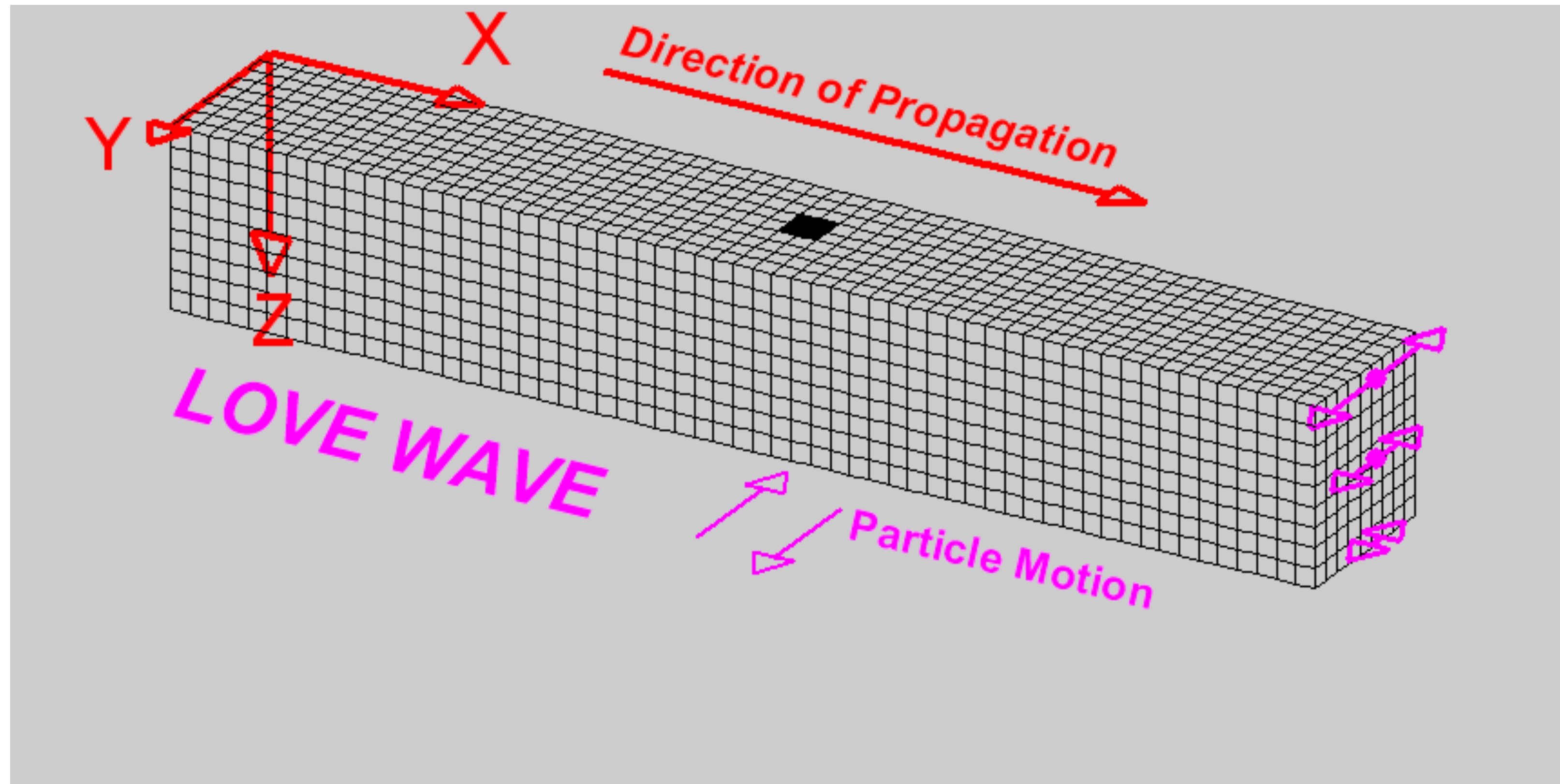
entre 2 a 5 km/s



Ondas Sísmicas

ondas de superfície

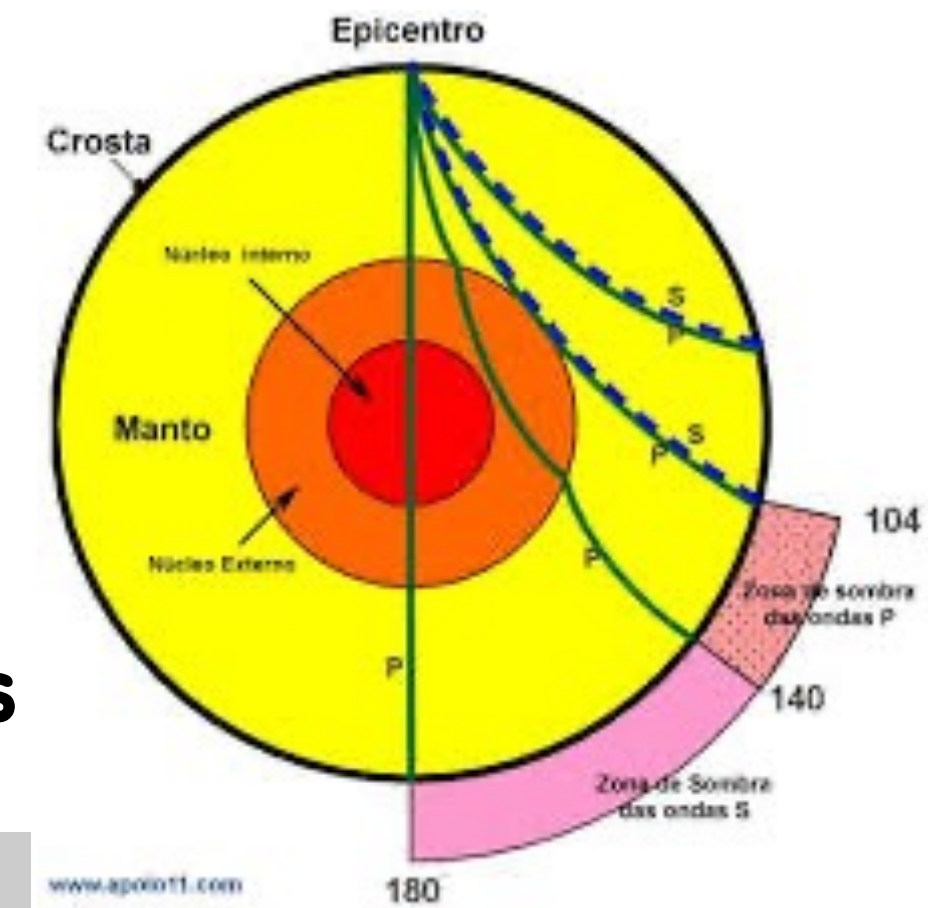
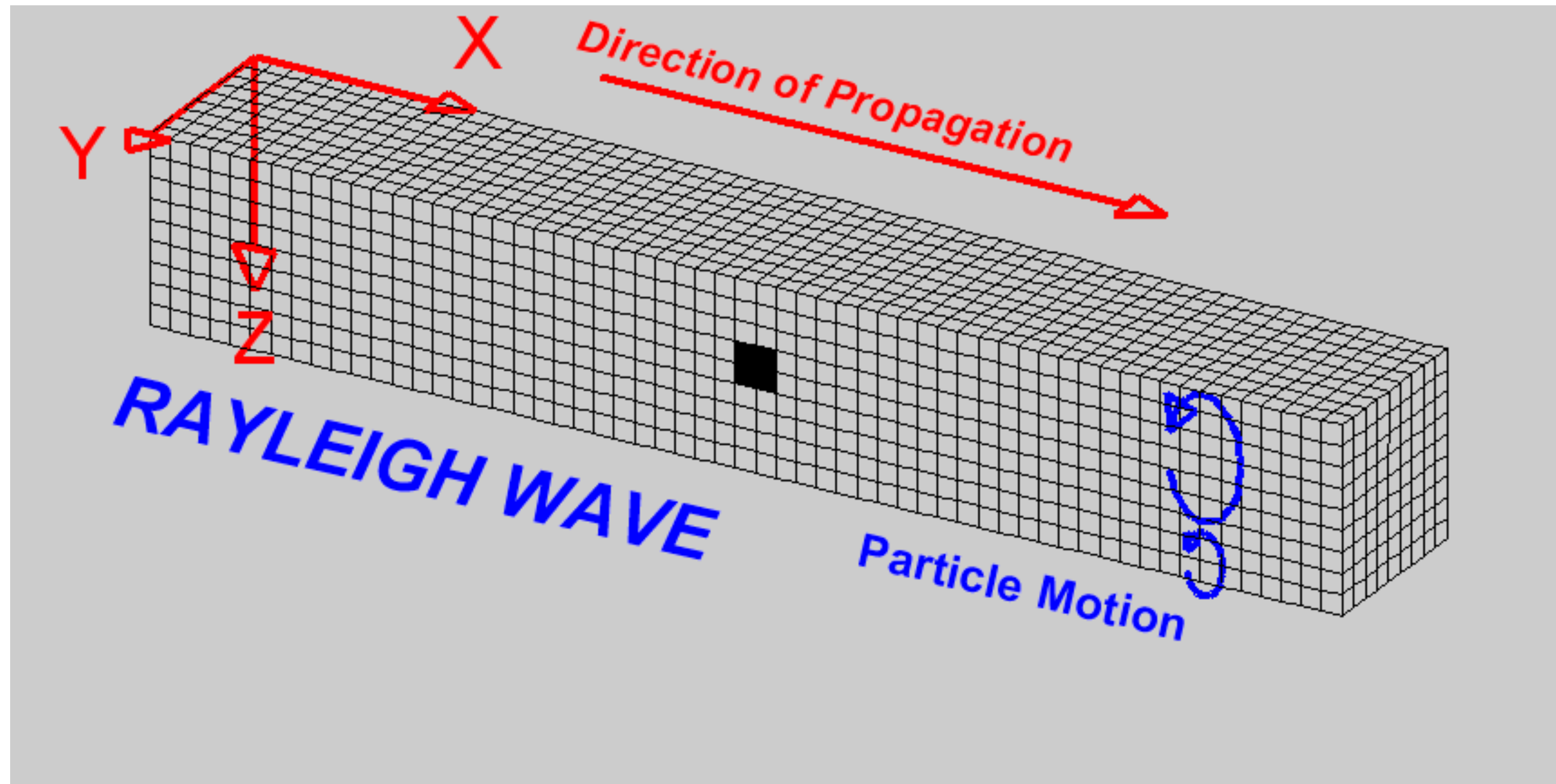
entre 2 a 4.5 km/s



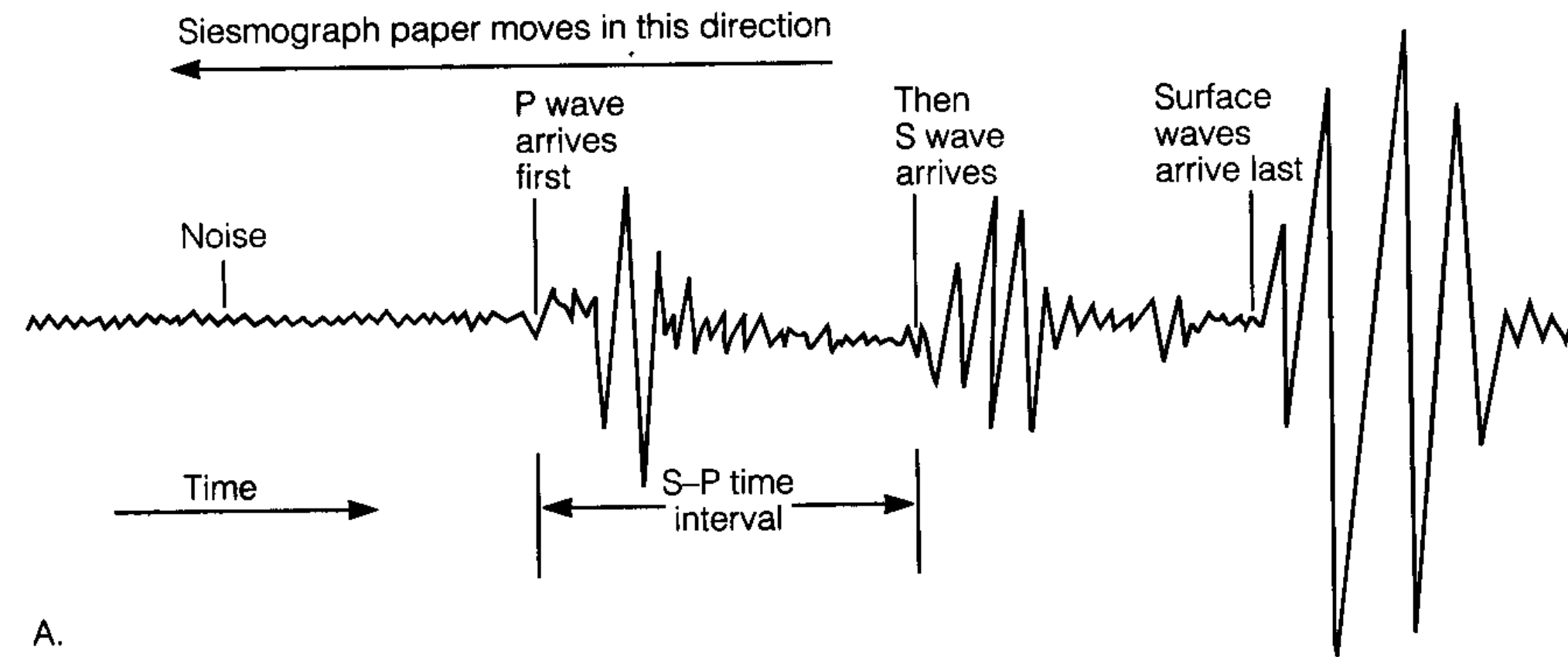
Ondas Sísmicas

ondas de superfície

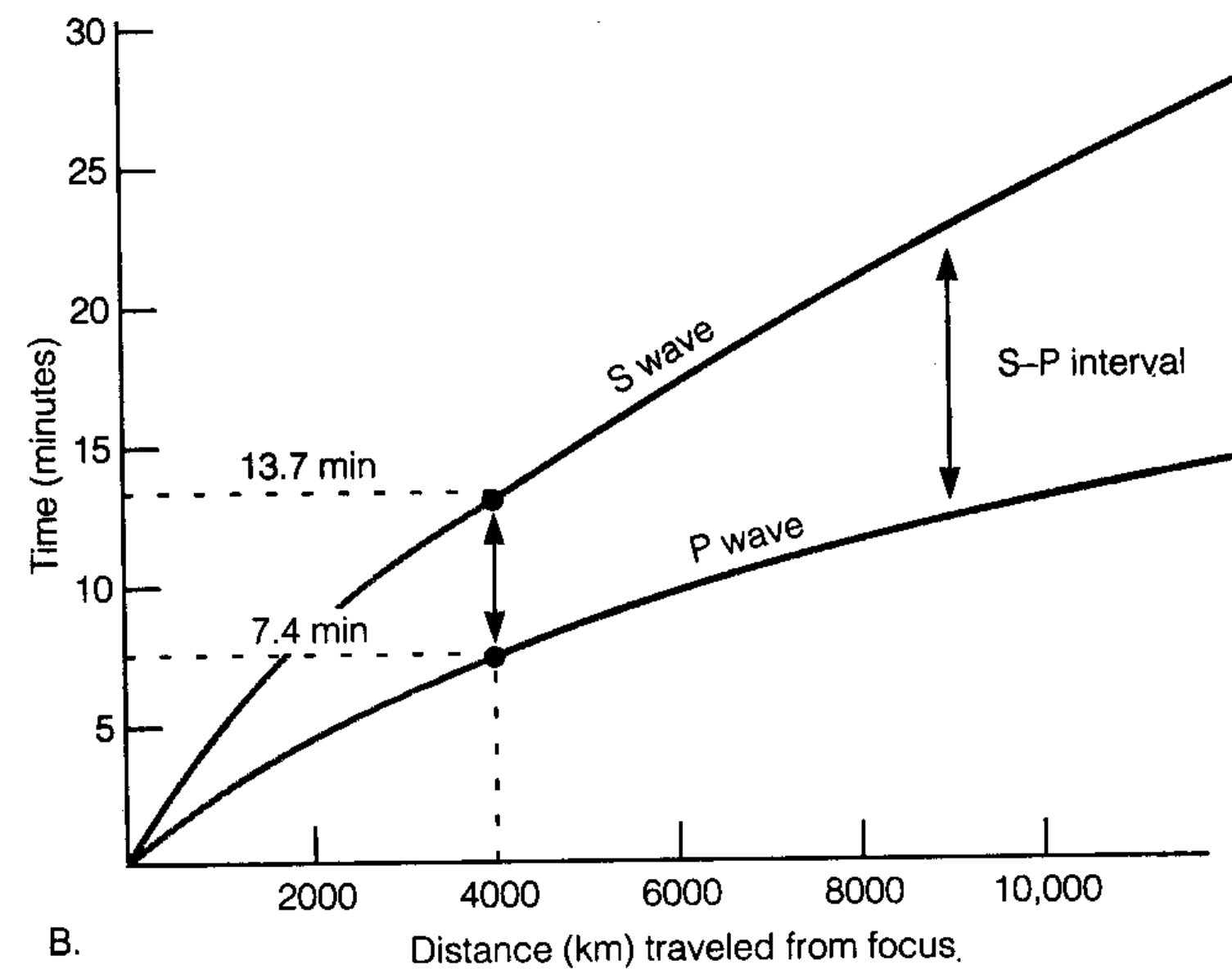
entre 2 a 4.5 km/s



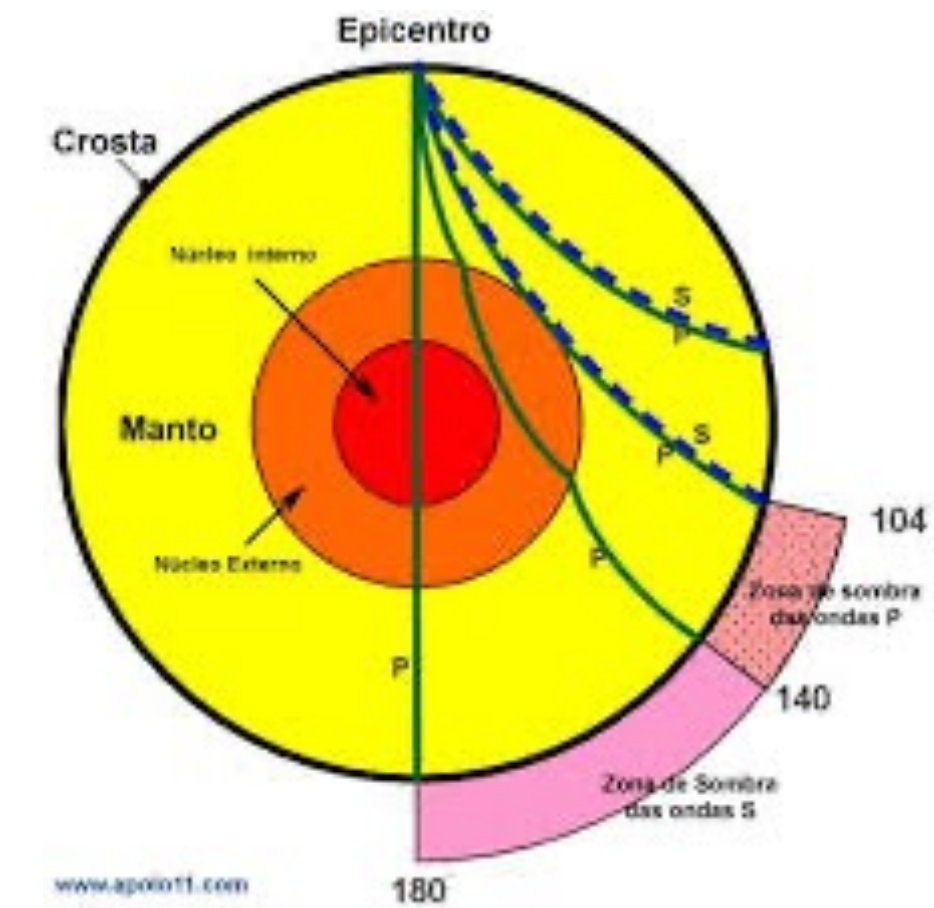
Ondas Sísmicas



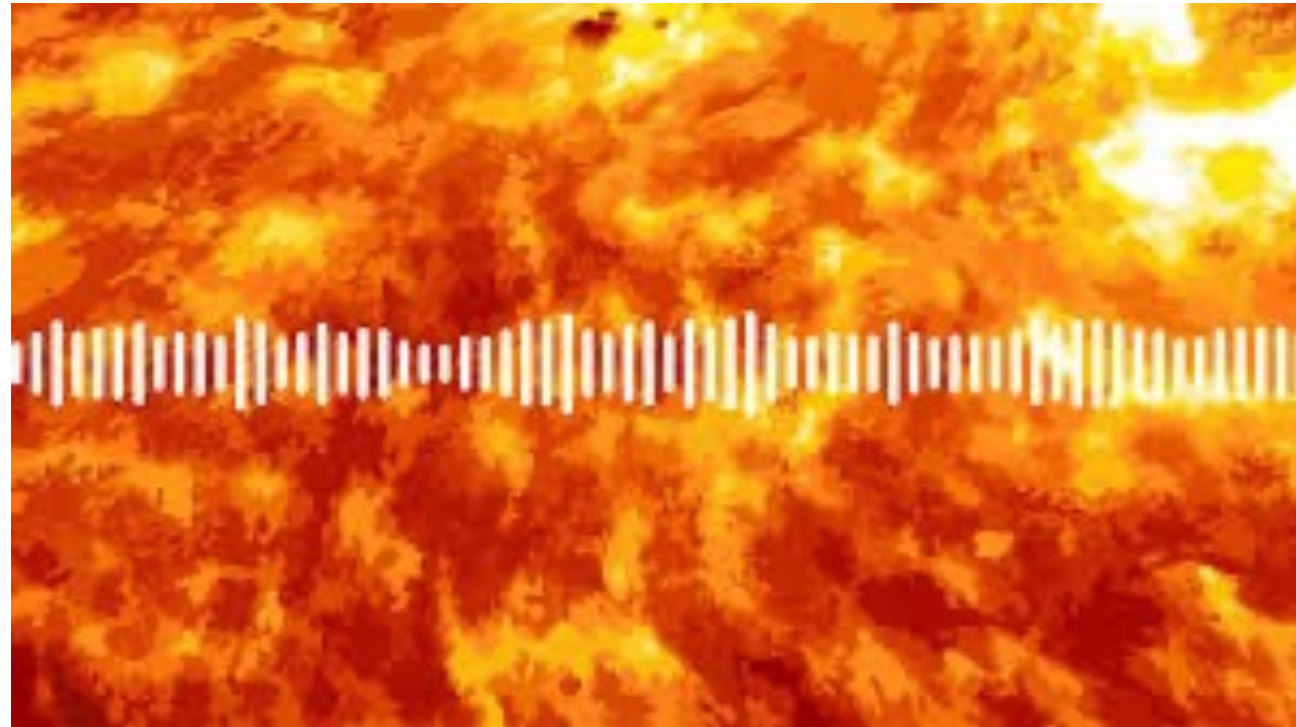
A.



B.

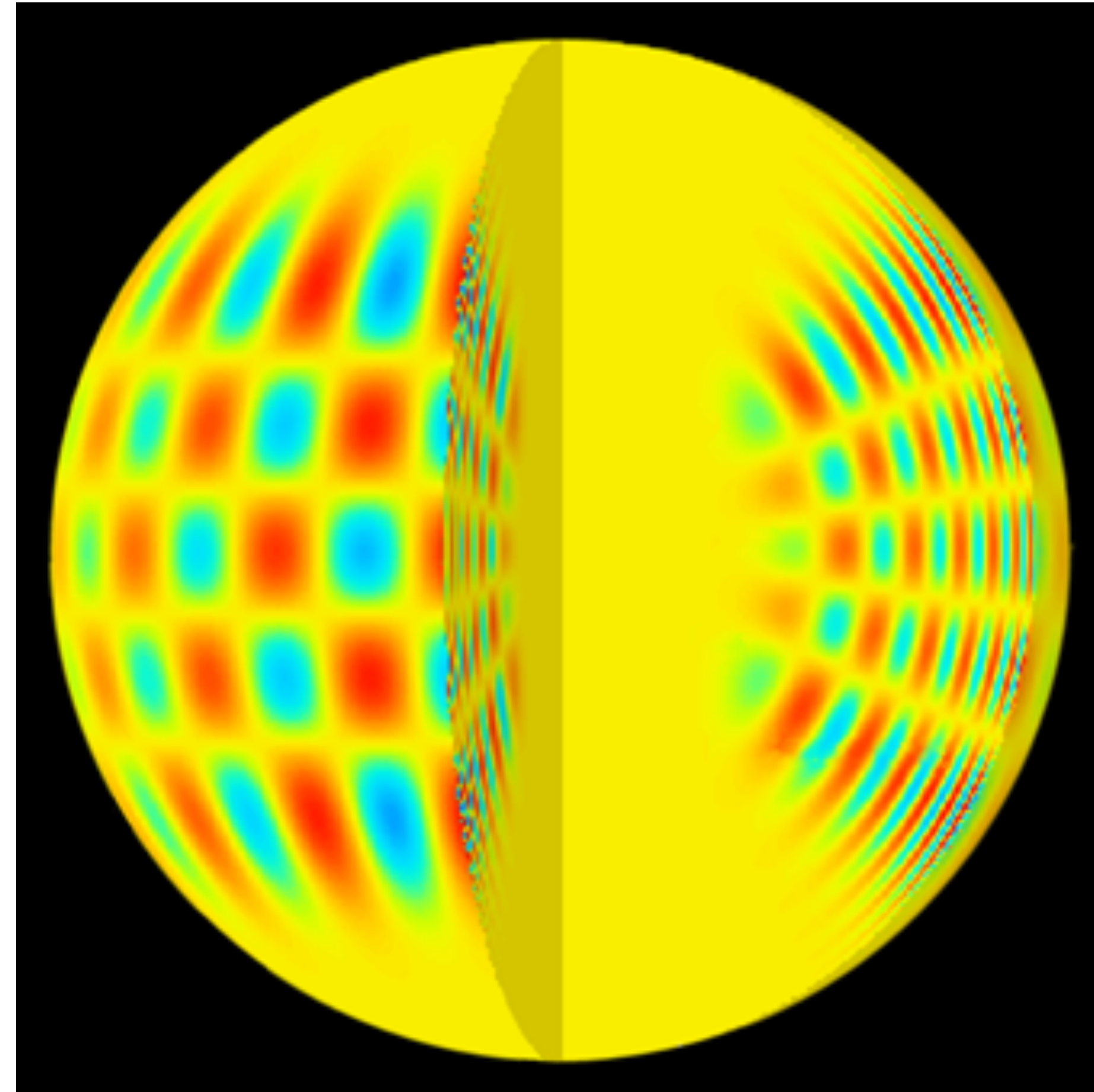


Heliosismologia



NASA

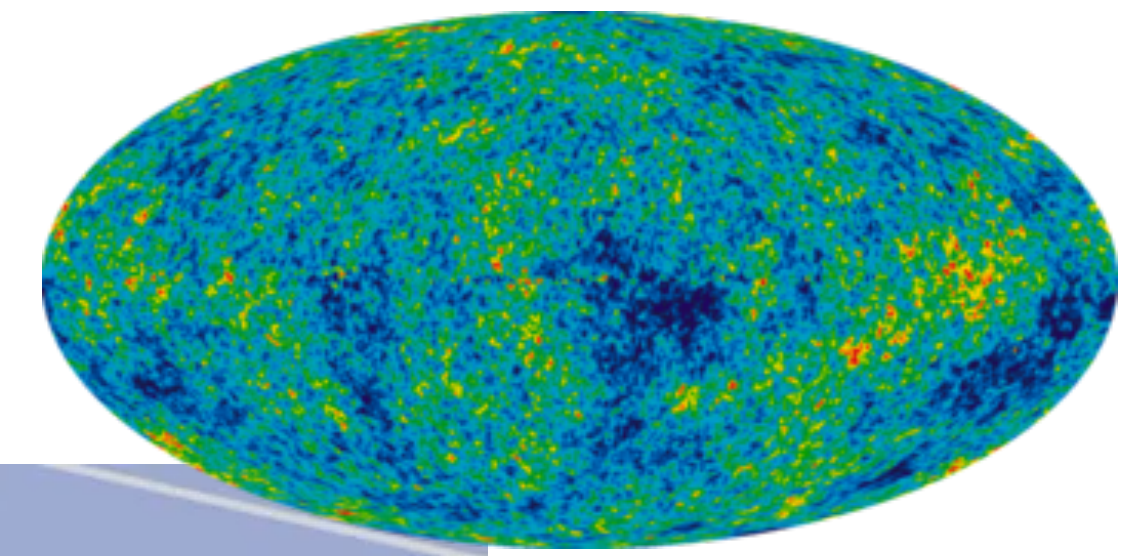
“vibrações do Sol”



Muito do que sabemos sobre o interior do nosso Sol vem do estudo de ondas sonoras que atravessam o Sol

<https://www.nasa.gov/feature/goddard/2018/sounds-of-the-sun>

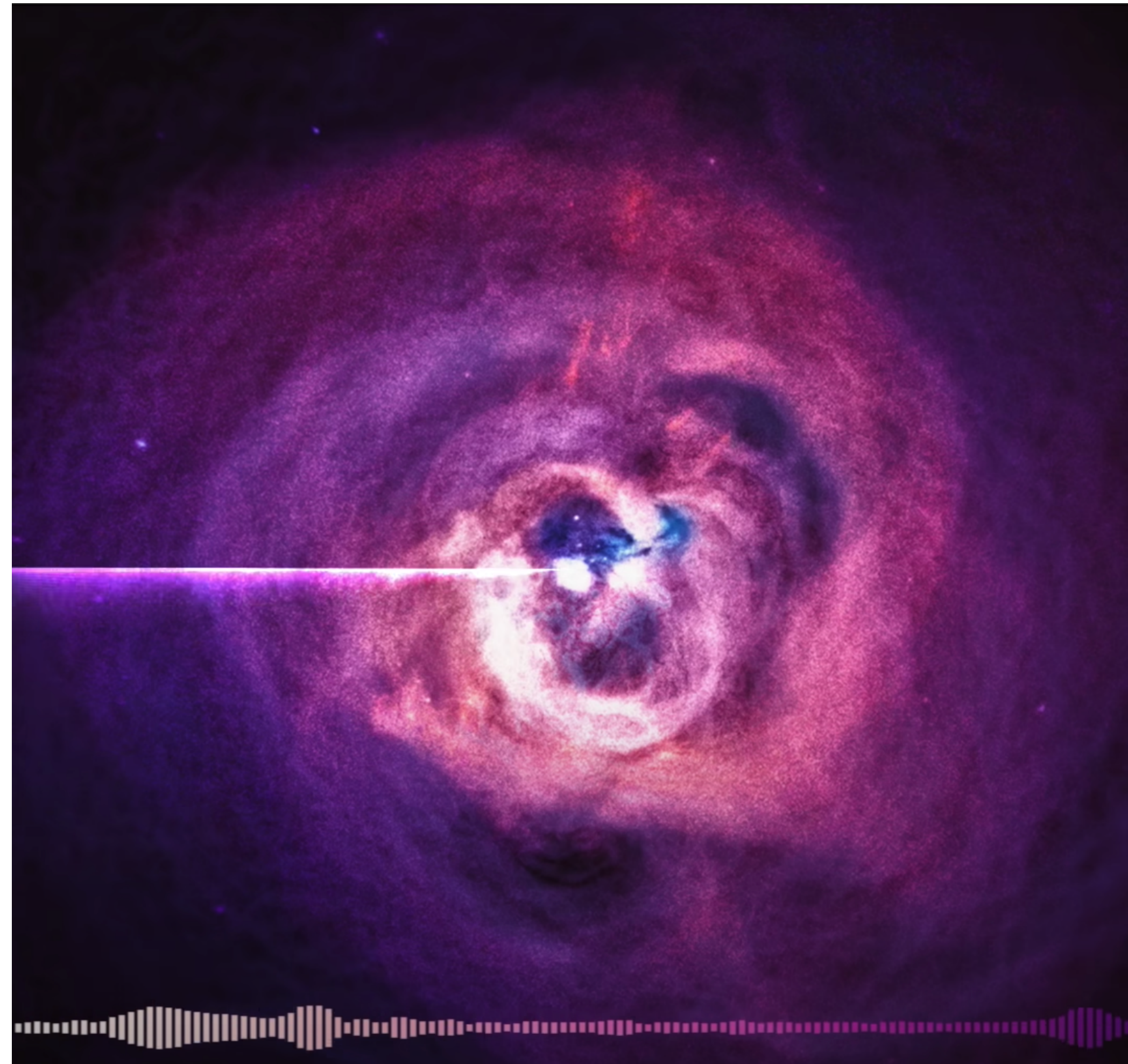
O "som" do cosmos



flutuações de densidade da matéria visível do Universo (oscilações acústicas)



O "som" de um buraco negro



<https://www.youtube.com/watch?v=ioR5np1fmEc&t=21s>

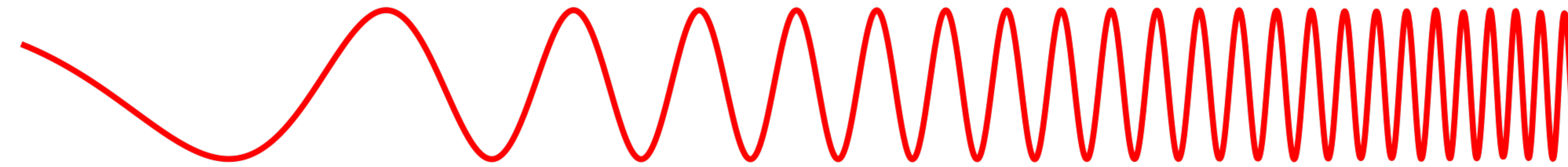
Eco e reverberação

TANQUE DE ÓLEO DE 237 M



Ondas Eletromagnética

Penetrates Earth's Atmosphere?



Radiation Type
Wavelength (m)

Radio
 10^3

Microwave
 10^{-2}

Infrared
 10^{-5}

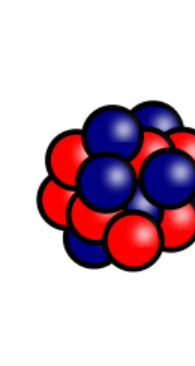
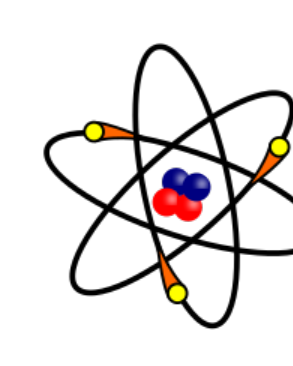
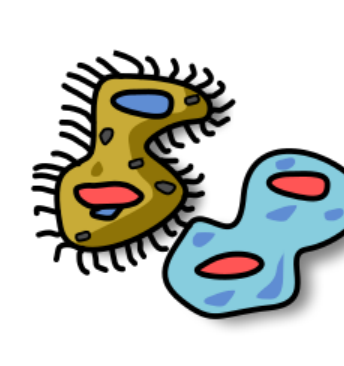
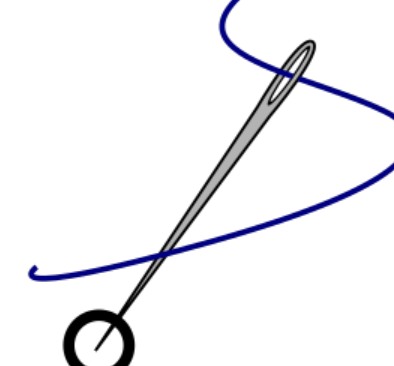
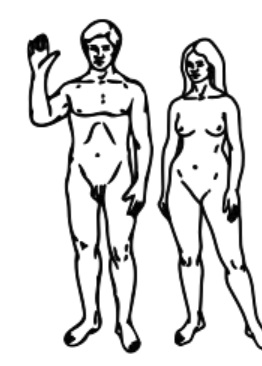
Visible
 0.5×10^{-6}

Ultraviolet
 10^{-8}

X-ray
 10^{-10}

Gamma ray
 10^{-12}

Approximate Scale
of Wavelength



Buildings

Humans

Butterflies

Needle Point

Protozoans

Molecules

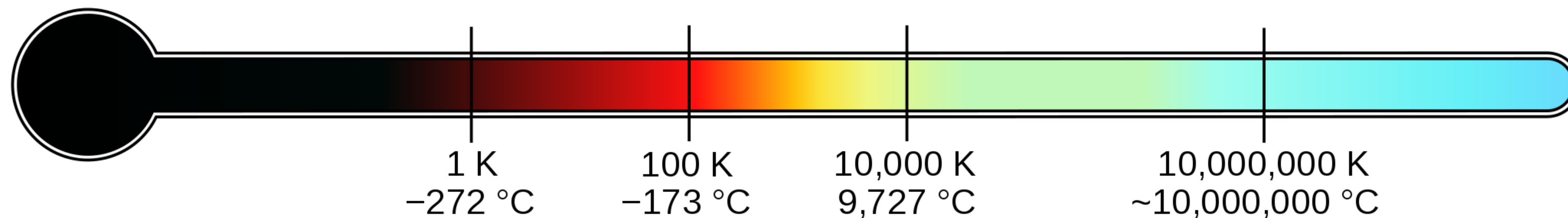
Atoms

Atomic Nuclei

Frequency (Hz)



Temperature of
objects at which
this radiation is the
most intense
wavelength emitted



Ondas Eletromagnética

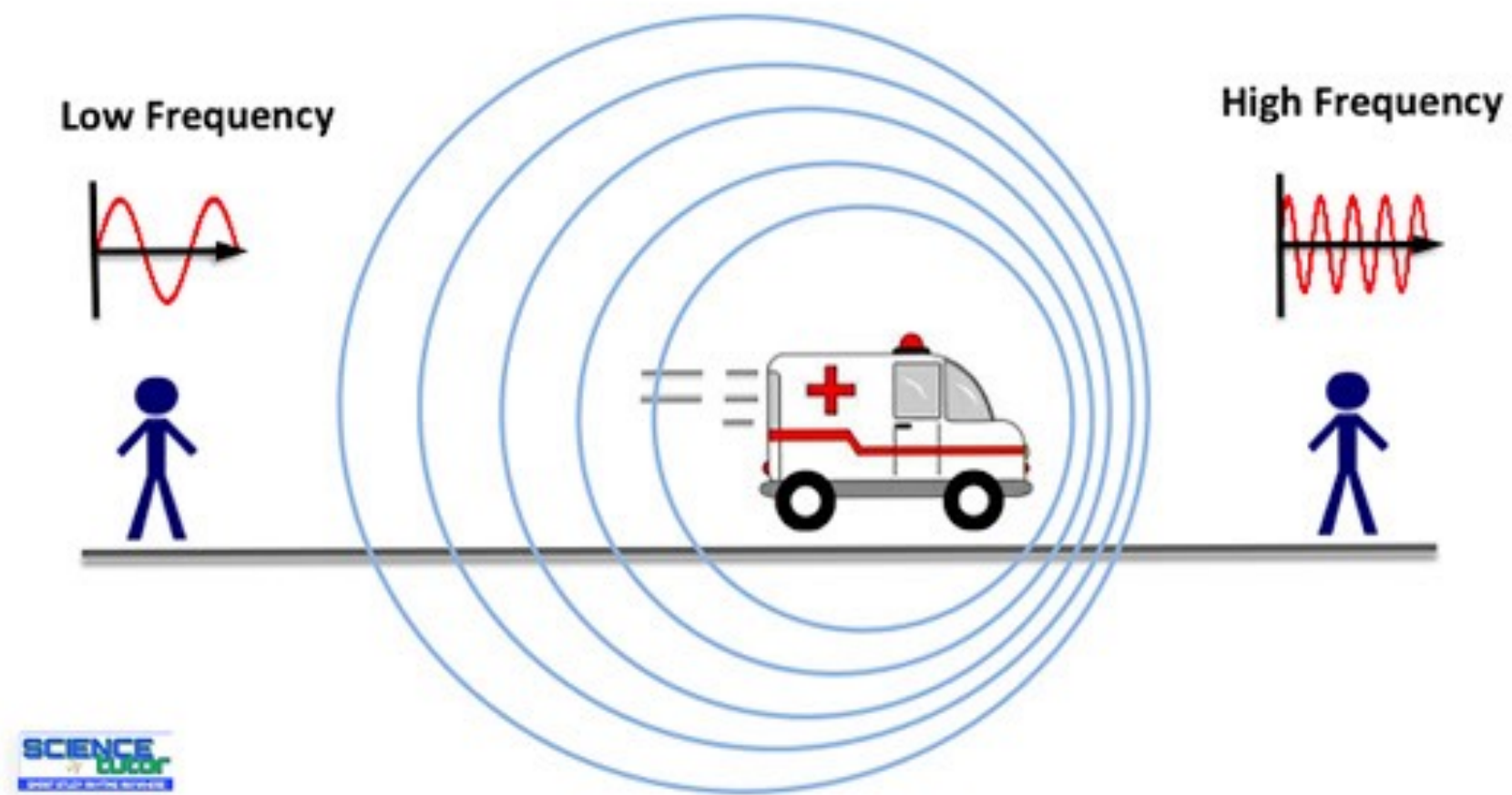
Interferência
fenda dupla

Double Slit Interference

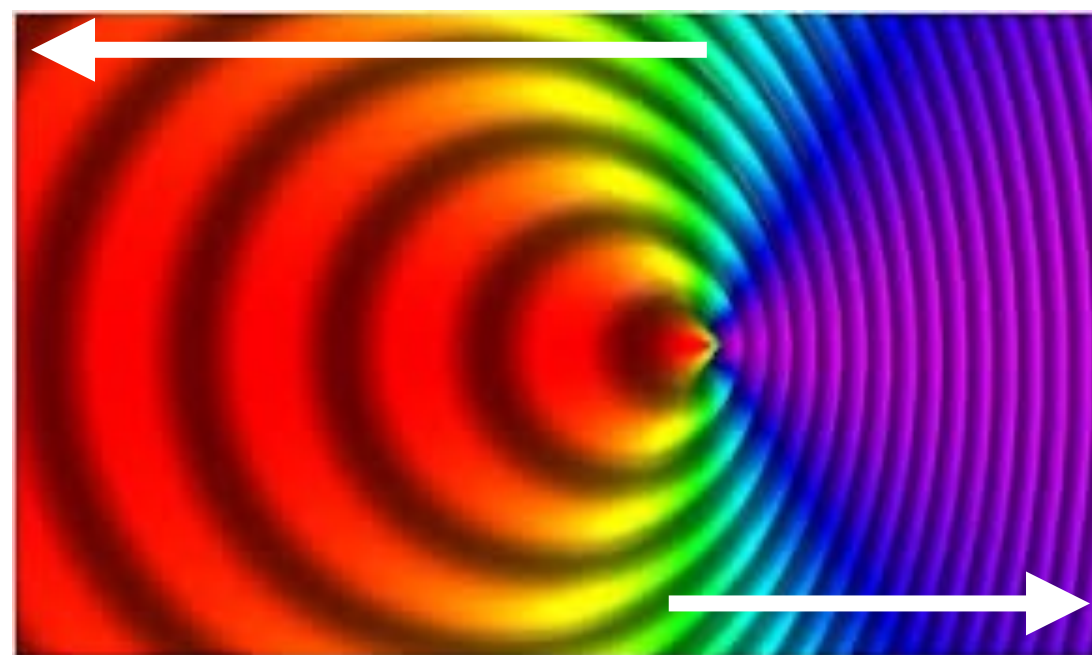
Shining three lasers
with different wavelengths
through two identical
small slits

Efeito Doppler

Doppler's Effect



desvio para o vermelho



desvio para o azul

Luz da estrela

Doppler Shift due to Stellar Wobble

The diagram shows a star (yellow sphere) and an 'Unseen planet' (small grey sphere) orbiting each other. A telescope on Earth is shown receiving light from the star. The light is depicted as a wavy line that is blue on the left and red on the right, representing the Doppler shift. The text 'Doppler Shift due to Stellar Wobble' is written above the diagram.

Unseen planet

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$

- Obs. Efeito Doppler da luz da estrela;
- Necessária grande precisão espectroscópica (m/s)
- Maioria dos planetas (92%)



Ultrapassando a Barreira do Som



Ondas na água, Ondas de Som, Ondas de Luz



@Fermilab

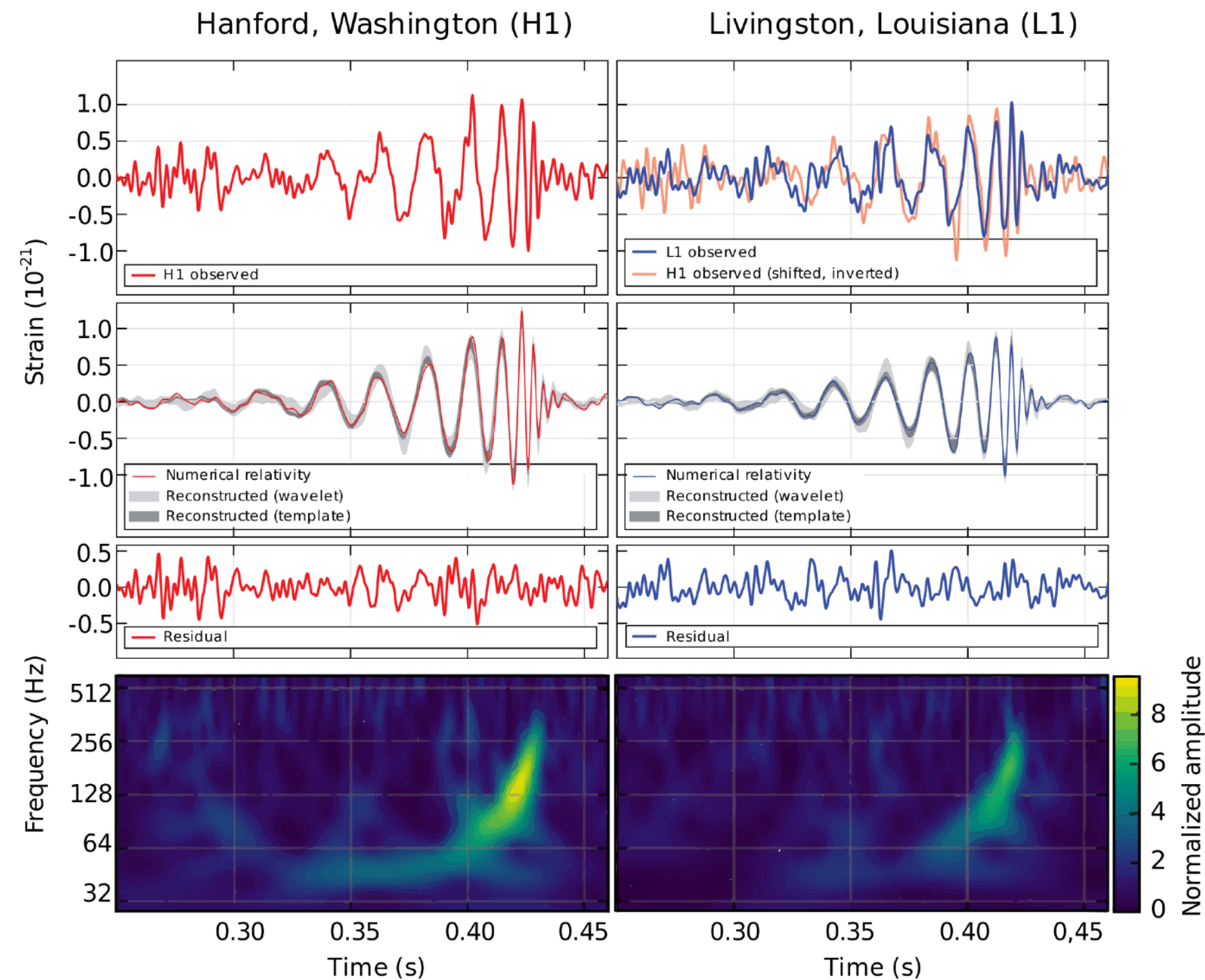
Ondas Gravitacionais



PHDCOMICS.COM/TV

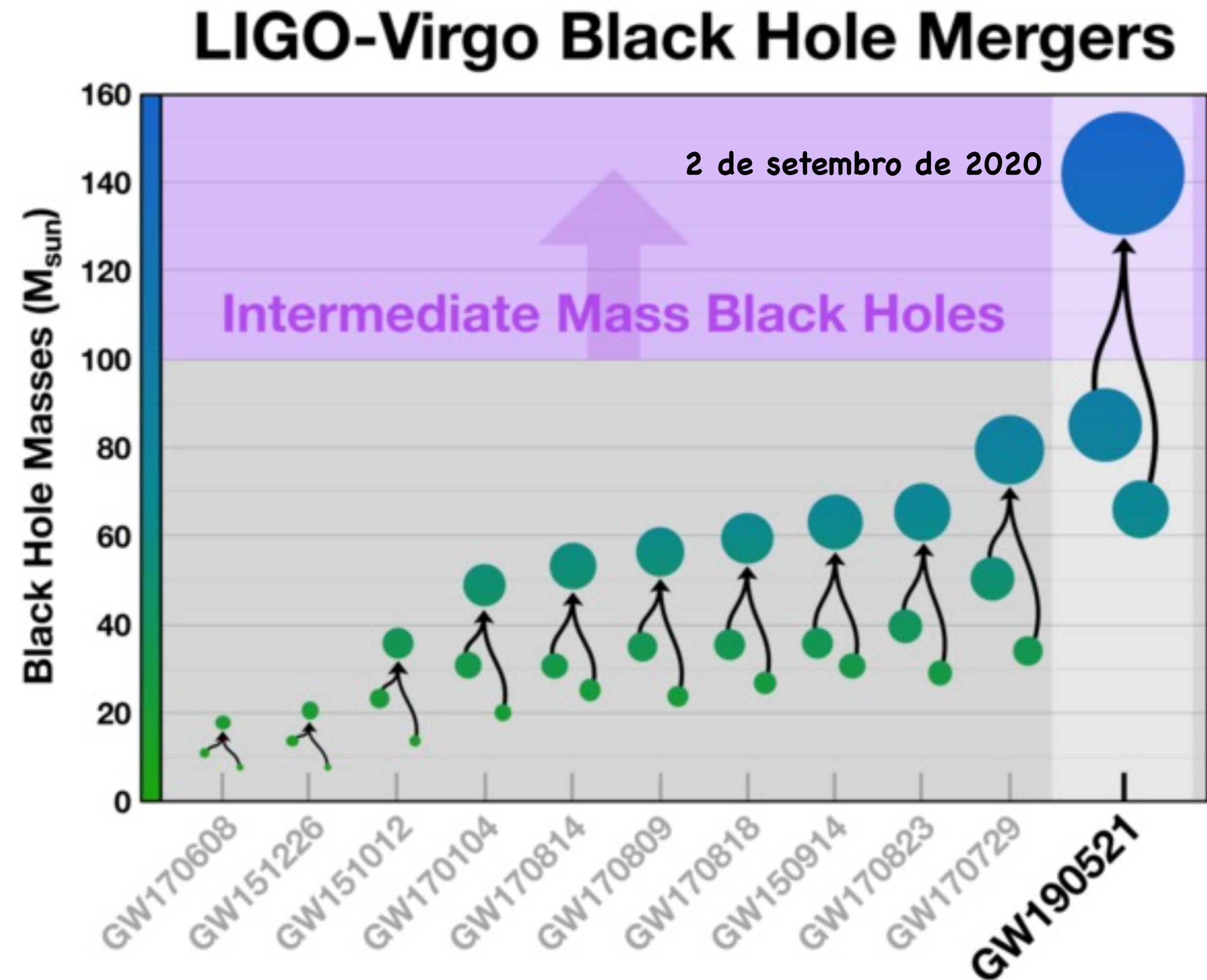
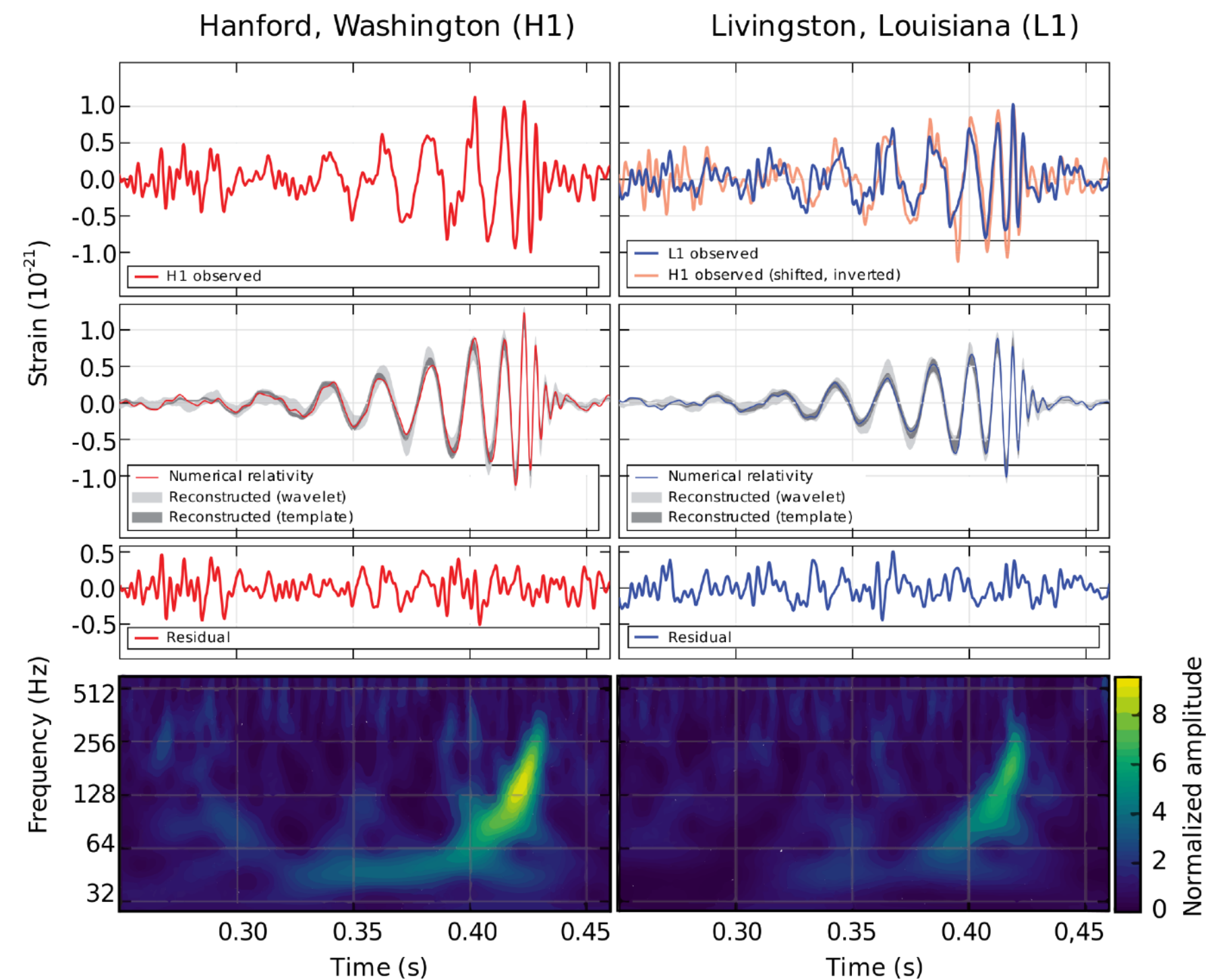
Ondas Gravitacionais

primeira observação
de uma onda gravitacional
14 September 2015



Ondas Gravitacionais

primeira observação
de uma onda gravitacional
14/Set/2015



UFO's, Roswell & Ondas

Acidente no Novo México 1947

Leased Wire
Associated Press

Roswell Daily Record

RECORD PHONES
Business Office 2288
News Department
2287

OL. 47, NUMBER 39, ESTABLISHED 1884

ROSWELL, NEW MEXICO, TUESDAY, JULY 8, 1947

4c PER COPY.

Movies as Usual

GRAND

FRIDAY-SATURDAY
LATEST MOVIES WESTERN
KID COMIC CARTOONS



Levees broke and flood waters rolled into the town of Grand Tower, Ill., but while the manager of the movie theater sweeps out the water that has entered the lobby these youngsters are standing in line for tickets for the night's performance. (AP Wirephoto).

**Some of Soviet Satellites
May Attend Paris Meeting**

Paris, July 8 (AP)—Indications mounted today that at least some of the nations within the Soviet bloc would attend the Paris conference on the Marshall plan.

A Sofia dispatch quoted an authoritative source as saying "probably Bulgaria will participate" in the conference, which opens in Paris Saturday. The dispatch said the Bulgarian council of ministers was meeting to reach a decision on the matter.

Despite a Moscow radio report that Yugoslavia had rejected the British-French invitation to participate, observers in Belgrade said the Yugoslavs still had not replied, and probably will not do so before Thursday — the deadline for an

Europe program, and a dispatch from Prague quoted Czechoslovak sources as suggesting Russia herself might try to get back into the game.

**Roswellians Have
Differing Opinions
On Flying Saurers**

Roswell is a bit uncertain about these flying disks, it would appear from interviews today with a number of local citizens, with about as many ideas concerning them as

**Claims Army
Is Stacking
Courts Martial**

Indiana Senator
Lays Protest
Before Patterson

Washington, July 8 (AP)—Senator Jenner (R-Ind.) contended today that "the high command in the European theatre is stacking courts martial defendants in court martial."

In a letter to Secretary of War Patterson demanding a full investigation of army military trial procedure, Jenner offered what he said was documentary proof that:

1. "Prisoners are not being permitted to employ either civilian or military counsel of their own choice in the preparation and presentation of their defense."

2. "Every effort is being made to prevent attorneys who were connected with the infamous Lindbergh prison case to practice in courts martial in the European theatre."

The Indiana senator made public a copy of an informal "routing slip" which he said was signed by Sirs. Gen. Cornelius E. Ryan, assistant deputy, military government headquarters for the military government for Germany, and written by Col. Francis H. Vanderveen, Jenner told newsmen that the routing slip substantiated his charges.

The slip, addressed to the chief of staff, USFET (presumably US forces, European theatre), was dated last Oct. 22.

It called attention to the impending arrival of Earl J. Carroll and Thomas Lester Foley, California attorneys, to act as special defense counsel for five prisoners then awaiting trial by general court martial at Frankfurt Am Main.

Jenner identified Carroll as counsel in the court martial of Col. James A. Kilian in the Lichfield trials after an alleged attempt at a diplomatic settlement.

Carroll, then an army captain, resigned as assistant prosecutor in the Lichfield trials after an alleged attempt at a diplomatic settlement.

The routing slip said that Carroll had received widespread publicity "by violent attacks on the system of military justice" and added "it is understood that Foley is an individual of similar connections, even

**RAAF Captures Flying Saucer
On Ranch in Roswell Region**

Lake Success, July 8 (AP)—The United Nations security council today approved an American blueprint for arms reduction discussions despite a Russian warning that the plan would bring about a collapse of arms regulation efforts.

The vote was 9 to 0, with Russia and Poland abstaining.

In view of Russia's firm stand against the U. S. plan it had been believed the might provoke the big power veto to block it.

Soviet Deputy Foreign Minister Andrei A. Gromyko gave his warning before the United Nations security council in a new effort to revive the Soviet wartime plan which already had been rejected by the commission for conventional armaments.

His challenge was taken up promptly by French Delegate Alexandre Parodi and U. S. Representative Marshall V. Johnson, who announced their opposition to any substitute for the American plan.

Gromyko insisted that no program for arms reduction could succeed unless the plan was linked directly with an absolute prohibition of atomic weapons.

He declared that the U. S. plan approved by the commission did not touch the problems of atomic weapons and the banning of atomic weapons and, for this reason, it offered no basis for a solution.

Gromyko opened debate on the arms question as delegates awaited another major declaration from him later in the day in reply to United States and British demands for action to restore order in the official Balkan situation.

Delegates agreed they were approaching perhaps the gravest moment in U. N. history.

Gromyko said the U. S. plan was not a plan to merely a document outlining a series of broad generalizations. The Soviet proposals on the other hand, he said, offered specific points which should be dealt with, such as the

**House Passes
Tax Slash by
Large Margin**

Defeat Amendment
By Demos to Remove
Many from Rolls

Washington, July 8 (AP)—The house passed today the Republican-backed bill to cut income taxes by \$4,000,000,000 annually for 45,000,000 taxpayers, beginning Jan. 1.

It goes to the senate where approval also is forecast.

The vote was 383 to 11, or more than the two-thirds majority needed to override a presidential veto.

The action, which may encourage another presidential veto, came after Speaker Martin (R-Ill.), personally appealed in the house to pass the bill by such a decisive vote as to persuade the president that the people should have this delayed justice.

The measure is identical with one vetoed by President Truman June 18 as "the wrong kind of tax reduction at the wrong time"—except that the effective date is changed from July 1, 1947 to Jan. 1, 1948.

Congress leaders expect to have the revised bill on Mr. Truman's desk before the week ends.

The house passed the bill after the Republicans had back a proposed Democratic substitute that would have reduced taxes by \$1,379,000,000 and removed 4,000,000 low-income persons from the tax rolls completely.

**Security Council
Paves Way to Talks
On Arms Reductions**

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**No Details of
Flying Disk
Are Revealed**

Roswell Hardware
Man and Wife
Report Disk Seen

The intelligence office of the South West command group at Roswell Army Air Field announced at noon today, that the field has come into possession of a flying saucer.

According to information released by the department, over authority of Maj. J. A. Marvel, intelligence officer, the disk was recovered on a ranch in the Roswell vicinity, after an unidentified rancher had notified Sheriff Groves, there, that he had found the instrument on his premises.

Maj. Marvel said a detail from his department went to the ranch and recovered the disk. It was stated.

After the intelligence office here had inspected the instrument it was flown to "higher headquarters."

The intelligence office stated that no details of the saucer's construction or its appearance had been revealed.

Mr. and Mrs. Dan Wilmet apparently were the only persons in Roswell who have seen what they thought was a flying disk.

They were sitting on their porch at 555 South First last Wednesday night at about ten minutes before ten o'clock when a large glowing object zoomed out of the sky from the southeast, came in a northeasterly direction at a high rate of speed.

Wilmet, called Mrs. Wilmet's attention to it and both ran down into the yard to watch. It was in front less than a minute, perhaps a few seconds, before it disappeared.

Wilmet said that it appeared to him to be about 1,200 feet high and moving fast. He estimated between 400 and 500 miles per hour. In appearance it looked oval in

Ex-King Carol Weds Mme. Lupescu



Former King Carol of Romania and Mrs. Elena Lupescu relax aboard the S. S. America bound for Cuba and Mexico in May, 1941. A member of Carol's household in Rio de Janeiro said the ex-king and his companion for 23 years in reign and exile were recently married at their hotel Copacabana Palace suite. (AP Wirephoto).

**Miners and Operators Sign
Highest Wage Pact in History**

Washington, July 8 (AP)—An agreement starting a nationwide coal strike was signed today by John L. Lewis and a majority of the bituminous operators.

In announcing the signing, Lewis told a news conference that it is "reasonable to assume" the entire industry will accept the agreement within a few days.

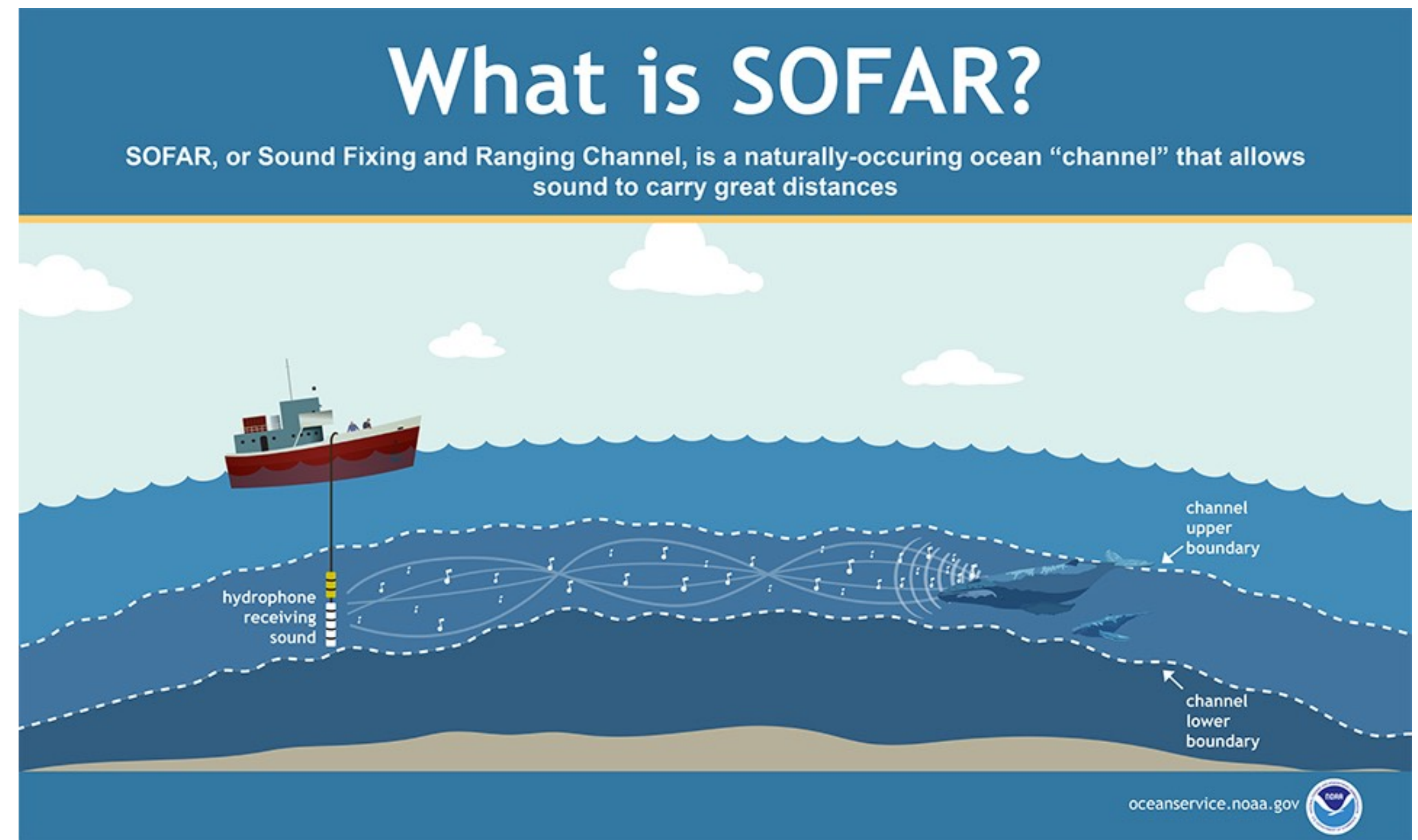
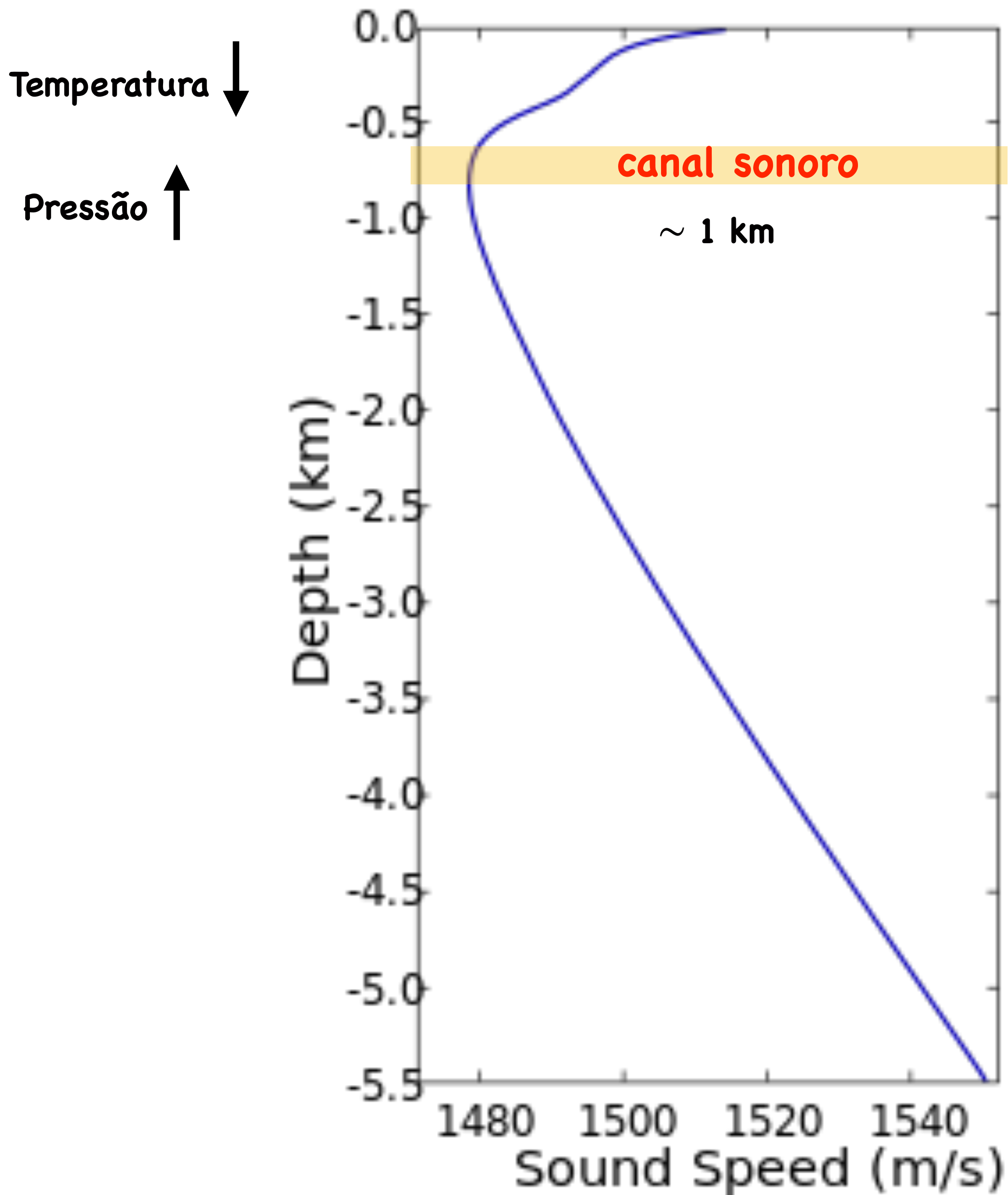
The men have been on a 12-2 variation, which ended last night. Lewis had told them to avoid a formal signing of the contract before going back to the pits.



Projeto Mogul ...

Sound Fixing and Ranging (SOFAR)

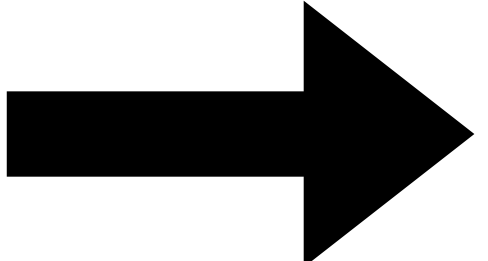
Maurice Ewing no final da WWII
esferas SOFAR

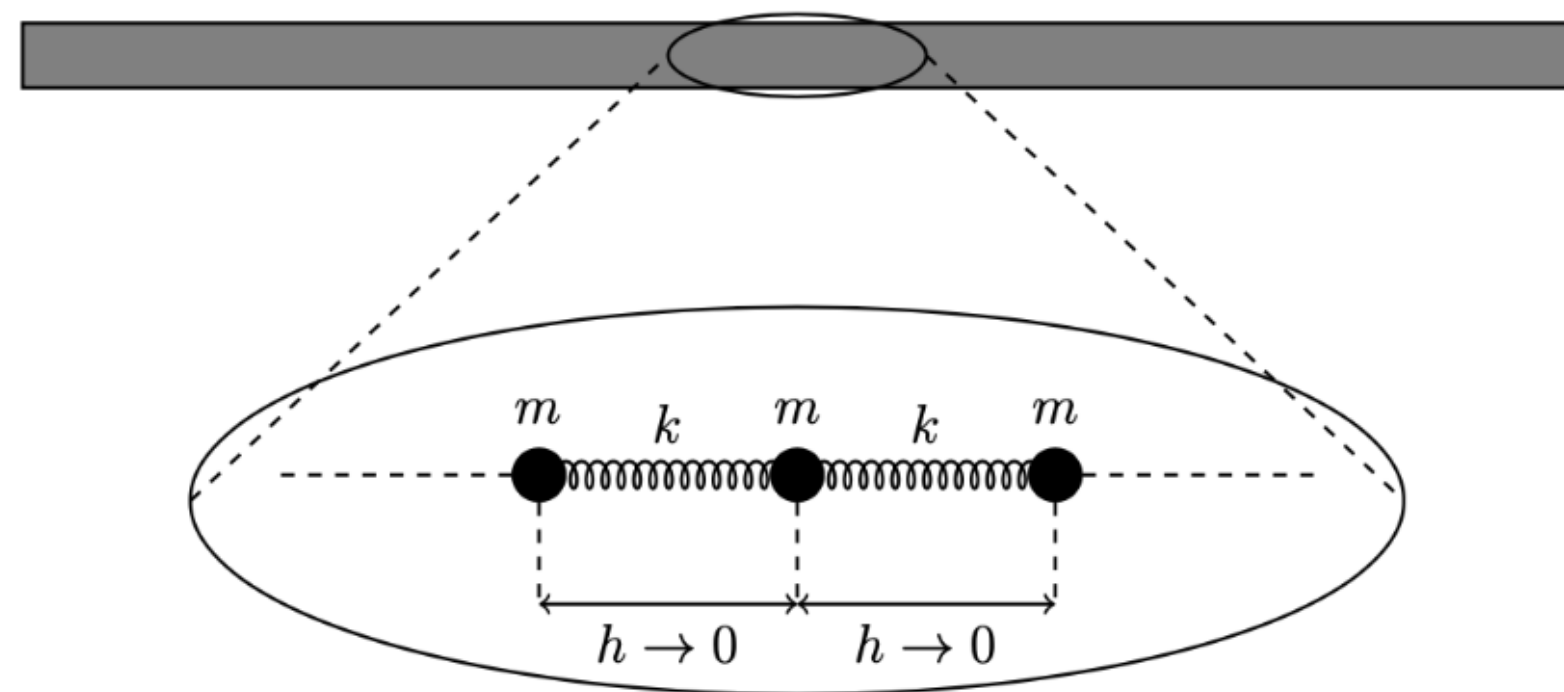


O que é isso ?

Shimizu, Shimizu and Takuma, Phys. Rev. A (1992)

Princípios Físicos Fundamentais

- linearidade  princípio da superposição
- interações locais
- invariância translacional



Lista 2 - Problema 10

Equação de Onda aparece em um sistema de osciladores lineares acoplados

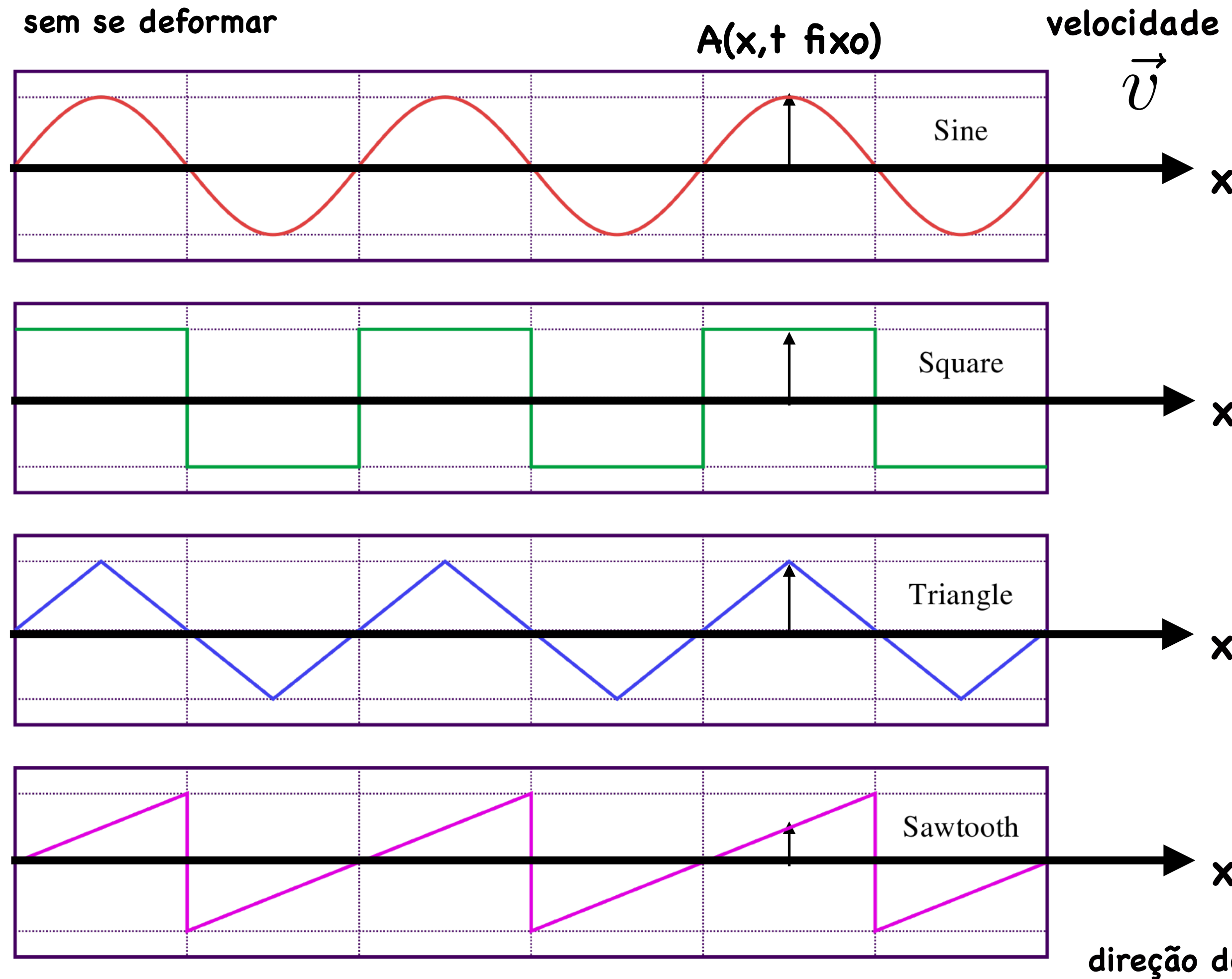
com invariância de translação e interações locais

Figura 3: Modelo simples para uma corda.

Equação de Onda

pulsos que se propagam
sem se deformar

meios não dispersivos



$$\frac{\partial^2 A(x, t)}{\partial t^2} = v^2 \frac{\partial^2 A(x, t)}{\partial x^2}$$

$A(x, t)$

é a amplitude de deslocamento da posição de equilíbrio
do ponto x no instante t

Equação de Onda

$$\frac{\partial^2 A(x, t)}{\partial t^2} = v^2 \frac{\partial^2 A(x, t)}{\partial x^2}$$

$$A(x, t) \sim e^{i\omega t} e^{ikx} = (\cos(\omega t) + i \sin(\omega t)) (\cos(kx) + i \sin(kx))$$

$$\frac{\partial A(x, t)}{\partial t} = i\omega A(x, t)$$

$$\frac{\partial^2 A(x, t)}{\partial t^2} = -\omega^2 A(x, t)$$

$$\frac{\partial A(x, t)}{\partial x} = ik A(x, t)$$

$$\frac{\partial^2 A(x, t)}{\partial x^2} = -k^2 A(x, t)$$



relação de dispersão linear

$$\omega^2 = v^2 k^2$$

$$\omega = vk$$

solução geral para uma frequência fixa $\omega = vk$

$$A(x, t) = a_k \cos(\omega t) \cos(kx) + b_k \cos(\omega t) \sin(kx) + c_k \sin(\omega t) \cos(kx) + d_k \sin(\omega t) \sin(kx)$$

Equação de Onda

$$A(x, t) = a_k \cos(\omega t) \cos(kx) + b_k \cos(\omega t) \sin(kx) + c_k \sin(\omega t) \cos(kx) + d_k \sin(\omega t) \sin(kx)$$

quais

$a_k, b_k, c_k, d_k \neq 0$ depende das condições de contorno, como veremos mais adiante

★ EXEMPLO I:

$$a_k = d_k = a \text{ \& \ } b_k = c_k = 0$$

$$A(x, t) = a(\cos(\omega t) \cos(kx) + \sin(\omega t) \sin(kx)) = a \cos(kx - \omega t) = a \cos(k(x - vt))$$

a amplitude na posição x no futuro é dada pela amplitude na posição à esquerda de x no instante atual

$$A(x, t + \Delta t) = A(x - v\Delta t, t)$$

a onda está se deslocando para a direita

onda progressiva para a direita

Equação de Onda

De maneira mais geral

$$A(x, t) = f(x - vt)$$

$$\frac{\partial^2}{\partial x^2} f(x - vt) = f''(x - vt)$$

$$\frac{\partial^2}{\partial t^2} f(x - vt) = v^2 f''(x - vt)$$



qualquer função $f(x-vt)$ é uma onda progressiva para a direita !

$$\frac{\partial^2}{\partial t^2} f(x - vt) = v^2 \frac{\partial^2}{\partial x^2} f(x - vt)$$

Mas não necessariamente associada a uma frequência fixa ...

Ondas da forma

$$A(x, t) = g(x + vt)$$



$$\frac{\partial^2}{\partial t^2} g(x + vt) = v^2 \frac{\partial^2}{\partial x^2} g(x + vt)$$

onda progressiva para a esquerda

qualquer função $g(x+vt)$ é uma onda progressiva para a esquerda !

Equação de Onda

De maneira mais geral

$$A(x, t) = f(x - vt)$$

$$\frac{\partial^2}{\partial x^2} f(x - vt) = f''(x - vt)$$

$$\frac{\partial^2}{\partial t^2} f(x - vt) = v^2 f''(x - vt)$$

qualquer função $f(x-vt)$ é uma onda progressiva para a direita !

$$\frac{\partial^2}{\partial t^2} f(x - vt)$$

Mas como qualquer solução poder ser escrita como a soma de modos normais (como veremos): qualquer onda progressiva (para esquerda ou direita) pode ser escrita como uma soma de soluções de frequência fixa! (Decomposição de Fourier)

Necessariamente associada a uma frequência fixa ...

Ondas da forma

$$A(x, t) = g(x + vt)$$

$$\frac{\partial^2}{\partial t^2} g(x + vt) = v^2 \frac{\partial^2}{\partial x^2} g(x + vt)$$

qualquer função $g(x+vt)$ é uma onda progressiva para a esquerda !

Equação de Onda

★ **EXEMPLO II:** outro exemplo de frequência fixa

$$A(x, t) = \cos(kx - \omega t) + \cos(kx + \omega t) = 2 \cos(kx) \cos(\omega t)$$

esta solução tem a propriedade que a amplitude em 2 pontos diferentes x_1 e x_2 sempre estão relacionadas por:

$$\frac{A(x_1, t)}{A(x_2, t)} = \frac{\cos(kx_1)}{\cos(kx_2)} \quad \text{onda estacionária}$$

Note que :

1. se ondas progressivas ou estacionárias são produzidas depende das condições iniciais
2. ondas estacionárias são somas de ondas progressivas para a direita e para a esquerda



Série de Fourier

Qualquer solução da equação de onda pode ser escrita como uma soma de modos normais. Esse é o princípio da superposição em ação. (Vimos isso para N osciladores em série !)

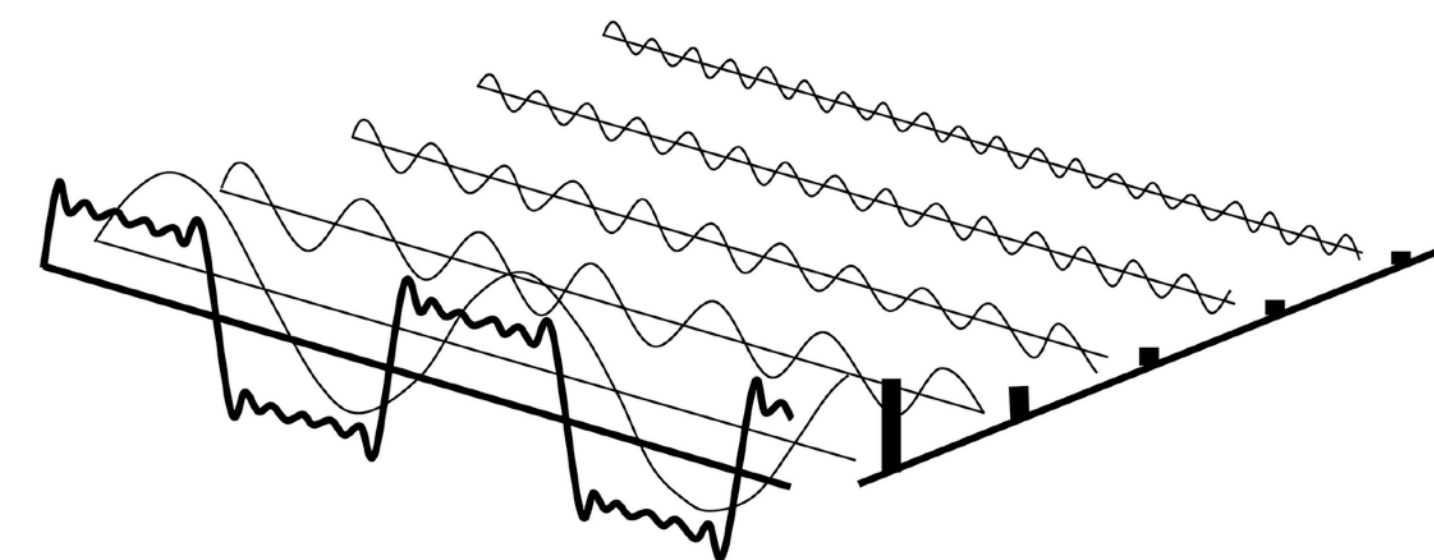
Jean Baptiste Fourier

se

$$f(x + vt) = \sum_k [a_k \cos(kx) \cos(\omega t) + b_k \sin(kx) \cos(\omega t) + c_k \cos(kx) \sin(\omega t) + d_k \sin(kx) \sin(\omega t)]$$

então $t = 0$

$$f(x) = \sum_k [a_k \cos(kx) + b_k \sin(kx)]$$



este é o conteúdo do chamado **Teorema de Fourier** — a prova requer certas propriedades para $f(x)$



Jean Baptiste Fourier

Teorema de Fourier

Qualquer função $f(x)$ para a qual a integral

$$\int_0^L dx f(x)^2 \quad \text{existe} \quad (\text{quadrado integrável})$$

com $f(x)$ periódica no intervalo $0 \leq x \leq L$ i.e. $f(x+L) = f(x)$ podemos escrever

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{L}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{L}x\right)$$



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Teorema de Fourier

com

$$a_0 = \frac{1}{L} \int_0^L dx f(x)$$

$$a_n = \frac{2}{L} \int_0^L dx f(x) \cos\left(\frac{2\pi n}{L}x\right)$$

$$b_n = \frac{2}{L} \int_0^L dx f(x) \sin\left(\frac{2\pi n}{L}x\right)$$

$$\int_0^L dx \cos\left(\frac{2\pi n}{L}x\right) = \int_0^L dx \sin\left(\frac{2\pi n}{L}x\right) = 0 \quad n > 0$$

base de funções ortogonais ("modos normais")

$$\int_0^L dx \cos\left(\frac{2\pi n}{L}x\right) \cos\left(\frac{2\pi m}{L}x\right) = \frac{1}{2} \int_0^L dx \left(\cos\left(\frac{2\pi(n+m)}{L}x\right) + \cos\left(\frac{2\pi(n-m)}{L}x\right) \right) = \frac{L}{2} \delta_{m,n}$$

delta de Kronecker

$$\int_0^L dx \sin\left(\frac{2\pi n}{L}x\right) \sin\left(\frac{2\pi m}{L}x\right) = \frac{L}{2} \delta_{m,n} \quad n > 0 \quad \delta_{m,n} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \quad \int_0^L dx \sin\left(\frac{2\pi n}{L}x\right) \cos\left(\frac{2\pi m}{L}x\right) = 0$$



Teorema de Fourier

Vamos mostrar que

$$a_0 = \frac{1}{L} \int_0^L dx f(x)$$

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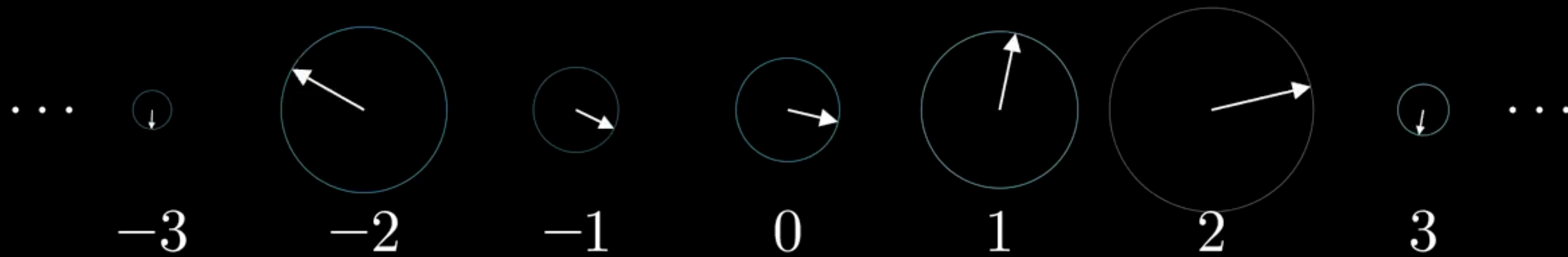
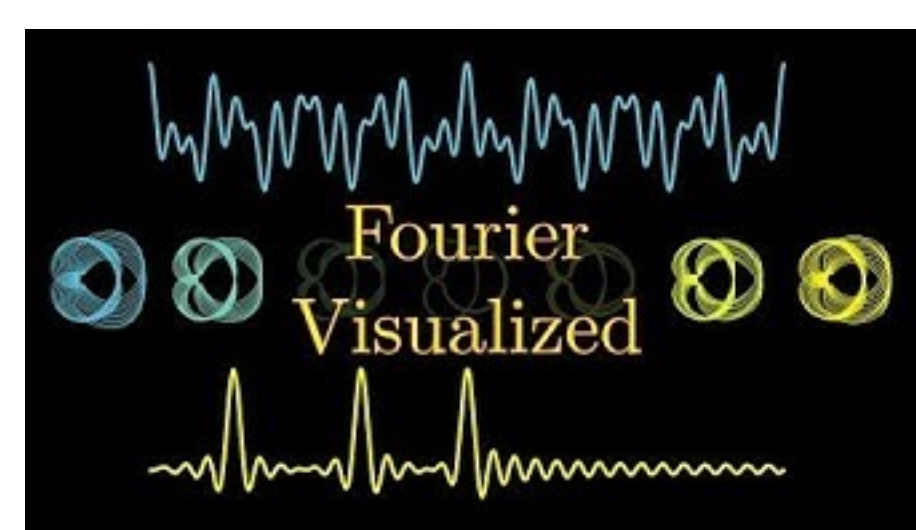
$$\int_0^L dx f(x) = a_0 \int_0^L dx + \sum_{n=1}^{\infty} a_n \int_0^L dx \cos\left(\frac{2\pi n}{L}x\right) + \sum_{n=1}^{\infty} b_n \int_0^L dx \sin\left(\frac{2\pi n}{L}x\right) = a_0 L$$

Você pode mostrar de forma análoga para a_n e b_n $n > 0$

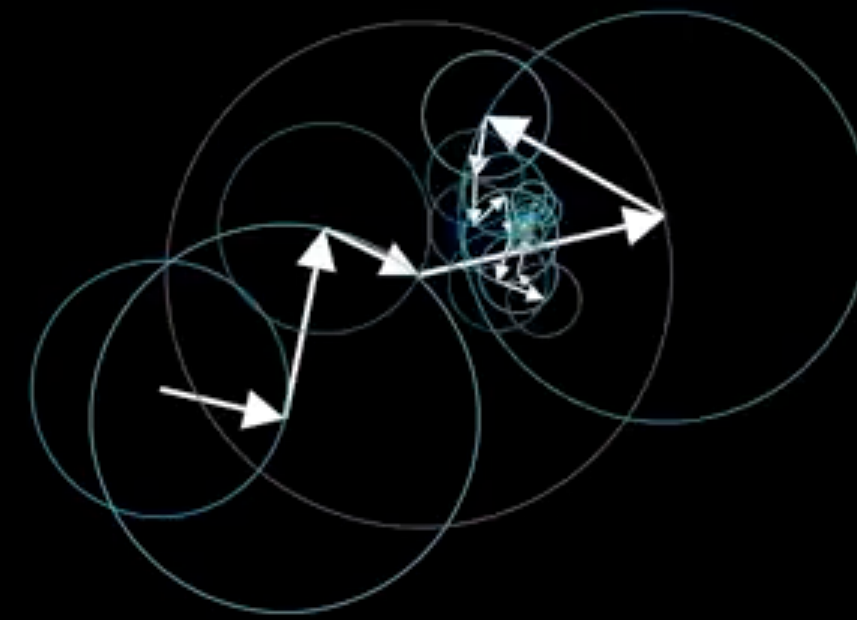
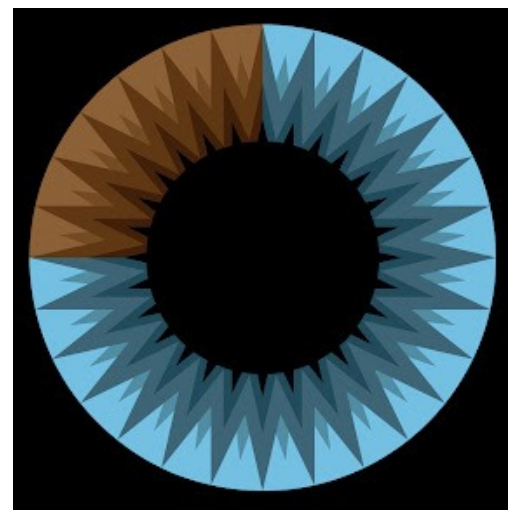
Note que :

1. usamos a série de Fourier de cossenos para funções pares i.e. $f(x) = f(L-x)$
2. usamos a série de Fourier de senos para funções ímpares i.e. $f(x) = -f(L-x)$
3. para funções que não são nem pares nem ímpares precisamos de ambos: senos & cossenos

Série de Fourier



3Blue1Brown



<https://www.youtube.com/watch?v=r6sGWTCMz2k>