UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE MATEMATICA E ESTATISTICA

1
Nota

Nome SUB P1 - MAP0125
Data __/__/__ Nº. USP
Disciplina Turma Prof.

Q1
Q2
Q3
Q4

Q1

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0.93 \\ 1 & +1.5 & -1 & -1.4 \\ 2 & 3 & -5 & 0.56 \end{array}\right) \quad L_1 \leftrightarrow L_3 \sim$$

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & 0.56 \\ 1 & 1.5 & -1 & -1.4 \\ 1 & -2 & 1 & 0,93 \end{array}\right) \quad L_2 \mapsto L_2 - \tfrac{1}{2}L_1 \quad L_3 \mapsto L_3 - \tfrac{1}{2}L_1 \sim$$

$-1.4 - \frac{1}{2} \times 0.56$

$0.93 - \frac{1}{2} \times 0.56$

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & 0.56 \\ 0 & 0 & 1.5 & -1.68 \\ 0 & -3.5 & 3.5 & 0.65 \end{array}\right) \quad L_2 \leftrightarrow L_3 \sim$$

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & 0.56 \\ 0 & -3.5 & 3.5 & 0.65 \\ 0 & 0 & 1.5 & -1.68 \end{array}\right)$$

$z = \frac{-1.68}{1.5} = -1.12$ $\quad y = \frac{0.65 - 3.5(-1.12)}{-3.5} = -1.31$

$x = \frac{0.56 + 5(-1.12) - 3(-1.31)}{2} \rightarrow -0.555$

$$\begin{pmatrix} -0.555 \\ -1.31 \\ -1.12 \end{pmatrix}$$

(b) Resíduo

$$0.93 - [1 \cdot (-0.555) - 2 \cdot (-1.31) + 1 \cdot (-1.12)] = -0.015$$

$$-1.4 - [1 \cdot (-0.555) + 1.5 \cdot (-1.31) - 1 \cdot (-1.12)] = 0$$

$$0.56 - [2 \cdot (-0.555) + 3 \cdot (-1.31) - 5 \cdot (-1.12)] = 0$$

Resíduo: $\begin{pmatrix} -0.015 \\ 0 \\ 0 \end{pmatrix}$

(2) $\log y = \log a + b \log x$ $\quad \log = \ln$, p. ex.

Sist. normal.

$$\begin{cases} \langle 1,1\rangle \log a + \langle 1, \log x\rangle b = \langle 1, \log y\rangle \\ \langle \log x, 1\rangle \log a + \langle \log x, \log x\rangle b = \langle \log x, \log y\rangle \end{cases}$$

x_i	$\log x_i$	y_i	$\log y_i$
0.1	-2.303	160	5.075
0.3	-1.204	10	2.303
0.6	-0.5108	1.85	0.6152
1.1	0.09531	0.4	-0.9163
1.5	0.4055	0.2	-1.609

$\langle 1,1\rangle = 5 \quad \langle 1, \log x\rangle = -3.517 \quad \langle 1, \log y\rangle = 5.468$

$\langle \log x, \log x\rangle = 7.188 \quad \langle \log x, \log y\rangle = -15.51$

$$\begin{cases} 5 \log a - 3.517\, b = 5.468 \\ -3.517 \log a + 7.188\, b = -15.51 \end{cases}$$

$$\left(\begin{array}{cc|c} 5 & -3.517 & 5.468 \\ -3.517 & 7.188 & -15.51 \end{array}\right) \qquad L_2 \mapsto L_2 + \frac{3.517}{5} L_1$$

$7.188 + \frac{3.517}{5}(-3.517) = 4.714$

$-15.51 + \frac{3.517}{5} \cdot 5.468 = -11.66$

$$\left(\begin{array}{cc|c} 5 & -3.517 & 5.468 \\ & 4.714 & -11.66 \end{array}\right) \Rightarrow \boxed{b = -2.473}$$

$\log a = \frac{5.468 + 3.517(-2.473)}{5}$

$= -0.6459$ $\quad \boxed{a = 0.5242}$

Caso de uso de $\log = \log_{10}$

x_i	$\log x_i$	y_i	$\log y_i$
0.1	-1	160	2.204
0.3	-0.5229	10	1
0.6	-0.2218	1.85	0.2672
1.1	0.04139	0.4	-0.3979
1.5	0.1761	0.2	-0.6990

$\langle 1,1 \rangle = 5 \quad \langle 1, \log x \rangle = -1.527 \quad \langle 1, \log y \rangle = 2.374$

$\langle \log x, \log x \rangle = 1.355 \quad \langle \log x, \log y \rangle = -2.926$

$$\begin{pmatrix} 5 & -1.527 & 2.374 \\ -1.527 & 1.355 & -2.926 \end{pmatrix}$$

$L_2 \mapsto L_2 + \frac{1.527}{5} \cdot L_1$

$1.355 + \frac{(1.527)(-1.527)}{5} = 0.8887$

$-2.926 + \frac{1.527}{5}(2.374) = -2.201$

$$\begin{pmatrix} 5 & -1.527 & 2.374 \\ 0 & 0.8887 & -2.201 \end{pmatrix}$$

$b = \frac{-2.201}{0.8887} = -2.477$

$\log a = \frac{2.374 + 1.527(-2.477)}{5} = -0.2817$

$a = 0.5228$

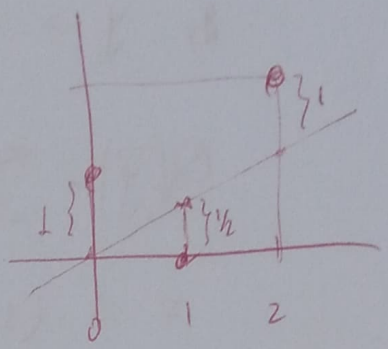
$0.5242\, x^{-2.473}$ ← usando ln

$0.5228\, x^{-2.477}$ ← usando $\log_{10}$

Q3

a)

x_i	y_i	p_i
0	1	1
1	0	4
2	2	1



gráfico

esperados: se fossem pesos iguais, seria "x" ($a=0, b=1$), mas c/ o peso maior no 1 deve puxar um pouco para baixo.

$$\begin{cases} \langle 1,1\rangle a + \langle 1,x\rangle b = \langle 1,y\rangle \\ \langle x,1\rangle a + \langle x,x\rangle b = \langle x,y\rangle \end{cases}$$

$\langle 1,1\rangle = \sum p_i = 6$

$\langle 1,x\rangle = \sum p_i x_i = 4+2 = 6$

$\langle x,x\rangle = \sum p_i x_i^2 = 4+4 = 8$

$\langle 1,y\rangle = \sum p_i y_i = 1+2 = 3$

$\langle x,y\rangle = \sum p_i x_i y_i = 4.$

$$\left(\begin{array}{cc|c} 6 & 6 & 3 \\ 6 & 8 & 4 \end{array}\right) \quad L_2 \mapsto L_2 - L_1 \quad \left(\begin{array}{cc|c} 6 & 6 & 3 \\ 0 & 2 & 1 \end{array}\right)$$

$\Rightarrow \quad 2b = 1 \qquad b = 1/2$

$$a = \frac{3-6b}{6} = \frac{1}{2} - b = 0$$

$0 + \frac{1}{2}x$

(b) R^2

$$\bar{y} = \frac{\sum p_i y_i}{\sum p_i} = \frac{3}{6} = 1/2 .$$

$$Q(\bar{y}) = \sum p_i (y_i - \bar{y})^2$$

$$= 1 \cdot \left(1 - \frac{1}{2}\right)^2 + 4 \cdot \left(0 - \frac{1}{2}\right)^2 + 1 \cdot \left(2 - \frac{1}{2}\right)^2$$

$$= \frac{1}{4} + 4 \times \frac{1}{4} + \frac{9}{4} = \frac{14}{4} = \frac{7}{2}$$

$$\sum p_i \left(y_i - \left(0 + \frac{1}{2} x_i\right)\right)^2$$

$$= 1 \cdot \left(1 - \frac{1}{2} \cdot 0\right)^2 + 4 \cdot \left(0 - \frac{1}{2} \cdot 1\right)^2 + 1 \cdot \left(2 - \frac{1}{2} \cdot 2\right)^2$$

$$= 1 + 1 + 1 = 3$$

$$R^2 = 1 - \frac{3}{7/2} = 1 - \frac{6}{7} = \text{ou } \frac{1}{7}$$ bem vim .