Lista Exercícios

sábado, 1 de julho de 2023 13:37

23) $\int_0^1 2x\,dx$ $\qquad F(x) = ?$

$$\int 2x\,dx = 2\int x\,dx = \cancel{2}\frac{x^2}{\cancel{2}} = x^2 + C$$

$$\int_0^1 2x\,dx = F(1) - F(0) = 1^2 - 0^2 = 1$$

24) $\int_2^7 3v\,dv$ $\qquad F(v) = ?$

$$\int 3v\,dv = 3\int v\,dv = \frac{3v^2}{2} + C$$

$$\int_2^7 3v\,dv = F(7) - F(3) = \frac{3(7)^2}{2} - \frac{3(2)^2}{2} = \frac{147}{2} - \frac{12}{2} = \frac{135}{2}$$

25) $\int_{-1}^0 (x-2)\,dx$ $\qquad F(x) = ?$

$$\int x - 2\,dx = \int x\,dx - \int 2\,dx = \int x\,dx - 2\int dx =$$

$$= \frac{x^2}{2} - 2x + C$$

$$\int_{-1}^0 (x-2)\cdot dx = F(0) - F(-1) = \left[\frac{0^2}{2} - 2(0)\right] - \left[\frac{(-1)^2}{2} - 2(-1)\right] = -\frac{1}{2} - 2 = -\frac{5}{2}$$

26) $\int_2^5 (-3x+4)\,dx$ $\qquad F(x) = ?$

$$\int (-3x+4)\,dx = -3\int x\,dx + 4\int dx = -\frac{3x^2}{2} + 4x + C$$

$$\int_2^5 (-3x+4)\,dx = F(5) - F(2) = \left[-\frac{3(5)^2}{2} + 4(5)\right] - \left[-\frac{3(2)^2}{2} + 4(2)\right] =$$

$$= \left[-\frac{75}{2} + 20\right] - \left[-\frac{12}{2} + 8\right] = \left[\frac{-75+40}{2}\right] - \left[\frac{-12+16}{2}\right]$$

$$= -\frac{35}{2} - 2 = \frac{-35-4}{2} = -\frac{39}{2}$$

27) $\int_{-1}^1 (2t-1)^2\,dt$ $\qquad F(t) = ?$

$$\int (2t-1)^2\,dt = \int 4t^2 - 4t + 1\,dt = 4\int t^2\,dt - 4\int t\,dt + \int dt$$

$$= \frac{4t^3}{3} - \frac{4t^2}{2} + t = \frac{4t^3}{3} - 2t^2 + t + C$$

$$\int_{-1}^1 (2t-1)^2\,dt = F(1) - F(-1) = \left[\frac{4(1)^3}{3} - 2(1)^2 + (1)\right] - \left[\frac{4(-1)^3}{3} - 2(-1) + (-1)\right]$$

$$= \frac{4}{3} - 2 + 1 + \frac{4}{3} - 2 + 1 = \frac{8}{3} - 2 = \frac{8-6}{3} = \frac{2}{3}$$

28) $\int^1 (1-2x)^2\,dx$ $\qquad F(x) = ?$

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28) $\int_0^1 (1-2x)^2 dx$ $\quad F(x) = ?$

$$\int (1-2x)^2 dx = \int 1 - 4x + 4x^2 dx = x - \frac{\cancel{4}^{2} x^2}{\cancel{2}} + \frac{4x^3}{3} + C$$

$$\int_0^1 (1-2x)^2 dx = F(1) - F(0) = \left[1 - 2(1)^2 + \frac{4(1)^3}{3}\right] - [0] = 1 - 2 + \frac{4}{3} = \frac{-3+4}{3} = \frac{1}{3}$$

29) $\int_0^3 (x-2)^3 dx$ $\quad F(x) = ?$

$$\int (\underbrace{x-2}_{u = x-2})^3 dx = \int u^3 du = \frac{u^4}{4} + C \Rightarrow F(x) = \frac{(x-2)^4}{4}$$

$$\frac{du}{dx} = 1 \Rightarrow du = dx$$

$$\int_0^3 (x-2)^3 dx = F(3) - F(0) = \left[\frac{(3-2)^4}{4}\right] - \left[\frac{(0-2)^4}{4}\right] = \frac{1}{4} - 4 = \frac{1-16}{4} = -\frac{15}{4}$$

30) $\int_2^2 (x-3)^4 dx = F(2) - F(2) = 0$ $\quad$ OBS: não sei quem é $F(x)$, mas como será a diferença no mesmo ponto (2) o resultado $= 0$

31) $\int_{-1}^1 (\sqrt[3]{t} - 2)\, dt$ $\quad F(t) = ?$

$$\int \sqrt[3]{t} - 2\, dt = \int \sqrt[3]{t}\, dt - \int 2\, dt = \int t^{1/3} dt - 2\int dt$$

$$= \frac{t^{1/3+1}}{1/3+1} - 2t = \frac{t^{4/3}}{4/3} - 2t = \frac{3\sqrt[3]{t^4}}{4} - 2t + C$$

$$\int_{-1}^1 \sqrt[3]{t} - 2\, dt = F(1) - F(-1) = \left[\frac{3\sqrt[3]{1^4}}{4} - 2(1)\right] - \left[\frac{3\sqrt[3]{(-1)^4}}{4} - 2(-1)\right]$$

$$= \frac{\cancel{3}}{\cancel{4}} - 2 - \frac{\cancel{3}}{\cancel{4}} - 2 = -4$$

32) $\int_1^4 \sqrt{\frac{2}{x}}\, dx \Rightarrow$ $\quad F(x) = ?$

$$\int \sqrt{\frac{2}{x}}\, dx = \int \frac{\sqrt{2}}{\sqrt{x}}\, dx = \sqrt{2}\int \frac{1}{\sqrt{x}}\, dx = \sqrt{2}\int \frac{1}{x^{1/2}}\, dx = \sqrt{2}\int x^{-1/2} dx =$$

$$= \sqrt{2} \cdot \frac{x^{-1/2+1}}{-1/2+1} = \frac{\sqrt{2}\, x^{1/2}}{1/2} = \frac{\sqrt{2} \cdot \sqrt{x}}{1/2} = 2\sqrt{2x} + C$$

$$\int_1^4 \sqrt{\frac{2}{x}}\, dx = F(4) - F(1) = \left[2\sqrt{2 \cdot 4}\right] - \left[2\sqrt{2 \cdot 1}\right] = 2\sqrt{8} - 2\sqrt{2}$$

$$= 2\sqrt{2^3} - 2\sqrt{2}$$

$$= 2\cdot\sqrt{2^2\cdot 2^1} - 2\sqrt{2}$$
$$= 2\sqrt{2^2}\sqrt{2} - 2\sqrt{2}$$
$$= 2\cdot 2\cdot\sqrt{2} - 2\sqrt{2}$$
$$= 4\sqrt{2} - 2\sqrt{2} = \underline{2\sqrt{2}}$$

33) $\displaystyle\int_1^4 \frac{u-2}{\sqrt{u}}\,du$ $\quad F(u) = ?$

$$\int \frac{u-2}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{2}{\sqrt{u}}\,du = \int u^1\cdot u^{-1/2}\,du - \int 2u^{-1/2}\,du =$$
$$= \int u^{1-1/2}\,du - 2\int u^{-1/2}\,du = \int u^{1/2}\,du - 2\int u^{-1/2}\,du =$$
$$= \frac{u^{1/2+1}}{1/2+1} - 2\,\frac{u^{-1/2+1}}{-1/2+1} = \frac{u^{3/2}}{3/2} - \frac{2u^{1/2}}{1/2} = \frac{2\sqrt{u^3}}{3} - 4\sqrt{u} + C$$

$$\int_1^4 \frac{u-2}{\sqrt{u}}\,du = F(4) - F(1) = \left[\frac{2\sqrt{4^3}}{3} - 4\sqrt{4}\right] - \left[\frac{2\sqrt{1^3}}{3} - 4\sqrt{1}\right] =$$
$$= \left[\frac{2\cdot\sqrt{64}}{3} - 8\right] - \left[\frac{2}{3} - 4\right] = \frac{16}{3} - 8 - \frac{2}{3} + 4 = \frac{14}{3} - 4 =$$
$$= \frac{14-12}{3} = \underline{\frac{2}{3}}$$

34) Semelhante ao 33

35) $\displaystyle\int_{-1}^{0} \left(t^{1/3} - t^{2/3}\right) dt$ $\quad F(t) = ?$

$$\int t^{1/3}\,dt - \int t^{2/3}\,dt = \frac{t^{1/3+1}}{1/3+1} - \frac{t^{2/3+1}}{2/3+1} = \frac{t^{4/3}}{4/3} - \frac{t^{5/3}}{5/3}$$
$$= \frac{3\sqrt[3]{t^4}}{4} - \frac{3\sqrt[3]{t^5}}{5} + C$$

$$\int_{-1}^{0} \left(t^{1/3} - t^{2/3}\right) dt = F(0) - F(-1) = \left[\cancel{\frac{3\sqrt[3]{0^4}}{4} - \frac{3\sqrt[3]{0^5}}{5}}^{\,0}\right] - \left[\frac{3\sqrt[3]{(-1)^4}}{4} - \frac{3\sqrt[3]{(-1)^5}}{5}\right]$$
$$= -\frac{3}{4} - \frac{3}{5} = \frac{-15-12}{20} = \underline{\frac{-27}{20}}$$

36) Semelhante ao 35

37) $\displaystyle\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = \int \frac{1}{\sqrt{u}}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du$

$$= \frac{1}{2}\cdot\frac{u^{-1/2+1}}{-1/2+1} = \frac{u^{1/2}}{2\cdot\frac{1}{2}} = \sqrt{u}$$

$u = 2x+1$

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$

$\sqrt{2x+1} + C$

$u = 2x+1$

$$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2} \qquad\qquad = \sqrt{2x+1} + C$$

$$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = F(4) - F(0) = \left[\sqrt{2(4)+1}\right] - \left[\sqrt{2(0)+1}\right] = \sqrt{9} - \sqrt{1} = 3 - 1 = 2$$

38) semelhante ao 37

39) $$\int_0^1 e^{-2x}\,dx = \int e^u \frac{du}{-2} = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u = -\frac{1}{2}e^{-2x} + C$$

$u = -2x$

$$\frac{du}{dx} = -2 \Rightarrow dx = \frac{du}{-2}$$

calculadora $\begin{cases} e^{-2} \approx 0{,}1353 \\ e^{0} = 1 \end{cases}$

$$\int_0^1 e^{-2x}\,dx = F(1) - F(0) = \left[-\frac{e^{-2(1)}}{2}\right] - \left[-\frac{e^{-2(0)}}{2}\right] = -\frac{e^{-2}}{2} + \frac{e^{0}}{2} = -\frac{e^{-2}}{2} + \frac{1}{2}$$

$$\approx 0{,}4323$$

40) Semelhante ao 39

$$u = \frac{3}{x} \qquad \frac{du}{dx} = -\frac{3}{x^2} \Rightarrow dx = -\frac{x^2}{3}\,du$$

41) $$\int_1^3 \frac{e^{3/x}}{x^2}\,dx = \int \frac{e^u}{\cancel{x^2}} \cdot -\frac{\cancel{x^2}}{3}\,du = -\frac{1}{3}\int e^u\,du = -\frac{e^u}{3} = -\frac{e^{3/x}}{3} + C$$

$$\int_1^3 \frac{e^{3/x}}{x^2}\,dx = F(3) - F(1) = \left[-\frac{e^{3/3}}{3}\right] - \left[-\frac{e^{3/1}}{3}\right] = -\frac{e^1}{3} + \frac{e^3}{3} \approx 5{,}7891$$

42) $$\int_{-1}^{1} (e^x - e^{-x})\,dx \qquad F(x) = ?$$

$$u = -x \qquad \frac{du}{dx} = -1 \qquad dx = -du$$

$$\int e^x - e^{-x}\,dx = \int e^x\,dx - \int e^{-x}\,dx$$

$$= \int e^x\,dx - \int e^u(-du) = \int e^x\,dx + \int e^u\,du$$

$$= e^x + e^u = e^x + e^{-x} + C$$

$$\int_{-1}^{1} (e^x - e^{-x})\,dx = F(1) - F(-1) = [e^1 + e^{-1}] - [e^{-1} + e^{-(-1)}] = \cancel{e} + \cancel{e^{-1}} - \cancel{e^{-1}} - \cancel{e} = 0$$

43) $$\int_0^1 e^{2x}\sqrt{e^{2x}+1}\,dx \qquad F(x) = ?$$

$u = e^{2x} + 1$

$$\frac{du}{dx} = 2e^{2x} \Rightarrow dx = \frac{1}{2e^{2x}}\,du$$

$\Rightarrow u = e^{2x} + 1$

$\frac{du}{dx} = 2e^{2x} \Rightarrow dx = \frac{1}{2e^{2x}} du$

$\int \cancel{e^{2x}} \sqrt{u} \, \frac{1}{2\cancel{e^{2x}}} du = \frac{1}{2} \int u^{1/2} du = \frac{1}{\cancel{2}} \frac{u^{3/2}}{3/\cancel{2}} = \frac{u^{3/2}}{3} = \frac{\sqrt{(e^{2x}+1)^3}}{3} + C$

$\int_0^1 e^{2x} \sqrt{e^{2x}+1} \, dx = F(1) - F(0) = \frac{\sqrt{(e^2+1)^3}}{3} - \frac{\sqrt{(e^0+1)^3}}{3} = \underline{\frac{\sqrt{(e^2+1)^3}}{3} - \sqrt{\frac{8}{3}}}$

calculadora

$\approx 7{,}1565$

44) $\int_0^1 \frac{e^{-x}}{\sqrt{e^{-x}+1}} dx = \int \frac{\cancel{e^{-x}}}{u^{1/2}} \left(-\frac{1}{\cancel{e^{-x}}} du \right) = -\int u^{-1/2} du$

$u = e^{-x} + 1$

$\frac{du}{dx} = -e^{-x}$

$dx = -\frac{1}{e^{-x}} du$

$= -\frac{u^{-1/2+1}}{-1/2+1} = -2u^{1/2} = -2\sqrt{e^{-x}+1} + C$

$\int_0^1 \frac{e^{-x}}{\sqrt{e^{-x}+1}} dx = F(1) - F(0) = \left[-2\sqrt{e^{-1}+1}\right] - \left[-2\sqrt{e^0+1}\right] \approx 0{,}4893$

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