# Solução problema 9

## Dados

$B = 5\,\text{Gb/seg}$

$V_0 = A = 2\times10^{-6}L$

$V_1 = B = 1 \cancel{+}\, 2\times10^{-6}L$

} fechamento da altura do olho devido à deformação do sinal devido à dispersão cromática

$\mathcal{R} = 1\,\text{A/W}$

$R_L = 50\,\Omega$

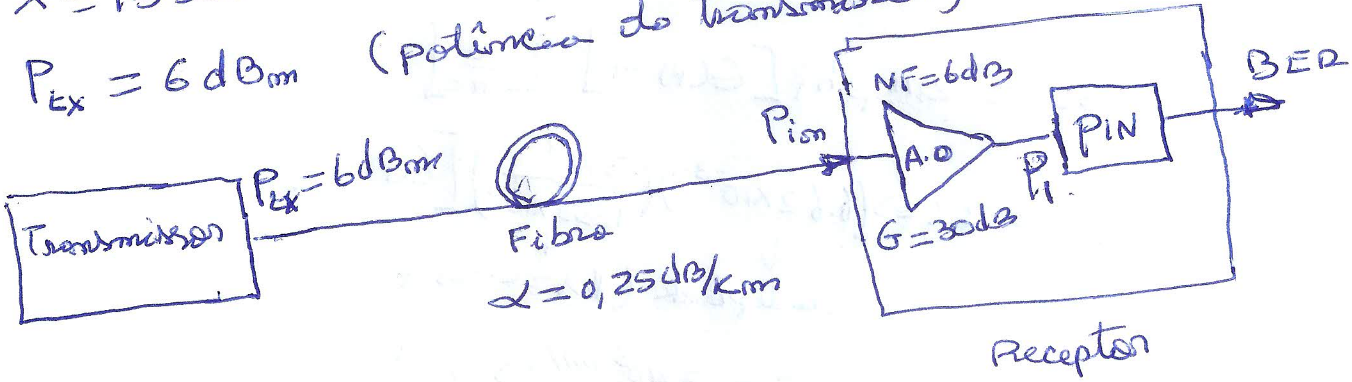
$B_e = 5\,\text{GHz}$

$F_n(\cancel{\text{ou}}\ NF) = 6\,\text{dB}$ (F: figura de ruído do amplificador óptico)

$G\ (\text{óptico}) = 30\,\text{dB}$

$\lambda = 1550\,\text{nm}$

$P_{Tx} = 6\,\text{dBm}$ (potência do transmissor)




A.O : amplificador óptico

PIN : fotodetector de diodo PIN

$P_1 = 2P_{Ave}$ ↳ potência media do nível "1"

a) Encontre $L_{max} = ?$ para que $BER = 10^{-12}$

$$\Rightarrow\ Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{(A-B)\,2\mathcal{R}P_{Ave}}{\sqrt{\left(\sigma_T^2 + \sigma_{dk}^2 + \sigma_{sh}^2 + \sigma_{s\text{-}ASE}^2\right)_1 B_e} + \sqrt{\left(\sigma_T^2 + \sigma_{dk}^2 + \sigma_{sh}^2 + \sigma_{s\text{-}ASE}^2\right)_0 B_e}}$$

$\sigma_T^2,\ \sigma_{dk}^2$ : independentes do sinal ($2P_{Ave}$)

$\sigma_{sh}^2,\ \sigma_{s\text{-}ASE}^2$ : dependentes do sinal

$P_1 = 2P_{Ave} \Rightarrow G = \dfrac{P_1}{P_{in}}$ ou $P_1 = G P_{in}$

②

- Para o sistema de Receptor com pre-amplificador óptico o ruido sinal-emissão spontanea (S-ASE) é dominante

$\Rightarrow$ Na eq. de Q, negligenciaremos $\sigma_T, \sigma_{dk}, \sigma_{SH}$

$$\Rightarrow \quad Q = \frac{(A-B)\mathcal{R}P_1}{\sqrt{(\sigma^2_{S-ASE})_1 B_e} + \sqrt{(\sigma^2_{S-ASE})_0 B_e}}$$

para o estado "1" — para o estado "0"

$$\sigma^2_{S-ASE} = M^2 F_M 2 \mathcal{R}^2 P_s P_{ASE}$$

$P_s$ → potência do sinal

$$P_{ASE} = 2n_{sp} h f [G(f)-1] \quad \left[\frac{W}{Hz}\right]$$

$$P_{ASE} = 2(2)(6.62\times10^{-34})\left(\frac{3\times10^8}{1550\times10^{-9}}\right)[G(f)-1] \quad [W/Hz]$$

cte de planck $(f\lambda = c \Rightarrow f = \frac{c}{\lambda})$

$c = 3\times10^8$ m/seg, $\lambda = 1550$ nm

$$P_{ASE} = 5{,}12\times10^{-19}[G(f)-1] \quad [W/Hz]$$

$$G(dB) = 30\,dB \Rightarrow G(f) = 10^{\frac{30dB}{10}} = 10^3 \gg 1$$

$$\boxed{P_{ASE} = 5{,}12\times10^{-16}\left[\frac{W}{Hz}\right]}$$

~~(nivel 1 para o sinal)~~

- $(\sigma^2_{S-ASE})_1 = 2F_M(\underbrace{AP_1}_{P_s})P_{ASE} = 2(\underbrace{10^{6dB/10}}_{F_M})AP_1P_{ASE}$

$= 8AP_1P_{ASE}$

$(\sigma^2_{S-ASE})_0 = 2F_M(BP_1)P_{ASE} = 8BP_1P_{ASE}$

$\Big\}\ \mathcal{R} = 1\,A/W$

$\Downarrow$

$\Rightarrow \quad \mathcal{R} = 1\ A/W$

$$Q = \frac{(A-B)P_i}{\sqrt{(8AP_i \rho_{ASE})B_e} + \sqrt{(8BP_i\rho_{ASE})B_e}}$$

$$Q = \frac{(A-B)\,P_i}{(\sqrt{A}+\sqrt{B})\sqrt{P_i}\sqrt{8\rho_{ASE}B_e}} = \frac{(A-B)\sqrt{P_i}}{(\sqrt{A}+\sqrt{B})\sqrt{8\rho_{ASE}B_e}}$$

$$A-B = ((\sqrt{A})^2 - (\sqrt{B})^2) = (\sqrt{A}+\sqrt{B})(\sqrt{A}-\sqrt{B})$$

$$\Rightarrow Q = (\sqrt{A} - \sqrt{B})\sqrt{\frac{P_i}{\rho_{ASE}B_e}}$$

$$(\sqrt{A}-\sqrt{B}) = Q\sqrt{\frac{\rho_{ASE}B_e}{P_i}}$$

$$\Rightarrow \boxed{(\sqrt{A}-\sqrt{B})^2 = Q^2\left(\frac{8\rho_{ASE}B_e}{P_i}\right)}$$

$A = 1 - aL$

$B = aL$

$L$: Comprimento da fibra

$a = 2\times10^{-6}$

-- Aproximado $P_i = 0\ dBm$
(Amplificadores compensam as perdas na fibra)

$$P_i = 1\ mW = 10^{-3}\ W$$

$$\Rightarrow \left(\frac{8\rho_{ASE}B_e}{P_i}\right) = \left(\frac{8(5\times10^{-16}\frac{W}{Hz})(5\times10^{9}\ Hz)}{10^{-3}\ W}\right) = 0{,}02$$

$$\Rightarrow (\sqrt{A}-\sqrt{B})^2 = (7)^2(0{,}02) = 0{,}98$$

$$= A + B - 2\sqrt{AB} = 0{,}98$$

$$(1-aL) + aL - 2\sqrt{(1-aL)aL} = 0{,}98$$

$$1 - 2\sqrt{(1-aL)aL} = 0{,}98 \Rightarrow \sqrt{(1-aL)aL} = 10^{-2}$$

$$\Rightarrow aL - (aL)^2 = 10^{-4} \qquad (aL)^2 - (aL) + 10^{-4} = 0$$

$$L \simeq a^{-1} = (2\times10^{-6})^{-1}\ m$$

$$L = 0{,}5\times10^{6} = 5\times10^{5}\ m$$

Assim $\boxed{L_{max} = 5 \times 10^5\, m = 500\, Km \quad \text{para } BER = 10^{-12}}$

b) Dispersão cromatica compensada

$\Rightarrow A = 1 \, , \, B = 0$

$$\Rightarrow Q = \frac{P_1}{\sqrt{(\sigma_{S-ASE})_1 B_e}} \qquad \left( (\sigma_{S-ASE})_0 = 0 \right) \quad (B = 0)$$

$$\Rightarrow Q = \frac{P_1}{\sqrt{8 P_1 \rho_{ASE} B_e}} = \frac{\sqrt{P_1}}{\sqrt{8 \rho_{ASE} B_e}}$$

$$\Rightarrow P_1 = Q^2 (8 \rho_{ASE} B_e)$$

Neste caso $\quad P_1 = G P_{in} \quad , \quad P_{in}(dB) = P_{tx} - \alpha L \quad (dB)$

$$\alpha = 0{,}25 \frac{dB}{Km}$$

$$Q^2 8 \rho_{ASE} B_e = (49)(8)\left(5 \times 10^{-16} \frac{W}{Hz}\right)(5 \times 10^9\, Hz)$$

$$= 7{,}84 \times 10^{-4}\, W \Rightarrow 0{,}784\, mW$$

$$= -1{,}06\, dBm$$

$$P_1 = G(P_{tx} - \alpha L) = -1{,}06\, dBm$$

$$P_{tx} - \alpha L = -\frac{1{,}06\, dBm}{30} = -0{,}0353\, dBm = -3{,}53 \times 10^{-2}\, dBm$$

$$\alpha L = P_{tx} + 3{,}53 \times 10^{-2}\, dB = 6{,}0353\, dBm$$

$$= 6 + 3{,}53 \times 10^{-2}\, dB$$

$$\boxed{L_{max} = \frac{6{,}035\, dBm}{0{,}25\, dB/Km} = 24\, Km}$$

a fibra ~~não~~ promovera distorção do sinal (fechamento do olho se L for 24 Km //

problema (6)

a)

$$BER = \frac{1}{4}\left[ erfc\left(\frac{Q_1}{\sqrt{2}}\right) + erfc\left(\frac{Q_0}{\sigma_0 \sqrt{2}}\right)\right]$$

$$Q_1 = \frac{V_1 - V_{th}}{\sigma_1} \quad , \quad Q_0 = \frac{V_{th} - V_0}{\sigma_0}$$

Diagrama do olho normalizado

$$\begin{matrix} V_1 = 1 \\ V_0 = 0 \end{matrix} \Rightarrow \begin{matrix} Q_0 = V_{th}/\sigma_0 \\ Q_1 = \frac{1 - V_{th}}{\sigma_1} \end{matrix}$$

$\Rightarrow$ otimizando o BER em relação a $V_{th}$

$$\frac{\partial BER}{\partial V_{th}} = \frac{1}{4}\left[\frac{\partial\, erfc(Q_1/\sqrt{2})}{\partial (Q_1/\sqrt{2})} \frac{\partial (Q_1/\sqrt{2})}{\partial V_{th}} + \frac{\partial\, erfc(Q_0/\sqrt{2})}{\partial (Q_0/\sqrt{2})} \frac{\partial (Q_0/\sqrt{2})}{\partial V_{th}}\right] = 0$$

$$\Rightarrow \frac{\partial\, erfc(x)}{\partial x} = \lim_{\Delta x \to 0} \frac{erfc(x+\Delta x) - erfc(x)}{\Delta x}$$

$$= \left(\lim_{\Delta x \to 0} \frac{\int_{x+\Delta x}^{\infty} e^{-y^2} dy - \int_{x}^{\infty} e^{-y^2} dy}{\Delta x}\right) \frac{2}{\sqrt{\pi}}$$

$$= \left(\lim_{\Delta x \to 0} \frac{\cancel{\int_{x+\Delta x}^{\infty} e^{-y^2}} - \int_{x}^{x+\Delta x} e^{-y^2} dy - \cancel{\int_{x+\Delta x}^{\infty} e^{-y^2} dy}}{\Delta x}\right) \frac{2}{\sqrt{\pi}}$$

$$= \frac{2}{\sqrt{\pi}} \lim_{\Delta x \to 0} \frac{-\int_{x}^{x+\Delta x} e^{-y^2} dy}{\Delta x} = \frac{2}{\sqrt{\pi}}\left(-e^{-x^2}\right)$$

$$= -\frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$\Rightarrow \frac{\partial BER}{\partial V_{th}} = \left(\frac{1}{4}\right)\left(-\frac{2}{\sqrt{\pi}}\right)\left[ e^{-Q_1^2/2}\left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sigma_1}\right) + e^{-Q_0^2/2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sigma_0}\right)\right]$$

$$= \left(\frac{1}{4}\right)\left(\frac{2}{\sqrt{\pi}}\right)\left(\frac{1}{\sqrt{2}}\right)\left[\frac{e^{-Q_1^2/2}}{\sigma_1} - \frac{e^{-Q_0^2/2}}{\sigma_0}\right] = 0$$

$$\frac{e^{-Q_1^2/2}}{\sigma_1} - \frac{e^{-Q_0^2/2}}{\sigma_0} = 0$$

$$\Rightarrow \frac{e^{-Q_1^2/2}}{\sigma_1} = \frac{e^{-Q_0^2/2}}{\sigma_0}$$

$$\frac{e^{-Q_1^2/2}}{e^{-Q_0^2/2}} = \frac{\sigma_1}{\sigma_0}$$

$$\Rightarrow e^{\frac{Q_0^2 - Q_1^2}{2}} = \frac{\sigma_1}{\sigma_0}$$

$$Q_0^2 - Q_1^2 = 2\ln\left(\frac{\sigma_1}{\sigma_0}\right)$$

$$\Rightarrow \frac{V_{th}^2}{\sigma_0^2} - \frac{(1-V_{th})^2}{\sigma_1^2} = 2\ln\left(\frac{\sigma_1}{\sigma_0}\right)$$

$$\sigma_1^2 V_{th}^2 - \sigma_0^2(1 - 2V_{th} + V_{th}^2) = 2\sigma_0^2\sigma_1^2 \ln\left(\frac{\sigma_1}{\sigma_0}\right)$$

$$V_{th}^2(\sigma_1^2 - \sigma_0^2) + 2V_{th}\sigma_0^2 - \sigma_0^2 = 2\sigma_0^2\sigma_1^2 \ln\left(\frac{\sigma_1}{\sigma_0}\right)$$

$$V_{th}^2 + \frac{2\sigma_0^2}{\sigma_1^2 - \sigma_0^2} V_{th} - \frac{\left(\sigma_0^2 + 2\sigma_0^2\sigma_1^2 \ln(\sigma_1/\sigma_0)\right)}{\sigma_1^2 - \sigma_0^2} = 0$$

$\sigma_1 = 0{,}15$

$\sigma_0 = 0{,}05$

$$\Rightarrow V_{th}^2 + 0{,}25\,V_{th} - 0{,}1312 = 0$$

$$V_{th} = -\left(\frac{0{,}25}{2}\right) \pm \sqrt{\left(\frac{0{,}25}{2}\right)^2 - (-0{,}1312)}$$

$$V_{th} = -0{,}125 \pm 0{,}3832$$

$$\boxed{V_{th} = 0{,}2582\ \text{V}}$$

b) $Q_0 = \frac{V_{th}}{\sigma_0} = \frac{0,2582V}{0,05} = 5,16$

$Q_1 = \frac{1 - V_{th}}{\sigma_1} = \frac{1 - 0,2582V}{0,15} = 4,95$

$Q_0 \simeq Q_1 = 5$

$\Rightarrow BER \simeq \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}} = \frac{\exp(-25/2)}{5\sqrt{2\pi}} = 3 \times 10^{-7}$ //

c) se $V_{th} = 0,5$

$\Rightarrow Q_0 = \frac{0,5}{0,05} = 10$

$Q_1 = \frac{1 - 0,5}{0,15} = \frac{0,5}{0,15} = 3,3$

$BER = \frac{1}{4}\left[\operatorname{erfc}(Q_1/\sqrt{2}) + \operatorname{erfc}(Q_0/\sqrt{2})\right]$

Como $Q_0 > Q_1 \Rightarrow \operatorname{erfc}(Q_1/\sqrt{2}) \gg \operatorname{erfc}(Q_0/\sqrt{2})$

$\Rightarrow BER = \frac{1}{4}\operatorname{erfc}(Q_1/\sqrt{2}) \approx \frac{\exp(-Q_1^2/2)}{2Q\sqrt{2\pi}}$

$= \frac{\exp(-(3,33)^2/2)}{2(3,33)\sqrt{2\pi}} = 2,34 \times 10^{-4}$ //

Exercício 5

a) $\beta(\omega) = A\omega_0 + B(\omega - \omega_0) + C(\omega - \omega_0)^2$

$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2$

∴ — $\beta_1 = v_g^{-1} = B \Rightarrow B = (2\times10^8 \, m/seg)^{-1} = 5\times10^{-9}\, seg/m$

$$\boxed{B = 5\, ns/m}$$

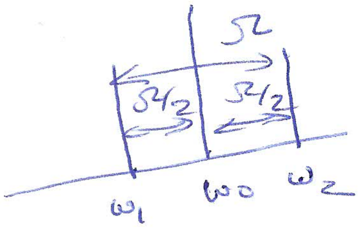
∴ — $C = \frac{1}{2}\beta_2 = \frac{1}{2}\left(-\frac{D\lambda^2}{2\pi c}\right) = -\frac{D\lambda^2}{4\pi c} = \frac{(15\, ps/nm\text{-}km)(1550\, nm)(1550\, nm)}{4\pi\times 3\times10^8\, m/seg}$

Com $D = -\frac{2\pi c}{\lambda^2}\beta_2$

$$\Rightarrow \boxed{C = 9.559\times10^{-27}\, seg^2/m}$$

$\omega = \frac{2\pi}{\lambda}c$

b) $v_g^{-1} = \frac{d\beta}{d\omega} = B + 2C(\omega - \omega_0)$




$(v_g^{-1})_{\omega_1} = B + 2C(\omega_1 - \omega_0)$

$(v_g^{-1})_{\omega_2} = B + 2C(\omega_2 - \omega_0)$

$\omega_0 \rightarrow 1550\, nm = \lambda_0$

$\Omega \rightarrow 3\, nm = \Delta\lambda$

$\omega_1 = \omega_0 - \frac{\Omega}{2}$

$\omega_2 = \omega_0 + \frac{\Omega}{2}$

∴ — separação entre os pulsos

$\Delta T = L\left[(v_g^{-1})_{\omega_1} - (v_g^{-1})_{\omega_2}\right]$

$= L\left[\cancel{B} + 2C\left(-\frac{\Omega}{2}\right) - \left(\cancel{B} + 2C\left(\frac{\Omega}{2}\right)\right)\right]$

$= L\left[-2C\Omega\right] = -2CL\Omega$

$= 2(20\times10^{3}\, m)(-9{,}559\times10^{-27}\, \frac{seg^2}{m})\left(\frac{2\pi\times3\times10^8\, \frac{m}{seg}\times3\times10^{-9}\, m}{(1550\times10^{-9}\, m)^2}\right)$

$\Omega = \omega_2 - \omega_1$

$= \frac{2\pi c}{\lambda_2} - \frac{2\pi c}{\lambda_1}$

$\Omega = \frac{2\pi c}{\lambda^2}(\lambda_1 - \lambda_2)$

$\Omega = \frac{2\pi c}{\lambda^2}\Delta\lambda$

$\Delta T = 8.999\times10^{-10}\, seg \simeq 9\times10^{-10}\, seg$

$$\boxed{\Delta T \simeq 900\, pseg}$$