Integral Definida - LCE0130 Aula30/06/2023

sexta-feira, 30 de junho de 2023 18:36

1) $\int_{-2}^{0} 2x+5 \; dx = \int 2x\,dx + \int 5\,dx = 2\int x\,dx + 5\int x^0 dx$

$= \cancel{2}\frac{x^2}{\cancel{2}} + 5x = x^2 + 5x = F(x)$

$F(x) = ?$

$\int_{-2}^{0} 2x+5 \; dx = F(0) - F(-2)$

$= 0 - (-6) = \underline{6}$ //

$F(0) = 0^2 + 5(0) = 0$

$F(-2) = (-2)^2 + 5(-2) = 4 - 10 = -6$

2) $\int_0^1 x^2 + \sqrt{x} \; dx = F(1) - F(0)$

$F(x) = ?$

$\int x^2 + \sqrt{x}\, dx = \int x^2 dx + \int \sqrt{x}\, dx$

$= \int x^2\, dx + \int x^{1/2} dx$

$= \frac{x^3}{3} + \frac{x^{1/2+1}}{1/2+1} = \frac{x^3}{3} + \frac{x^{3/2}}{\frac{3}{2}} = \frac{x^3}{3} + \frac{2}{3}x^{3/2}$

$1/2+1 = 3/2$

$= \frac{x^3}{3} + \frac{2\sqrt[2]{x^3}}{3} = F(x)$

$F(1) = \frac{1^3}{3} + \frac{2\sqrt{1^3}}{3} = \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1$

$= 1$

$F(0) = \frac{0^3}{3} + \frac{2\sqrt{0^3}}{3} = 0 + 0 = 0$

F(b) F(a)

$\int_0^1 x + \sqrt{x}\; dx = F(1) - F(0) = 1 - 0 = \underline{1}$ //

3) $\int_{-2}^{-1} \frac{2}{x^2} dx = 2\int \frac{1}{x^2} dx = 2\int x^{-2} dx = 2\,\frac{x^{-2+1}}{-2+1} = \frac{2x^{-1}}{-1} = \boxed{-\frac{2}{x^1}}$

$\hookrightarrow F(x)$

$F(-1) = -\frac{2}{(-1)} = 2$

$F(-2) = -\frac{2}{(-2)} = 1$

$\int_{-2}^{-1} \frac{2}{x^2} dx = F(-1) - F(-2)$

$= 2 - 1 = \boxed{1}$ //

4) $\int_{-1}^{1} (r+1)^2 dr = \int r^2 + 2r + 1 \; dr$

$= \int r^2 dr + \int 2r\,dr + \int 1\,dr$ r^0

$= \frac{r^3}{3} + \cancel{2}\frac{r^2}{\cancel{2}} + r = F(r)$

$F(1) = \frac{1^3}{3} + 1^2 + 1 = \frac{1}{3} + \frac{2}{1} = \frac{1+6}{3} = \frac{7}{3}$

$F(-1) = \frac{(-1)^3}{3} + (-1)^2 + (-1) = -\frac{1}{3} + \cancel{1} - \cancel{1} = -\frac{1}{3}$

$\int_{-1}^{1} (r+1)^2 dr = F(1) - F(-1)$

$= \frac{7}{3} - \left(-\frac{1}{3}\right) = \frac{7}{3} + \frac{1}{3} = \boxed{\frac{8}{3}}$ //

$$F(-1) = \frac{(-1)^3}{3} + (-1)^2 + (-1) = -\frac{1}{3} + \cancel{1} - \cancel{1} = -\frac{1}{3}$$

$$= \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

5) $$\int_{-1}^{1} 3y^2 \sqrt{y^3+1}\, dy = \int \cancel{3y^2}\, u^{1/2} \boxed{\frac{1}{\cancel{3y^2}} du} = \int u^{1/2} du = \frac{u^{1/2+1}}{1/2+1} = \frac{u^{3/2}}{3/2} = \frac{2\sqrt{u^3}}{3}$$

$y^3+1 \to u$

$dy \to$ trocar por $du \Rightarrow \frac{du}{\boxed{dy}} = \frac{3y^2}{1} \Rightarrow dy = \frac{1}{3y^2} \cdot du$

$F(u)$

$u = y^3 + 1$

$u = y^3+1$

$\frac{du}{dy} = u'(y)$

$\frac{du}{dy} = 3y^2$

$\frac{a}{b} = \frac{c}{d} \Rightarrow bc = ad \Rightarrow c = \frac{ad}{b}$

$du = 3y^2 dy \Rightarrow dy = \frac{du}{3y^2}$

$$\boxed{\frac{du}{3y^2}} = dy$$

$F(u) = \frac{2\sqrt{u^3}}{3} \xrightarrow{\text{voltar para } y} F(y) = \frac{2\sqrt{(y^3+1)^3}}{3}$ → $F(1)$, $F(-1)$

$$F(1) = \frac{2\sqrt{(1^3+1)^3}}{3} = \frac{2\sqrt{2^3}}{3} = \frac{2\sqrt{8}}{3}$$

$$F(-1) = \frac{2\sqrt{[(-1)^3+1]^3}}{3} = \frac{2\sqrt{0}}{3} = 0$$

$$\int_{-1}^{1} 3y^2\sqrt{y^3+1}\, dy = F(1) - F(-1) = \frac{2\sqrt{8}}{3} - 0 = \frac{2\sqrt{8}}{3}$$

6) $$\int_0^1 \frac{5t}{(4+t^2)^2} dt = \int \frac{5\cancel{t}}{u^2} \frac{du}{2\cancel{t}} = \frac{5}{2}\int \frac{1}{u^2} du$$

$4+t^2 \to du$

$u = 4 + t^2$

$\frac{du}{dt} = u'(t)$

$\frac{du}{\boxed{dt}} = 2t \Rightarrow \frac{du}{2t} = dt$

$$= \frac{5}{2}\int u^{-2} du = \frac{5}{2} \cdot \frac{u^{-2+1}}{-2+1} = \frac{5}{2} \cdot \frac{u^{-1}}{-1} = \frac{+5u^{-1}}{-2} = -\frac{5}{2u}$$

$$= -\frac{5}{2(4+t^2)} = F(t)$$

$$F(1) = -\frac{5}{2(4+1^2)} = -\frac{5}{10} = -\frac{1}{2}$$

$$F(0) = -\frac{5}{2(4+0^2)} = -\frac{5}{8}$$

$$F(1) - F(0) = -\frac{1}{2} - \left(-\frac{5}{8}\right) = -\frac{1}{2} + \frac{5}{8} = \frac{-4+5}{8}$$

$$= \boxed{\frac{1}{8}}$$

$$\boxed{= \frac{1}{8}}\ u$$

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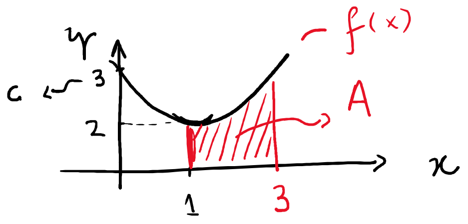
1) $f(x) = x^2 - 2x + 3$ $\qquad I = [1;3]$

Báskara $\to x = ?$ $\quad f(x) = 0$

$$V\left(-\frac{b}{2a} ; -\frac{\Delta}{4a}\right) = \left(-\frac{-2}{2} ; -\frac{-8}{4}\right) = (1;2)$$

$$x^2 - 2x + 3 = 0$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(3) = \overbrace{4 - 12}^{-8} < 0$$



$$A = \int_1^3 x^2 - 2x + 3\, dx$$

$$F(x) = \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 3x = \frac{x^3}{3} - x^2 + 3x$$

$$A = F(3) - F(1)$$

$$F(3) = \frac{3^3}{3} - \cancel{(3)^2} + \cancel{3(3)} = 9$$

$$F(1) = \frac{1^3}{3} - (1)^2 + 3(1) = \frac{1}{3} - 1 + 3 = \frac{1}{3} - 2 = \frac{1-6}{3} = -\frac{5}{3}$$

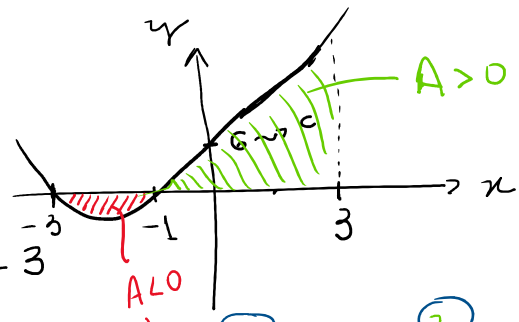
$$A = 9 - \left(-\frac{5}{3}\right) = \frac{9}{1} + \frac{5}{3} = \frac{27+5}{3} = \boxed{\frac{32}{3}\ u.a.}$$

2) $f(x) = 2x^2 + 8x + 6$ $\qquad I = [-3;3]$

$$2x^2 + 8x + 6 = 0$$

$$\Delta = 8^2 - 4(2)(6) = 64 - 48 = 16$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \begin{cases} x_1 = \dfrac{-8-4}{4} = -3 \\ x_2 = \dfrac{-8+4}{4} = -1 \end{cases}$$



$$AT = -\underbrace{\int_{-3}^{-1} f(x)\,dx}_{(-)} + \int_{-1}^{3} f(x)\,dx$$

$$\int 2x^2 + 8x + 6\, dx = \frac{2x^3}{3} + \frac{8x^2}{2} + 6x$$

$$= \frac{2x^3}{3} + 4x^2 + 6x = F(x) \to F(-3),\ F(-1),\ F(3)$$

$$F(-3) = \frac{2(-3)^3}{3} + 4(-3)^2 + 6(-3) = -\frac{54}{3} + 36 - 18 = -\frac{54}{3} + 18 = \frac{-54+54}{3} = 0$$

$$F(-1) = \frac{2(-1)^3}{3} + 4(-1)^2 + 6(-1) = -\frac{2}{3} + 4 - 6 = -\frac{2}{3} - 2 = \frac{-2-6}{3} = -\frac{8}{}$$

$$F(-1) = \frac{2(-1)^3}{3} + 4(-1)^2 + 6(-1) = -\frac{2}{3} + 4 - 6 = -\frac{2}{3} - 2 = \frac{-2-6}{3} = -\frac{8}{3}$$

$= 0$

$$F(3) = \frac{2(3)^3}{3} + 4(3)^2 + 6(3) = \frac{54}{3} + 36 + 18 = 72$$

$$AT = -\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx$$

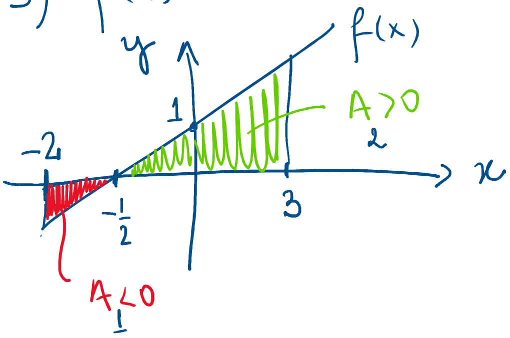
$$= -\left[F(-1) - F(-3)\right] + \left[F(3) - F(-1)\right]$$

inicial lle <0

$$= -\left[-\frac{8}{3} - 0\right] + \left[72 - \left(-\frac{8}{3}\right)\right] = \frac{8}{3} + \frac{216+8}{3} = \frac{8}{3} + \frac{224}{3} = \frac{232}{3}$$

$\approx 77{,}3$ u.a.

3) $f(x) = 2x + 1 \quad I = [-2; 3]$



$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

$$\int 2x + 1\,dx = \cancel{2}\frac{x^2}{\cancel{2}} + x$$
$$= \boxed{x^2 + x}$$

$$AT = -A_1 + A_2$$
$$= -\int_{-2}^{-1/2} f(x)\,dx + \int_{-1/2}^{3} f(x)\,dx$$

$$F(x) = x^2 + x$$

$$F(-2) = (-2)^2 + (-2) = 2$$

$$F(-1/2) = (-1/2)^2 + (-1/2) = \frac{1}{4} - \frac{1}{2} = \frac{1-2}{4} = -\frac{1}{4}$$

$$F(3) = 3^2 + 3 = 9 + 3 = 12$$

$$AT = -\left[F(-1/2) - F(-2)\right] + \left[F(3) - F(-1/2)\right]$$
$$= -\left[-1/4 - 2\right] + \left[12 - (-1/4)\right] = \frac{1}{4} + 2 + 12 + \frac{1}{4} = \boxed{14{,}5 \text{ u.a.}}$$