Aprendizado Profundo

Prof. Clodoaldo A M Lima

Aprendizado Profundo

- A aprendizagem profunda é um subcampo de aprendizado de máquina que tenta aprender abstrações de alto nível nos dados utilizando arquitetura hierárquica.
- É uma abordagem emergente e tem sido amplamente aplicada em domínios tradicionais de inteligência artificial
  - análise semântica, transferência de aprendizado, processamento de linguagem natural e visão computacional
- Três razões importantes para o crescimento da aprendizagem profunda
  - capacidades de processamento dos chips aumentaram drasticamente
  - custo significativamente reduzido de hardware para computação
  - avanços nos algoritmos de aprendizado de máquina

Aprendizado Profundo

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Redes Neurais Convolucionais

Prof. Clodoaldo A M Lima

Introdução

- Redes convolucionais (LeCun, 1989), também conhecidas como neurais redes convolucionais, ou CNNs, são um tipo especializado de rede neural para processamento de dados que possui uma topologia conhecida como grid
- O nome “**Rede neural convolucional** ”indica que a rede emprega uma operação matemática chamada **convolução**.
- Convolução é um tipo especializado de operação linear.
- Redes Convolucionais são simplesmente redes neurais que usam convolução no lugar da matriz geral multiplicação em pelo menos uma de suas camadas

Introdução

- **Usualmente, a operação usada em uma rede neural convolucional não corresponde exatamente à definição de convolução usada em outros campos, como engenharia ou matemática pura**.
- A operação de convolução é normalmenteindicado com um asterisco:

$$s(t) = (x * w)(t).$$

- Na terminologia de rede convolucional, o primeiro argumento (a função x) é geralmente chamada de entrada e o segundo argumento (a função w) como o kernel. A saída é às vezes referido como o mapa de características
- Podemos definir a convolução como

$$s(t) = (x * w)(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$

Introdução

- Qualquer algoritmo de rede neural que funcione com multiplicação da matriz e não depende de propriedades específicas da matriz estrutura deve funcionar com convolução, sem exigir mais alterações para a rede neural
- As redes neurais convolucionais típicas fazem uso de especializações adicionais para lidar com grandes entradas de forma eficiente, mas esses são estritamente necessário do ponto de vista teórico
- Camadas de redes neurais tradicionais usam multiplicação matriz de parâmetros por um parâmetro separado que descreve a interação entre cada unidade de entrada e cada unidade de saída
  - Isso significa que toda unidade de saída interage com todas as unidades de entrada.
- Redes convolucionais, no entanto, geralmente têm escassez de interações (também conhecidas como conectividade esparsa ou pesos esparsos). Isso é feito adotando o kernel com dimensões menores que a entrada

Introdução

- Geralmente, uma CNN consiste em três camadas neurais principais, que são camadas convolucionais, camadas de pooling e camadas totalmente conectadas.

- Os diferentes tipos de camadas desempenham papéis diferentes

- Existem duas etapas para o treinamento da rede neural:
  - **Forward** - visa representar a imagem de entrada com os parâmetros atuais (pesos e viés) em cada camada. Em seguida, a saída de previsão é usada para calcular o custo de perda com base nos rótulo.
  - **Backward** - com base no custo da perda, calcula a gradientes de cada parâmetro. Todos os parâmetros são atualizados com base nos gradientes

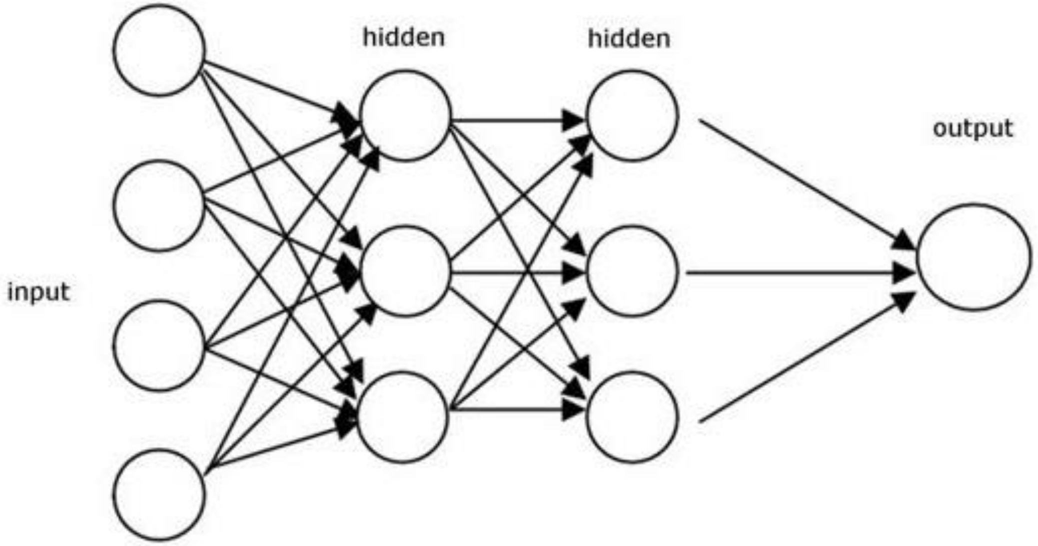
Redes Neurais Profundas

Cat? (0/1)

64×64×3 = 12288

1000×1000×3 = 3 million

Explosão de parâmetros

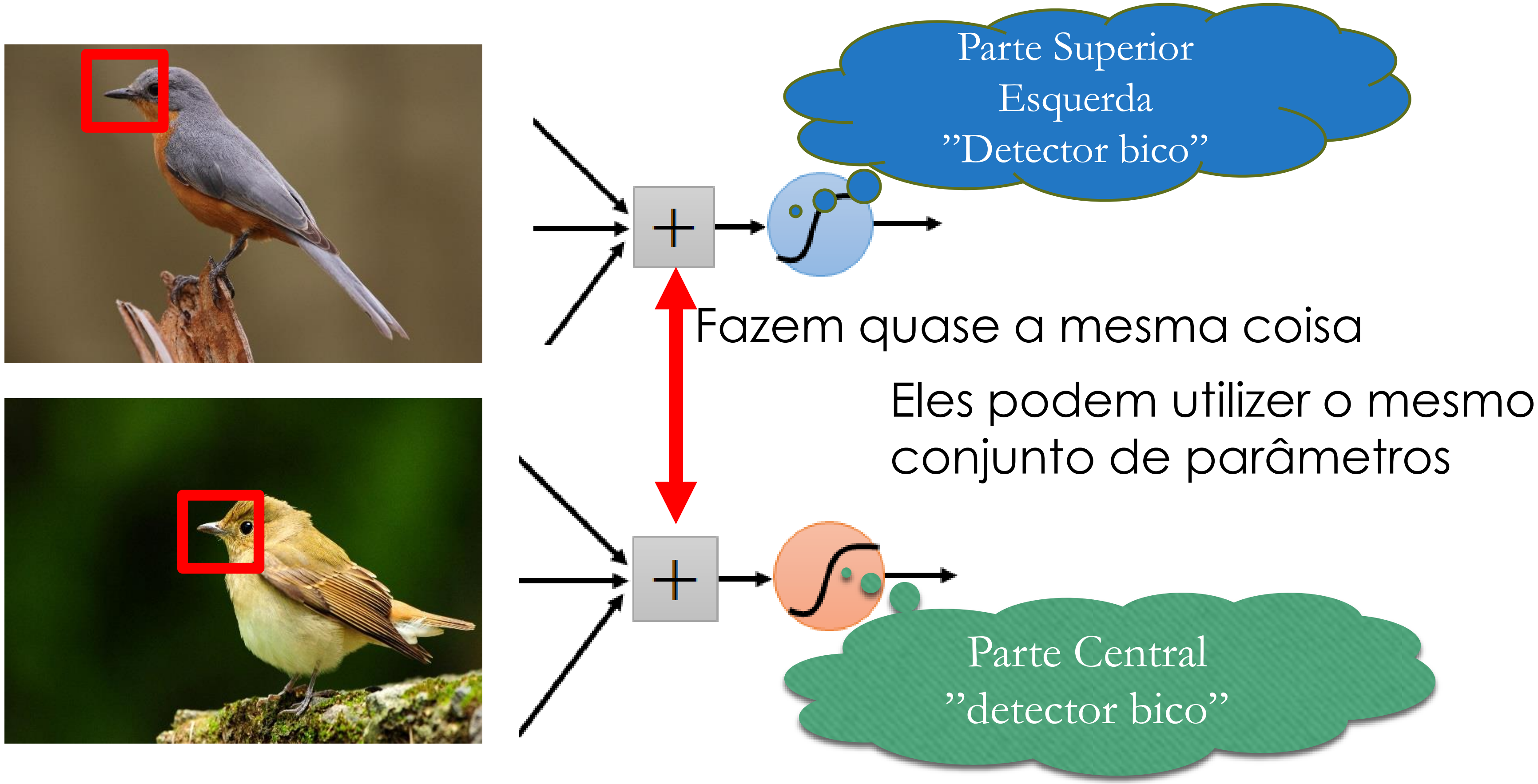


Motivação

- Alguns padrões são muito menores que a imagem de entrada
- Um neurônio não precisa analisar toda a imagem para descobrir o padrão
- Conecta pequenas regiões com menos parâmetros

Motivação

- O mesmo padrão pode aparecer em regiões diferentes.



Motivação

- Subamostragem dos pixels não muda o objeto

passáro

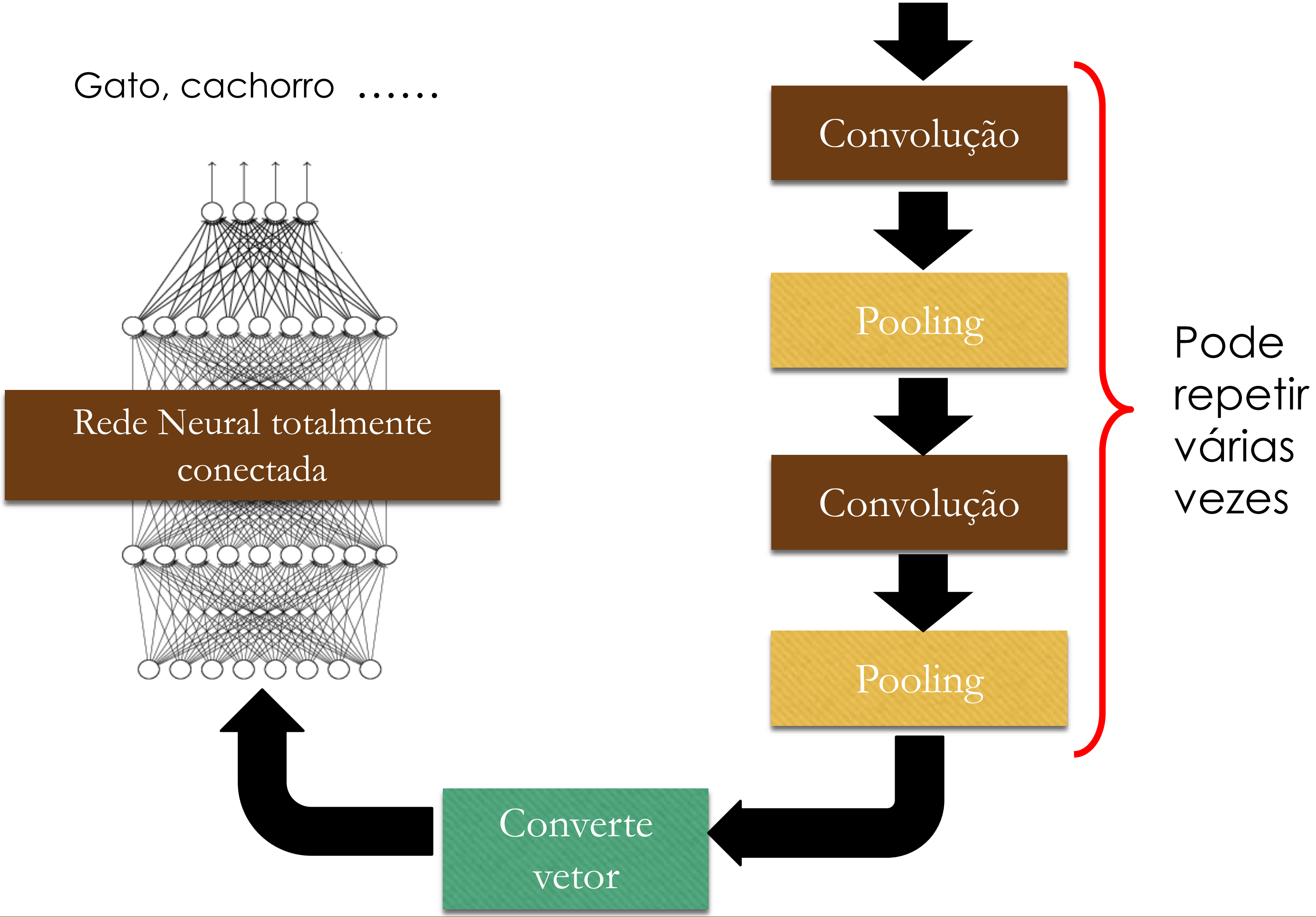
Subamostragem

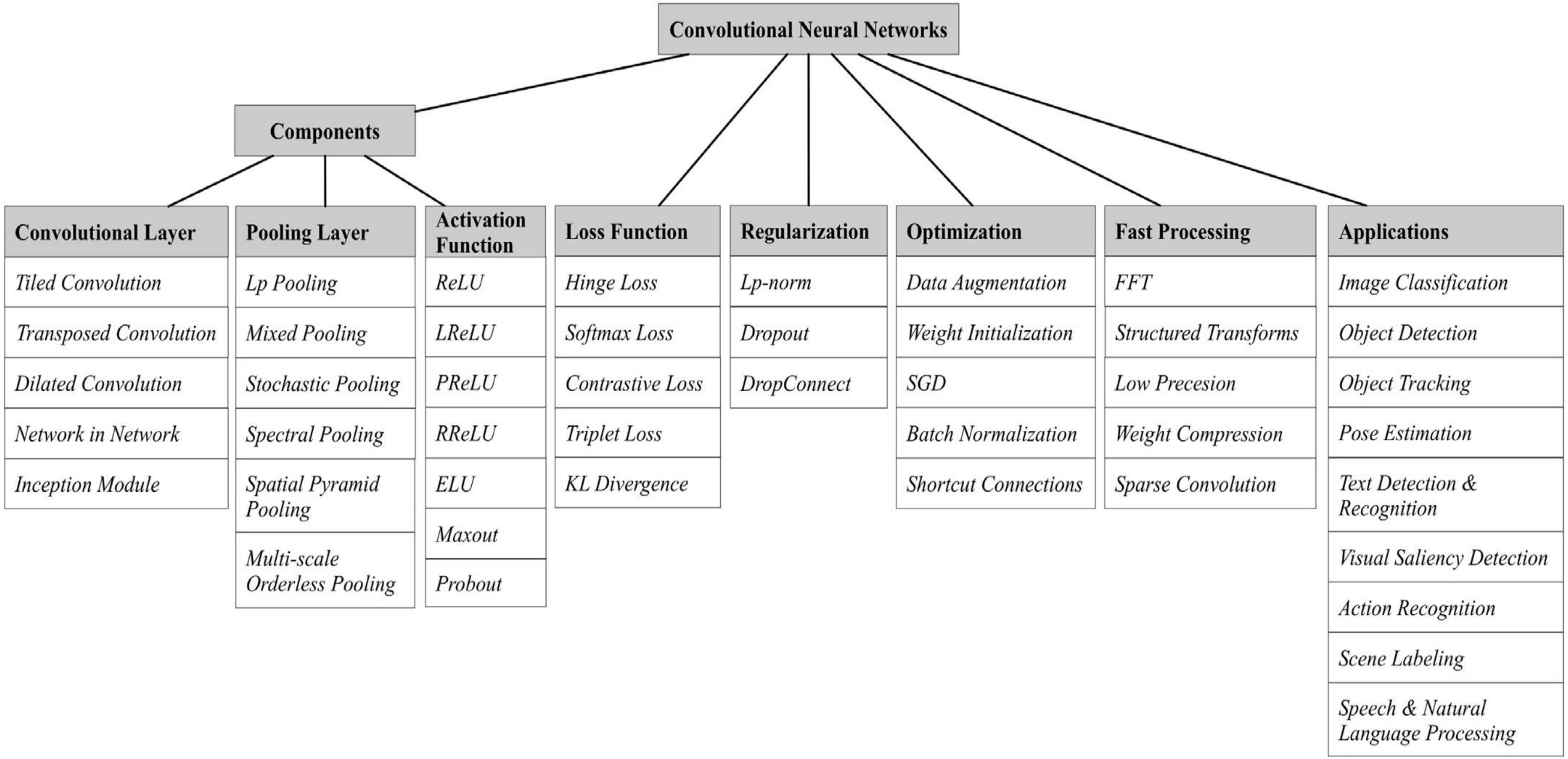
passáro

Nós podemos subamostrar os pixels para tornar a imagem menor

➡ Menos parâmetro para a rede processor a imagem

Estrutura da CNN





- Recent advances in convolutional neural network

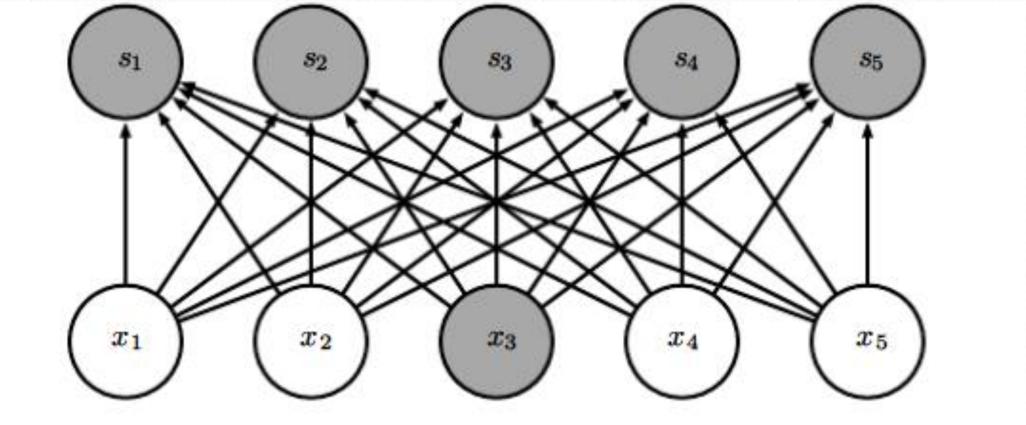
Camada Pooling

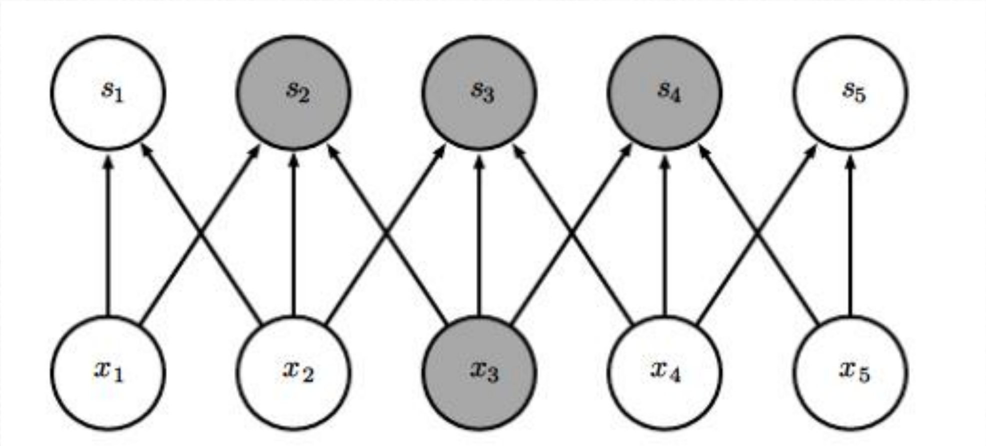
- A função da camada de pooling é reduzir progressivamente o tamanho espacial da representação para reduzir a quantidade de parâmetros e computação na rede e, portanto, controla também o sobre ajuste. Nenhum aprendizado ocorre nas camadas de pooling.
- As unidades de pool são obtidas usando funções como pooling máximo, pooling médio e mesmo pool de norma L2.
- Na camada de pooling, a propagação direta resulta em um bloco de pooling $N \times N$ sendo reduzido a um valor único - o valor da "unidade vencedora".
- A retropropagação da camada de pool calcula o erro que é adquirido por esse valor único "unidade vencedora".

Compartilhamento de Pesos

- O compartilhamento de parâmetros refere-se ao uso do mesmo parâmetro para mais de uma função em um modelo
- Em uma rede neural tradicional, cada elemento da matriz de peso é usado exatamente uma vez ao calcular a saída de uma camada
- Como sinônimo de parâmetro compartilhamento, pode-se dizer que uma rede “amarrou” pesos, porque o valor do peso aplicado a uma entrada está vinculado ao valor de um peso aplicado em outro local.
- Em uma rede neural convolucional, cada membro do núcleo é usado em todas as posições da entrada (exceto talvez alguns dos pixels de limite, dependendo das decisões de projeto relacionadas aos limites).
- O compartilhamento de parâmetro usado pela operação de convolução significa que, em vez de aprender um conjunto separado de parâmetros para cada local, aprendemos apenas um conjunto para toda entrada

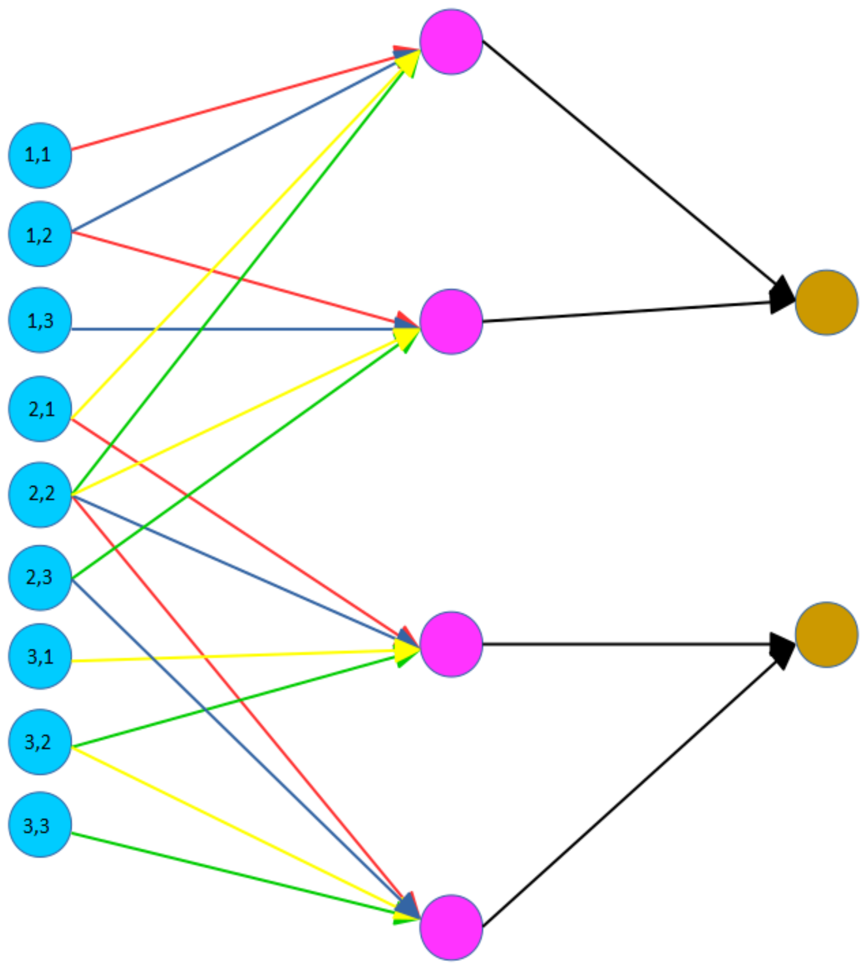
Compartilhamento de Pesos

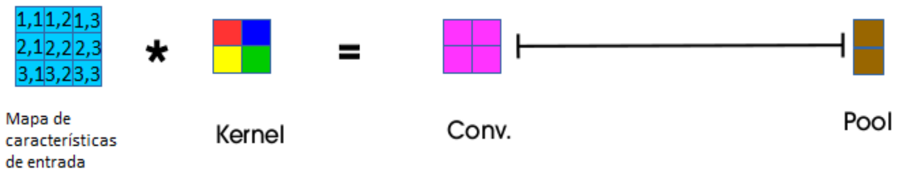




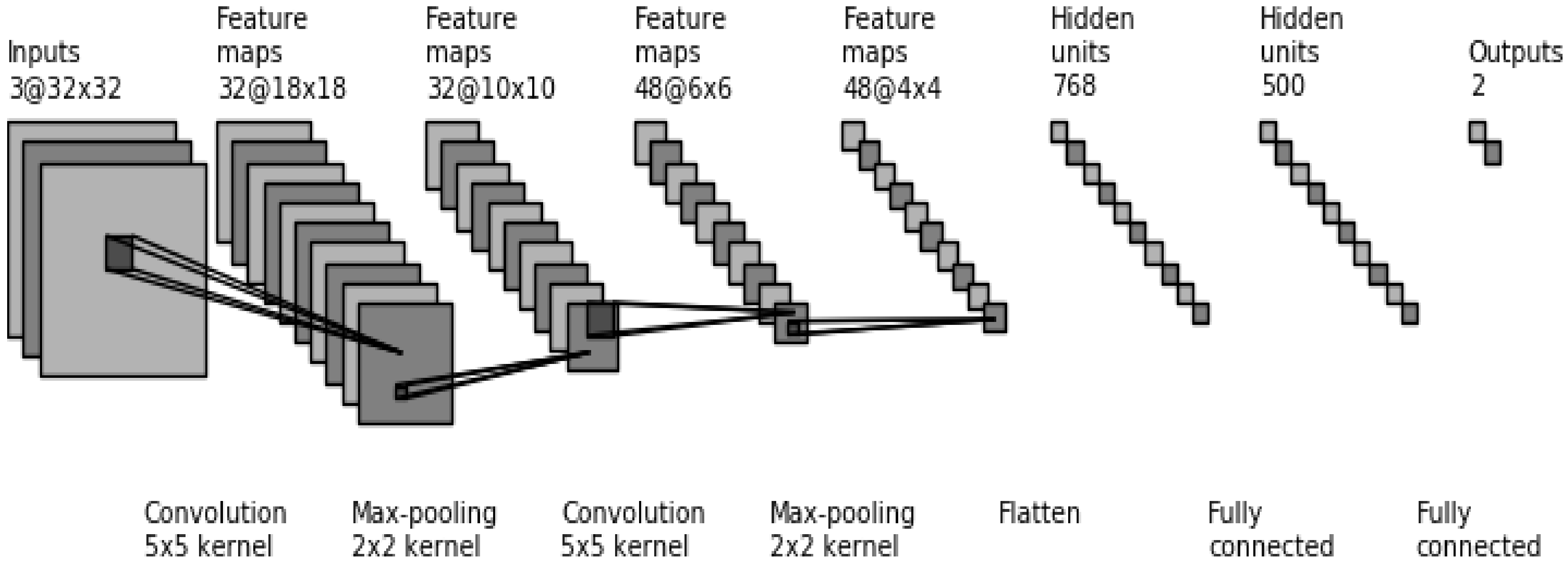
- Destacamos uma unidade de entrada x_3,
  - A esquerda afeta somente todos os valores de saída
  - A direita afeta somente 03 saídas
- Quando s é formado por convolução com um kernel de largura 3, apenas três saídas são afetadas por x.
- Quando s é formado pela multiplicação da matriz, a conectividade não é mais esparsa, portanto, todas as saídas são afetados por x3.

Compartilhamento de Pesos

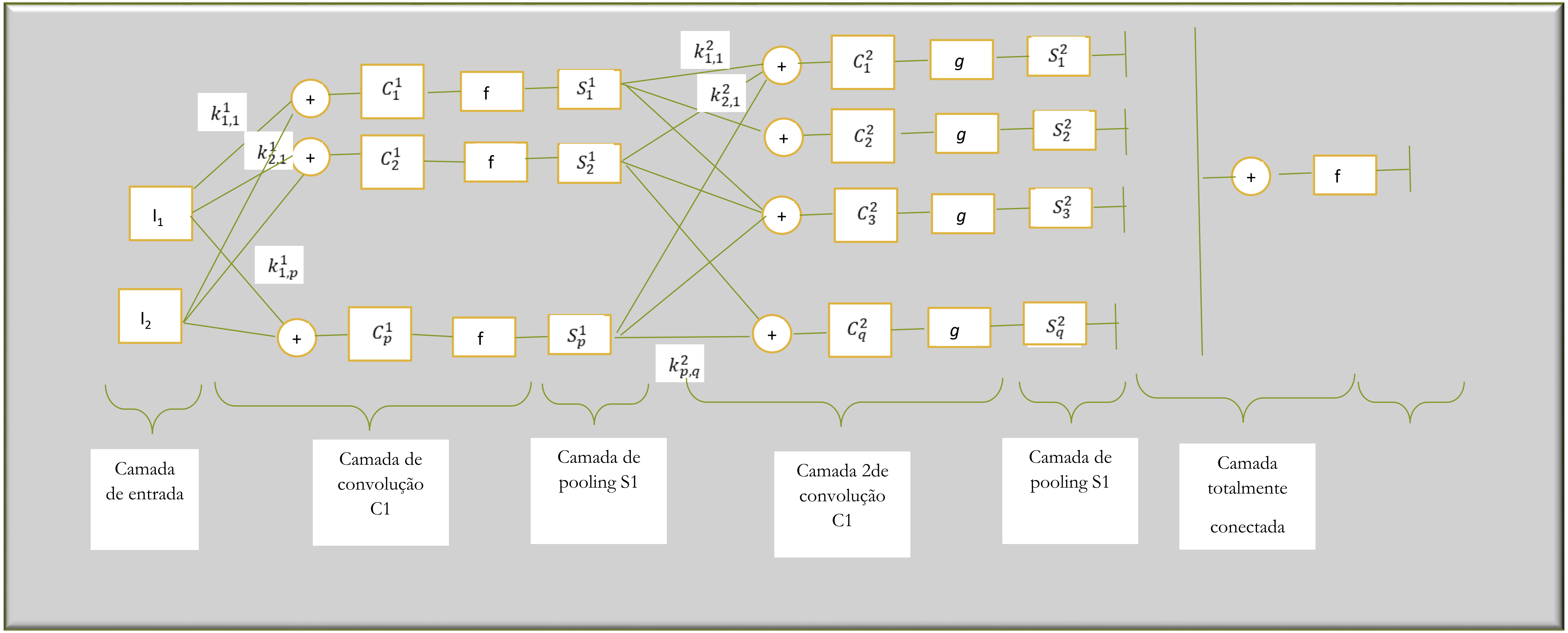




Exemplo


Inputs
3@32x32
Feature
maps
32@18x18
Feature
maps
32@10x10
Feature
maps
48@6x6
Feature
maps
48@4x4
Hidden
units
768
Hidden
units
500
Outputs
2
Convolution
5x5 kernel
Max-pooling
2x2 kernel
Convolution
5x5 kernel
Max-pooling
2x2 kernel
Flatten
Fully
connected
Fully
connected

Exemplo didático de CNN



Propagação Forward na CNN

- Assuma que a imagem I, possuí as seguintes componentes I_1, I_2, I_3

- Parâmetros a serem ajustados
  - Pesos da camada de convolução C1

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u, j+v).\, k_{p,k}^1(u,v) + b_p^1\right)$$

  - Pesos da camada de convolução C2

$$C_q^2(i,j) = f\left(\sum_{p=1}^{6}\sum_{u=-2}^{2}\sum_{v=-2}^{2} S_p^1(i+u, j+v) k_{q,p}^2(u,v) + b_q^1\right)$$

Propagação Forward na CNN

- 1º Passo) Cálculo da camada de convolução C1
- 2º Passo) Cálculo do Pooling C1
- 3º Passo) Cálculo da camada de convolução C2
- 4º Passo) Cálculo do Pooling C2
- 5º Passo) Concatenação da saída
- 6º Passo) Cálculo da camada de saída

Cálculo da saída da Camada de convolução C1

strider =1

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u, j+v).\, k_{p,k}^1(u,v) + b_p^1\right)$$

$$\begin{bmatrix} C_p^1(1,1) & C_p^1(1,2) & \cdots & C_p^1(1,10) \\ C_p^1(2,1) & C_p^1(2,2) & \cdots & C_p^1(2,10) \\ \vdots & \vdots & \vdots & \vdots \\ C_p^1(10,1) & C_p^1(10,2) & \cdots & C_p^1(10,10) \end{bmatrix} = f\left(\text{Correlação}\left(\begin{bmatrix} \boxed{\begin{matrix} I_1(1,1) & I_1(1,2) \\ I_1(2,1) & I_1(2,2) \end{matrix}} & \begin{matrix} I_1(1,3) \\ I_1(2,3) \end{matrix} & \begin{matrix} \cdots \\ \cdots \end{matrix} & \begin{matrix} I_1(1,11) \\ I_1(2,11) \end{matrix} \\ I_1(3,1) \quad I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots \quad\quad \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) \quad I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix}, \boxed{\begin{bmatrix} k_{p,1}^1(1,1) & k_{p,1}^1(1,2) \\ k_{p,1}^1(2,1) & k_{p,1}^1(2,2) \end{bmatrix}}\right) + \right.$$

$$\text{Correlação}\left(\begin{bmatrix} \boxed{\begin{matrix} I_2(1,1) & I_2(1,2) \\ I_2(2,1) & I_2(2,2) \end{matrix}} & \begin{matrix} I_2(1,3) \\ I_2(2,3) \end{matrix} & \begin{matrix} \cdots \\ \cdots \end{matrix} & \begin{matrix} I_2(1,11) \\ I_2(2,11) \end{matrix} \\ I_2(3,1) \quad I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots \quad\quad \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) \quad I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \boxed{\begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix}}\right) +$$

$$\left.\text{Correlação}\left(\begin{bmatrix} \boxed{\begin{matrix} I_3(1,1) & I_3(1,2) \\ I_3(2,1) & I_3(2,2) \end{matrix}} & \begin{matrix} I_3(1,3) \\ I_3(2,3) \end{matrix} & \begin{matrix} \cdots \\ \cdots \end{matrix} & \begin{matrix} I_3(1,11) \\ I_3(2,11) \end{matrix} \\ I_3(3,1) \quad I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots \quad\quad \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) \quad I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \boxed{\begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix}}\right)\right)$$

Cálculo da saída da Camada de convolução C1

strider =1

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

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$$\text{Correlação}\left(\begin{bmatrix} I_2(1,1) & I_2(1,2) & I_2(1,3) & \cdots & I_2(1,11) \\ I_2(2,1) & I_2(2,2) & I_2(2,3) & \cdots & I_2(2,11) \\ I_2(3,1) & I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) & I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix} \right) +$$

$$\left. \text{Correlação}\left(\begin{bmatrix} I_3(1,1) & I_3(1,2) & I_3(1,3) & \cdots & I_3(1,11) \\ I_3(2,1) & I_3(2,2) & I_3(2,3) & \cdots & I_3(2,11) \\ I_3(3,1) & I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) & I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix} \right) \right)$$

Cálculo da saída da Camada de convolução C1

strider =1

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

$$\begin{bmatrix} C_p^1(1,1) & C_p^1(1,2) & \cdots & C_p^1(1,10) \\ C_p^1(2,1) & C_p^1(2,2) & \cdots & C_p^1(2,10) \\ \vdots & \vdots & \vdots & \vdots \\ C_p^1(10,1) & C_p^1(10,2) & \cdots & C_p^1(10,10) \end{bmatrix} = f\left(\text{Correlação}\left(\begin{bmatrix} I_1(1,1) & I_1(1,2) & I_1(1,3) & \cdots & I_1(1,11) \\ I_1(2,1) & I_1(2,2) & I_1(2,3) & \cdots & I_1(2,11) \\ I_1(3,1) & I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) & I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,1}^1(1,1) & k_{p,1}^1(1,2) \\ k_{p,1}^1(2,1) & k_{p,1}^1(2,2) \end{bmatrix}\right) + \right.$$

$$\text{Correlação}\left(\begin{bmatrix} I_2(1,1) & I_2(1,2) & I_2(1,3) & \cdots & I_2(1,11) \\ I_2(2,1) & I_2(2,2) & I_2(2,3) & \cdots & I_2(2,11) \\ I_2(3,1) & I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) & I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix}\right) +$$

$$\left.\text{Correlação}\left(\begin{bmatrix} I_3(1,1) & I_3(1,2) & I_3(1,3) & \cdots & I_3(1,11) \\ I_3(2,1) & I_3(2,2) & I_3(2,3) & \cdots & I_3(2,11) \\ I_3(3,1) & I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) & I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix}\right)\right)$$

Cálculo da saída da Camada de convolução C1

strider =2

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

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$$\text{Correlação}\left(\begin{bmatrix} I_2(1,1) & I_2(1,2) & I_2(1,3) & \cdots & I_2(1,11) \\ I_2(2,1) & I_2(2,2) & I_2(2,3) & \cdots & I_2(2,11) \\ I_2(3,1) & I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) & I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix}\right) +$$

$$\left. \text{Correlação}\left(\begin{bmatrix} I_3(1,1) & I_3(1,2) & I_3(1,3) & \cdots & I_3(1,11) \\ I_3(2,1) & I_3(2,2) & I_3(2,3) & \cdots & I_3(2,11) \\ I_3(3,1) & I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) & I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix}\right)\right)$$

Cálculo da saída da Camada de convolução C1

strider =2

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

$$\begin{bmatrix} C_p^1(1,1) & C_p^1(1,2) & \cdots & C_p^1(1,10) \\ C_p^1(2,1) & C_p^1(2,2) & \cdots & C_p^1(2,10) \\ \vdots & \vdots & \vdots & \vdots \\ C_p^1(10,1) & C_p^1(10,2) & \cdots & C_p^1(10,10) \end{bmatrix} = f\left(\text{Correlação}\left(\begin{bmatrix} I_1(1,1) & I_1(1,2) & I_1(1,3) & \cdots & I_1(1,11) \\ I_1(2,1) & I_1(2,2) & I_1(2,3) & \cdots & I_1(2,11) \\ I_1(3,1) & I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) & I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,1}^1(1,1) & k_{p,1}^1(1,2) \\ k_{p,1}^1(2,1) & k_{p,1}^1(2,2) \end{bmatrix} \right) + \right.$$

$$\text{Correlação}\left(\begin{bmatrix} I_2(1,1) & I_2(1,2) & I_2(1,3) & \cdots & I_2(1,11) \\ I_2(2,1) & I_2(2,2) & I_2(2,3) & \cdots & I_2(2,11) \\ I_2(3,1) & I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) & I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix} \right) +$$

$$\left. \text{Correlação}\left(\begin{bmatrix} I_3(1,1) & I_3(1,2) & I_3(1,3) & \cdots & I_3(1,11) \\ I_3(2,1) & I_3(2,2) & I_3(2,3) & \cdots & I_3(2,11) \\ I_3(3,1) & I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) & I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix} \right) \right)$$

Cálculo da saída da Camada de convolução C1

strider =2

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

$$\begin{bmatrix} C_p^1(1,1) & C_p^1(1,2) & \cdots & C_p^1(1,10) \\ C_p^1(2,1) & C_p^1(2,2) & \cdots & C_p^1(2,10) \\ \vdots & \vdots & \vdots & \vdots \\ C_p^1(10,1) & C_p^1(10,2) & \cdots & C_p^1(10,10) \end{bmatrix} = f\left(\text{Correlação}\left(\begin{bmatrix} I_1(1,1) & I_1(1,2) & I_1(1,3) & \cdots & I_1(1,11) \\ I_1(2,1) & I_1(2,2) & I_1(2,3) & \cdots & I_1(2,11) \\ I_1(3,1) & I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) & I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,1}^1(1,1) & k_{p,1}^1(1,2) \\ k_{p,1}^1(2,1) & k_{p,1}^1(2,2) \end{bmatrix}\right) + \right.$$

$$\text{Correlação}\left(\begin{bmatrix} I_2(1,1) & I_2(1,2) & I_2(1,3) & \cdots & I_2(1,11) \\ I_2(2,1) & I_2(2,2) & I_2(2,3) & \cdots & I_2(2,11) \\ I_2(3,1) & I_2(3,2) & I_2(3,3) & \cdots & I_2(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_2(11,1) & I_2(11,2) & I_2(11,3) & \cdots & I_2(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,2}^1(1,1) & k_{p,2}^1(1,2) \\ k_{p,2}^1(2,1) & k_{p,2}^1(2,2) \end{bmatrix}\right) +$$

$$\left.\text{Correlação}\left(\begin{bmatrix} I_3(1,1) & I_3(1,2) & I_3(1,3) & \cdots & I_3(1,11) \\ I_3(2,1) & I_3(2,2) & I_3(2,3) & \cdots & I_3(2,11) \\ I_3(3,1) & I_3(3,2) & I_3(3,3) & \cdots & I_3(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_3(11,1) & I_3(11,2) & I_3(11,3) & \cdots & I_3(11,11) \end{bmatrix}, \begin{bmatrix} k_{p,3}^1(1,1) & k_{p,3}^1(1,2) \\ k_{p,3}^1(2,1) & k_{p,3}^1(2,2) \end{bmatrix}\right)\right)$$

Cálculo da saída da Camada de convolução C1

$$C_p^1(i,j) = f\left(\sum_{k=1}^{3}\sum_{u=-2}^{2}\sum_{v=-2}^{2} I_k(i+u,j+v).\,k_{p,k}^1(u,v) + b_p^1\right)$$

Escrevendo as equações anteriores, considerando strider igual a 1

$$C_p^1(1,1) = f\Big(I_1(1,1)k_{p,1}^1(1,1) + I_1(1,2)k_{p,1}^1(1,2) + I_1(2,1)k_{p,1}^1(2,1) + I_1(2,2)k_{p,1}^1(2,2) + \cdots + I_3(1,1)k_{p,3}^1(1,1) + I_3(1,2)k_{p,3}^1(1,2) + I_3(2,1)k_{p,3}^1(2,1) + I_3(2,2)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(1,2) = f\Big(I_1(1,2)k_{p,1}^1(1,1) + I_1(1,3)k_{p}^1(1,2) + I_1(2,2)k_{p,1}^1(2,1) + I_1(2,3)k_{p,1}^1(2,2) + \cdots + I_3(1,2)k_{p,3}^1(1,1) + I_3(1,3)k_{p,3}^1(1,2) + I_3(2,2)k_{p,3}^1(2,1) + I_3(2,3)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(2,1) = f\Big(I_1(2,1)k_{p,1}^1(1,1) + I_1(2,2)k_{p,1}^1(1,2) + I_1(3,1)k_{p,1}^1(2,1) + I_1(3,2)k_{p,1}^1(2,2) + \cdots + I_3(2,1)k_{p,3}^1(1,1) + I_3(2,2)k_{p,3}^1(1,2) + I_3(3,1)k_{p,3}^1(2,1) + I_3(3,2)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(2,2) = f\Big(I_1(2,2)k_{p,1}^1(1,1) + I_1(2,3)k_{p,1}^1(1,2) + I_1(3,2)k_{p,1}^1(2,1) + I_1(3,3)k_{p,1}^1(2,2) + \cdots + I_3(2,2)k_{p,3}^1(1,1) + I_3(2,3)k_{p,3}^1(1,2) + I_3(3,2)k_{p,3}^1(2,1) + I_3(3,3)k_{p,3}^1(2,2)\Big)$$

Cálculo da saída do Pooling C1

$$S_p^1(i,j) = \frac{1}{4}\sum_{u=0}^{1}\sum_{v=0}^{1} C_p^1(2i-u, 2j-v), \qquad i,j = 1,2,\cdots,12$$

$$\begin{bmatrix} S_p^1(1,1) & S_p^1(1,2) & S_p^1(1,3) & S_p^1(1,4) & S_p^1(1,5) \\ S_p^1(2,1) & S_p^1(2,2) & S_p^1(2,3) & S_p^1(2,4) & S_p^1(2,5) \\ S_p^1(3,1) & S_p^1(3,2) & S_p^1(3,3) & S_p^1(3,4) & S_p^1(3,5) \\ S_p^1(4,1) & S_p^1(4,2) & S_p^1(4,3) & S_p^1(4,4) & S_p^1(4,5) \\ S_p^1(5,1) & S_p^1(5,2) & S_p^1(5,3) & S_p^1(5,4) & S_p^1(5,5) \end{bmatrix} = \text{Correlação}\left(\begin{bmatrix} C_p^1(1,1) & C_p^1(1,2) & \cdots & C_p^1(1,10) \\ C_p^1(2,1) & C_p^1(2,2) & \cdots & C_p^1(2,10) \\ \vdots & \vdots & \vdots & \vdots \\ C_p^1(10,1) & C_p^1(10,2) & \cdots & C_p^1(10,10) \end{bmatrix}, \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix} \right)$$

$$S_p^1(1,1) = \frac{1}{4}(C_p^1(1,1) + C_p^1(1,2) + C_p^1(2,1) + C_p^1(2,2))$$

$$S_p^1(1,2) = \frac{1}{4}(C_p^1(1,3) + C_p^1(1,4) + C_p^1(2,3) + C_p^1(2,4))$$

$$S_p^1(2,1) = \frac{1}{4}(C_p^1(3,1) + C_p^1(3,2) + C_p^1(4,1) + C_p^1(4,2))$$

$$S_p^1(2,2) = \frac{1}{4}(C_p^1(3,3) + C_p^1(3,4) + C_p^1(4,3) + C_p^1(4,4))$$

Cálculo da saída da Camada de convolução C2

$$
\begin{aligned}
\begin{bmatrix}
C_q^2(1,1) & C_q^2(1,2) & C_q^2(1,4) & C_q^2(1,4) \\
C_q^2(2,1) & C_q^2(2,2) & C_q^2(2,3) & C_q^2(2,4) \\
C_q^2(3,1) & C_q^2(3,2) & C_q^2(3,4) & C_q^2(3,4) \\
C_q^2(4,1) & C_q^2(4,2) & C_q^2(4,3) & C_q^2(4,4)
\end{bmatrix}
= f\Bigg(& \text{Correlação}\left(
\begin{bmatrix}
S_1^1(1,1) & S_1^1(1,2) & S_1^1(1,3) & S_1^1(1,4) & S_1^1(1,5) \\
S_1^1(2,1) & S_1^1(2,2) & S_1^1(2,3) & S_1^1(2,4) & S_1^1(2,5) \\
S_1^1(3,1) & S_1^1(3,2) & S_1^1(3,3) & S_1^1(3,4) & S_1^1(3,5) \\
S_1^1(4,1) & S_1^1(4,2) & S_1^1(4,3) & S_1^1(4,4) & S_1^1(4,5) \\
S_1^1(5,1) & S_1^1(5,2) & S_1^1(5,3) & S_1^1(5,4) & S_1^1(5,5)
\end{bmatrix},
\begin{bmatrix}
k_{q,1}^2(1,1) & k_{q,1}^2(1,2) \\
k_{q,1}^2(2,1) & k_{q,1}^2(2,2)
\end{bmatrix}\right) + \\
& \text{Correlação}\left(
\begin{bmatrix}
S_2^1(1,1) & S_2^1(1,2) & S_2^1(1,3) & S_2^1(1,4) & S_2^1(1,5) \\
S_2^1(2,1) & S_2^1(2,2) & S_2^1(2,3) & S_2^1(2,4) & S_2^1(2,5) \\
S_2^1(3,1) & S_2^1(3,2) & S_2^1(3,3) & S_2^1(3,4) & S_2^1(3,5) \\
S_2^1(4,1) & S_2^1(4,2) & S_2^1(4,3) & S_2^1(4,4) & S_2^1(4,5) \\
S_2^1(5,1) & S_2^1(5,2) & S_2^1(5,3) & S_2^1(5,4) & S_2^1(5,5)
\end{bmatrix},
\begin{bmatrix}
k_{q,2}^2(1,1) & k_{q,2}^2(1,2) \\
k_{q,2}^2(2,1) & k_{q,2}^2(2,2)
\end{bmatrix}\right) + \\
& \cdots\ \text{Correlação}\left(
\begin{bmatrix}
S_p^1(1,1) & S_p^1(1,2) & S_p^1(1,3) & S_p^1(1,4) & S_p^1(1,5) \\
S_p^1(2,1) & S_p^1(2,2) & S_p^1(2,3) & S_p^1(2,4) & S_p^1(2,5) \\
S_p^1(3,1) & S_p^1(3,2) & S_p^1(3,3) & S_p^1(3,4) & S_p^1(3,5) \\
S_p^1(4,1) & S_p^1(4,2) & S_p^1(4,3) & S_p^1(4,4) & S_p^1(4,5) \\
S_p^1(5,1) & S_p^1(5,2) & S_p^1(5,3) & S_p^1(5,4) & S_p^1(5,5)
\end{bmatrix},
\begin{bmatrix}
k_{q,p}^2(1,1) & k_{q,p}^2(1,2) \\
k_{q,p}^2(2,1) & k_{q,p}^2(2,2)
\end{bmatrix}\right)\Bigg)
\end{aligned}
$$

Cálculo da saída da Camada de convolução C2

Escrevendo as equações anteriores

$$C_p^2(1,1) = f\left(S_p^1(1,1)k_{q,1}^2(1,1) + S_p^1(1,2)k_{q,1}^2(1,2) + S_p^1(2,1)k_{q,1}^2(2,1) + S_p^1(2,2)k_{q,1}^2(2,2) + \cdots + S_p^1(1,1)k_{q,p}^2(1,1) + S_p^1(1,2)k_{q,p}^2(1,2) + S_p^1(2,1)k_{q,p}^2(2,1) + S_p^1(2,2)k_{q,p}^2(2,2)\right)$$

$$C_p^2(1,2) = f\left(S_p^1(1,2)k_{q,1}^2(1,1) + S_p^1(1,3)k_{q,1}^2(1,2) + S_p^1(2,2)k_{q,1}^2(2,1) + S_p^1(2,2)k_{q,1}^2(2,2) + \cdots + S_p^1(1,2)k_{q,p}^2(1,1) + S_p^1(1,3)k_{q,p}^2(1,2) + S_p^1(2,2)k_{q,p}^2(2,1) + S_p^1(2,2)k_{q,p}^2(2,2)\right)$$

$$C_p^2(1,3) = f\left(S_p^1(1,3)k_{q,1}^2(1,1) + S_p^1(1,4)k_{q,1}^2(1,2) + S_p^1(2,3)k_{q,1}^2(2,1) + S_p^1(2,4)k_{q,1}^2(2,2) + \cdots + S_p^1(1,3)k_{q,p}^2(1,1) + S_p^1(1,4)k_{q,p}^2(1,2) + S_p^1(2,3)k_{q,p}^2(2,1) + S_p^1(2,4)k_{q,p}^2(2,2)\right)$$

$$C_p^2(1,4) = f\left(S_p^1(1,4)k_{q,1}^2(1,1) + S_p^1(1,5)k_{q,1}^2(1,2) + S_p^1(2,4)k_{q,1}^2(2,1) + S_p^1(2,5)k_{q,1}^2(2,2) + \cdots + S_p^1(1,4)k_{q,p}^2(1,1) + S_p^1(1,5)k_{q,p}^2(1,2) + S_p^1(2,4)k_{q,p}^2(2,1) + S_p^1(2,5)k_{q,p}^2(2,2)\right)$$

Cálculo da saída do Pooling C2

$$S_q^2(i,j) = \frac{1}{4}\sum_{u=0}^{1}\sum_{v=0}^{1} C_q^2(2i-u, 2j-v), \qquad i,j = 1,2,\cdots,4$$

$$\begin{bmatrix} S_q^2(1,1) & S_q^2(1,2) \\ S_q^2(2,1) & S_q^2(2,2) \end{bmatrix} = \text{Correlação}\left(\begin{bmatrix} C_q^2(1,1) & C_q^2(1,2) & C_q^2(1,3) & C_q^2(1,4) \\ C_q^2(2,1) & C_q^2(2,2) & C_q^2(2,3) & C_q^2(2,4) \\ C_q^2(3,1) & C_q^2(3,2) & C_q^2(3,3) & C_q^2(3,4) \\ C_q^2(4,1) & C_q^2(4,2) & C_q^2(4,3) & C_q^2(4,4 \end{bmatrix}, \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix} \right)$$

$$S_q^2(1,1) = \frac{1}{4}(C_q^2(1,1) + C_q^2(1,2) + C_q^2(2,1) + C_q^2(2,2))$$

$$S_q^2(1,2) = \frac{1}{4}(C_q^2(1,3) + C_q^2(1,4) + C_q^2(2,3) + C_q^2(2,4))$$

$$S_q^2(2,1) = \frac{1}{4}(C_q^2(3,1) + C_q^2(3,2) + C_q^2(4,1) + C_q^2(4,2))$$

$$S_q^2(2,2) = \frac{1}{4}(C_q^2(3,3) + C_q^2(3,4) + C_q^2(4,3) + C_q^2(4,4))$$

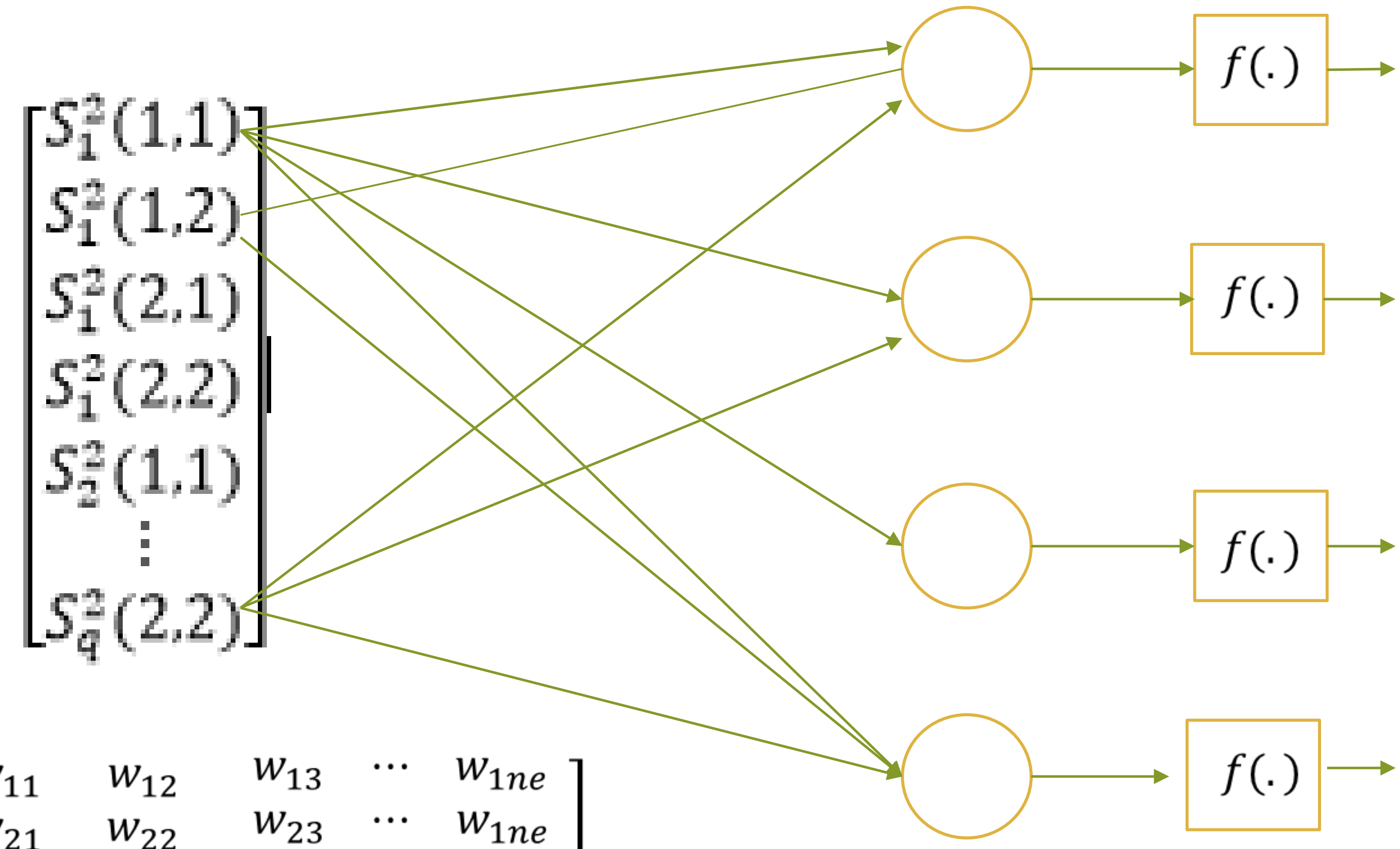
Concatenação da saída

$$\begin{bmatrix} S_1^2(1,1) & S_1^2(1,2) \\ S_1^2(2,1) & S_1^2(2,2) \end{bmatrix} \begin{bmatrix} S_2^2(1,1) & S_2^2(1,2) \\ S_2^2(2,1) & S_2^2(2,2) \end{bmatrix} \begin{bmatrix} S_3^2(1,1) & S_3^2(1,2) \\ S_3^2(2,1) & S_3^2(2,2) \end{bmatrix} \cdots \begin{bmatrix} S_q^2(1,1) & S_q^2(1,2) \\ S_q^2(2,1) & S_q^2(2,2) \end{bmatrix}$$

$$\begin{bmatrix} S_1^2(1,1) \\ S_1^2(1,2) \\ S_1^2(2,1) \\ S_1^2(2,2) \\ S_2^2(1,1) \\ \vdots \\ S_q^2(2,2) \end{bmatrix}$$

Cálculo da camada de saída

- Pode ter apenas a camada de saída ou uma ou mais camadas de escondida
- Neste exemplo, apenas a camada de saída



$$W = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \cdots & w_{1ne} \\ w_{21} & w_{22} & w_{23} & \cdots & w_{1ne} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{ns1} & w_{ns2} & w_{ns3} & \cdots & w_{nsne} \end{bmatrix}$$

Código Matlab

Número de Neurônios

- Estrutura de dados

```
cnn.camadas = {
    struct('tipo', 'c', 'numFiltros', 6,'strider',1,'dimFiltros', 2) %camada convolução
    struct('tipo', 'p', 'strider',1,'dimPool', 2)                     %camada subamostragem
    struct('tipo', 'c', 'numFiltros',8,'strider',1, 'dimFiltros', 2) %camada convolução
    struct('tipo', 'p', 'strider',1, 'dimPool',2)                     %camada subamostragem
};
```

Parâmetros utilizados na CNN

```
opts.alpha = 1e-1;
opts.batchsize = 150; %tamanho do conjunto de treinamento 150
opts.numepocas = 3;   %Numero de epocas
opts.imageDimX = 28;  %Dimensão do eixo X da imagem
opts.imageDimY = 28;  %Dimensão do eixo X da imagem
opts.imageCanal = 1;  %Quantidade de canais da imagem de entrada
opts.numClasses = 10; %Numero de classes
opts.lambda = 0.0001; %fator de decaimento dos pesos
opts.ratio=0.1;       % fator de congelamento dos pesos
opts.momentum = .95;  %fator do momento
opts.mom = 0.5;
opts.momIncrease = 20; %Numero de epocas para incrementar momento
```

Propagação Forward

Determina os neurônios
Que serão desligados da rede

Número de épocas ou EQM

```
numCamadas = numel(cnn.camadas); %Numero de camadas
for e = 1:epocas
    % Congela alguns neuronios para cada epoca
    for l=1:numCamadas,
        camada = cnn.camadas{l};
         if (strcmp(camada.tipo,'c'))
            numFiltros = camada.numFiltros;
            camada.indcongFiltros=find(rand(numFiltros,1)<=ratio); %Indice dos filtros congelados
            camada.txcongFiltros = length(Idx)/numFiltros; %Taxa de filtros congelados
            sprintf('Na camada %i, foram congelados efetivamente %d filtros',l,length(Idx));
            cnn.camadas{l}=camada;
         else % Camada pooling segue o mesmo congelamento da camada convolução
            sprintf('Na camada %i, foram congelados efetivamente %d filtros',l,length(Idx));
         end
    end
```

Propagação Forward

```
% Permuta randomicamente os indices de entrada
p = randperm(N);
% Separa os dados em minibatch
for s=1:minibatch:(N-minibatch+1)
    it = it + 1;
    %incrementa o momento
    if it == momIncrease
        mom = momentum;
    end;
    % Gera o subconjunto de treinamen
    X = imagens(:,:,:,p(s:s+minibatch-1)); %Seleciona as entrada para treinamento
    Yd = labels(p(s:s+minibatch-1)); % seleciona os rotulos para treinamento
    numImagens = size(X,4); % [dimX, dimY, canal, numero de imagens]
```

Propagação Forward

```
%Realiza feedforward
ativacao = X; %Ativação para camada entrada
for l = 1:numCamadas
    camada = cnn.camadas{l};
    strider = camada.strider;
    if (strcmp(camada.tipo,'c')) %Camada convolução
        ativacao = cnnConv(ativacao,camada.W,camada.b,strider);
    else  % Camada de Pooling - Pooling medio
        ativacao = cnnPool(camada.dimPool,ativacao,strider);
    end
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    indcong=camada.indcongFiltros; %individuos congelados
    txcong=camada.txcongFiltros; %taxa de individuos
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ativacao= dropout(ativacao,indcong,txcong); %ativação para proxima camada
    camada.ativacao = ativacao;
    cnn.camadas{l} = camada;
end
```

Propagação Forward

```
%Concatena a saida anterior para as proximas camadas
ativacao = reshape(ativacao,[],numImagens);

%camada saida
probs = exp(bsxfun(@plus, cnn.Wd * ativacao, cnn.bd)); %exp(W*ativ + b)
sumProbs = sum(probs, 1); % calcula a soma das exponenciais
probs = bsxfun(@times, probs, 1 ./ sumProbs); % exp(W*ativ + b)/soma exponencial

%% ---------- Calcuo do Custo -----------
% Calcula a entropia cruzada
logp = log(probs); %Numero de classes x Numero de amostras
index = sub2ind(size(logp),Yd',1:size(probs,2)); %busca o indice que correponde a classe
Custo = -sum(logp(index)); %Equivalente -Yd.*log(Probs)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Propagação Forward

Termo de Regularização

```
% Calcula a soma dos pesos ao quadrado
wCusto = 0;
for l = 1:numCamadas
    camada = cnn.camadas{l};
    if(strcmp(camada.tipo,'c'))
        wCusto = wCusto + sum(camada.W(:) .^ 2);
    end
end
% Adiciona a soma dos pesos ao quadrado
wCusto = lambda/2*(wCusto + sum(cnn.Wd(:) .^ 2));
CustoTotal = Custo + wCusto;    %dciona os pesos na função custo
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

BackPropagation para CNN

Entropia Cruzada Regularizada
Pode ser EQM

- Função objetiva a ser maximizada

$$J_T = -\sum_{n=1}^{N}\sum_{k=1}^{ns} Yd_k(n) log Y_k(n) + \frac{\lambda}{2}\|w\|^2$$

- w – vetor de pesos da camada de saída e dos filtros

- Cálculo da velocidade

$$v_w^{new} = \mu v_w^{old} + \alpha \frac{\partial J_T}{\partial w}$$

- Atualização dos pesos

$$w_{new} = w_{new} - v_w^{new}$$

$$Yin_k(n) = \sum_{m=1}^{ne} w_{km} z_m(n) \quad \Longrightarrow \quad Y_k(n) = \frac{\exp\left(Yin_k(n)\right)}{\sum_{j=1}^{ns} \exp\left(Yin_j(n)\right)}$$

$$J = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \log Y_k(n) \quad \Longrightarrow \quad J_T = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \log Y_k(n) + \frac{\lambda}{2} \|w\|^2$$

$$\frac{\partial J_T}{\partial w_{ij}} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \frac{1}{Y_k(n)} \left(\delta_{ki} - Y_i(n)\right) Y_k(n)\, z_j(n) + \lambda w_{ij}$$

$$Z = \begin{bmatrix} S_1^2(1,1) \\ S_1^2(1,2) \\ S_1^2(2,1) \\ S_1^2(2,2) \\ S_2^2(1,1) \\ \vdots \\ S_q^2(2,2) \end{bmatrix}$$

$$\frac{\partial J_T}{\partial w_{ij}} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \left(\delta_{ki} - Y_i(n)\right) z_j(n) + \lambda w_{ij}$$

$$\frac{\partial J_T}{\partial w_{ij}} = -\sum_{n=1}^{N} \left(\sum_{k=1}^{ns} Yd_k(n) \delta_{ki} - \sum_{k=1}^{ns} Yd_k(n) Y_i(n) \right) z_j(n) + \lambda w_{ij}$$

$$\frac{\partial J_T}{\partial w_{ij}} = -\sum_{n=1}^{N} \left(Yd_i - Y_i(n)\right) z_j(n) + \lambda w_{ij}$$

$$\boxed{\frac{\partial J_T}{\partial w_{ij}} = \sum_{n=1}^{N} \left(Y_i(n) - Yd_i\right) z_j(n) + \lambda w_{ij}}$$

$$Yin_k(n) = \sum_{m=1}^{ne} w_{km} z_m(n) \Rightarrow Y_k(n) = \frac{\exp(Yin_k(n))}{\sum_{j=1}^{ns} \exp(Yin_j(n))} \Rightarrow J = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \log Y_k(n)$$

$$\frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \frac{\partial}{\partial z_j} \log Y_k(n) \Rightarrow \frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \frac{1}{Y_k(n)} \frac{\partial Y_k(n)}{\partial z_j}$$

$$\frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \frac{1}{Y_k(n)} \sum_{i=1}^{ns} \frac{\partial Y_k(n)}{\partial Yin_i(n)} \frac{\partial Yin_i(n)}{\partial z_j} \Rightarrow \frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \frac{1}{Y_k(n)} \sum_{i=1}^{ns} (\delta_{ki} - Y_i(n)) Y_k(n) w_{ij}$$

$$\frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{k=1}^{ns} Yd_k(n) \sum_{i=1}^{ns} (\delta_{ki} - Y_i(n)) w_{ij} \Rightarrow \frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{i=1}^{ns} \sum_{k=1}^{ns} Yd_k(n) (\delta_{ki} - Y_i(n)) w_{ij}$$

$$\frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{i=1}^{ns} \left(\sum_{k=1}^{ns} Yd_k(n) \delta_{ki} - \sum_{k=1}^{ns} Yd_k(n) Y_i(n) \right) w_{ij} \Rightarrow \frac{\partial J}{\partial z_j} = -\sum_{n=1}^{N} \sum_{i=1}^{ns} (Yd_i(n) - Y_i(n)) w_{ij}$$

$$\boxed{\frac{\partial J}{\partial z_j} = \sum_{n=1}^{N} \sum_{i=1}^{ns} (Y_i(n) - Yd_i(n)) w_{ij}}$$

Sabemos que $$\frac{\partial J}{\partial z_j} = \sum_{n=1}^{N} \sum_{i=1}^{ns} (Y_i(n) - Yd_i(n)) w_{ij}$$

$$Z = \begin{bmatrix} S_1^2(1,1) \\ S_1^2(1,2) \\ S_1^2(2,1) \\ S_1^2(2,2) \\ S_2^2(1,1) \\ \vdots \\ S_q^2(2,2) \end{bmatrix}$$

$$\frac{\partial J}{\partial Z} = \begin{bmatrix} \frac{\partial J}{\partial z_1} \\ \frac{\partial J}{\partial z_2} \\ \frac{\partial J}{\partial z_3} \\ \frac{\partial J}{\partial z_4} \\ \frac{\partial J}{\partial z_5} \\ \vdots \\ \frac{\partial J}{\partial z_{ne}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial S_1^2(1,1)} \\ \frac{\partial J}{\partial S_1^2(1,2)} \\ \frac{\partial J}{\partial S_1^2(2,1)} \\ \frac{\partial J}{\partial S_1^2(2,2)} \\ \frac{\partial J}{\partial S_2^2(1,1)} \\ \vdots \\ \frac{\partial J}{\partial S_q^2(2,2)} \end{bmatrix}$$

Derivada em relação a saída da camada de pooling

Sabemos que

$$\begin{bmatrix} S_q^2(1,1) & S_q^2(1,2) \\ S_q^2(2,1) & S_q^2(2,2) \end{bmatrix} = \text{Correlação}\left(\begin{bmatrix} C_q^2(1,1) & C_q^2(1,2) & C_q^2(1,3) & C_q^2(1,4) \\ C_q^2(2,1) & C_q^2(2,2) & C_q^2(2,3) & C_q^2(2,4) \\ C_q^2(3,1) & C_q^2(3,2) & C_q^2(3,3) & C_q^2(3,4) \\ C_q^2(4,1) & C_q^2(4,2) & C_q^2(4,3) & C_q^2(4,4 \end{bmatrix}, \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}\right)$$

Saída da camada pooling C2

$$S_q^2(1,1) = \frac{1}{4}(C_q^2(1,1) + C_q^2(1,2) + C_q^2(2,1) + C_q^2(2,2))$$

$$S_q^2(1,2) = \frac{1}{4}(C_q^2(1,3) + C_q^2(1,4) + C_q^2(2,3) + C_q^2(2,4))$$

$$S_q^2(2,1) = \frac{1}{4}(C_q^2(3,1) + C_q^2(3,2) + C_q^2(4,1) + C_q^2(4,2))$$

$$S_q^2(2,2) = \frac{1}{4}(C_q^2(3,3) + C_q^2(3,4) + C_q^2(4,3) + C_q^2(4,4))$$

Cálculo da derivada em relação a entrada da camada de pooling

$$\frac{\partial J}{\partial C_q^2(1,1)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{\partial S_2^2(1,1)}{\partial C_q^2(1,1)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(1,2)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{\partial S_2^2(1,2)}{\partial C_q^2(1,1)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(1,3)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{\partial S_2^2(1,2)}{\partial C_q^2(1,3)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(1,4)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{\partial S_2^2(1,2)}{\partial C_q^2(1,4)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(2,1)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{\partial S_2^2(1,1)}{\partial C_q^2(2,1)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(2,2)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{\partial S_2^2(1,1)}{\partial C_q^2(2,2)} = \frac{\partial J}{\partial S_2^2(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(2,3)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{\partial S_2^2(1,2)}{\partial C_q^2(2,3)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2(2,4)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{\partial S_2^2(1,2)}{\partial C_q^2(2,4)} = \frac{\partial J}{\partial S_2^2(1,2)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_q^2} = \begin{bmatrix} \frac{\partial J}{\partial C_q^2(1,1)} & \frac{\partial J}{\partial C_q^2(1,2)} & \frac{\partial J}{\partial C_q^2(1,3)} & \frac{\partial J}{\partial C_q^2(1,4)} \\ \frac{\partial J}{\partial C_q^2(2,1)} & \frac{\partial J}{\partial C_q^2(2,2)} & \frac{\partial J}{\partial C_q^2(2,3)} & \frac{\partial J}{\partial C_q^2(2,4)} \\ \frac{\partial J}{C_q^2(3,1)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,4)} \\ \frac{\partial J}{C_q^2(4,1)} & \frac{\partial J}{C_q^2(4,2)} & \frac{\partial J}{C_q^2(4,3)} & \frac{\partial J}{C_q^2(4,4)} \end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2} = \begin{bmatrix} \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} \\ \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} \\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} \\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} \end{bmatrix}$$

Cálculo da derivada em relação a entrada da camada de polling

$$\frac{\partial J}{\partial Z} = \begin{bmatrix} \frac{\partial J}{\partial z_1} \\ \frac{\partial J}{\partial z_2} \\ \frac{\partial J}{\partial z_3} \\ \frac{\partial J}{\partial z_4} \\ \frac{\partial J}{\partial z_5} \\ \vdots \\ \frac{\partial J}{\partial z_{ne}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial S_1^2(1,1)} \\ \frac{\partial J}{\partial S_1^2(1,2)} \\ \frac{\partial J}{\partial S_1^2(2,1)} \\ \frac{\partial J}{\partial S_1^2(2,2)} \\ \frac{\partial J}{\partial S_2^2(1,1)} \\ \vdots \\ \frac{\partial J}{\partial S_q^2(2,2)} \end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2} = \frac{1}{4}\begin{bmatrix} \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)} \\ \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)} \\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)} \\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)} \end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2}=\begin{bmatrix}\frac{\partial J}{\partial C_q^2(1,1)} & \frac{\partial J}{\partial C_q^2(1,2)} & \frac{\partial J}{\partial C_q^2(1,3)} & \frac{\partial J}{\partial C_q^2(1,4)}\\ \frac{\partial J}{\partial C_q^2(2,1)} & \frac{\partial J}{\partial C_q^2(2,2)} & \frac{\partial J}{\partial C_q^2(2,3)} & \frac{\partial J}{\partial C_q^2(2,4)}\\ \frac{\partial J}{C_q^2(3,1)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,4)}\\ \frac{\partial J}{C_q^2(4,1)} & \frac{\partial J}{C_q^2(4,2)} & \frac{\partial J}{C_q^2(4,3)} & \frac{\partial J}{C_q^2(4,4)}\end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2}=\begin{bmatrix}\frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4}\end{bmatrix}$$

$$\frac{\partial J}{\partial Z}=\begin{bmatrix}\frac{\partial J}{\partial z_1}\\ \frac{\partial J}{\partial z_2}\\ \frac{\partial J}{\partial z_3}\\ \frac{\partial J}{\partial z_4}\\ \frac{\partial J}{\partial z_5}\\ \vdots\\ \frac{\partial J}{\partial z_{ne}}\end{bmatrix}=\begin{bmatrix}\frac{\partial J}{\partial S_1^2(1,1)}\\ \frac{\partial J}{\partial S_1^2(1,2)}\\ \frac{\partial J}{\partial S_1^2(2,1)}\\ \frac{\partial J}{\partial S_1^2(2,2)}\\ \frac{\partial J}{\partial S_2^2(1,1)}\\ \vdots\\ \frac{\partial J}{\partial S_q^2(2,2)}\end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2}=\frac{1}{4}\begin{bmatrix}\frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)}\\ \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)}\\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)}\\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)}\end{bmatrix}$$

Camada de Convolução C2

$$C_q^2(1{,}1) = f\left(S_1^1(1{,}1)k_{q,1}^2(1{,}1) + S_1^1(1{,}2)k_{q,1}^2(1{,}2) + S_1^1(2{,}1)k_{q,1}^2(2{,}1) + S_1^1(2{,}2)k_{q,1}^2(2{,}2) + \cdots + S_p^1(1{,}1)k_{q,p}^2(1{,}1) + S_p^1(1{,}2)k_{q,p}^2(1{,}2) + S_p^1(2{,}1)k_{q,p}^2(2{,}1) + S_p^1(2{,}2)k_{q,p}^2(2{,}2)\right)$$

$$C_q^2(1{,}2) = f\left(S_1^1(1{,}2)k_{q,1}^2(1{,}1) + S_1^1(1{,}3)k_{q,1}^2(1{,}2) + S_1^1(2{,}2)k_{q,1}^2(2{,}1) + S_1^1(2{,}2)k_{q,1}^2(2{,}2) + \cdots + S_p^1(1{,}2)k_{q,p}^2(1{,}1) + S_p^1(1{,}3)k_{q,p}^2(1{,}2) + S_p^1(2{,}2)k_{q,p}^2(2{,}1) + S_p^1(2{,}2)k_{q,p}^2(2{,}2)\right)$$

$$C_q^2(1{,}3) = f\left(S_1^1(1{,}3)k_{q,1}^2(1{,}1) + S_1^1(1{,}4)k_{q,1}^2(1{,}2) + S_1^1(2{,}3)k_{q,1}^2(2{,}1) + S_1^1(2{,}4)k_{q,1}^2(2{,}2) + \cdots + S_p^1(1{,}3)k_{q,p}^2(1{,}1) + S_p^1(1{,}4)k_{q,p}^2(1{,}2) + S_p^1(2{,}3)k_{q,p}^2(2{,}1) + S_p^1(2{,}4)k_{q,p}^2(2{,}2)\right)$$

$$C_q^2(1{,}4) = f\left(S_1^1(1{,}4)k_{q,1}^2(1{,}1) + S_1^1(1{,}5)k_{q,1}^2(1{,}2) + S_1^1(2{,}4)k_{q,1}^2(2{,}1) + S_1^1(2{,}5)k_{q,1}^2(2{,}2) + \cdots + S_p^1(1{,}4)k_{q,p}^2(1{,}1) + S_p^1(1{,}5)k_{q,p}^2(1{,}2) + S_p^1(2{,}4)k_{q,p}^2(2{,}1) + S_p^1(2{,}5)k_{q,p}^2(2{,}2)\right)$$

$$\frac{\partial J}{\partial k_{q,p}^2(1{,}1)} = \frac{\partial J}{\partial C_q^2(1{,}1)}\frac{\partial C_q^2(1{,}1)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(1{,}2)}\frac{\partial C_q^2(1{,}2)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(1{,}3)}\frac{\partial C_q^2(1{,}3)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(1{,}4)}\frac{\partial C_q^2(1{,}4)}{\partial k_{q,p}^2(1{,}1)} + \cdots$$

$$\frac{\partial J}{\partial C_q^2(2{,}1)}\frac{\partial C_q^2(2{,}1)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(2{,}2)}\frac{\partial C_q^2(1{,}2)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(2{,}3)}\frac{\partial C_p^2(1{,}3)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{\partial C_q^2(2{,}4)}\frac{\partial C_q^2(1{,}4)}{\partial k_{q,p}^2(1{,}1)}$$

$$\frac{\partial J}{\partial C_p^2(3{,}1)}\frac{\partial C_p^2(3{,}1)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{C_p^2(3{,}2)}\frac{\partial C_p^2(3{,}2)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{C_p^2(3{,}3)}\frac{\partial C_p^2(3{,}3)}{\partial k_{q,p}^2(1{,}1)} + \frac{\partial J}{C_p^2(3{,}4)}\frac{\partial C_p^2(3{,}4)}{\partial k_{q,p}^2(1{,}1)}$$

Cálculo da derivada em relação aos pesos da camada convolução C2

$$\frac{\partial J}{\partial k_{q,p}^2(1{,}1)}$$
$$= \frac{\partial J}{\partial C_q^2(1{,}1)}\grave{f}(1{,}1)S_p^1(1{,}1) + \frac{\partial J}{\partial C_q^2(1{,}2)}\grave{f}(1{,}2)S_p^1(1{,}2) + \frac{\partial J}{\partial C_q^2(1{,}3)}\grave{f}(1{,}3)S_p^1(1{,}3) + \frac{\partial J}{\partial C_q^2(1{,}4)}\grave{f}(1{,}4)S_p^1(1{,}4) + \cdots$$

$$\frac{\partial J}{\partial C_q^2(2{,}1)}\grave{f}(2{,}1)S_p^1(2{,}1) + \frac{\partial J}{\partial C_q^2(2{,}2)}\grave{f}(2{,}2)S_p^1(2{,}2) + \frac{\partial J}{\partial C_q^2(2{,}3)}\grave{f}(2{,}3)S_p^1(2{,}3) + \frac{\partial J}{\partial C_q^2(2{,}4)}\grave{f}(2{,}4)S_p^1(2{,}4)$$

$$\frac{\partial J}{\partial C_q^2(3{,}1)}\grave{f}(3{,}1)S_p^1(3{,}1) + \frac{\partial J}{\partial C_q^2(3{,}2)}\grave{f}(3{,}2)S_p^1(3{,}2) + \frac{\partial J}{\partial C_q^2(3{,}3)}\grave{f}(3{,}3)S_p^1(3{,}3) + \frac{\partial J}{\partial C_q^2(3{,}4)}\grave{f}(3{,}4)S_p^1(3{,}4)$$

$$\frac{\partial J}{\partial C_q^2(4{,}1)}\grave{f}(4{,}1)S_p^1(4{,}1) + \frac{\partial J}{\partial C_q^2(4{,}2)}\grave{f}(4{,}2)S_p^1(4{,}2) + \frac{\partial J}{\partial C_q^2(4{,}3)}\grave{f}(4{,}3)S_p^1(4{,}3) + \frac{\partial J}{\partial C_q^2(4{,}4)}\grave{f}(4{,}4)S_p^1(4{,}4)$$

$$\frac{\partial J}{\partial k_{q,p}^{2}(1,2)} = \frac{\partial J}{C_p^1(1,1)}\grave{f}(1,1)S_p^1(1,2) + \frac{\partial J}{C_p^1(1,2)}\grave{f}(1,2)S_p^1(1,3) + \frac{\partial J}{C_p^1(1,3)}\grave{f}(1,3)S_p^1(1,4) + \frac{\partial J}{C_p^1(1,4)}\grave{f}(1,4)S_p^1(1,5) + \cdots$$

$$\frac{\partial J}{C_p^1(2,1)}\grave{f}(2,1)S_p^1(2,2) + \frac{\partial J}{C_p^1(2,2)}\grave{f}(2,2)S_p^1(2,3) + \frac{\partial J}{C_p^1(2,3)}\grave{f}(2,3)S_p^1(2,4) + \frac{\partial J}{C_p^1(2,4)}\grave{f}(2,4)S_p^1(2,5)$$

$$\frac{\partial J}{C_p^1(3,1)}\grave{f}(3,1)S_p^1(3,2) + \frac{\partial J}{C_p^1(3,2)}\grave{f}(3,2)S_p^1(3,3) + \frac{\partial J}{C_p^1(3,3)}\grave{f}(3,3)S_p^1(3,4) + \frac{\partial J}{C_p^1(3,4)}\grave{f}(3,4)S_p^1(3,5)$$

$$\frac{\partial J}{C_p^1(4,1)}\grave{f}(4,1)S_p^1(4,2) + \frac{\partial J}{C_p^1(4,2)}\grave{f}(4,2)S_p^1(4,3) + \frac{\partial J}{C_p^1(4,3)}\grave{f}(4,3)S_p^1(4,4) + \frac{\partial J}{C_p^1(4,4)}\grave{f}(4,4)S_p^1(4,5)$$

Derivada de J em relação aos pesos da Convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial k_{q,p}^{2}(1,1)} & \frac{\partial J}{\partial k_{q,p}^{2}(1,2)} \\ \frac{\partial J}{\partial k_{q,p}^{2}(2,1)} & \frac{\partial J}{\partial k_{q,p}^{2}(2,2)} \end{bmatrix} = \text{correlação}\left(\begin{pmatrix} S_1^1(1,1) & S_1^1(1,2) & S_1^1(1,3) & S_1^1(1,4) & S_1^1(1,5) \\ S_1^1(2,1) & S_1^1(2,2) & S_1^1(2,3) & S_1^1(2,4) & S_1^1(2,5) \\ S_1^1(3,1) & S_1^1(3,2) & S_1^1(3,3) & S_1^1(3,4) & S_1^1(3,5) \\ S_1^1(4,1) & S_1^1(4,2) & S_1^1(4,3) & S_1^1(4,4) & S_1^1(4,5) \\ S_1^1(5,1) & S_1^1(5,2) & S_1^1(5,3) & S_1^1(5,4) & S_1^1(5,5) \end{pmatrix}, \begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \right)$$

$$\frac{\partial J}{\partial Z}=\begin{bmatrix}\frac{\partial J}{\partial z_1}\\ \frac{\partial J}{\partial z_2}\\ \frac{\partial J}{\partial z_3}\\ \frac{\partial J}{\partial z_4}\\ \frac{\partial J}{\partial z_5}\\ \vdots\\ \frac{\partial J}{\partial z_{ne}}\end{bmatrix}=\begin{bmatrix}\frac{\partial J}{\partial S_1^2(1,1)}\\ \frac{\partial J}{\partial S_1^2(1,2)}\\ \frac{\partial J}{\partial S_1^2(2,1)}\\ \frac{\partial J}{\partial S_1^2(2,2)}\\ \frac{\partial J}{\partial S_2^2(1,1)}\\ \vdots\\ \frac{\partial J}{\partial S_q^2(2,2)}\end{bmatrix}$$

$$\frac{\partial J}{\partial C_q^2}=\begin{bmatrix}\frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(1,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4}\\ \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,1)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4} & \frac{\partial J}{\partial S_2^2(2,2)}\frac{1}{4}\end{bmatrix}$$

$$\begin{bmatrix}\frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4)\\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4)\\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4)\\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4)\end{bmatrix}$$

$$=\begin{bmatrix}\frac{\partial J}{\partial C_q^2(1,1)} & \frac{\partial J}{\partial C_q^2(1,2)} & \frac{\partial J}{\partial C_q^2(1,3)} & \frac{\partial J}{\partial C_q^2(1,4)}\\ \frac{\partial J}{\partial C_q^2(2,1)} & \frac{\partial J}{\partial C_q^2(2,2)} & \frac{\partial J}{\partial C_q^2(2,3)} & \frac{\partial J}{\partial C_q^2(2,3)}\\ \frac{\partial J}{C_q^2(3,1)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,3)} & \frac{\partial J}{C_q^2(3,4)}\\ \frac{\partial J}{C_q^2(4,1)} & \frac{\partial J}{C_q^2(4,2)} & \frac{\partial J}{C_q^2(4,3)} & \frac{\partial J}{C_q^2(4,4)}\end{bmatrix}.*\begin{bmatrix}\grave{f}(1,1) & \grave{f}(1,2) & \grave{f}(1,3) & \grave{f}(1,4)\\ \grave{f}(2,1) & \grave{f}(2,2) & \grave{f}(2,3) & \grave{f}(2,4)\\ \grave{f}(3,1) & \grave{f}(3,2) & \grave{f}(3,3) & \grave{f}(3,4)\\ \grave{f}(4,1) & \grave{f}(4,2) & \grave{f}(4,3) & \grave{f}(4,4)\end{bmatrix}$$

$$\begin{bmatrix}\frac{\partial J}{\partial k_{q,p}^2(1,1)} & \frac{\partial J}{\partial k_{q,p}^2(1,2)}\\ \frac{\partial J}{\partial k_{q,p}^2(2,1)} & \frac{\partial J}{\partial k_{q,p}^2(2,2)}\end{bmatrix}=\text{correlação}\left(\begin{pmatrix}S_1^1(1,1) & S_1^1(1,2) & S_1^1(1,3) & S_1^1(1,4) & S_1^1(1,5)\\ S_1^1(2,1) & S_1^1(2,2) & S_1^1(2,3) & S_1^1(2,4) & S_1^1(2,5)\\ S_1^1(3,1) & S_1^1(3,2) & S_1^1(3,3) & S_1^1(3,4) & S_1^1(3,5)\\ S_1^1(4,1) & S_1^1(4,2) & S_1^1(4,3) & S_1^1(4,4) & S_1^1(4,5)\\ S_1^1(5,1) & S_1^1(5,2) & S_1^1(5,3) & S_1^1(5,4) & S_1^1(5,5)\end{pmatrix},\begin{pmatrix}\frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4)\\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4)\\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4)\\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4)\end{pmatrix}\right)$$

Retroprogando o erro para saída da camada de pooling C1

$$C_p^2(1,1) = f\Big(S_p^1(1,1)k_{q,1}^2(1,1) + S_p^1(1,2)k_{q,1}^2(1,2) + S_p^1(2,1)k_{q,1}^2(2,1) + S_p^1(2,2)k_{q,1}^2(2,2) + \cdots + S_p^1(1,1)k_{q,p}^2(1,1) + S_p^1(1,2)k_{q,p}^2(1,2) + S_p^1(2,1)k_{q,p}^2(2,1) + S_p^1(2,2)k_{q,p}^2(2,2)\Big)$$

$$C_p^2(1,2) = f\Big(S_p^1(1,2)k_{q,1}^2(1,1) + S_p^1(1,3)k_{q,1}^2(1,2) + S_p^1(2,2)k_{q,1}^2(2,1) + S_p^1(2,2)k_{q,1}^2(2,2) + \cdots + S_p^1(1,2)k_{q,p}^2(1,1) + S_p^1(1,3)k_{q,p}^2(1,2) + S_p^1(2,2)k_{q,p}^2(2,1) + S_p^1(2,2)k_{q,p}^2(2,2)\Big)$$

$$C_p^2(1,3) = f\Big(S_p^1(1,3)k_{q,1}^2(1,1) + S_p^1(1,4)k_{q,1}^2(1,2) + S_p^1(2,3)k_{q,1}^2(2,1) + S_p^1(2,4)k_{q,1}^2(2,2) + \cdots + S_p^1(1,3)k_{q,p}^2(1,1) + S_p^1(1,4)k_{q,p}^2(1,2) + S_p^1(2,3)k_{q,p}^2(2,1) + S_p^1(2,4)k_{q,p}^2(2,2)\Big)$$

$$C_p^2(1,4) = f\Big(S_p^1(1,4)k_{q,1}^2(1,1) + S_p^1(1,5)k_{q,1}^2(1,2) + S_p^1(2,4)k_{q,1}^2(2,1) + S_p^1(2,5)k_{q,1}^2(2,2) + \cdots + S_p^1(1,4)k_{q,p}^2(1,1) + S_p^1(1,5)k_{q,p}^2(1,2) + S_p^1(2,4)k_{q,p}^2(2,1) + S_p^1(2,5)k_{q,p}^2(2,2)\Big)$$

$$\frac{\partial J}{\partial S_p^1(1,1)} = \frac{\partial J}{C_p^1(1,1)}\frac{\partial C_p^1(1,1)}{\partial S_p^1(1,1)} = \frac{\partial J}{C_p^1(1,1)} f(1,1)k_{q,1}^2(1,1) + \cdots \frac{\partial J}{C_p^1(1,1)} f(1,1)k_{q,p}^2(1,1)$$

$$\frac{\partial J}{\partial S_p^1(1,2)} = \frac{\partial J}{C_p^1(1,1)}\frac{\partial C_p^{1\grave{}}(1,1)}{\partial S_p^1(1,2)} + \frac{\partial J}{C_p^1(1,2)}\frac{\partial C_p^{1\grave{}}(1,2)}{\partial S_p^1(1,2)}$$
$$= \frac{\partial J}{C_p^1(1,1)} f(1,1)k_{q,1}^2(1,2) + \frac{\partial J}{C_p^1(1,2)} f(1,2)k_{q,1}^{\grave{2}}(1,1) + \cdots \frac{\partial J}{C_p^1(1,1)} f(1,1)k_{q,p}^2(1,2) + \frac{\partial J}{C_p^1(1,2)} f(1,2)k_{q,p}^{\grave{2}}(1,1)$$

$$\frac{\partial J}{\partial S_p^1(1,3)} = \frac{\partial J}{C_p^1(1,2)}\frac{\partial C_p^{1\grave{}}(1,2)}{\partial S_p^1(1,3)} + \frac{\partial J}{C_p^1(1,3)}\frac{\partial C_p^{1\grave{}}(1,3)}{\partial S_p^1(1,3)}$$
$$= \frac{\partial J}{C_p^1(1,2)} f(1,2)k_{q,1}^2(1,2) + \frac{\partial J}{C_p^1(1,3)} f(1,3)k_{q,1}^{\grave{2}}(1,1) + \cdots \frac{\partial J}{C_p^1(1,2)} f(1,2)k_{q,p}^2(1,2) + \frac{\partial J}{C_p^1(1,3)} f(1,3)k_{q,p}^{\grave{2}}(1,1)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\dots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix}\right)$$

$$+\ldots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix}\right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+ \ldots \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\ldots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\ldots.\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\ldots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Retropropagando o erro para entrada da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\ldots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Saida da Camada de Pooling C1

$$S_p^1(1,1) = \frac{1}{4}(C_p^1(1,1) + C_p^1(1,2) + C_p^1(2,1) + C_p^1(2,2))$$

$$S_p^1(1,2) = \frac{1}{4}(C_p^1(1,3) + C_p^1(1,4) + C_p^1(2,3) + C_p^1(2,4))$$

$$S_p^1(2,1) = \frac{1}{4}(C_p^1(3,1) + C_p^1(3,2) + C_p^1(4,1) + C_p^1(4,2))$$

$$S_p^1(2,2) = \frac{1}{4}(C_p^1(3,3) + C_p^1(3,4) + C_p^1(4,3) + C_p^1(4,4))$$

Cálculo da derivada em relação a entrada da Camada de Pooling C1

$$\frac{\partial J}{\partial C_p^1(1,1)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{\partial S_p^1(1,1)}{\partial C_p^1(1,1)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_p^1(1,2)} = \frac{\partial J}{\partial S_p^1(1,2)} \frac{\partial S_p^1(1,2)}{\partial C_p^1(1,2)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_p^1(2,1)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{\partial S_p^1(1,1)}{\partial C_p^1(2,1)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{1}{4}$$

$$\frac{\partial J}{\partial C_p^1(2,2)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{\partial S_p^1(1,1)}{\partial C_p^1(2,2)} = \frac{\partial J}{\partial S_p^1(1,1)} \frac{1}{4}$$

Cálculo da derivada em relação a entrada da Camada de Pooling C1

$$\begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)} & \frac{\partial J}{\partial C_p^1(1,2)} & \frac{\partial J}{\partial C_p^1(1,3)} & \frac{\partial J}{\partial C_p^1(1,4)} & \cdots & \frac{\partial J}{\partial C_p^1(1,10)} \\ \frac{\partial J}{\partial C_p^1(2,1)} & \frac{\partial J}{\partial C_p^1(2,2)} & \frac{\partial J}{\partial C_p^1(2,3)} & \frac{\partial J}{\partial C_p^1(2,4)} & \cdots & \frac{\partial J}{\partial C_p^1(2,10)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)} & \frac{\partial J}{\partial C_p^1(10,2)} & \frac{\partial J}{\partial C_p^1(10,3)} & \frac{\partial J}{\partial C_p^1(10,4)} & \cdots & \frac{\partial J}{\partial C_p^1(10,10)} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,2)} & \cdots & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,2)} & \cdots & \frac{\partial J}{\partial S_p^1(1,5)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,2)} & \cdots & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix}$$

Saída da camada de convolução C1

$$C_p^1(1,1) = f\left(I_1(1,1)k_{p,1}^1(1,1) + I_1(1,2)k_{p,1}^1(1,2) + I_1(2,1)k_{p,1}^1(2,1) + I_1(2,2)k_{p,1}^1(2,2) + \cdots + I_3(1,1)k_{p,3}^1(1,1) + I_3(1,2)k_{p,3}^1(1,2) + I_3(2,1)k_{p,3}^1(2,1) + I_3(2,2)k_{p,3}^1(2,2)\right)$$

$$C_p^1(1,2) = f\left(I_1(1,2)k_{p,1}^1(1,1) + I_1(1,3)k_p^1(1,2) + I_1(2,2)k_{p,1}^1(2,1) + I_1(2,3)k_{p,1}^1(2,2) + \cdots + I_3(1,2)k_{p,3}^1(1,1) + I_3(1,3)k_{p,3}^1(1,2) + I_3(2,2)k_{p,3}^1(2,1) + I_3(2,3)k_{p,3}^1(2,2)\right)$$

$$C_p^1(2,1) = f\left(I_1(2,1)k_{p,1}^1(1,1) + I_1(2,2)k_{p,1}^1(1,2) + I_1(3,1)k_{p,1}^1(2,1) + I_1(3,2)k_{p,1}^1(2,2) + \cdots + I_3(2,1)k_{p,3}^1(1,1) + I_3(2,2)k_{p,3}^1(1,2) + I_3(3,1)k_{p,3}^1(2,1) + I_3(3,2)k_{p,3}^1(2,2)\right)$$

$$C_p^1(2,2) = f\left(I_1(2,2)k_{p,1}^1(1,1) + I_1(2,3)k_{p,1}^1(1,2) + I_1(3,2)k_{p,1}^1(2,1) + I_1(3,3)k_{p,1}^1(2,2) + \cdots + I_3(2,2)k_{p,3}^1(1,1) + I_3(2,3)k_{p,3}^1(1,2) + I_3(3,2)k_{p,3}^1(2,1) + I_3(3,3)k_{p,3}^1(2,2)\right)$$

Cálculo da derivada dos peso da camada convolução C1

$$\frac{\partial J}{\partial k_{p,1}^1(1,1)} = \frac{\partial J}{\partial C_p^1(1,1)}\frac{\partial C_p^1(1,1)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,2)}\frac{\partial C_p^1(1,2)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,3)}\frac{\partial C_p^1(1,3)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,4)}\frac{\partial C_p^1(1,4)}{\partial k_{p,1}^1(1,1)} + \cdots$$

$$\frac{\partial J}{\partial C_p^1(1,5)}\frac{\partial C_p^1(1,5)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,6)}\frac{\partial C_p^1(1,6)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,7)}\frac{\partial C_p^1(1,7)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,8)}\frac{\partial C_p^1(1,8)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,9)}\frac{\partial C_p^1(1,9)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(1,10)}\frac{\partial C_p^1(1,10)}{\partial k_{p,1}^1(1,1)} \cdots$$

$$\frac{\partial J}{\partial C_p^1(10,1)}\frac{\partial C_p^1(10,1)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,2)}\frac{\partial C_p^1(10,2)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,3)}\frac{\partial C_p^1(10,3)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(10,4)}\frac{\partial C_p^1(10,4)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,5)}\frac{\partial C_p^1(10,5)}{\partial k_{p,1}^1(1,1)} + \cdots$$

$$+ \frac{\partial J}{\partial C_p^1(10,6)}\frac{\partial C_p^1(10,6)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,7)}\frac{\partial C_p^1(10,7)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,8)}\frac{\partial C_p^1(10,8)}{\partial k_{p,1}^1(1,1)} + \frac{\partial J}{\partial C_p^1(10,10)}\frac{\partial C_p^1(10,10)}{\partial k_{p,1}^1(1,1)}$$

Anteriormente

$$\frac{\partial J}{\partial k^1_{p,1}(1,1)} = \frac{\partial J}{\partial C^1_p(1,1)}\frac{\partial C^1_p(1,1)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,2)}\frac{\partial C^1_p(1,2)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,3)}\frac{\partial C^1_p(1,3)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,4)}\frac{\partial C^1_p(1,4)}{\partial k^1_{p,1}(1,1)} + \cdots$$

$$\frac{\partial J}{\partial C^1_p(1,5)}\frac{\partial C^1_p(1,5)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,6)}\frac{\partial C^1_p(1,6)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,7)}\frac{\partial C^1_p(1,7)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,8)}\frac{\partial C^1_p(1,8)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,9)}\frac{\partial C^1_p(1,9)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(1,10)}\frac{\partial C^1_p(1,10)}{\partial k^1_{p,1}(1,1)} \cdots$$

$$\frac{\partial J}{\partial C^1_p(10,1)}\frac{\partial C^1_p(10,1)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,2)}\frac{\partial C^1_p(10,2)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,3)}\frac{\partial C^1_p(10,3)}{\partial k^1_{p,1}(1,2)} + \frac{\partial J}{\partial C^1_p(10,4)}\frac{\partial C^1_p(10,4)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,5)}\frac{\partial C^1_p(10,5)}{\partial k^1_{p,1}(1,1)} + \cdots$$

$$+ \frac{\partial J}{\partial C^1_p(10,6)}\frac{\partial C^1_p(10,6)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,7)}\frac{\partial C^1_p(10,7)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,8)}\frac{\partial C^1_p(10,8)}{\partial k^1_{p,1}(1,1)} + \frac{\partial J}{\partial C^1_p(10,10)}\frac{\partial C^1_p(10,10)}{\partial k^1_{p,1}(1,1)}$$

$$\frac{\partial J}{\partial k^1_{p,1}(1,1)} = \frac{\partial J}{\partial C^1_p(1,1)}\acute{f}(1,1)I_1(1,1) + \frac{\partial J}{\partial C^1_p(1,2)}\acute{f}(1,2)I_1(1,2) + \frac{\partial J}{\partial C^1_p(1,3)}\acute{f}(1,3)I_1(1,3) + \frac{\partial J}{\partial C^1_p(1,4)}\acute{f}(1,4)I_1(1,4) + \cdots$$

$$\frac{\partial J}{\partial C^1_p(1,5)}\acute{f}(1,5)I_1(1,5) + \frac{\partial J}{\partial C^1_p(1,6)}\acute{f}(1,6)I_1(1,6) + \frac{\partial J}{\partial C^1_p(1,7)}\acute{f}(1,7)I_1(1,7) + \frac{\partial J}{\partial C^1_p(1,8)}\acute{f}(1,8)I_1(1,8) + \frac{\partial J}{\partial C^1_p(1,9)}\acute{f}(1,9)I_1(1,9) + \frac{\partial J}{\partial C^1_p(1,10)}\acute{f}(1,10)I_1(1,10)$$

$$\frac{\partial J}{\partial C^1_p(10,1)}\acute{f}(10,1)I_1(10,1) + \frac{\partial J}{\partial C^1_p(10,2)}\acute{f}(10,2)I_1(10,2) + \frac{\partial J}{\partial C^1_p(10,3)}\acute{f}(10,3)I_1(10,3) + \frac{\partial J}{\partial C^1_p(10,4)}\acute{f}(10,4)I_1(10,4) + \frac{\partial J}{\partial C^1_p(10,5)}\acute{f}(10,5)I_1(10,5) + \cdots$$

$$+ \frac{\partial J}{\partial C^1_p(10,6)}\acute{f}(10,6)I_1(10,6) + \frac{\partial J}{\partial C^1_p(10,7)}\acute{f}(10,7)I_1(10,7) + \frac{\partial J}{\partial C^1_p(10,8)}\acute{f}(10,8)I_1(10,8) + \frac{\partial J}{\partial C^1_p(10,9)}\acute{f}(10,9)I_1(10,9) + \frac{\partial J}{\partial C^1_p(10,10)}\acute{f}(10,10)I_1(10,10)$$

Saída da camada de convolução C1

$$C_p^1(1,1) = f\Big(I_1(1,1)k_{p,1}^1(1,1) + I_1(1,2)k_{p,1}^1(1,2) + I_1(2,1)k_{p,1}^1(2,1) + I_1(2,2)k_{p,1}^1(2,2) + \cdots + I_3(1,1)k_{p,3}^1(1,1) + I_3(1,2)k_{p,3}^1(1,2) + I_3(2,1)k_{p,3}^1(2,1) + I_3(2,2)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(1,2) = f\Big(I_1(1,2)k_{p,1}^1(1,1) + I_1(1,3)k_p^1(1,2) + I_1(2,2)k_{p,1}^1(2,1) + I_1(2,3)k_{p,1}^1(2,2) + \cdots + I_3(1,2)k_{p,3}^1(1,1) + I_3(1,3)k_{p,3}^1(1,2) + I_3(2,2)k_{p,3}^1(2,1) + I_3(2,3)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(2,1) = f\Big(I_1(2,1)k_{p,1}^1(1,1) + I_1(2,2)k_{p,1}^1(1,2) + I_1(3,1)k_{p,1}^1(2,1) + I_1(3,2)k_{p,1}^1(2,2) + \cdots + I_3(2,1)k_{p,3}^1(1,1) + I_3(2,2)k_{p,3}^1(1,2) + I_3(3,1)k_{p,3}^1(2,1) + I_3(3,2)k_{p,3}^1(2,2)\Big)$$

$$C_p^1(2,2) = f\Big(I_1(2,2)k_{p,1}^1(1,1) + I_1(2,3)k_{p,1}^1(1,2) + I_1(3,2)k_{p,1}^1(2,1) + I_1(3,3)k_{p,1}^1(2,2) + \cdots + I_3(2,2)k_{p,3}^1(1,1) + I_3(2,3)k_{p,3}^1(1,2) + I_3(3,2)k_{p,3}^1(2,1) + I_3(3,3)k_{p,3}^1(2,2)\Big)$$

Cálculo da derivada dos peso da camada convolução C1

$$\frac{\partial J}{\partial k_{p,1}^1(1,2)} = \frac{\partial J}{\partial C_p^1(1,1)}\frac{\partial C_p^1(1,1)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,2)}\frac{\partial C_p^1(1,2)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,3)}\frac{\partial C_p^1(1,3)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,4)}\frac{\partial C_p^1(1,4)}{\partial k_{p,1}^1(1,2)} + \cdots$$

$$\frac{\partial J}{\partial C_p^1(1,5)}\frac{\partial C_p^1(1,5)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,6)}\frac{\partial C_p^1(1,6)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,7)}\frac{\partial C_p^1(1,7)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,8)}\frac{\partial C_p^1(1,8)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,9)}\frac{\partial C_p^1(1,9)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,10)}\frac{\partial C_p^1(1,10)}{\partial k_{p,1}^1(1,2)} \cdots$$

$$\frac{\partial J}{\partial C_p^1(9,1)}\frac{\partial C_p^1(9,1)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,2)}\frac{\partial C_p^1(9,2)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,3)}\frac{\partial C_p^1(9,3)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,4)}\frac{\partial C_p^1(9,4)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,5)}\frac{\partial C_p^1(9,5)}{\partial k_{p,1}^1(1,2)} + \cdots$$

$$+ \frac{\partial J}{\partial C_p^1(9,6)}\frac{\partial C_p^1(9,6)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,7)}\frac{\partial C_p^1(9,7)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,8)}\frac{\partial C_p^1(9,8)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,9)}\frac{\partial C_p^1(9,9)}{\partial k_{p,1}^1(1,2)}$$

Anteriormente

$$\frac{\partial J}{\partial k_{p,1}^1(1,2)} = \frac{\partial J}{\partial C_p^1(1,1)}\frac{\partial C_p^1(1,1)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,2)}\frac{\partial C_p^1(1,2)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,3)}\frac{\partial C_p^1(1,3)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,4)}\frac{\partial C_p^1(1,4)}{\partial k_{p,1}^1(1,2)} + \cdots$$

$$\frac{\partial J}{\partial C_p^1(1,5)}\frac{\partial C_p^1(1,5)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,6)}\frac{\partial C_p^1(1,6)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,7)}\frac{\partial C_p^1(1,7)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,8)}\frac{\partial C_p^1(1,8)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,9)}\frac{\partial C_p^1(1,9)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(1,10)}\frac{\partial C_p^1(1,10)}{\partial k_{p,1}^1(1,2)} \cdots$$

$$\frac{\partial J}{\partial C_p^1(9,1)}\frac{\partial C_p^1(9,1)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,2)}\frac{\partial C_p^1(9,2)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,3)}\frac{\partial C_p^1(9,3)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,4)}\frac{\partial C_p^1(9,4)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,5)}\frac{\partial C_p^1(9,5)}{\partial k_{p,1}^1(1,2)} + \cdots$$

$$+ \frac{\partial J}{\partial C_p^1(9,6)}\frac{\partial C_p^1(9,6)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,7)}\frac{\partial C_p^1(9,7)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,8)}\frac{\partial C_p^1(9,8)}{\partial k_{p,1}^1(1,2)} + \frac{\partial J}{\partial C_p^1(9,9)}\frac{\partial C_p^1(9,9)}{\partial k_{p,1}^1(1,2)}$$

$$\frac{\partial J}{\partial k_{p,1}^1(1,2)} = \frac{\partial J}{\partial C_p^1(1,1)}\acute{f}(1,1)I_1(1,2) + \frac{\partial J}{\partial C_p^1(1,2)}\acute{f}(1,2)I_1(1,3) + \frac{\partial J}{\partial C_p^1(1,3)}\acute{f}(1,3)I_1(1,4) + \frac{\partial J}{\partial C_p^1(1,4)}\acute{f}(1,4)I_1(1,5) + \cdots$$

$$\frac{\partial J}{\partial C_p^1(1,5)}\acute{f}(1,5)I_1(1,6) + \frac{\partial J}{\partial C_p^1(1,6)}\acute{f}(1,6)I_1(1,7) + \frac{\partial J}{\partial C_p^1(1,7)}\acute{f}(1,7)I_1(1,8) + \frac{\partial J}{\partial C_p^1(1,8)}\acute{f}(1,8)I_1(1,9) + \frac{\partial J}{\partial C_p^1(1,9)}\acute{f}(1,9)I_1(1,10) + \frac{\partial J}{\partial C_p^1(1,10)}\acute{f}(1,10)I_1(1,11)$$

$$\frac{\partial J}{\partial C_p^1(10,1)}\acute{f}(10,1)I_1(10,2) + \frac{\partial J}{\partial C_p^1(10,2)}\acute{f}(10,2)I_1(10,3) + \frac{\partial J}{\partial C_p^1(10,3)}\acute{f}(10,3)I_1(10,4) + \frac{\partial J}{\partial C_p^1(10,4)}\acute{f}(10,4)I_1(10,5) + \frac{\partial J}{\partial C_p^1(10,5)}\acute{f}(10,5)I_1(10,6) + \cdots$$

$$+ \frac{\partial J}{\partial C_p^1(10,6)}\acute{f}(10,6)I_1(10,7) + \frac{\partial J}{\partial C_p^1(10,7)}\acute{f}(10,7)I_1(10,8) + \frac{\partial J}{\partial C_p^1(10,8)}\acute{f}(10,8)I_1(10,9) + \frac{\partial J}{\partial C_p^1(10,9)}\acute{f}(10,9)I_1(10,10) + \frac{\partial J}{\partial C_p^1(10,10)}\acute{f}(10,10)I_1(10,11)$$

$$\begin{bmatrix} \frac{\partial J}{\partial k_{p,1}^{1}(1,1)} & \frac{\partial J}{\partial k_{p,1}^{1}(1,2)} \\ \frac{\partial J}{\partial k_{p,1}^{2}(2,1)} & \frac{\partial J}{\partial k_{p,1}^{2}(2,2)} \end{bmatrix} = \text{correlação}$$

$$\left(\left(\begin{bmatrix} I_1(1,1) & I_1(1,2) & I_1(1,3) & \cdots & I_1(1,11) \\ I_1(2,1) & I_1(2,2) & I_1(2,3) & \cdots & I_1(2,11) \\ I_1(3,1) & I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) & I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix} \right), \begin{pmatrix} \frac{\partial J}{\partial C_p^1(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_p^1(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_p^1(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_p^1(1,4)}\grave{f}(1,4) & \cdots & \frac{\partial J}{\partial C_p^1(1,10)}\grave{f}(1,10) \\ \frac{\partial J}{\partial C_p^1(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_p^1(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_p^1(2,3)}\grave{f}(3,3) & \frac{\partial J}{\partial C_p^1(2,4)}\grave{f}(3,4) & \cdots & \frac{\partial J}{\partial C_p^1(2,10)}\grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)}\grave{f}(10,1) & \frac{\partial J}{\partial C_p^1(10,2)}\grave{f}(10,2) & \frac{\partial J}{\partial C_p^1(10,3)}\grave{f}(10,3) & \frac{\partial J}{\partial C_p^1(10,4)}\grave{f}(10,4) & \cdots & \frac{\partial J}{\partial C_p^1(10,10)}\grave{f}(10,10) \end{pmatrix} \right)$$

$$\begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_p^1(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_p^1(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_p^1(1,4)}\grave{f}(1,4) & \cdots & \frac{\partial J}{\partial C_p^1(1,10)}\grave{f}(1,10) \\ \frac{\partial J}{\partial C_p^1(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_p^1(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_p^1(2,3)}\grave{f}(3,3) & \frac{\partial J}{\partial C_p^1(2,4)}\grave{f}(3,4) & \cdots & \frac{\partial J}{\partial C_p^1(2,10)}\grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)}\grave{f}(10,1) & \frac{\partial J}{\partial C_p^1(10,2)}\grave{f}(10,2) & \frac{\partial J}{\partial C_p^1(10,3)}\grave{f}(10,3) & \frac{\partial J}{\partial C_p^1(10,4)}\grave{f}(10,4) & \cdots & \frac{\partial J}{\partial C_p^1(10,10)}\grave{f}(10,10) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)} & \frac{\partial J}{\partial C_p^1(1,2)} & \frac{\partial J}{\partial C_p^1(1,3)} & \frac{\partial J}{\partial C_p^1(1,4)} & \cdots & \frac{\partial J}{\partial C_p^1(1,10)} \\ \frac{\partial J}{\partial C_p^1(2,1)} & \frac{\partial J}{\partial C_p^1(2,2)} & \frac{\partial J}{\partial C_p^1(2,3)} & \frac{\partial J}{\partial C_p^1(2,4)} & \cdots & \frac{\partial J}{\partial C_p^1(2,10)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)} & \frac{\partial J}{\partial C_p^1(10,2)} & \frac{\partial J}{\partial C_p^1(10,3)} & \frac{\partial J}{\partial C_p^1(10,4)} & \cdots & \frac{\partial J}{\partial C_p^1(10,10)} \end{bmatrix} .* \begin{bmatrix} \grave{f}(1,1) & \grave{f}(1,2) & \grave{f}(1,3) & \grave{f}(1,4) & \cdots & \grave{f}(1,10) \\ \grave{f}(2,1) & \grave{f}(2,2) & \grave{f}(3,3) & \grave{f}(3,4) & \cdots & \grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \grave{f}(10,1) & \grave{f}(10,2) & \grave{f}(10,3) & \grave{f}(10,4) & \cdots & \grave{f}(10,10) \end{bmatrix}$$

Algoritmo

- 1º Passo) Cálculo da derivada em relação a saída da camada de pooling P2

$$\frac{\partial J}{\partial z_j} = \sum_{n=1}^{N} \sum_{i=1}^{ns} (Y_i(n) - Yd_i(n))w_{ij}$$

$$\frac{\partial J}{\partial Z} = \begin{bmatrix} \frac{\partial J}{\partial z_1} \\ \frac{\partial J}{\partial z_2} \\ \frac{\partial J}{\partial z_3} \\ \frac{\partial J}{\partial z_4} \\ \frac{\partial J}{\partial z_5} \\ \vdots \\ \frac{\partial J}{\partial z_{ne}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial S_1^2(1,1)} \\ \frac{\partial J}{\partial S_1^2(1,2)} \\ \frac{\partial J}{\partial S_1^2(2,1)} \\ \frac{\partial J}{\partial S_1^2(2,2)} \\ \frac{\partial J}{\partial S_2^2(1,1)} \\ \vdots \\ \frac{\partial J}{\partial S_q^2(2,2)} \end{bmatrix}$$

- 2º Passo) Cálculo da derivada em relação a entrada da camada de pooling P2

$$\frac{\partial J}{\partial C_q^2} = \frac{1}{4} \begin{bmatrix} \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)} \\ \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,1)} & \frac{\partial J}{\partial S_2^2(1,2)} & \frac{\partial J}{\partial S_2^2(1,2)} \\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)} \\ \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,1)} & \frac{\partial J}{\partial S_2^2(2,2)} & \frac{\partial J}{\partial S_2^2(2,2)} \end{bmatrix}$$

Algoritmo

- 3º Passo) Cálculo da derivada em relação a ativação da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial J}{\partial C_q^2(1,1)} & \frac{\partial J}{\partial C_q^2(1,2)} & \frac{\partial J}{\partial C_q^2(1,3)} & \frac{\partial J}{\partial C_q^2(1,4)} \\ \frac{\partial J}{\partial C_q^2(2,1)} & \frac{\partial J}{\partial C_q^2(2,2)} & \frac{\partial J}{\partial C_q^2(2,3)} & \frac{\partial J}{\partial C_q^2(2,3)} \\ \frac{\partial J}{C_q^2(3,1)} & \frac{\partial J}{C_q^2(3,2)} & \frac{\partial J}{C_q^2(3,3)} & \frac{\partial J}{C_q^2(3,4)} \\ \frac{\partial J}{C_q^2(4,1)} & \frac{\partial J}{C_q^2(4,2)} & \frac{\partial J}{C_q^2(4,3)} & \frac{\partial J}{C_q^2(4,4)} \end{bmatrix} .* \begin{bmatrix} \grave{f}(1,1) & \grave{f}(1,2) & \grave{f}(1,3) & \grave{f}(1,4) \\ \grave{f}(2,1) & \grave{f}(2,2) & \grave{f}(2,3) & \grave{f}(2,4) \\ \grave{f}(3,1) & \grave{f}(3,2) & \grave{f}(3,3) & \grave{f}(3,4) \\ \grave{f}(4,1) & \grave{f}(4,2) & \grave{f}(4,3) & \grave{f}(4,4) \end{bmatrix}$$

Algoritmo

- 4º Passo) Cálculo da derivada em relação aos pesos da camada convolução C2

$$\begin{bmatrix} \frac{\partial J}{\partial k_{q,p}^2(1,1)} & \frac{\partial J}{\partial k_{q,p}^2(1,2)} \\ \frac{\partial J}{\partial k_{q,p}^2(2,1)} & \frac{\partial J}{\partial k_{q,p}^2(2,2)} \end{bmatrix} = \text{correlação}\left(\begin{pmatrix} S_1^1(1,1) & S_1^1(1,2) & S_1^1(1,3) & S_1^1(1,4) & S_1^1(1,5) \\ S_1^1(2,1) & S_1^1(2,2) & S_1^1(2,3) & S_1^1(2,4) & S_1^1(2,5) \\ S_1^1(3,1) & S_1^1(3,2) & S_1^1(3,3) & S_1^1(3,4) & S_1^1(3,5) \\ S_1^1(4,1) & S_1^1(4,2) & S_1^1(4,3) & S_1^1(4,4) & S_1^1(4,5) \\ S_1^1(5,1) & S_1^1(5,2) & S_1^1(5,3) & S_1^1(5,4) & S_1^1(5,5) \end{pmatrix}, \begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix} \right)$$

5º Passo) Retropropaga o erro

$$\begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,3)} & \frac{\partial J}{\partial S_p^1(1,4)} & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(2,1)} & \frac{\partial J}{\partial S_p^1(2,2)} & \frac{\partial J}{\partial S_p^1(2,3)} & \frac{\partial J}{\partial S_p^1(2,4)} & \frac{\partial J}{\partial S_p^1(2,5)} \\ \frac{\partial J}{\partial S_p^1(3,1)} & \frac{\partial J}{\partial S_p^1(3,2)} & \frac{\partial J}{\partial S_p^1(3,3)} & \frac{\partial J}{\partial S_p^1(3,4)} & \frac{\partial J}{\partial S_p^1(3,5)} \\ \frac{\partial J}{\partial S_p^1(4,1)} & \frac{\partial J}{\partial S_p^1(4,2)} & \frac{\partial J}{\partial S_p^1(4,3)} & \frac{\partial J}{\partial S_p^1(4,4)} & \frac{\partial J}{\partial S_p^1(4,5)} \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,3)} & \frac{\partial J}{\partial S_p^1(5,5)} & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix} = \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,1}^2(2,2) & k_{q,1}^2(2,1) \\ k_{q,1}^2(1,2) & k_{q,1}^2(1,1) \end{pmatrix} \right)$$

$$+\ldots\ \text{correlação\_full}\left(\begin{pmatrix} \frac{\partial J}{\partial C_q^2(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_q^2(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_q^2(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_q^2(1,4)}\grave{f}(1,4) \\ \frac{\partial J}{\partial C_q^2(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_q^2(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,3) & \frac{\partial J}{\partial C_q^2(2,3)}\grave{f}(2,4) \\ \frac{\partial J}{C_q^2(3,1)}\grave{f}(3,1) & \frac{\partial J}{C_q^2(3,2)}\grave{f}(3,2) & \frac{\partial J}{C_q^2(3,3)}\grave{f}(3,3) & \frac{\partial J}{C_q^2(3,4)}\grave{f}(3,4) \\ \frac{\partial J}{C_q^2(4,1)}\grave{f}(4,1) & \frac{\partial J}{C_q^2(4,2)}\grave{f}(4,2) & \frac{\partial J}{C_q^2(4,3)}\grave{f}(4,3) & \frac{\partial J}{C_q^2(4,4)}\grave{f}(4,4) \end{pmatrix}, \begin{pmatrix} k_{q,p}^2(2,2) & k_{q,p}^2(2,1) \\ k_{q,p}^2(1,2) & k_{q,p}^2(1,1) \end{pmatrix} \right)$$

Algoritmo

6º Passo) Cálculo da derivada em relação a entrada da camada de pooling P1 (similar 2º Passo)

$$\begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)} & \frac{\partial J}{\partial C_p^1(1,2)} & \frac{\partial J}{\partial C_p^1(1,3)} & \frac{\partial J}{\partial C_p^1(1,4)} & \cdots & \frac{\partial J}{\partial C_p^1(1,10)} \\ \frac{\partial J}{\partial C_p^1(2,1)} & \frac{\partial J}{\partial C_p^1(2,2)} & \frac{\partial J}{\partial C_p^1(2,3)} & \frac{\partial J}{\partial C_p^1(2,4)} & \cdots & \frac{\partial J}{\partial C_p^1(2,10)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)} & \frac{\partial J}{\partial C_p^1(10,2)} & \frac{\partial J}{\partial C_p^1(10,3)} & \frac{\partial J}{\partial C_p^1(10,4)} & \cdots & \frac{\partial J}{\partial C_p^1(10,10)} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,2)} & \cdots & \frac{\partial J}{\partial S_p^1(1,5)} \\ \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,1)} & \frac{\partial J}{\partial S_p^1(1,2)} & \frac{\partial J}{\partial S_p^1(1,2)} & \cdots & \frac{\partial J}{\partial S_p^1(1,5)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,1)} & \frac{\partial J}{\partial S_p^1(5,2)} & \frac{\partial J}{\partial S_p^1(5,2)} & \cdots & \frac{\partial J}{\partial S_p^1(5,5)} \end{bmatrix}$$

Algoritmo

- 7º Passo) Cálculo da derivada em relação a ativação da camada convolução C1 (similar ao 3º Passo)

$$\begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C_p^1(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C_p^1(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C_p^1(1,4)}\grave{f}(1,4) & \cdots & \frac{\partial J}{\partial C_p^1(1,10)}\grave{f}(1,10) \\ \frac{\partial J}{\partial C_p^1(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C_p^1(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C_p^1(2,3)}\grave{f}(3,3) & \frac{\partial J}{\partial C_p^1(2,4)}\grave{f}(3,4) & \cdots & \frac{\partial J}{\partial C_p^1(2,10)}\grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)}\grave{f}(10,1) & \frac{\partial J}{\partial C_p^1(10,2)}\grave{f}(10,2) & \frac{\partial J}{\partial C_p^1(10,3)}\grave{f}(10,3) & \frac{\partial J}{\partial C_p^1(10,4)}\grave{f}(10,4) & \cdots & \frac{\partial J}{\partial C_p^1(10,10)}\grave{f}(10,10) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial J}{\partial C_p^1(1,1)} & \frac{\partial J}{\partial C_p^1(1,2)} & \frac{\partial J}{\partial C_p^1(1,3)} & \frac{\partial J}{\partial C_p^1(1,4)} & \cdots & \frac{\partial J}{\partial C_p^1(1,10)} \\ \frac{\partial J}{\partial C_p^1(2,1)} & \frac{\partial J}{\partial C_p^1(2,2)} & \frac{\partial J}{\partial C_p^1(2,3)} & \frac{\partial J}{\partial C_p^1(2,4)} & \cdots & \frac{\partial J}{\partial C_p^1(2,10)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C_p^1(10,1)} & \frac{\partial J}{\partial C_p^1(10,2)} & \frac{\partial J}{\partial C_p^1(10,3)} & \frac{\partial J}{\partial C_p^1(10,4)} & \cdots & \frac{\partial J}{\partial C_p^1(10,10)} \end{bmatrix} .* \begin{bmatrix} \grave{f}(1,1) & \grave{f}(1,2) & \grave{f}(1,3) & \grave{f}(1,4) & \cdots & \grave{f}(1,10) \\ \grave{f}(2,1) & \grave{f}(2,2) & \grave{f}(3,3) & \grave{f}(3,4) & \cdots & \grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \grave{f}(10,1) & \grave{f}(10,2) & \grave{f}(10,3) & \grave{f}(10,4) & \cdots & \grave{f}(10,10) \end{bmatrix}$$

Algoritmo

- 8º Passo) Cálculo da derivada em relação aos pesos da camada convolução C1 (similar ao 4º Passo)

$$
\begin{bmatrix} \frac{\partial J}{\partial k^1_{p,1}(1,1)} & \frac{\partial J}{\partial k^1_{p,1}(1,2)} \\ \frac{\partial J}{\partial k^2_{p,1}(2,1)} & \frac{\partial J}{\partial k^2_{p,1}(2,2)} \end{bmatrix} = \text{correlação}
$$

$$
\left(\left(\begin{bmatrix} I_1(1,1) & I_1(1,2) & I_1(1,3) & \cdots & I_1(1,11) \\ I_1(2,1) & I_1(2,2) & I_1(2,3) & \cdots & I_1(2,11) \\ I_1(3,1) & I_1(3,2) & I_1(3,3) & \cdots & I_1(3,11) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1(11,1) & I_1(11,2) & I_1(11,3) & \cdots & I_1(11,11) \end{bmatrix} \right), \left(\begin{matrix} \frac{\partial J}{\partial C^1_p(1,1)}\grave{f}(1,1) & \frac{\partial J}{\partial C^1_p(1,2)}\grave{f}(1,2) & \frac{\partial J}{\partial C^1_p(1,3)}\grave{f}(1,3) & \frac{\partial J}{\partial C^1_p(1,4)}\grave{f}(1,4) & \cdots & \frac{\partial J}{\partial C^1_p(1,10)}\grave{f}(1,10) \\ \frac{\partial J}{\partial C^1_p(2,1)}\grave{f}(2,1) & \frac{\partial J}{\partial C^1_p(2,2)}\grave{f}(2,2) & \frac{\partial J}{\partial C^1_p(2,3)}\grave{f}(3,3) & \frac{\partial J}{\partial C^1_p(2,4)}\grave{f}(3,4) & \cdots & \frac{\partial J}{\partial C^1_p(2,10)}\grave{f}(3,10) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial J}{\partial C^1_p(10,1)}\grave{f}(10,1) & \frac{\partial J}{\partial C^1_p(10,2)}\grave{f}(10,2) & \frac{\partial J}{\partial C^1_p(10,3)}\grave{f}(10,3) & \frac{\partial J}{\partial C^1_p(10,4)}\grave{f}(10,4) & \cdots & \frac{\partial J}{\partial C^1_p(10,10)}\grave{f}(10,10) \end{matrix} \right) \right)
$$

Código

1º Passo)

```
%softmax layer
output = zeros(size(probs));
output(index) = 1; %Aciona o valor de 1 a saida desejada
DeltaSoftmax = (probs - output); %Calcula o erro
```

Cálculo do erro

```
%Transforma o erro em matriz
numFiltros2 = cnn.camadas{numCamadas-1}.numFiltros; %Numero de filtros da ultima camada
dimSaidaX= size(cnn.camadas{numCamadas}.ativacao,1); % Numero de saida
dimSaidaY= size(cnn.camadas{numCamadas}.ativacao,2); % Numero de saida
cnn.camadas{numCamadas}.delta = reshape(cnn.Wd' * DeltaSoftmax,dimSaidaX,dimSaidaY,numFiltros2,numImagens);
```

Transforma em Matriz

Código

```
%Outras camadas
for l = numCamadas-1:-1:1
    camada = cnn.camadas{l};
    if strcmp(camada.tipo,'c') % camada convolucional
        numFiltros = cnn.camadas{l}.numFiltros; %Numero de filtros
        dimSaida1 = size(cnn.camadas{l+1}.ativacao,1); %dimensao da saida
        dimSaida2 = size(cnn.camadas{l+1}.ativacao,2); %dimensao da saida
        strider = cnn.camadas{l+1}.strider;
        dimPool = cnn.camadas{l+1}.dimPool; %dimensão do poolin
        convDim1 = dimSaida1*strider + dimPool-1;
        convDim2 = dimSaida2*strider + dimPool-1;
        DeltaPool = cnn.camadas{l+1}.delta;
        DeltaUnpool = zeros(convDim1,convDim2,numFiltros,numImagens);  %Cria
        for imNum = 1:numImagens
            for FilterNum = 1:numFiltros
                unpool = DeltaPool(:,:,FilterNum,imNum)/dimPool ^ 2;
                DeltaUnpool(:,:,FilterNum,imNum)= replica(unpool,dimPool,strider,convDim1,convDim2);
            end
        end
        ativacao = camada.ativacao;
        DeltaConv = DeltaUnpool .* ativacao .* (1 - ativacao);
        camada.delta = DeltaConv;
        cnn.camadas{l} = camada;
    else
        numFiltros1 = cnn.camadas{l-1}.numFiltros; %Numero de filtros da camada anterior
        numFiltros2 = cnn.camadas{l+1}.numFiltros; %Numero de filtrso da camada posterior
        dimSaida1 = size(camada.ativacao,1); %Numero de saida da camada atual
        dimSaida2 = size(camada.ativacao,2); %Numero de saida da camada atual
        DeltaPooled = zeros(dimSaida1,dimSaida2,numFiltros1,numImagens); % Matriz de derivada com zero
        DeltaConv = cnn.camadas{l+1}.delta; %copia a derivada da camada da frente
        Wc = cnn.camadas{l+1}.W; %pesos da camada posterior
        strider = cnn.camadas{l+1}.strider;
        for i = 1:numImagens
            for f1 = 1:numFiltros1
                for f2 = 1:numFiltros2
                    DeltaPooled(:,:,f1,i) = DeltaPooled(:,:,f1,i) +...
                        conv3pad(DeltaConv(:,:,f2,i),Wc(:,:,f1,f2),strider);
                end
            end
        end
        camada.delta = DeltaPooled;
        cnn.camadas{l} = camada; %armazena na camada
    end
end
```

2º Passo)

3º Passo)

4º Passo) Foi implementado na atualização

5º Passo)

Código

Realiza atualização

```
%gradients
ativacaoPool = cnn.camadas{numCamadas}.ativacao; %ativacao da ultima camada
ativacaoPool = reshape(ativacaoPool,[],numImagens); % Transforma em vetor
Wd_grad = DeltaSoftmax*(ativacaoPool)'; %dJdw
bd_grad = sum(DeltaSoftmax,2);%dIdb

cnn.Wd_velocidade = mom*cnn.Wd_velocidade + alpha * (Wd_grad/minibatch+lambda*cnn.Wd);
cnn.bd_velocidade = mom*cnn.bd_velocidade + alpha * (bd_grad/minibatch);
cnn.Wd = cnn.Wd - cnn.Wd_velocidade; %atualiza os pesos da camada de saida
cnn.bd = cnn.bd - cnn.bd_velocidade; %atualiza o bias
```

Código

4º Passo)

```
%Realiza a atualização
for l = numCamadas:-1:1
    camada = cnn.camadas{l};
    if(strcmp(camada.tipo,'c'))%if this is a convolutional layer
        numFiltros2 = camada.numFiltros;
        if(l == 1)
            numFiltros1 = cnn.imageCanal;
            ativacaoPool = X;
        else
            numFiltros1 = cnn.camadas{l-2}.numFiltros;
            ativacaoPool = cnn.camadas{l-1}.ativacao;
        end
        Wc_grad = zeros(size(camada.W));
        bc_grad = zeros(size(camada.b));
        DeltaConv = camada.delta;
        strider=camada.strider;
    for fil2 = 1:numFiltros2
        for fil1 = 1:numFiltros1
            for im = 1:numImagens
                Wc_grad(:,:,fil1,fil2) = Wc_grad(:,:,fil1,fil2) +...
                conv3aux(ativacaoPool(:,:,fil1,im),rot90(DeltaConv(:,:,fil2,im),2),strider);
            end
        end
        temp = DeltaConv(:,:,fil2,:);
        bc_grad(fil2) = sum(temp(:));
    end
    camada.W_velocidade = mom*camada.W_velocidade + alpha*(Wc_grad/numImagens+lambda*camada.W);
    camada.b_velocidade = mom*camada.b_velocidade + alpha*(bc_grad/numImagens);
    camada.W = camada.W - camada.W_velocidade;
    camada.b = camada.b - camada.b_velocidade;
  end
  cnn.camadas{l} = camada;
end
fprintf('Epoca %d: Custo na iteracao %d is %f\n',e,it,cost);
C(length(C)+1) = cost;
```