EXAMPLE SQP

Minimize

$$f(\mathbf{x}) = (x_1 - 1)^2 + (x_2 - 2)^2$$

subject to

$$h_1(\mathbf{x}) = 2x_1 - x_2 = 0$$

$$g_1(\mathbf{x}) = x_1 \le 5$$

from a starting point (10, –5).
Iteration 1

$$f(\mathbf{x}) = 130; \quad \nabla f(\mathbf{x}) = \begin{bmatrix} 18 \\ -14 \end{bmatrix}; \quad \nabla h = \begin{bmatrix} 2 \\ -1 \end{bmatrix}; \quad \nabla g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \nabla^2 L = \begin{bmatrix} 12 & -4 \\ -4 & 4 \end{bmatrix}$$

ITERATION 1 - Write the quadratic problem, obtain the solution and the new point:

The quadratic problem is

Minimize

$$Q = \Delta\mathbf{x}^T \begin{bmatrix} 18 \\ -14 \end{bmatrix} + \frac{1}{2} \Delta\mathbf{x}^T \begin{bmatrix} 12 & -4 \\ -4 & 4 \end{bmatrix} \Delta\mathbf{x}$$

subject to

$$25 + [2 \quad -1]\Delta\mathbf{x} = 0$$

$$5 + [1 \quad 0]\Delta\mathbf{x} = 0$$

The solution of the quadratic problem is

$$\Delta x = \begin{bmatrix} -7.5 \\ 10 \end{bmatrix}$$

Now x is updated as

$$x = x + \Delta x = \begin{bmatrix} 10 \\ -5 \end{bmatrix} + \begin{bmatrix} -7.5 \\ 10 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 5 \end{bmatrix}$$

ITERATION 2 - Write the quadratic problem, obtain the solution and the new point:

$$f(x) = 11.25; \quad \nabla f(x) = \begin{bmatrix} 3 \\ 6 \end{bmatrix}; \quad \nabla h = \begin{bmatrix} 2 \\ -1 \end{bmatrix}; \quad \nabla g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \nabla^2 L = \begin{bmatrix} 10 & -4 \\ -4 & 4 \end{bmatrix}$$

The quadratic problem is

Minimize

$$Q = \Delta x^T \begin{bmatrix} 3 \\ 6 \end{bmatrix} + \frac{1}{2} \Delta x^T \begin{bmatrix} 10 & -4 \\ -4 & 4 \end{bmatrix} \Delta x$$

subject to

$$0 + [2 \quad -1]\Delta x = 0$$

$$-2.5 + [1 \quad 0]\Delta x = 0$$

The solution of the quadratic problem is

$$\Delta x = \begin{bmatrix} -1.5 \\ -3.0 \end{bmatrix}$$

Now $\boldsymbol{x}$ is updated as

$$\boldsymbol{x} = \boldsymbol{x} + \Delta\boldsymbol{x} = \begin{bmatrix} 2.5 \\ 5 \end{bmatrix} + \begin{bmatrix} -1.5 \\ -3.0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

ITERATION 3 - Write the quadratic problem, obtain the solution and the new point:

$$f(\boldsymbol{x}) = 0; \quad \nabla f(\boldsymbol{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \nabla h = \begin{bmatrix} 2 \\ -1 \end{bmatrix}; \quad \nabla g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \nabla^2 L = \begin{bmatrix} 10 & -4 \\ -4 & 4 \end{bmatrix}$$

Thus minimum of the function is at $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The value of multiplier is zero for the inequality constraint. That is, the inequality constraint is inactive at the optimum point. The MATLAB code *sqp.m* solves the constrained optimization problem using SQP method.