



Redes de primeira ordem

Experiência 7

Prof. Dr. Laisa Costa De Biase | Prof. Dr. Elisabete Galeazzo



Objetivos da Experiência

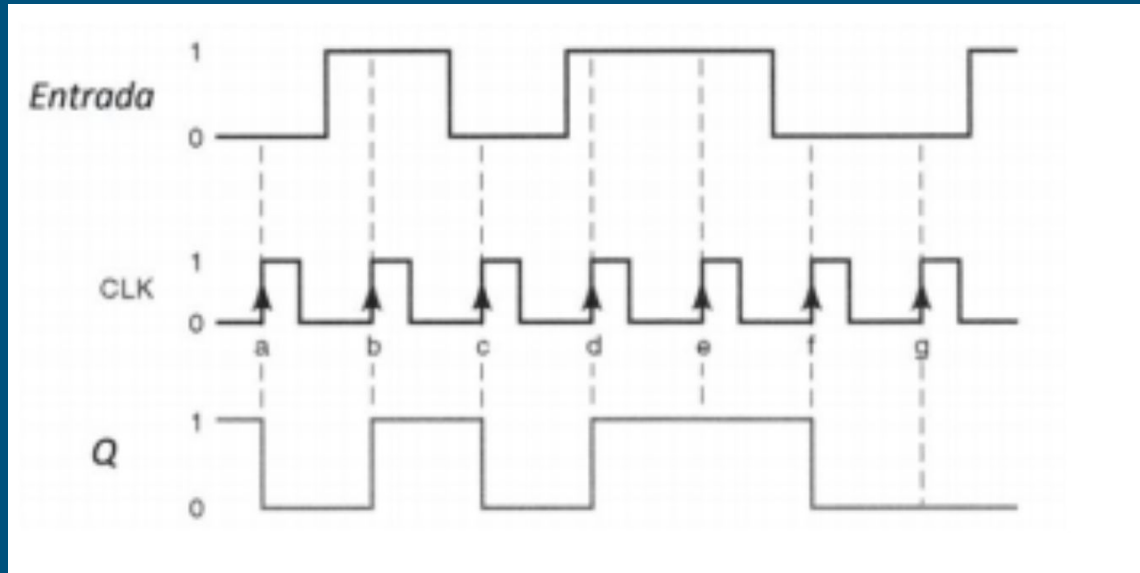
- Estudo no domínio do tempo de redes de 1ª ordem
- Tipos de resposta de redes de 1ª ordem
- Caracterização de redes de 1ª ordem



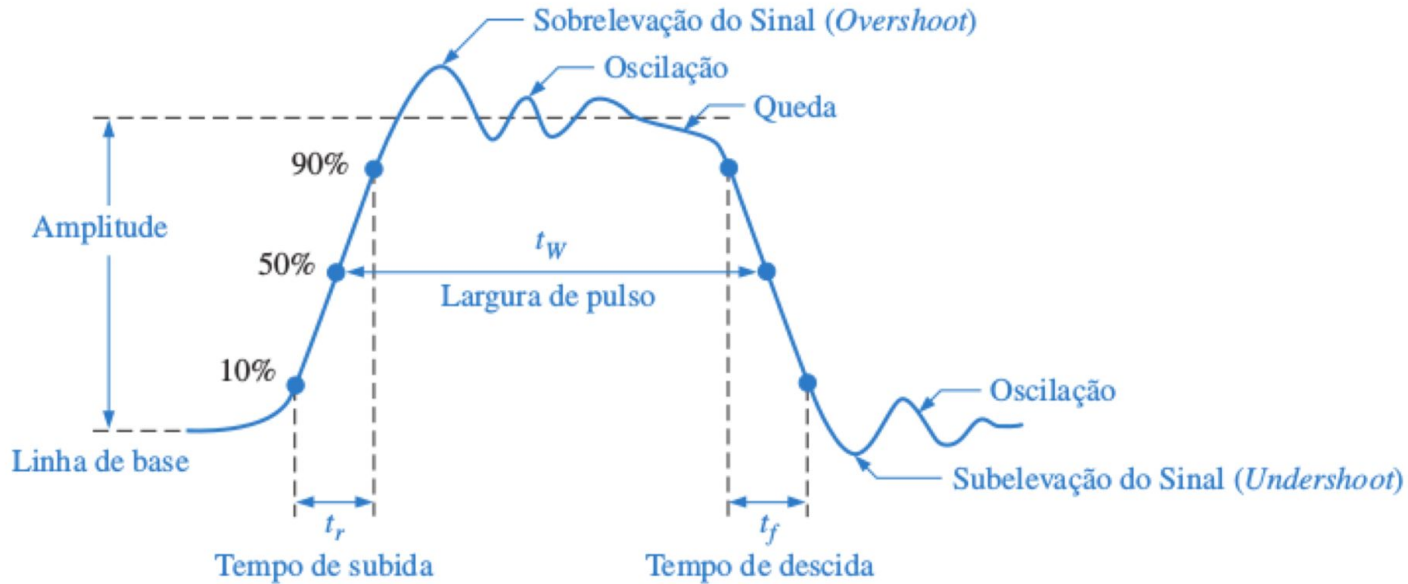
Funcionamento de circuito digital ideal



Funcionamento de circuito digital ideal

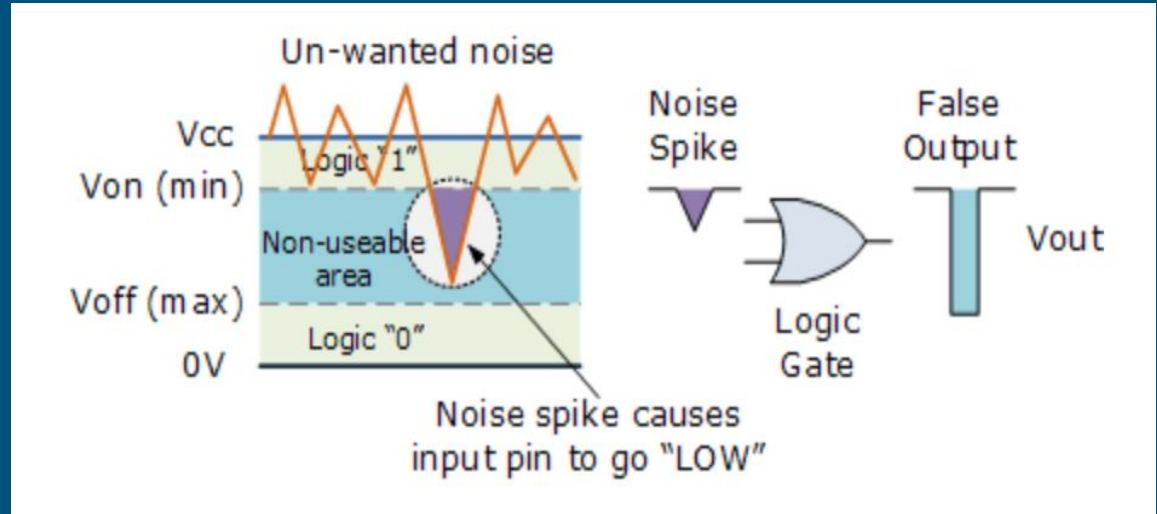
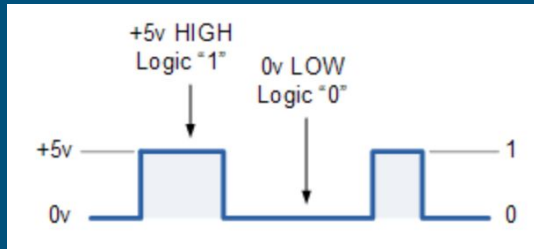


Sinal real



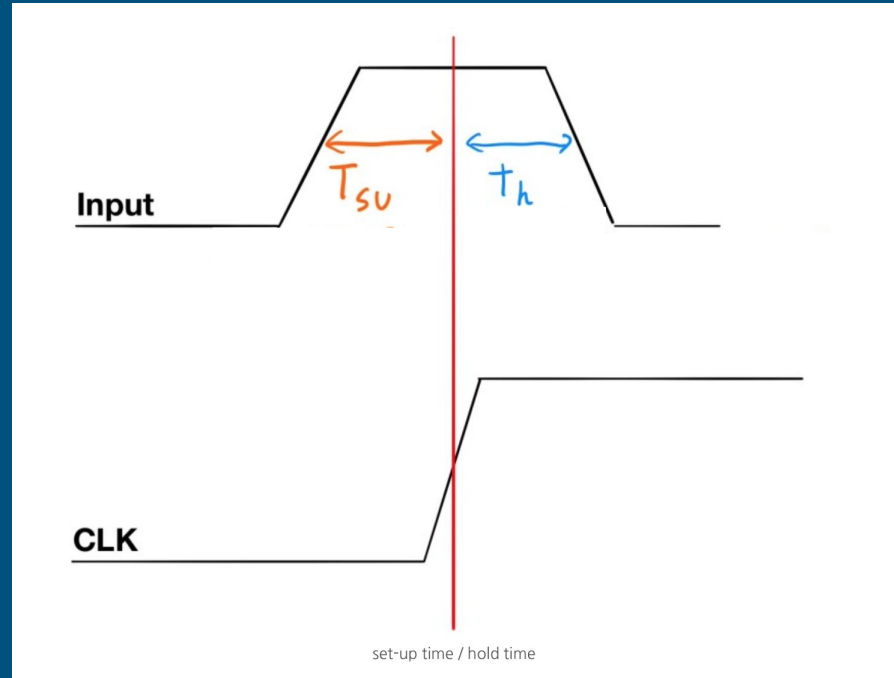
Efeitos de respostas oscilatórias

- Podem causar respostas "instáveis" em circuitos digitais

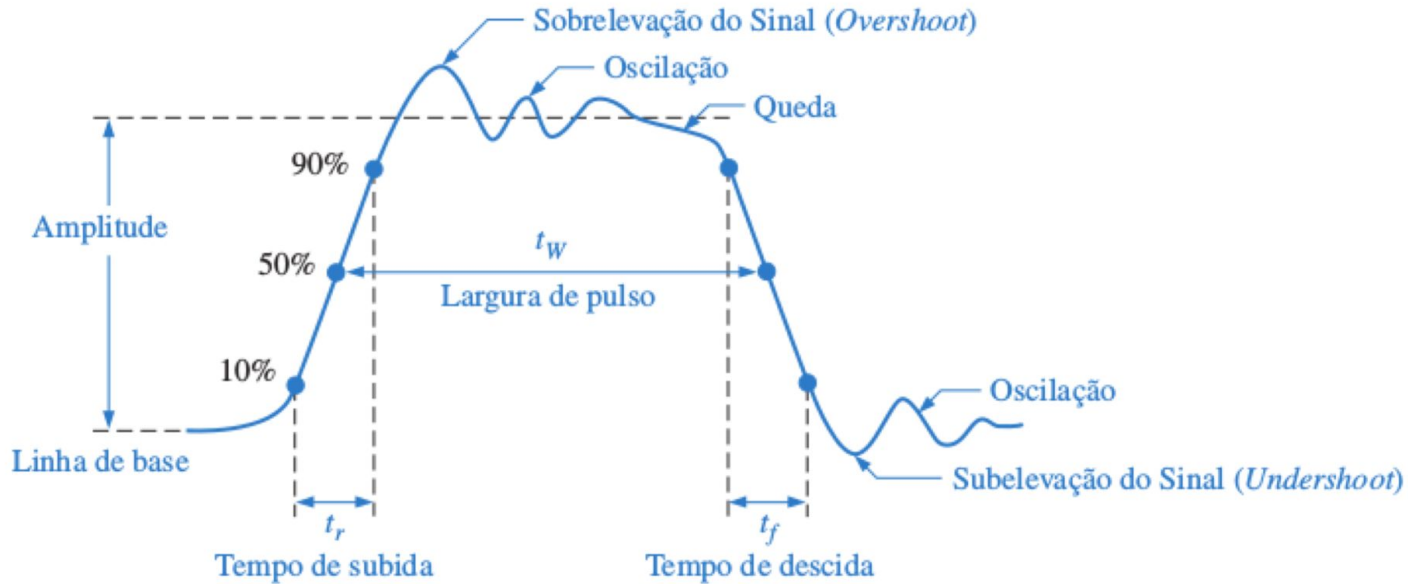


Efeito dos tempos de subida e de descida

- Podem causar frequência de operação mais baixa (tempo de resposta mais lento)



Sinal real





Resposta no tempo

$$\text{Resposta} = \text{Transitória} + \text{Permanente}$$

$\lim_{t \rightarrow \infty} = 0$



Resposta no tempo

$$\text{Resposta} = \underset{\lim_{t \rightarrow \infty} = 0}{\text{Transitória}} + \text{Permanente}$$

- Qual tipo de excitação poderia ser utilizada para apresentar o regime transitório e o permanente?

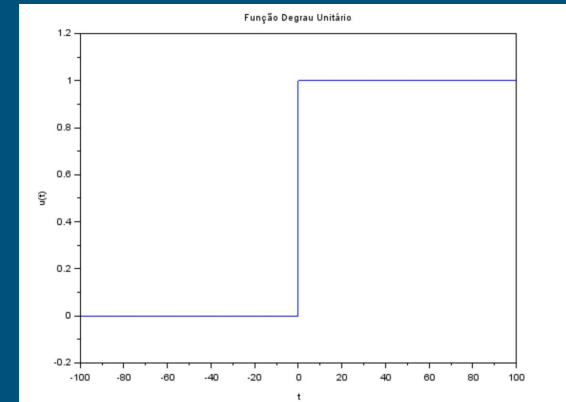


Resposta no tempo

$$\text{Resposta} = \text{Transitória} + \text{Permanente}$$

$\lim_{t \rightarrow \infty} = 0$

- Qual tipo de excitação poderia ser utilizada para apresentar o regime transitório e o permanente?



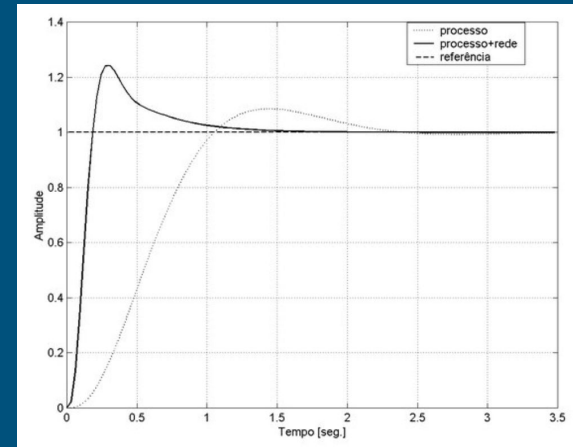


Resposta no tempo

$$\text{Resposta} = \text{Transitória} + \text{Permanente}$$

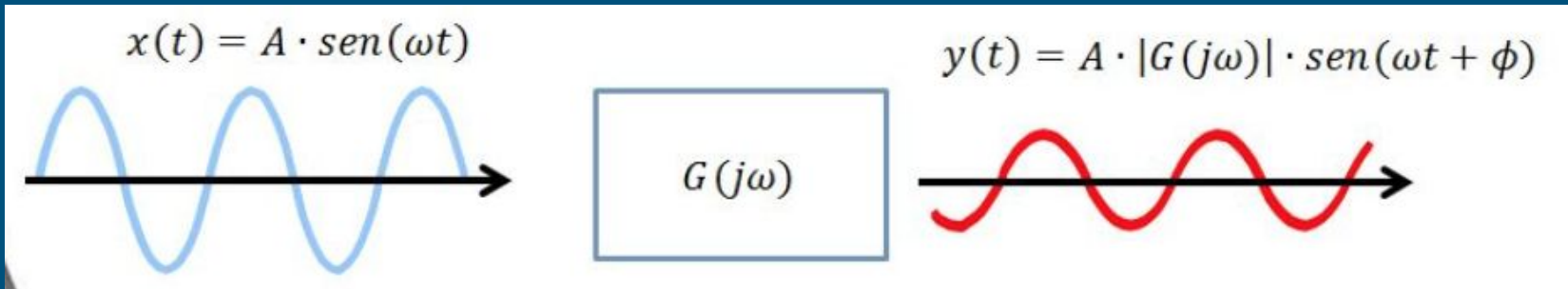
$\lim_{t \rightarrow \infty} = 0$

- Qual tipo de excitação poderia ser utilizada para apresentar o regime transitório e o permanente?



Experiência de resposta em frequência

- Estudamos os mesmos circuitos, mas com excitação por cossenoides de várias frequências





Caracterização de circuitos RC e RLC

Resposta em frequência

- Caracterização
 - Frequência de corte
 - Frequência de ressonância
 - Faixa de passagem (intervalo: de f_1 a f_2)
- Excitação
 - Ondas cossenoidais com várias frequências

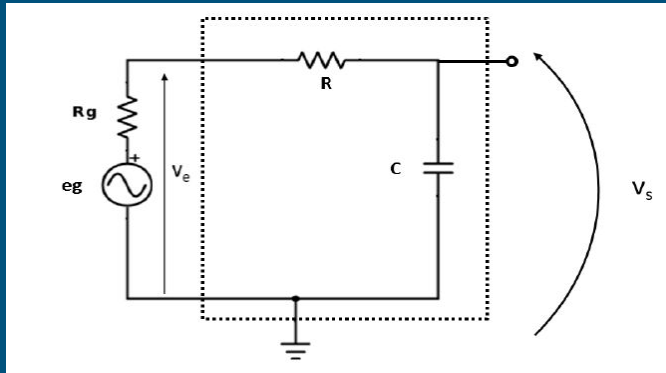
Resposta temporal

- Caracterização
 - Tempo de subida e de descida, constante de tempo
 - Oscilação -> apenas 2ª ordem
- Excitação
 - Degrau -> onda quadrada com período suficiente para atingir o regime permanente

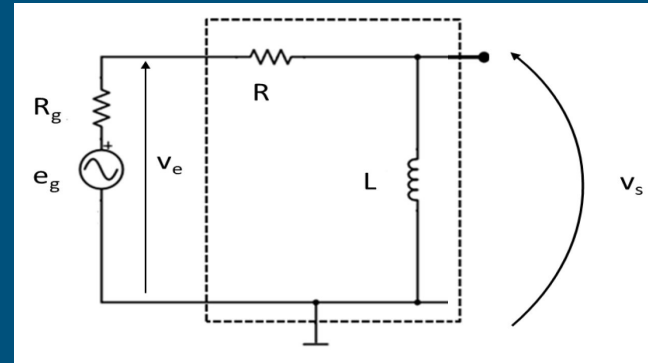
Redes de primeira ordem

1. Possui um único elemento armazenador de energia (capacitor ou indutor)
2. Descrito por uma equação diferencial de primeira ordem (primeira derivada)

Circuito RC

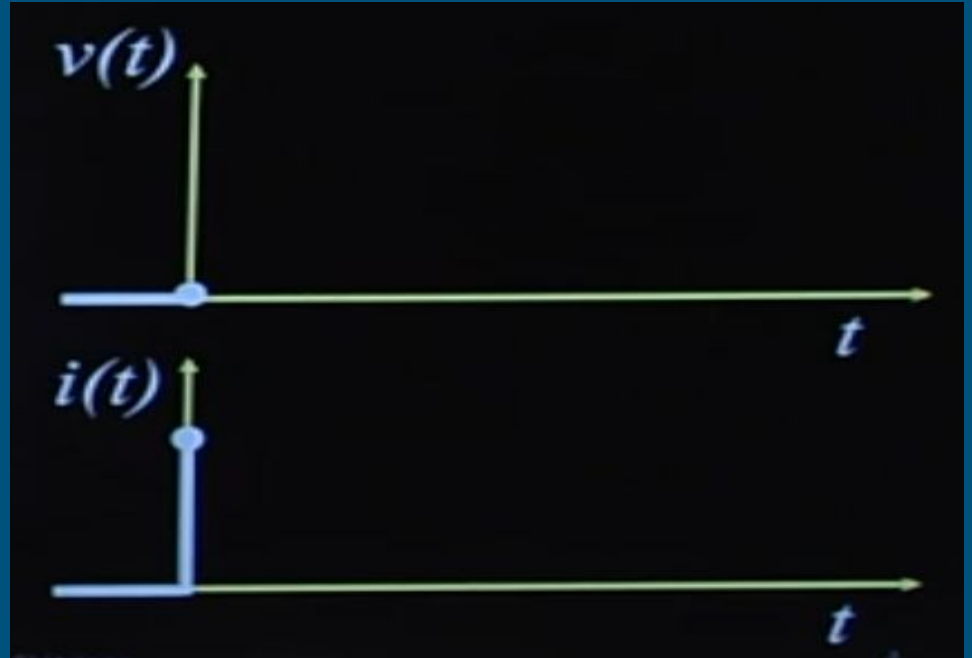
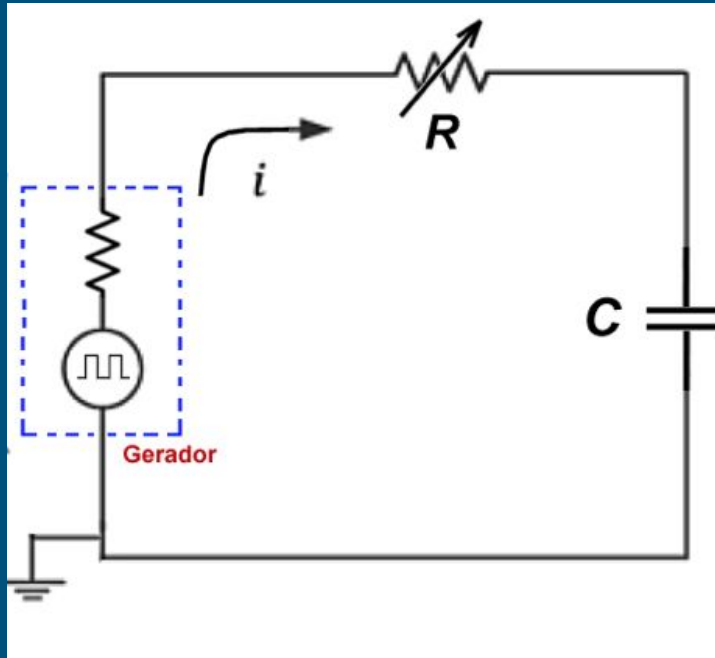


Circuito RL



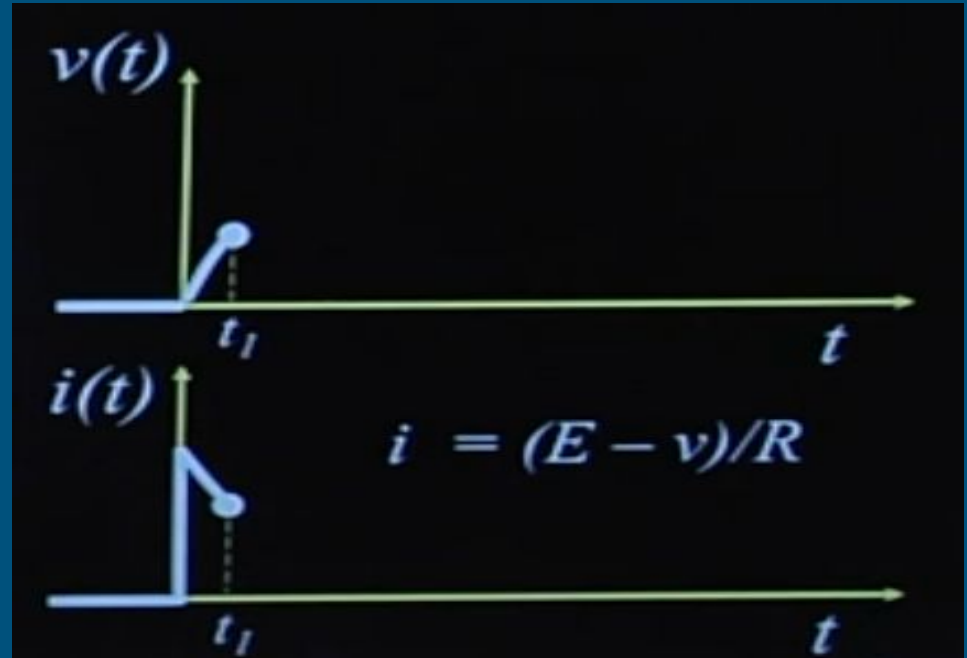
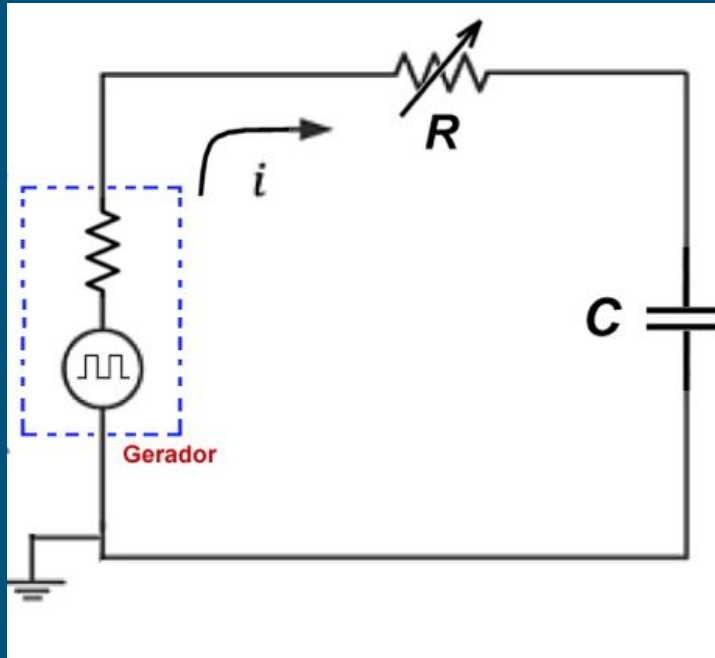
Circuito RC

Resposta no domínio do tempo



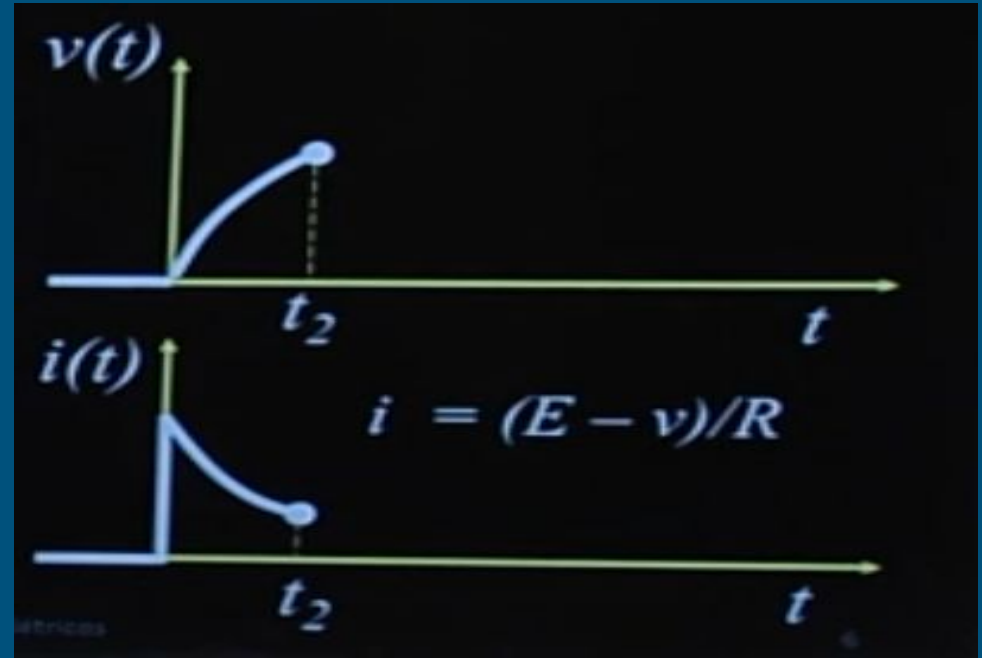
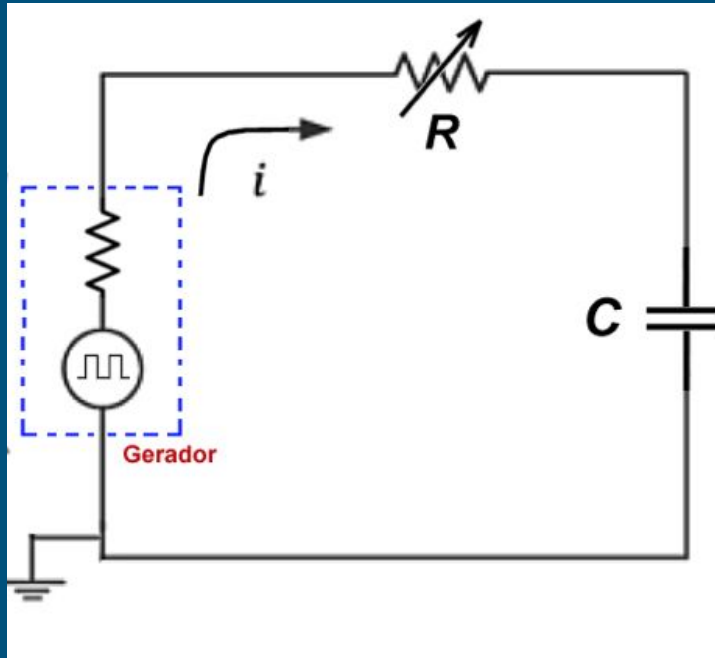
Circuito RC

Resposta no domínio do tempo



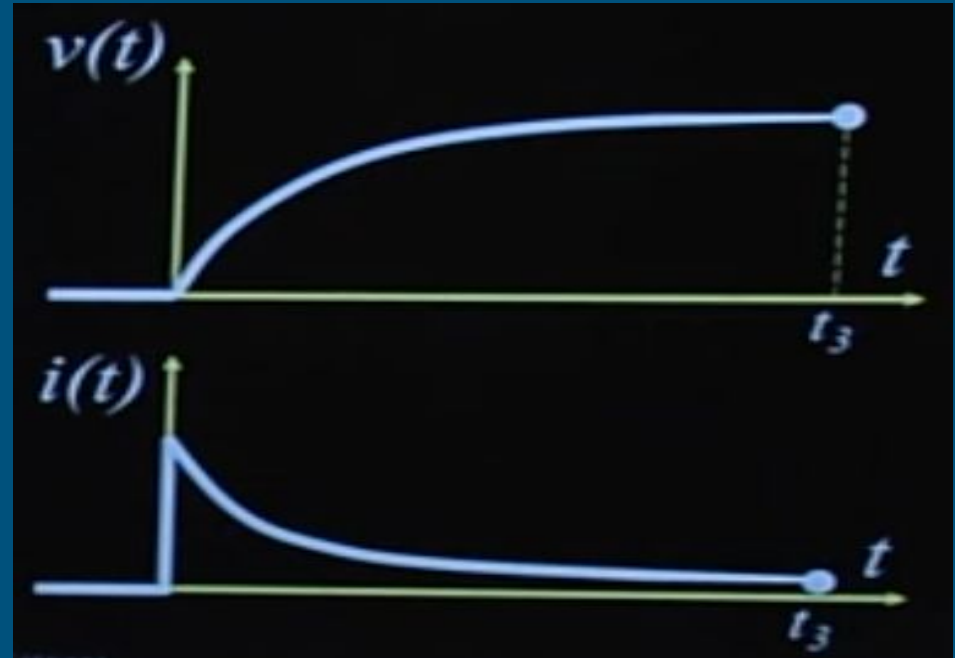
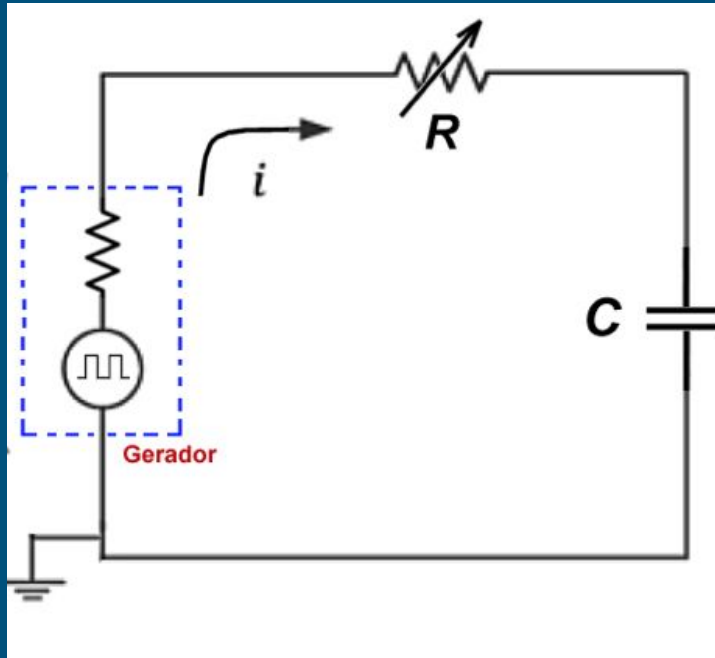
Circuito RC

Resposta no domínio do tempo



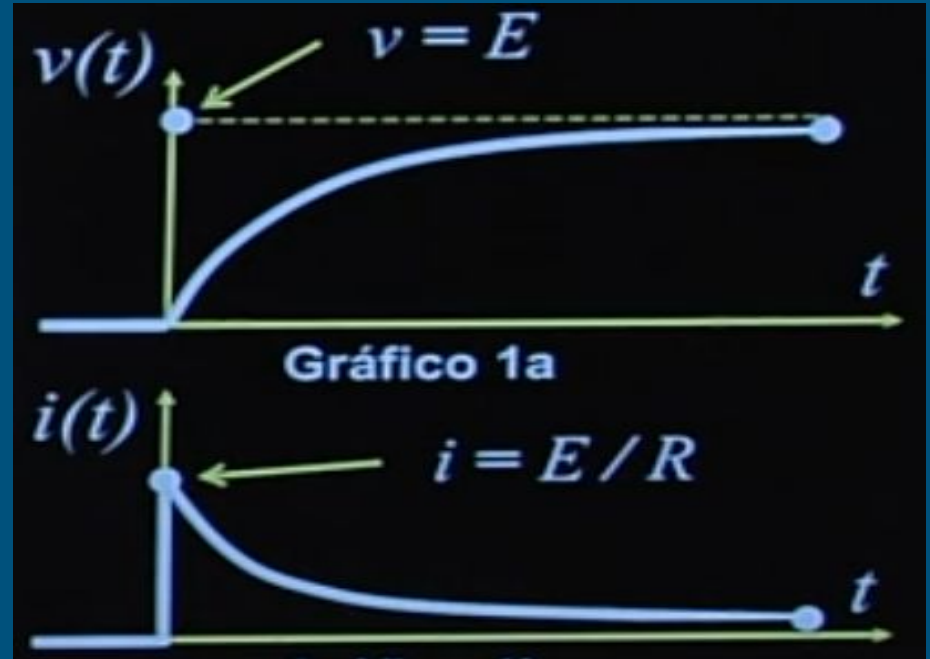
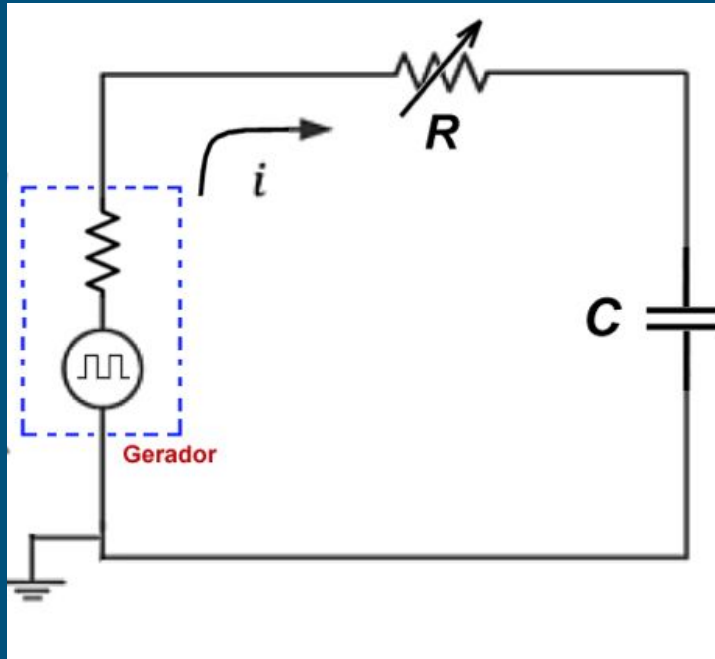
Circuito RC

Resposta no domínio do tempo



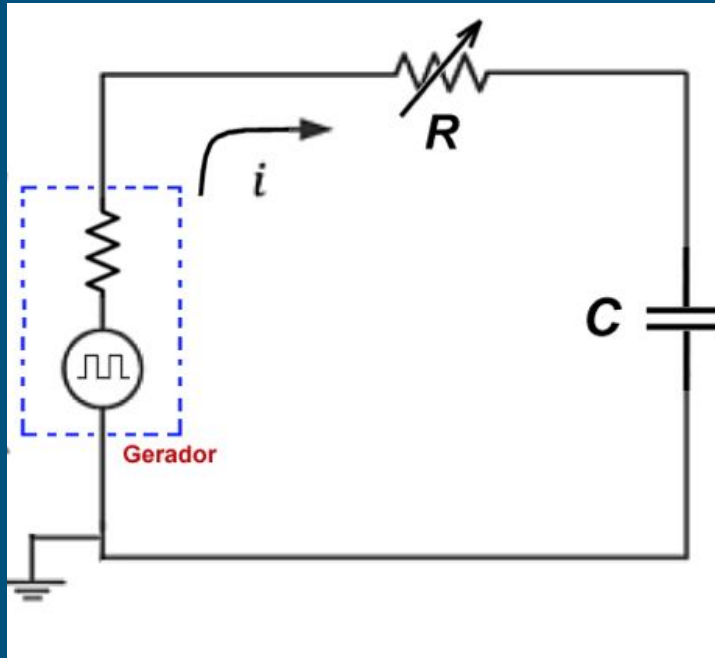
Circuito RC

Resposta no domínio do tempo

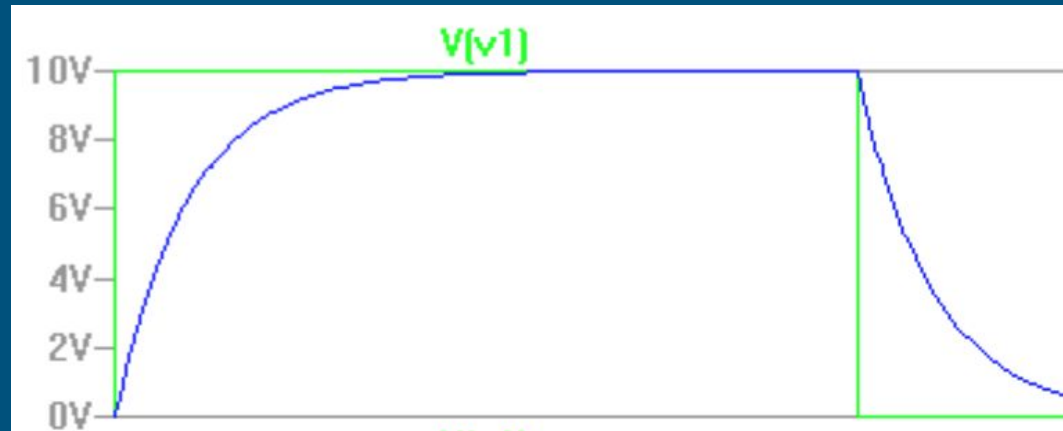


Circuito RC

Resposta no domínio do tempo



$$v_C(t) = E(1 - e^{-t/\tau}), \text{ onde } \tau = \mathbf{RC}.$$

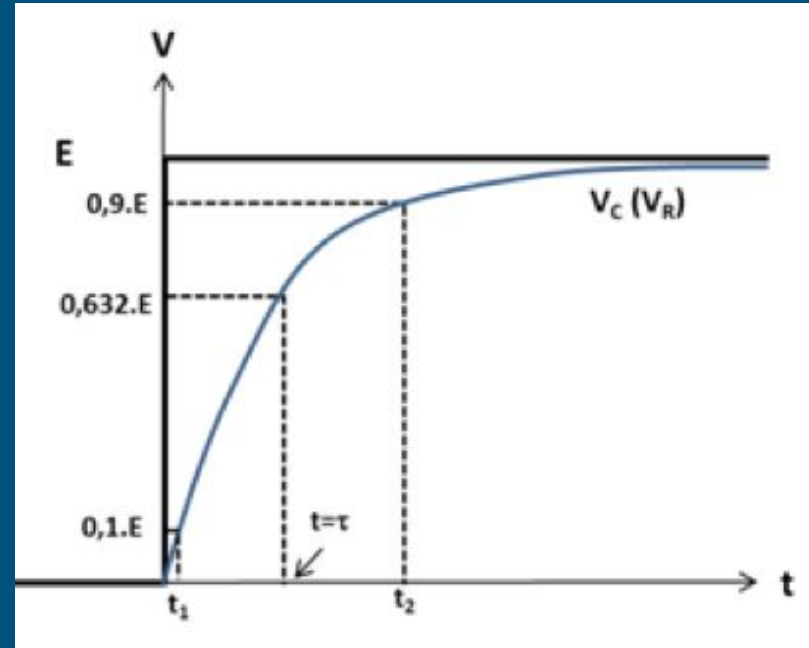


Caracterização dos circuitos RC

- Constante de tempo (τ)
 - Tempo para a extinção (parcial) do seu regime natural
 - Reduzidas a -37%

- Tempo de subida (t_r) e descida (t_f)
 - intervalo de tempo onde a tensão de saída encontra-se entre 10% (t_1) e 90% (t_2) do valor final,

$$t_r = t_2 - t_1$$





Caracterização dos circuitos

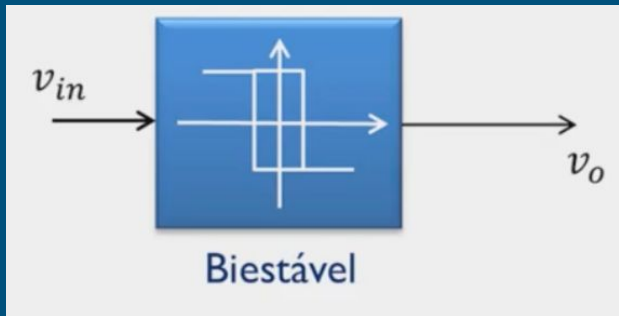
- Frequência de corte

$$f_c t_r = \frac{\ln(9)}{2\pi}$$

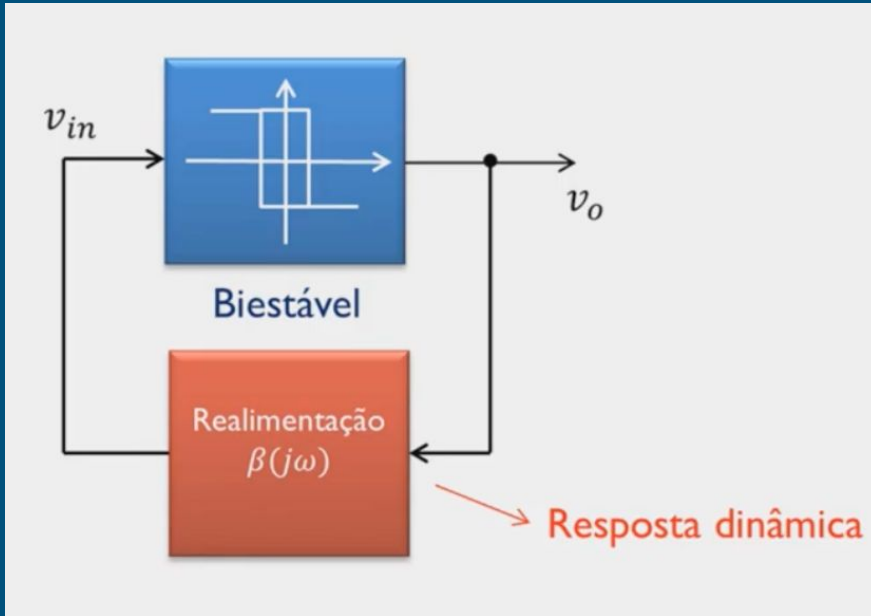
$$f_c = 1/(2\pi R.C)$$



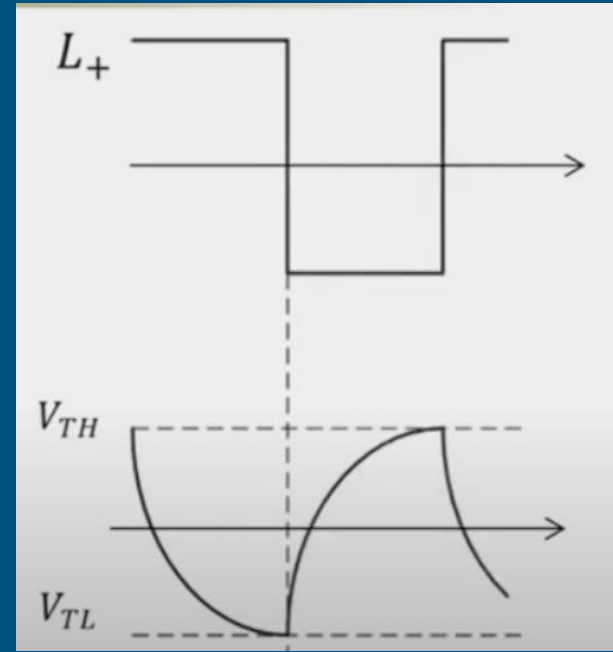
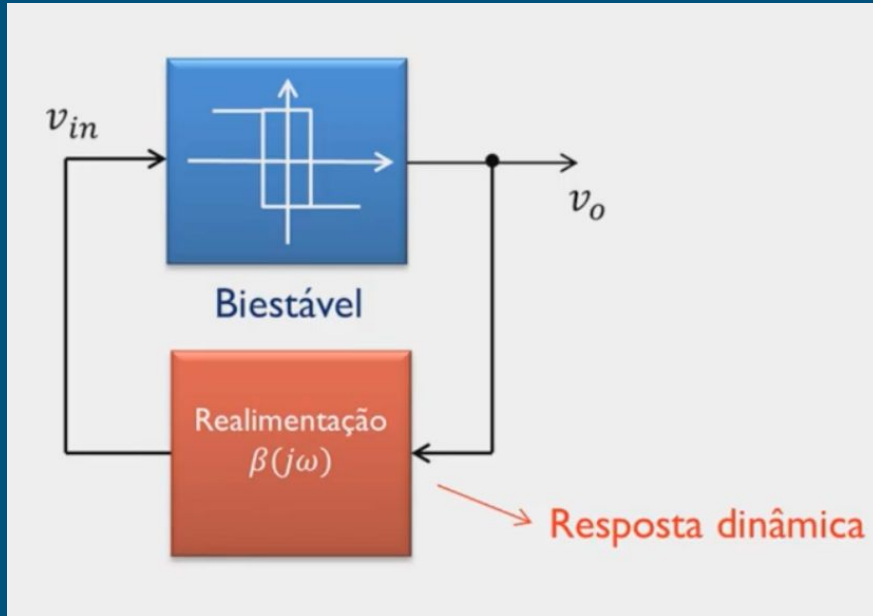
Aplicação: Oscilador



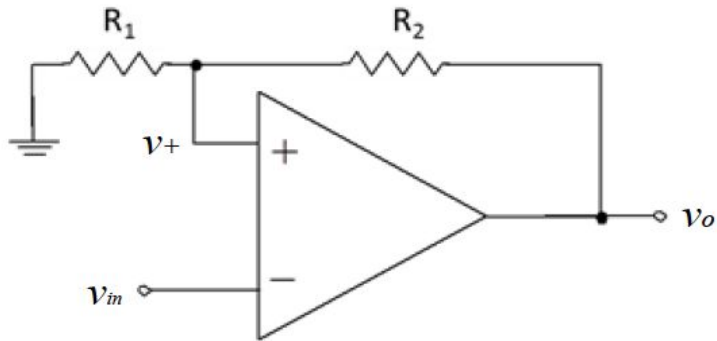
Aplicação: Oscilador



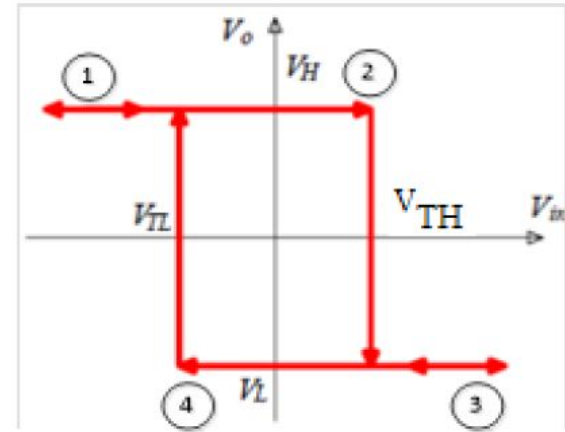
Aplicação: Oscilador



Amplificador Operacional como Comparador

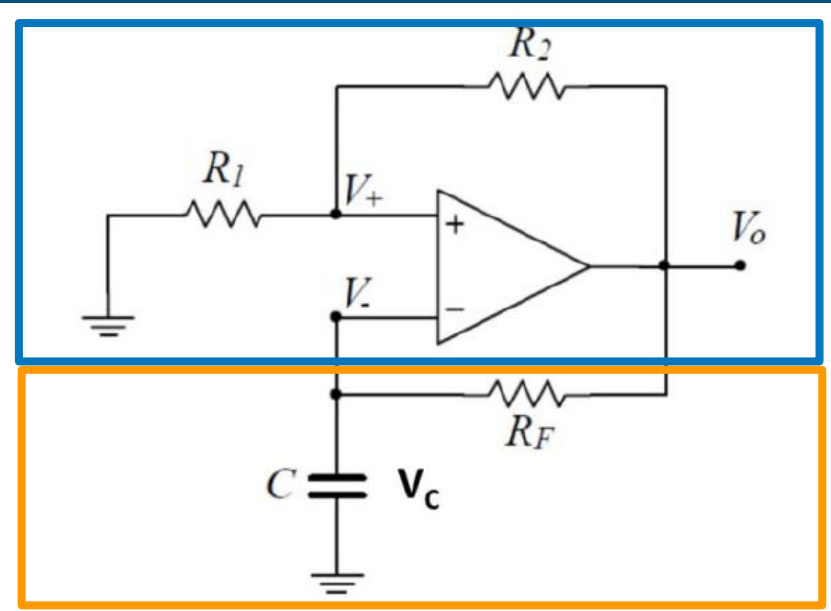
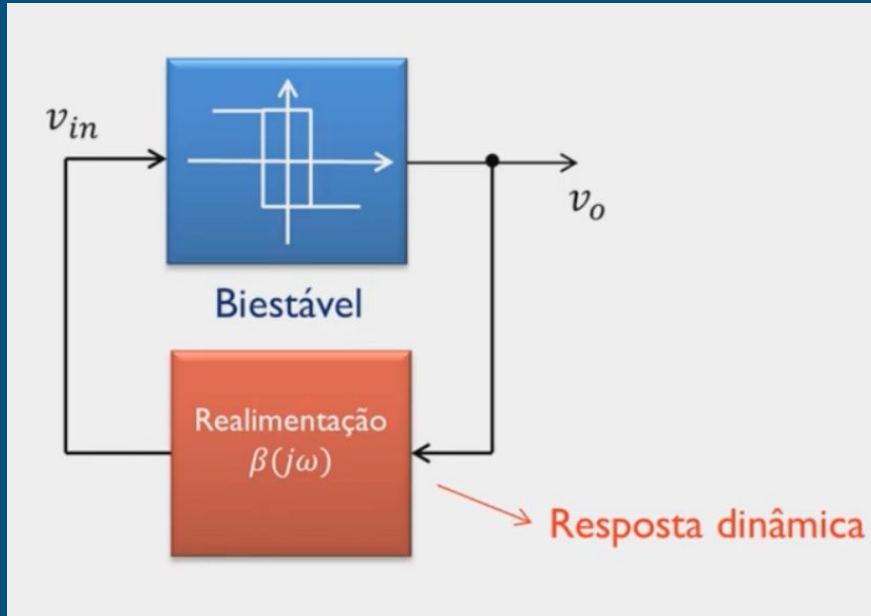


Circuito de um amplificador na configuração "comparador".

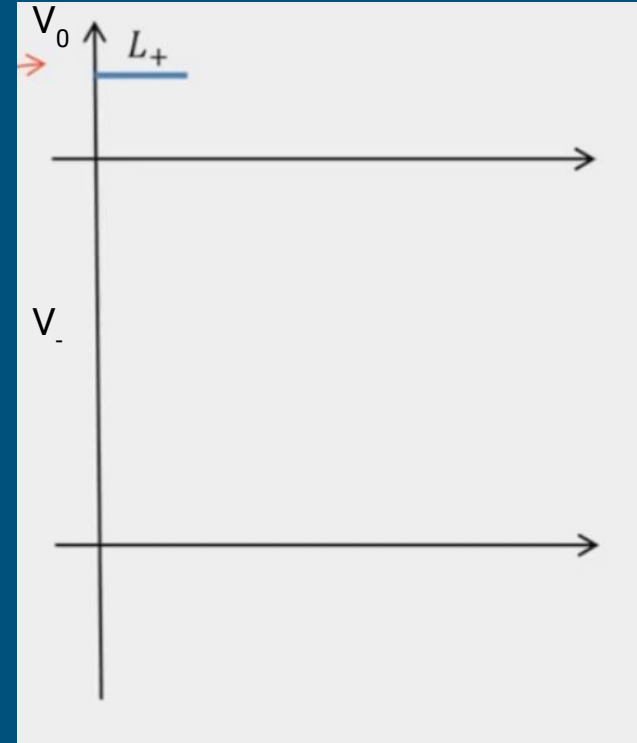
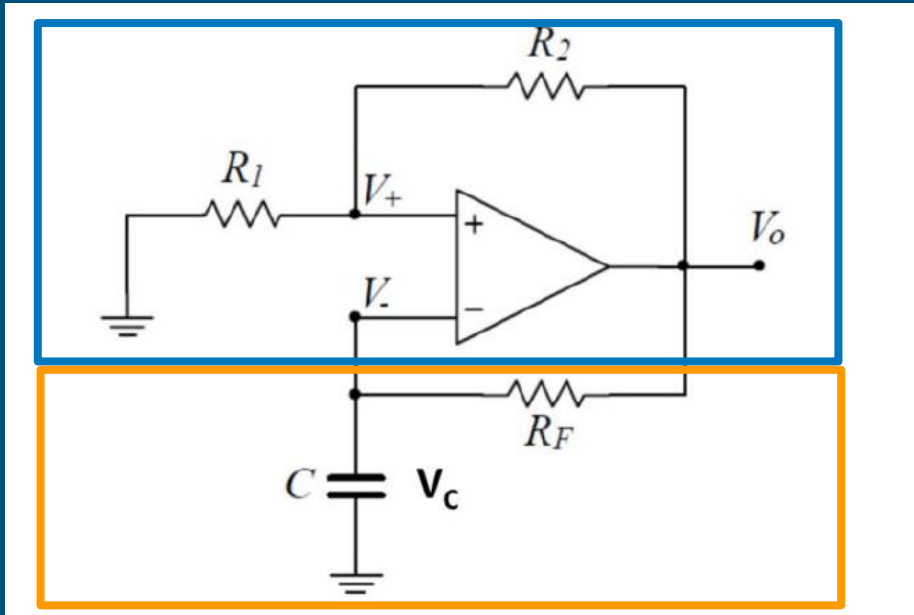


Curva de histerese do "comparador".

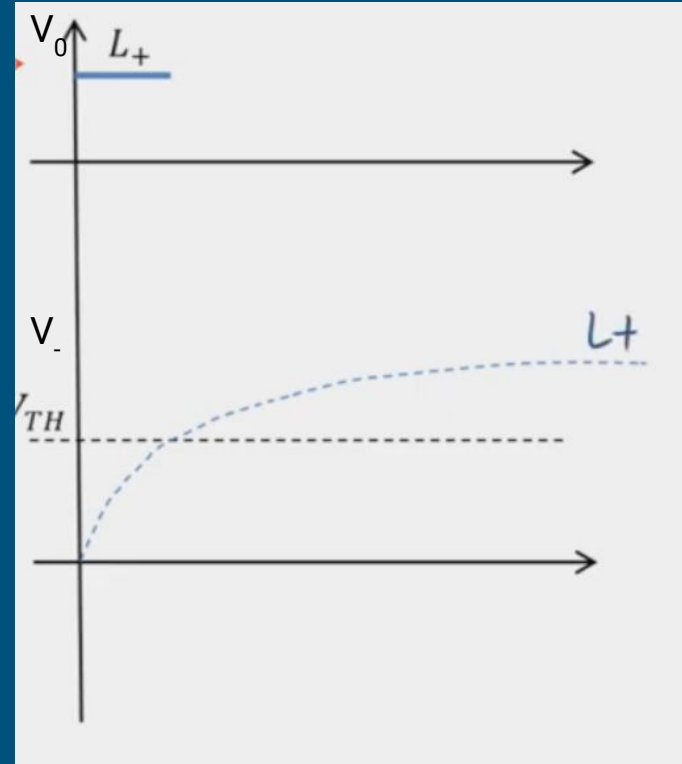
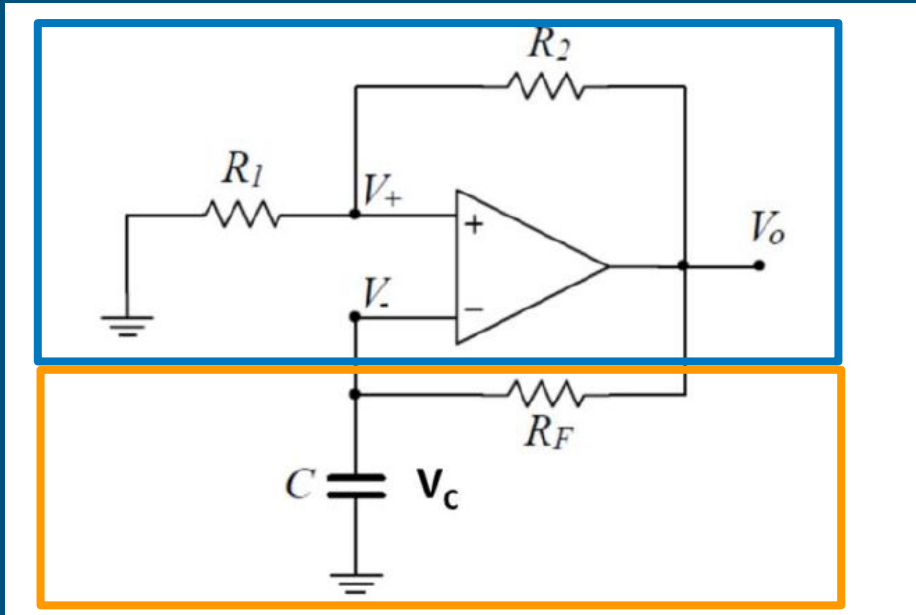
Oscilador com AmpOp + RC



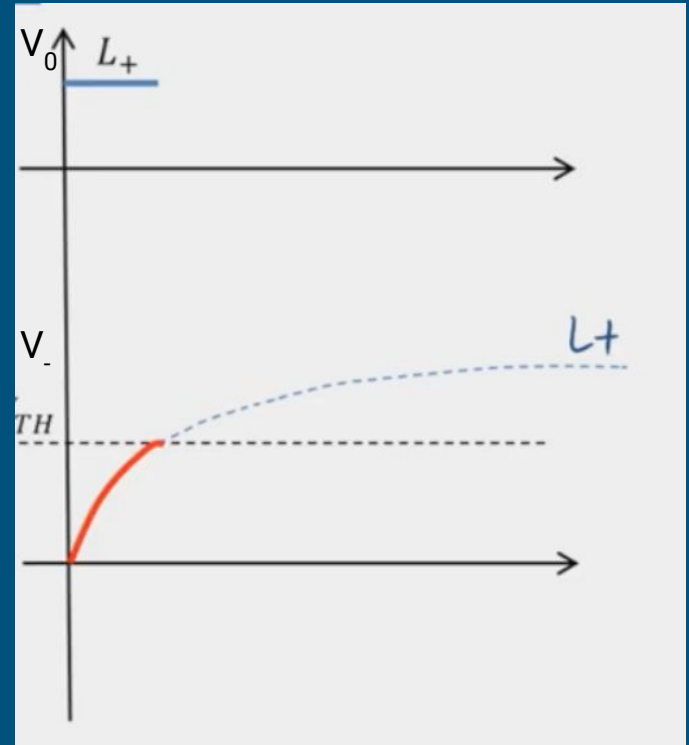
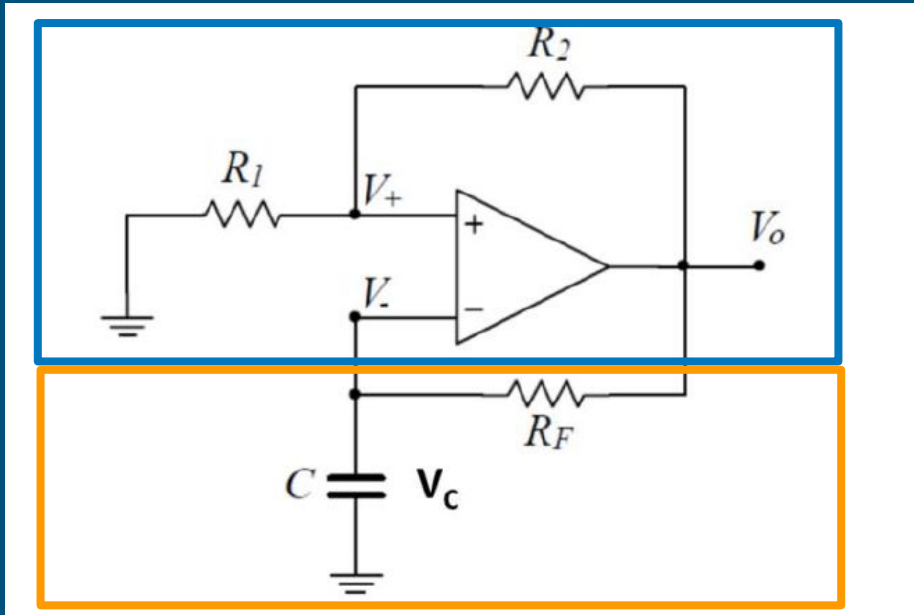
Aplicação: Oscilador



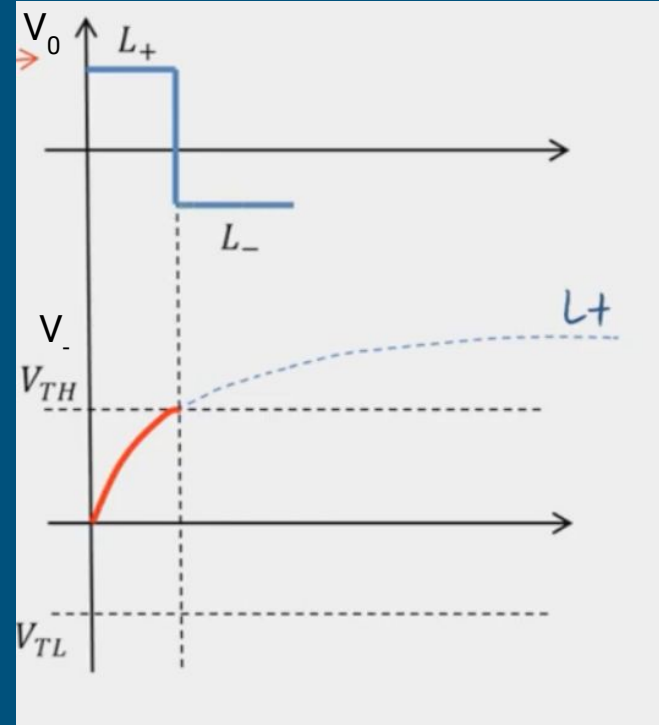
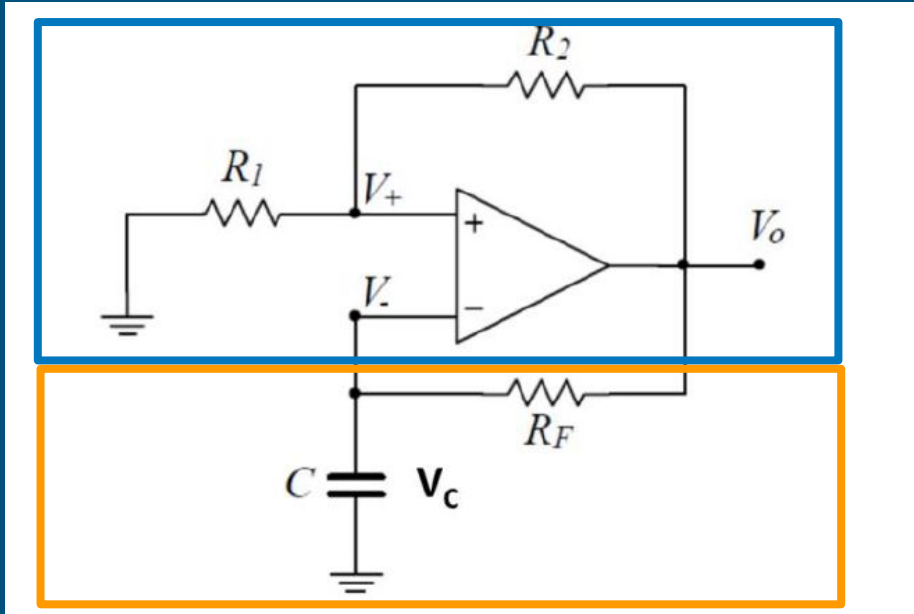
Aplicação: Oscilador



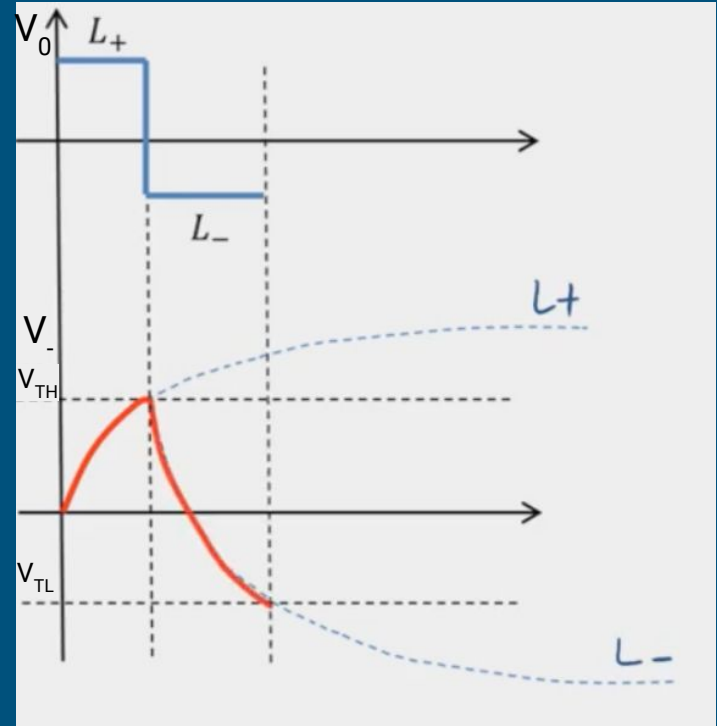
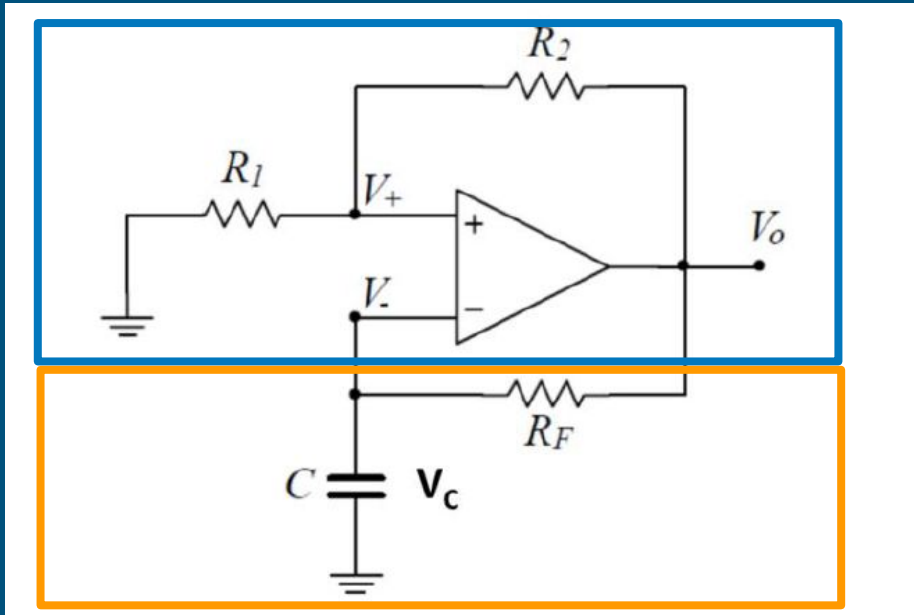
Aplicação: Oscilador



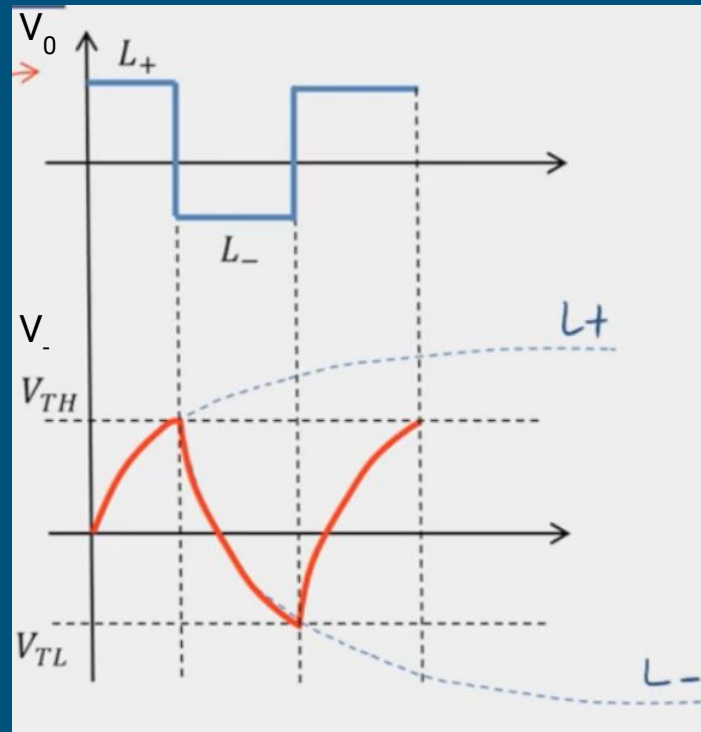
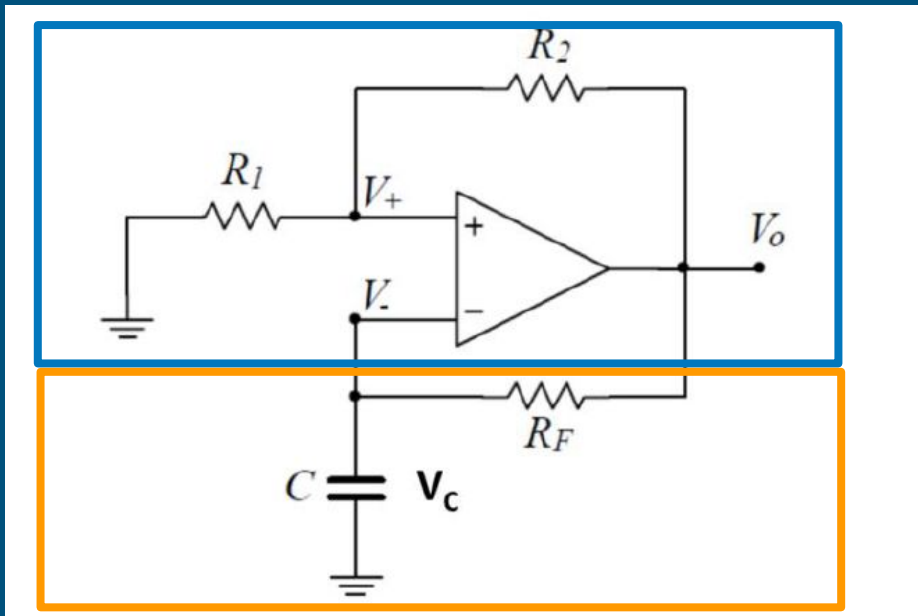
Aplicação: Oscilador



Aplicação: Oscilador



Aplicação: Oscilador





Análise teórica



Análise teórica

Solução
Completa

=

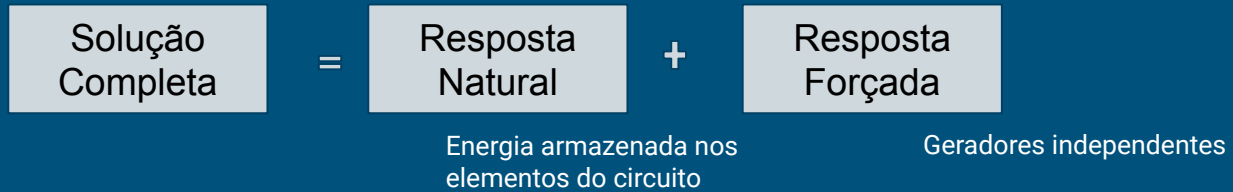
Resposta
Natural

+

Resposta
Forçada



Análise teórica





Análise teórica

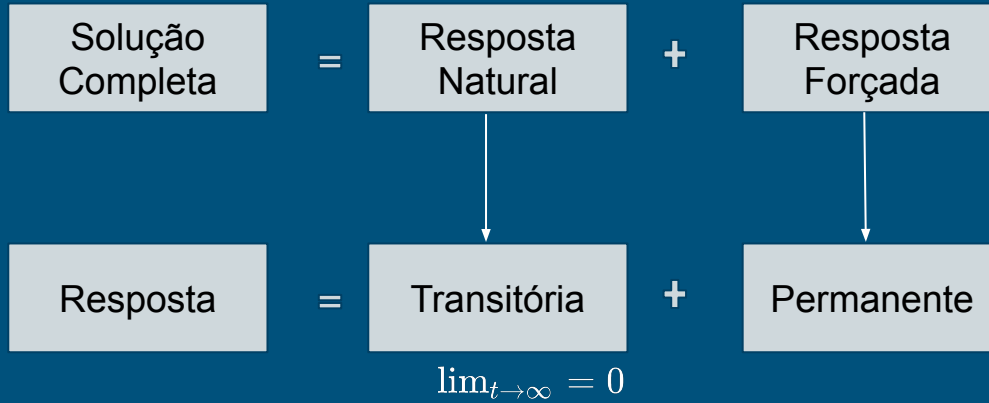
$$\text{Solução Completa} = \text{Resposta Natural} + \text{Resposta Forçada}$$

$$\text{Resposta} = \text{Transitória} + \text{Permanente}$$

$\lim_{t \rightarrow \infty} = 0$



Análise teórica





Resolução de equações diferenciais

#1 Aplicar 2a Lei de Kirchhoff

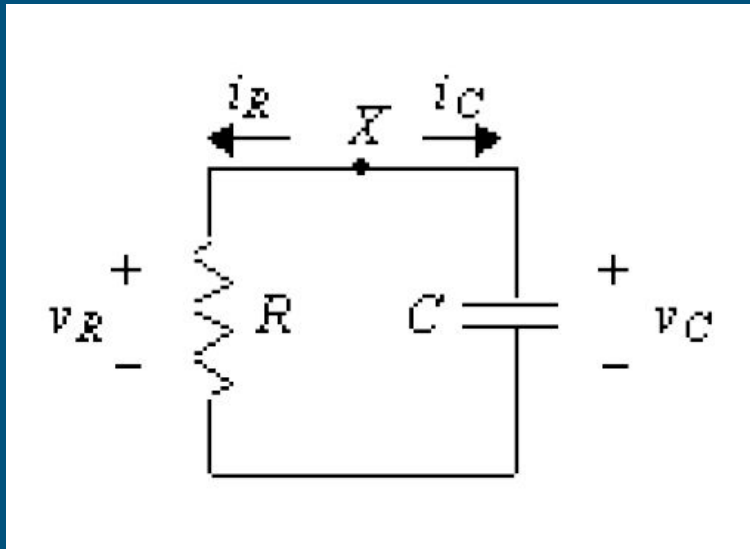
#2 Aplicar relação constitutiva dos elementos da rede

#3 Resolver Equação Diferencial Ordinária



Análise teórica do circuito RC

Resposta Natural do Circuito RC

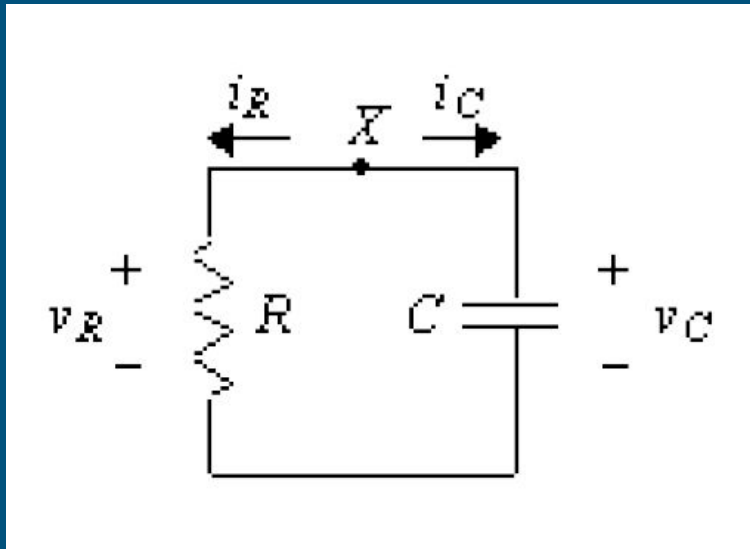


#1 Aplicar 2a Lei de Kirchhoff

#2 Aplicar relação constitutiva da capacitor

#3 Resolver Equação Diferencial Ordinária

Resposta Natural do Circuito RC



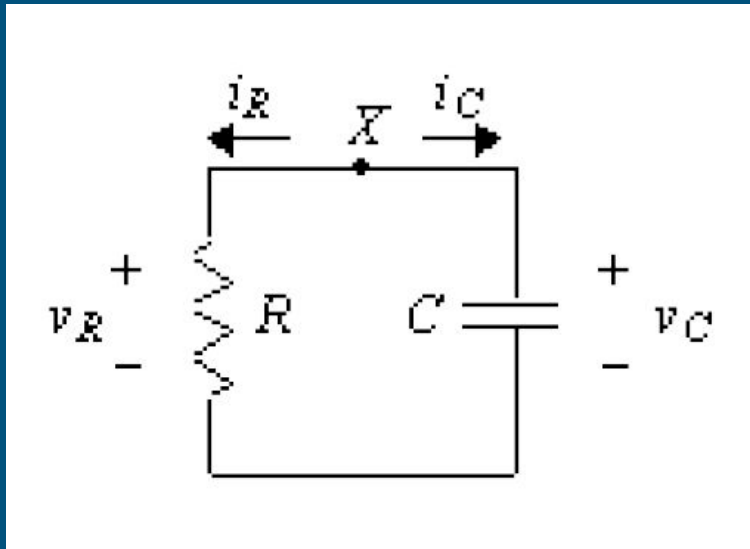
#1 Aplicar 2a Lei de Kirchhoff

$$R \cdot i(t) + v(t) = 0$$

#2 Aplicar relação constitutiva da capacitor

#3 Resolver Equação Diferencial Ordinária

Resposta Natural do Circuito RC



#1 Aplicar 2a Lei de Kirchhoff

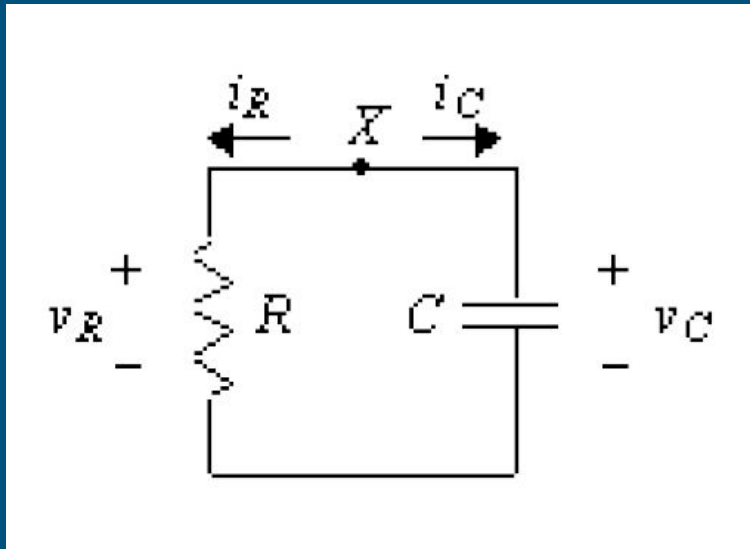
$$R \cdot i(t) + v(t) = 0$$

#2 Aplicar relação constitutiva da capacitor

$$i(t) = C \frac{dv(t)}{dt}$$

#3 Resolver Equação Diferencial Ordinária

Resposta Natural do Circuito RC



#1 Aplicar 2a Lei de Kirchhoff

$$R \cdot i(t) + v(t) = 0$$

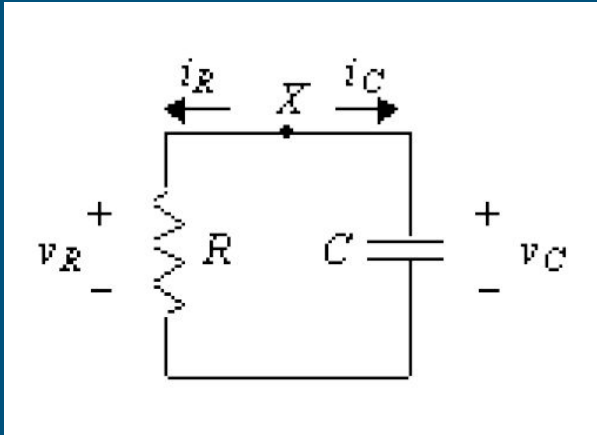
#2 Aplicar relação constitutiva da capacitor

$$i(t) = C \frac{dv(t)}{dt}$$

#3 Resolver Equação Diferencial Ordinária

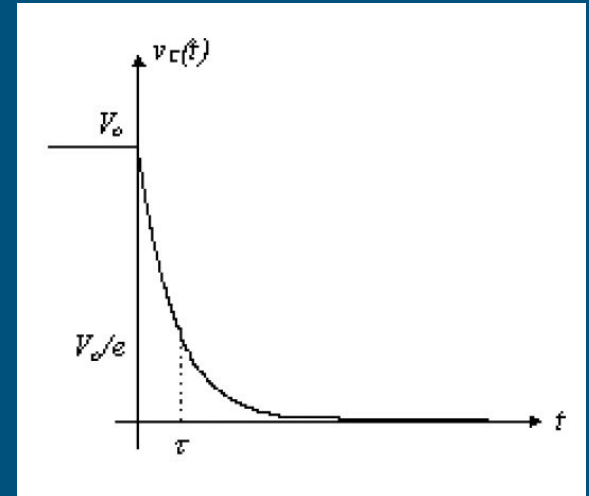
$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = 0$$

Resposta Natural do Circuito RC



$$v_C(t) = A e^{-\frac{t}{\tau}}$$

$$\tau = RC$$





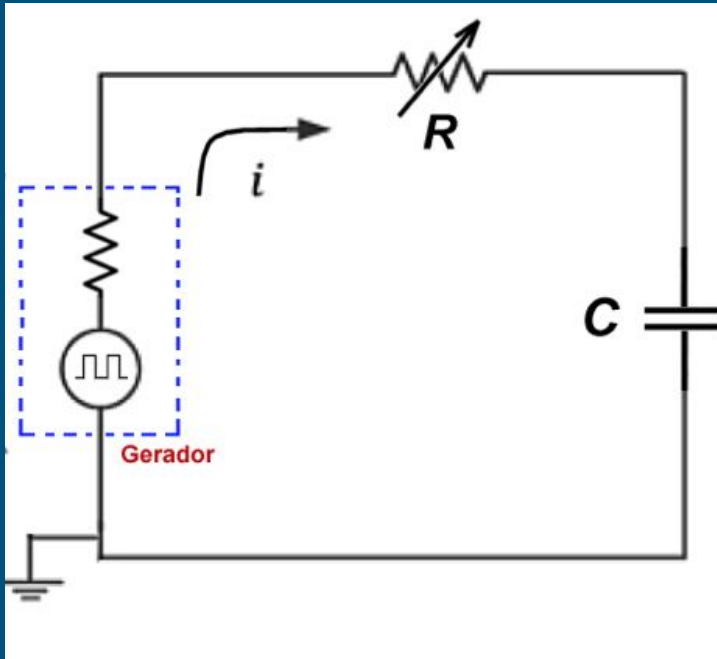
Solução teórica do Sistema de Primeira Ordem: Resposta forçada

$$\text{Solução Completa} = \text{Resposta Natural} + \text{Resposta Forçada}$$

Energia armazenada nos elementos do circuito

Geradores independentes

Resposta teórica do RC ao degrau

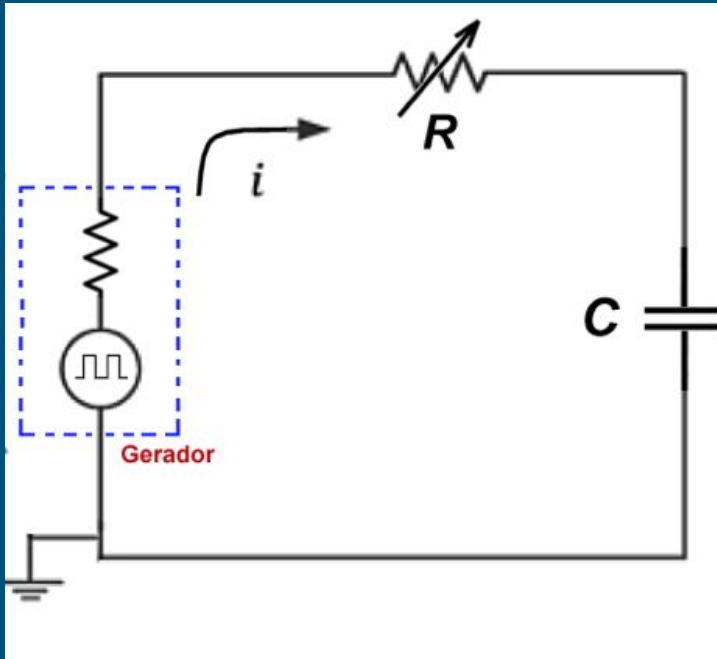


#1 Aplicar 2a Lei de Kirchhoff

#2 Aplicar relação constitutiva da capacitor

#3 Resolver Equação Diferencial Ordinária

Resposta teórica do RC ao degrau



#1 Aplicar 2a Lei de Kirchhoff

$$R \cdot i(t) + v(t) = E$$

#2 Aplicar relação constitutiva da capacitor

$$v(t) = \frac{1}{C} \int i(t) dt$$

#3 Resolver Equação Diferencial Ordinária

$$\frac{di(t)}{dt} + \frac{1}{RC} i(t) = 0$$

Resolvendo a EDO

$$\frac{di(t)}{dt} + \frac{1}{\tau} i(t) = 0$$

$$i(t) = Ae^{pt}$$

$$\frac{d}{dt} (Ae^{pt}) + \frac{A}{\tau} e^{pt} = 0$$

$$Ae^{pt} \left(p + \frac{1}{\tau} \right) = 0$$

$$p = -\frac{1}{\tau}$$

$$A = \frac{E}{R} \quad (\text{C.I.})$$



Resolvendo a EDO

Obtemos:

$$i(t) = \frac{E}{R} e^{-t/\tau}$$

Mas sabemos que:

$$v(t) = \frac{1}{C} \int i(t) dt$$

$$v(t) = \frac{1}{C} \int \frac{E}{R} e^{-t/\tau} dt$$

$$v(t) = \frac{1}{RC} (-\tau) (e^{-t/\tau}) \Big|_0^t$$

Finalmente:

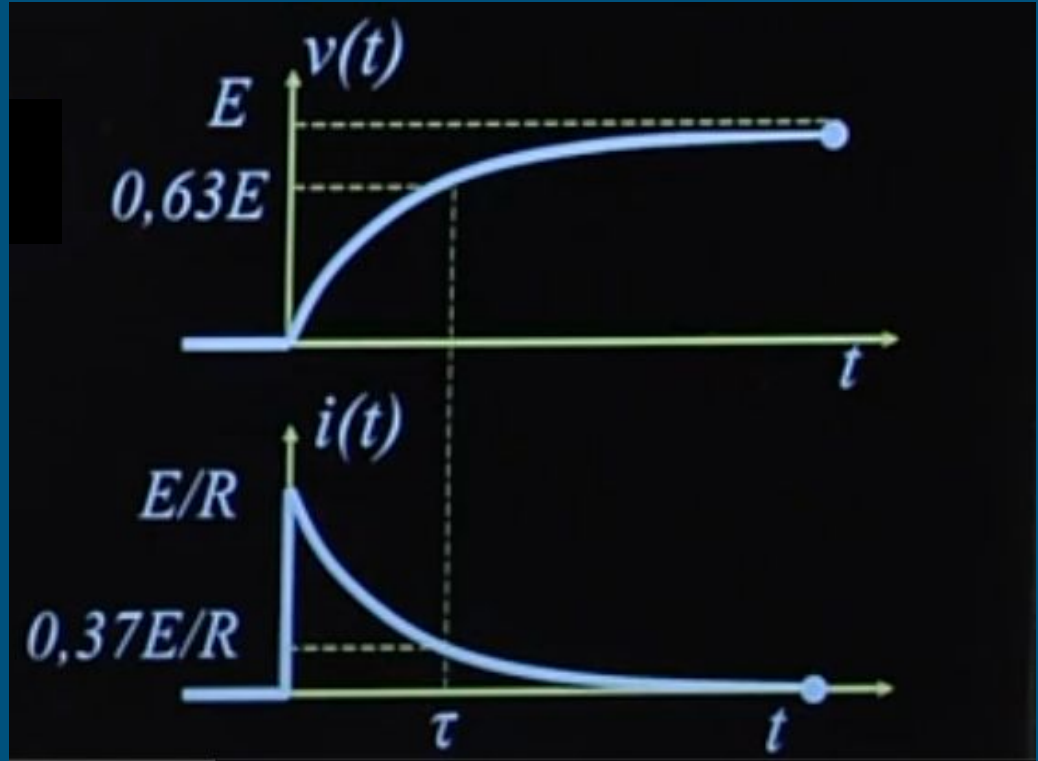
$$v(t) = E(1 - e^{-t/\tau})$$

Resposta teórica do RC ao degrau

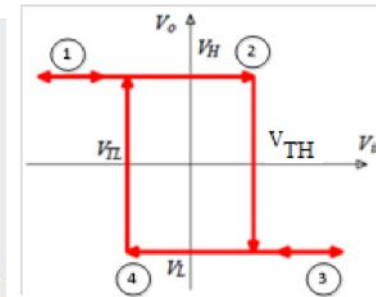
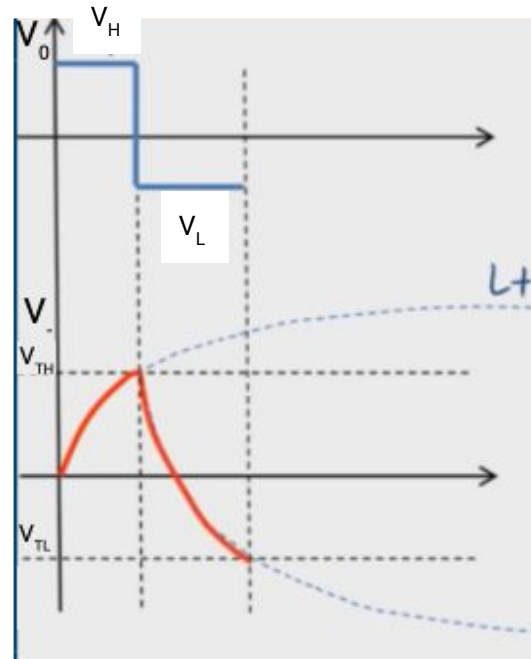
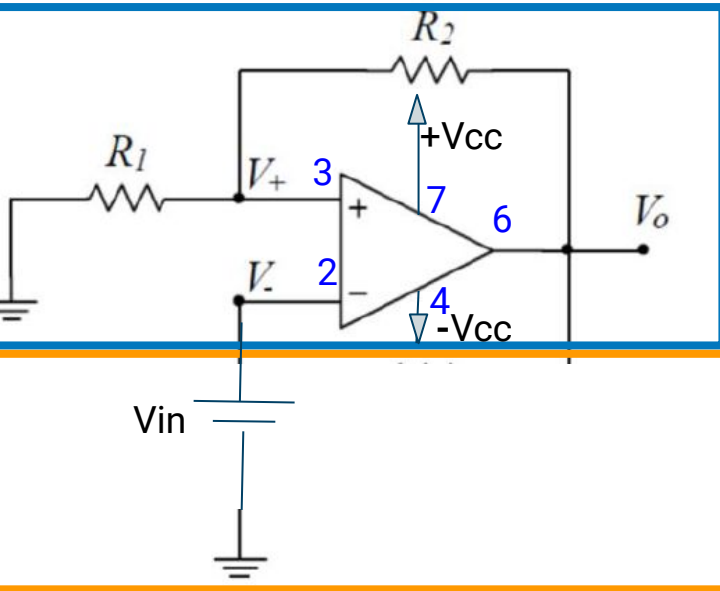
$$v(t) = E(1 - e^{-\frac{t}{\tau}})$$

$$i(t) = \frac{E}{R}e^{-\frac{t}{\tau}}$$

$$e^{-1} = 0,37$$



Experiência - Parte 2 Gerador de onda quadrada com circuito RC e AmpOp

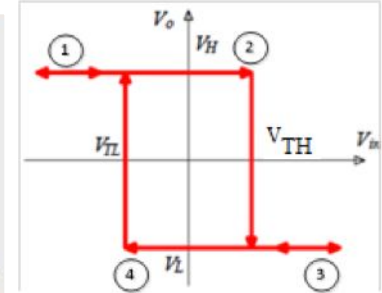
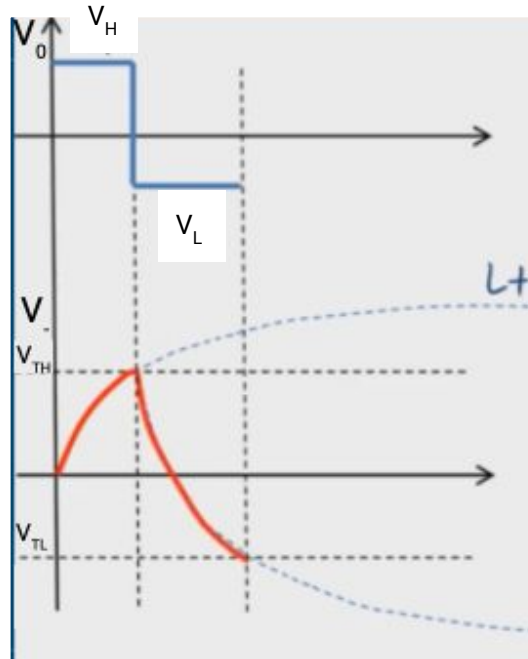
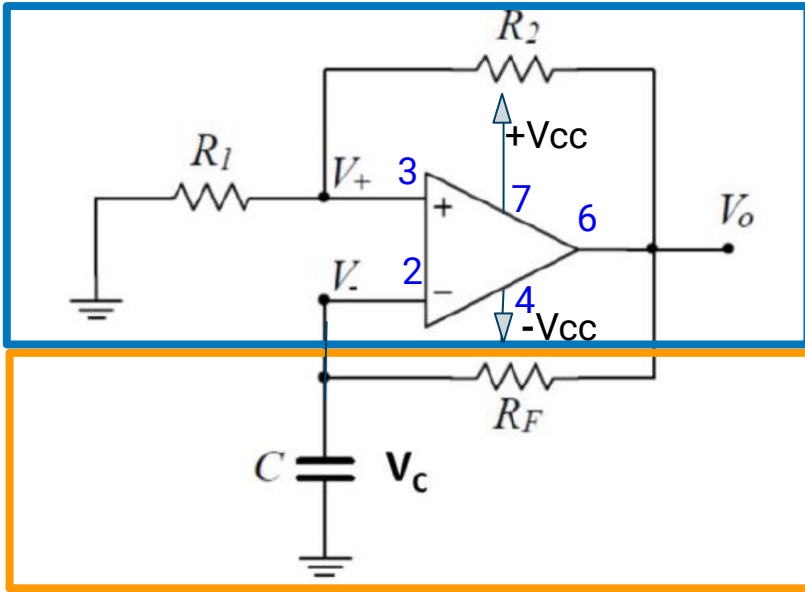


$$V_{+} = \frac{R_1}{R_1 + R_2} V_0$$

$$V_{in} = V_{TH} = \frac{R_1}{R_1 + R_2} V_H$$

$$V_{in} = V_{TL} = \frac{R_1}{R_1 + R_2} V_L$$

Experiência - Parte 2 Gerador de onda quadrada com circuito RC e AmpOp



$$V_+ = \frac{R_1}{R_1 + R_2} V_0$$

$$V_{in} = V_{TH} = \frac{R_1}{R_1 + R_2} V_H$$

$$V_{in} = V_{TL} = \frac{R_1}{R_1 + R_2} V_L$$