Introdução à Física das Partículas Elementares

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(buscar: física das partículas elementares)

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Plano do Curso

14/03	Cap. 1	25/04	Cap. 4	25/05	Cap. 9
16/03	Cap. 1	27/04	Cap. 5	30/05	Cap. 9
21/03	Cap. 2	02/05	Cap. 6	01/06	Cap. 9
23/03	Cap. 2	04/05	Cap. 6	06/06	
28/03	Cap. 3	09/05	Cap. 7	08/06	
30/03	Cap. 3	11/05	Cap. 7	13/06	Cap. 10
04/04		16/05	Cap. 8	15/06	Cap. 10
06/04		18/05	Cap. 8	20/06	Cap. 10
11/04	Cap. 4	23/05	P2	22/06	Cap. 11
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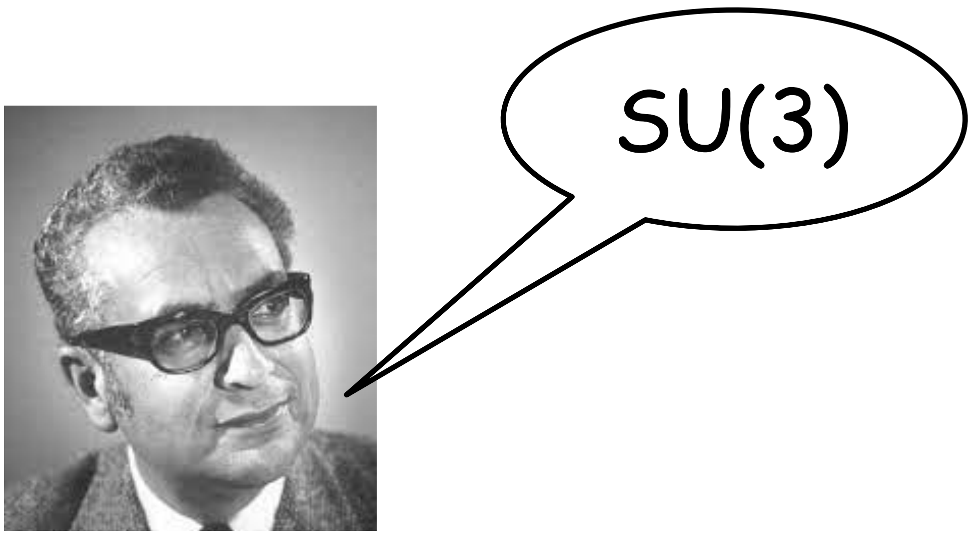
Aula 20

Capítulo 9

Simetrias e o Modelo de Quarks



Murray Gell-Mann

SU(3) de sabor

Vamos estender SU(2) para incluir o quark estranho s

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \mathrm{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathrm{d} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \mathrm{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Para transformar o quark q precisamos de matrizes unitárias 3 X 3 :

$$q' = Uq \qquad \begin{pmatrix} \mathrm{u}' \\ \mathrm{d}' \\ \mathrm{s}' \end{pmatrix} = \hat{U} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix}$$

Elas formam o grupo SU(3) com elementos:

$$\hat{U} = e^{i\boldsymbol{\alpha}\cdot\hat{\mathbf{T}}}$$

Geradores : $\hat{\mathbf{T}} = \frac{1}{2}\lambda$

Matrizes de Gell-Mann

SU(2)

$$\lambda_1 = \begin{pmatrix} \boxed{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} \boxed{\begin{matrix} 0 & -i \\ i & 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \longrightarrow$$

Diagonais e comutam

Hermitianas com traço nulo

Componente I_3 do Isospin :

$$\hat{T}_3 = \tfrac{1}{2}\lambda_3 \quad \begin{cases} \hat{T}_3 \mathrm{u} = +\tfrac{1}{2}\mathrm{u} \\ \hat{T}_3 \mathrm{d} = -\tfrac{1}{2}\mathrm{d} \\ \hat{T}_3 \mathrm{s} = 0 \end{cases}$$

Isospin total :

$$\hat{T}^2 = \sum_{i=1}^{8} \hat{T}_i^2 = \tfrac{1}{4}\sum_{i=1}^{8} \lambda_i^2 = \frac{4}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{aligned} T_3 &= \tfrac{1}{2}\lambda_3 \\ T_8 &= \tfrac{1}{2}\lambda_8 \end{aligned}\right\}$$ Comutam e descrevem duas grandezas compatíveis

$$\boxed{\hat{T}_3 = \tfrac{1}{2}\lambda_3}$$

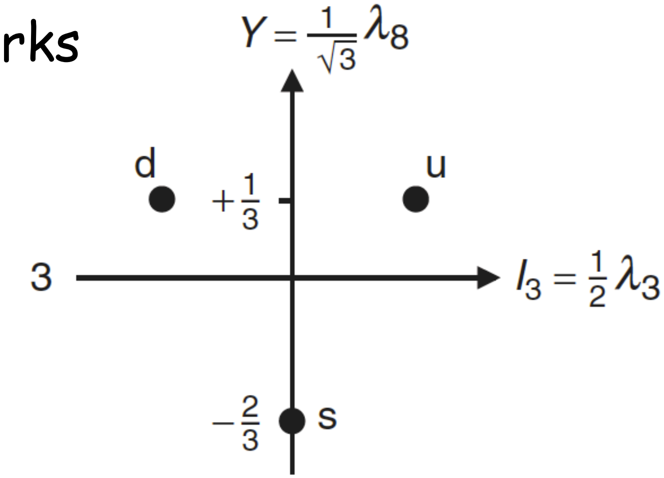
3ª componente do isospin

$$\boxed{\hat{Y} = \frac{1}{\sqrt{3}}\lambda_8} \quad \begin{cases} \hat{Y}\mathrm{u} = +\tfrac{1}{3}\mathrm{u}, \\ \hat{Y}\mathrm{d} = +\tfrac{1}{3}\mathrm{d}, \\ \hat{Y}\mathrm{s} = -\tfrac{2}{3}\mathrm{s}. \end{cases}$$

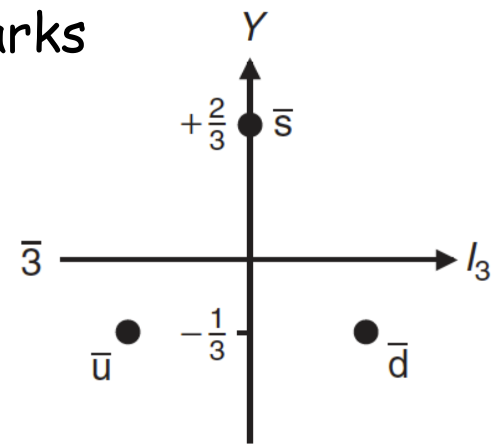
hipercarga

I_3 e Y são colocados em dois eixos perpendiculares e definem um plano

Quarks



Antiquarks



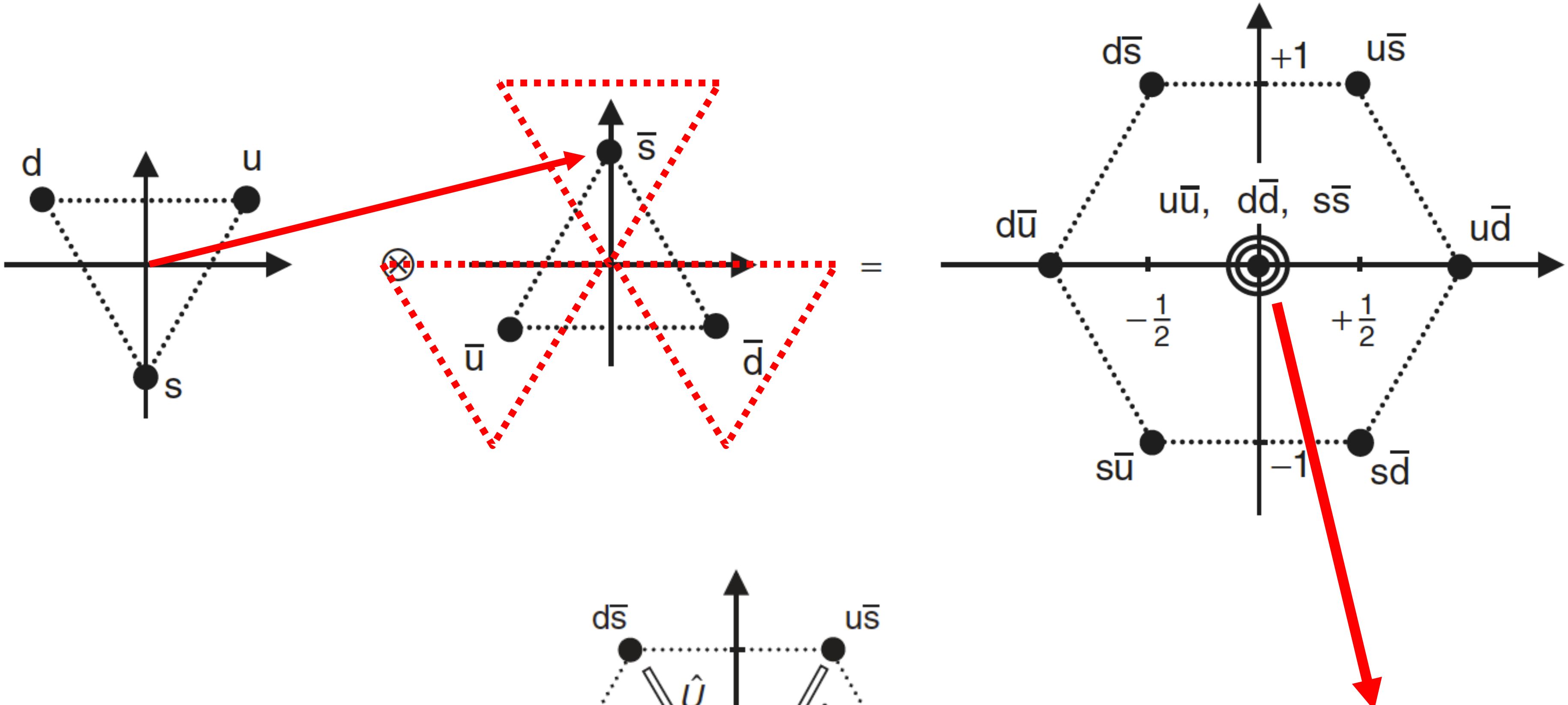
Neste plano andamos com três operadores "escada" :

$$\hat{T}_{\pm} = \frac{1}{2}(\lambda_1 \pm i\lambda_2), \qquad \mathrm{d} \leftrightarrow \mathrm{u} \qquad \begin{array}{ll} \hat{T}_+\mathrm{d} = +\mathrm{u} & \hat{T}_+\bar{\mathrm{u}} = -\bar{\mathrm{d}} \\ \hat{T}_-\mathrm{u} = +\mathrm{d}, & \hat{T}_-\bar{\mathrm{d}} = -\bar{\mathrm{u}}. \end{array}$$

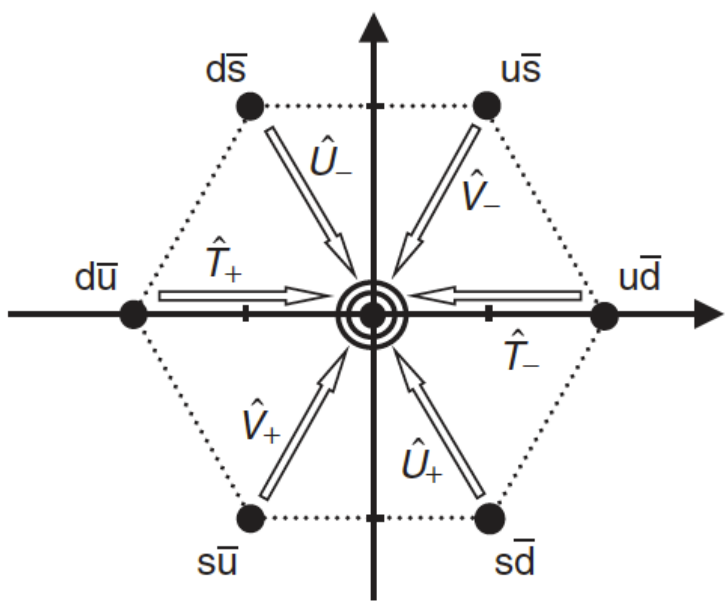
$$\hat{V}_{\pm} = \frac{1}{2}(\lambda_4 \pm i\lambda_5), \qquad \mathrm{s} \leftrightarrow \mathrm{u} \qquad \begin{array}{ll} \hat{V}_+\mathrm{s} = +\mathrm{u}, & \hat{V}_+\bar{\mathrm{u}} = -\bar{\mathrm{s}}, \\ \hat{V}_-\mathrm{u} = +\mathrm{s}, & \hat{V}_-\bar{\mathrm{s}} = -\bar{\mathrm{u}}, \end{array}$$

$$\hat{U}_{\pm} = \frac{1}{2}(\lambda_6 \pm i\lambda_7), \qquad \mathrm{d} \leftrightarrow \mathrm{s} \qquad \begin{array}{ll} \hat{U}_+\mathrm{s} = +\mathrm{d}, & \hat{U}_+\bar{\mathrm{d}} = -\bar{\mathrm{s}}, \\ \hat{U}_-\mathrm{d} = +\mathrm{s}, & \hat{U}_-\bar{\mathrm{s}} = -\bar{\mathrm{d}}, \end{array}$$

Mesons



Usamos os operadores escada



Os três estados no centro são agrupados

$$
\begin{cases}
T_+|d\bar{u}\rangle = |u\bar{u}\rangle - |d\bar{d}\rangle & T_-|u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle, \\
V_+|s\bar{u}\rangle = |u\bar{u}\rangle - |s\bar{s}\rangle & V_-|u\bar{s}\rangle = |s\bar{s}\rangle - |u\bar{u}\rangle, \\
U_+|s\bar{d}\rangle = |d\bar{d}\rangle - |s\bar{s}\rangle & U_-|d\bar{s}\rangle = |s\bar{s}\rangle - |d\bar{d}\rangle.
\end{cases}
$$

Só dois estados são linearmente independentes

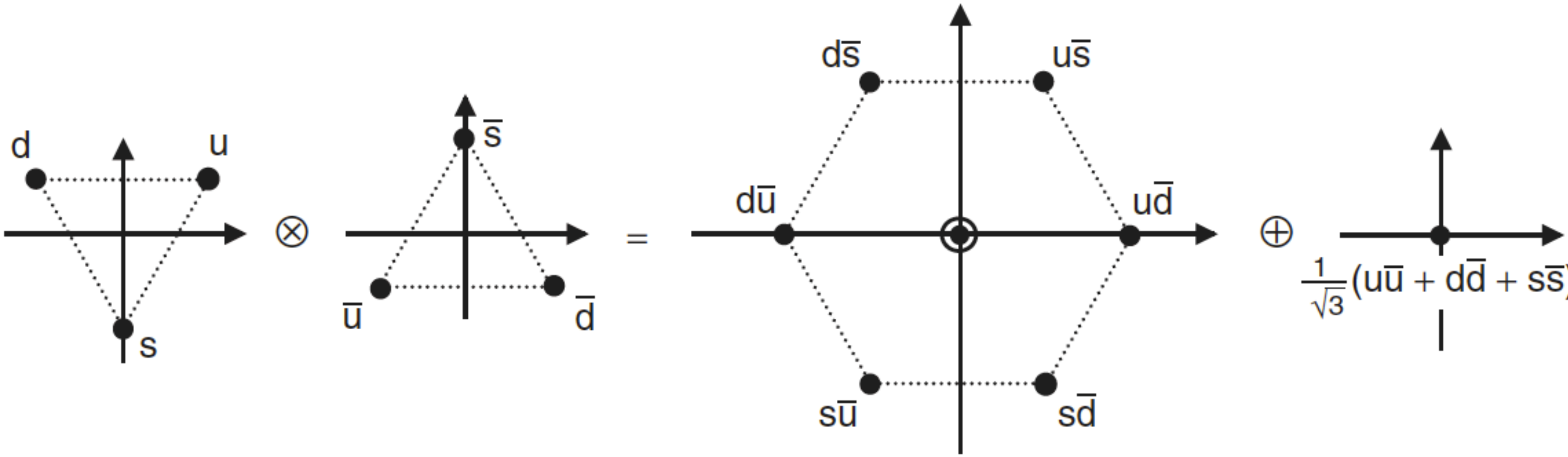
Estes dois e mais os seis do contorno hexagonal formam um octeto

O estado que sobra forma um singleto

$$
|\psi_S\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})
$$

Quando aplicamos algum operador escada dá zero !

$$
\begin{aligned}
T_+\psi_S &= \frac{1}{\sqrt{3}}([T_+u]\bar{u} + u[T_+\bar{u}] + [T_+d]\bar{d} + d[T_+\bar{d}] + [T_+s]\bar{s} + s[T_+\bar{s}]) \\
&= \frac{1}{\sqrt{3}}(0 - u\bar{d} + u\bar{d} + 0 + 0 + 0) = 0,
\end{aligned}
$$



Em grupês: $$3 \otimes \bar{3} = 8 \oplus 1$$

Função de onda: $$\psi(\text{meson}) = \phi_{\text{flavour}}\, \chi_{\text{spin}}\, \xi_{\text{colour}}\, \eta_{\text{space}}$$

Paridade do sistema quark-antiquark :

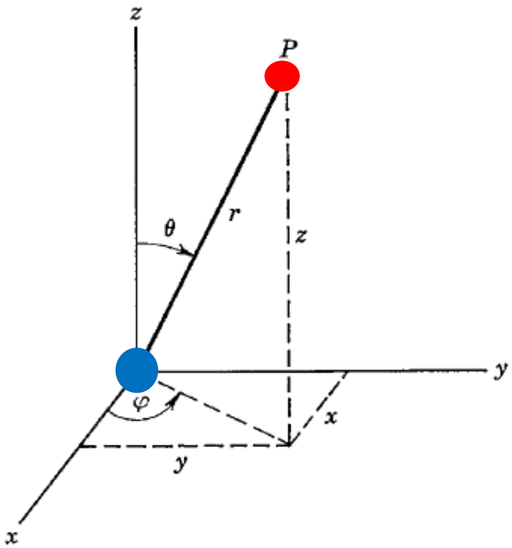
$$P(q\bar{q}) = P(q)P(\bar{q}) \times (-1)^{\ell} = (+1)(-1)(-1)^{\ell}$$

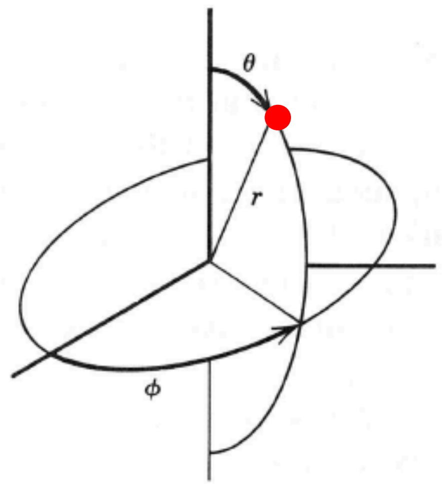
l é o momento angular orbital

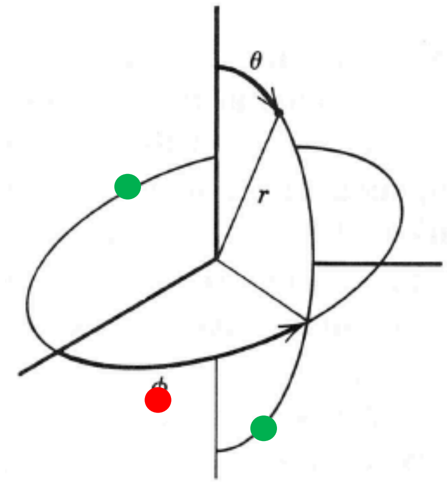
Coords. Esféricas: Solução da Eq. Schrödinger: Transformação de paridade :

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$$

$$r \rightarrow r;\ \theta \rightarrow \pi - \theta;\ \text{e}\ \phi \rightarrow \pi + \phi.$$







$$Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^{\ell} Y_{\ell m}(\theta, \phi)$$

Table 7-2 Some Eigenfunctions for the One-Electron Atom

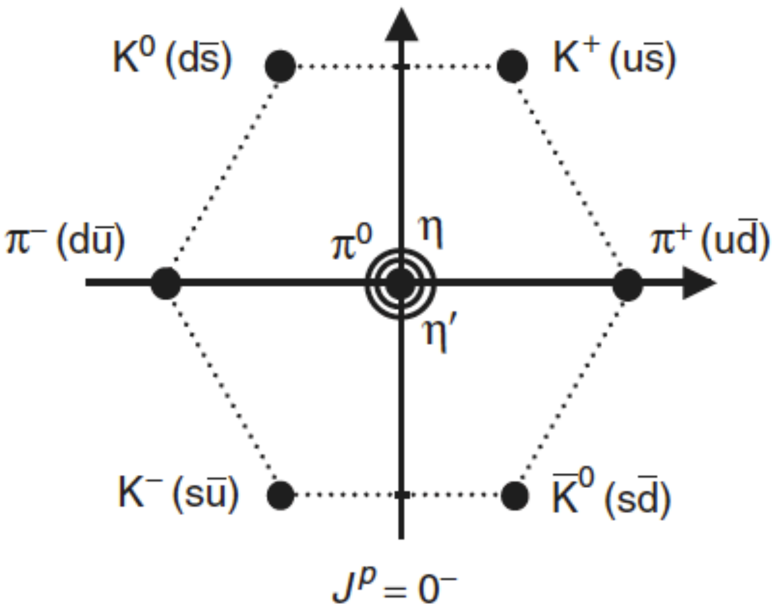
Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2 - \frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \cos\theta$

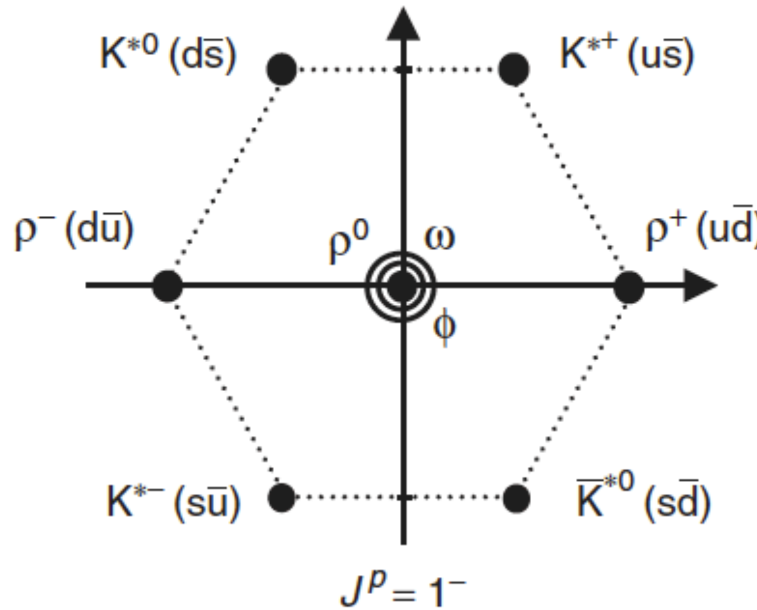
$$\cos(\pi - \theta) = -\cos\theta$$

Paridade = P = - 1

Mesons com momento angular L = 0

$$P = -(-1)^l = -1$$

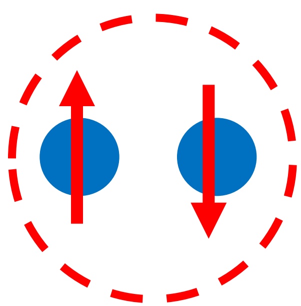




S = 0

L = 0

J = L + S = 0



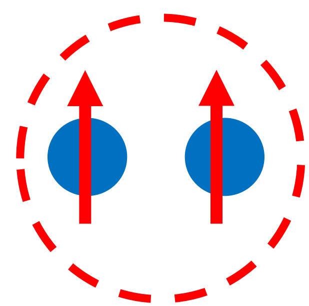
$$J^P = 0^-$$

mesons pseudoescalares

S = 1

L = 0

J = L + S = 1



$$J^P = 1^-$$

mesons vetoriais

Pseudoescalar singleto

$$|\eta'\rangle \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).$$

Os dois membros do octeto com $I_3 = 0$ e $Y = 0$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$|\eta\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

Os mesons vetoriais com $I_3 = 0$ e $Y = 0$

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),$$

$$|\omega\rangle \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),$$

$$|\phi\rangle \approx s\bar{s}.$$

Aqui há mistura entre octeto e singleto !

As massas dos mesons

Table 9.1 The $L = 0$ pseudoscalar and vector meson masses.

Pseudoscalar mesons		Vector mesons	
π^0	135 MeV	ρ^0	775 MeV
$\pi^\pm$	140 MeV	$\rho^\pm$	775 MeV
$K^\pm$	494 MeV	$K^{*\pm}$	892 MeV
$K^0, \overline{K}^0$	498 MeV	$K^{*0}/\overline{K}^{*0}$	896 MeV
η	548 MeV	ω	783 MeV
η'	958 MeV	ϕ	1020 MeV

As massas deveriam ser iguais mas não são !

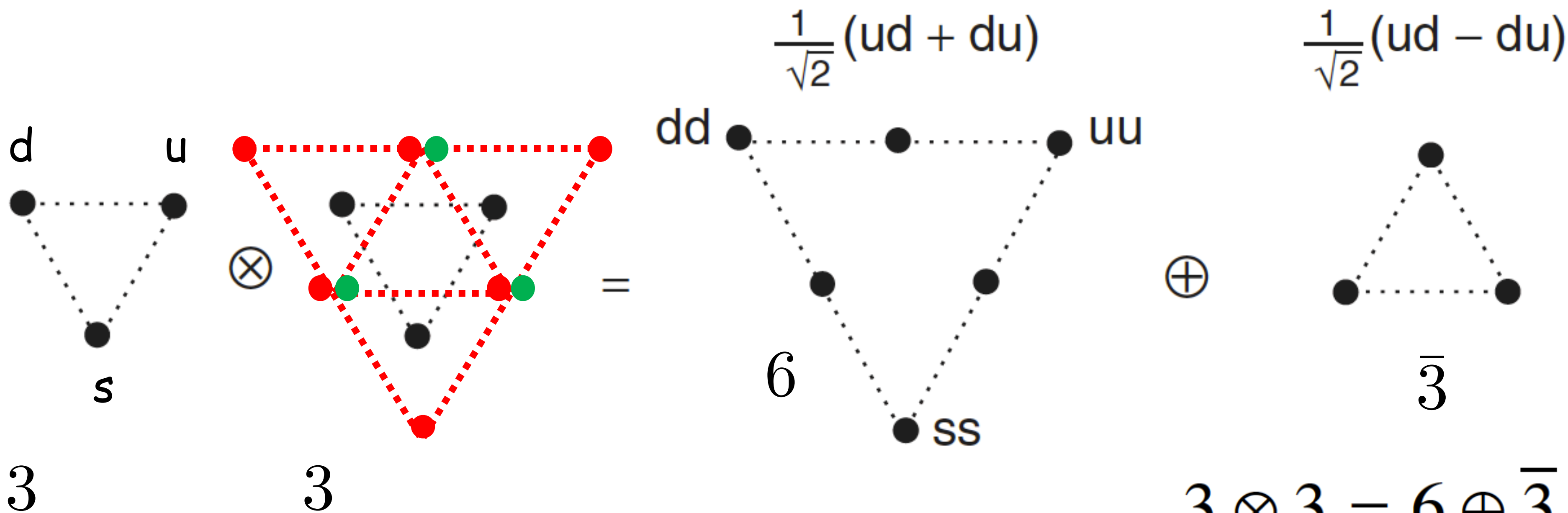
O quark estranho tem massa bem maior.

SU(3) é apenas aproximada !

A diferença de massa entre pseudoescalares e vetores vem do spin !

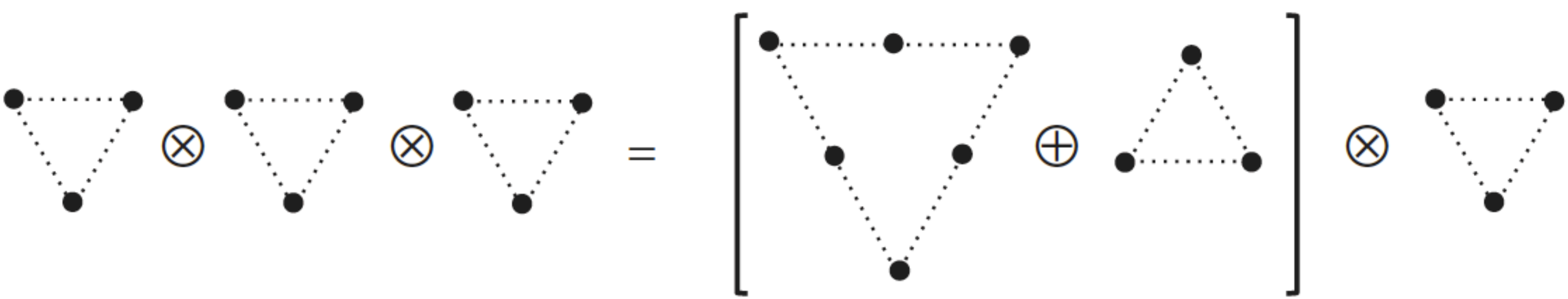
Barions

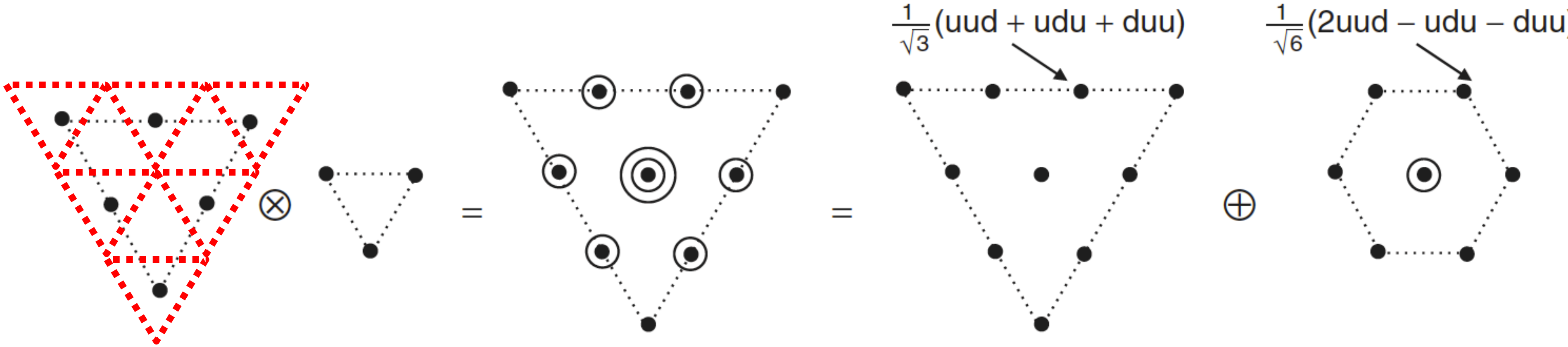
Primeiro combinamos dois quarks :



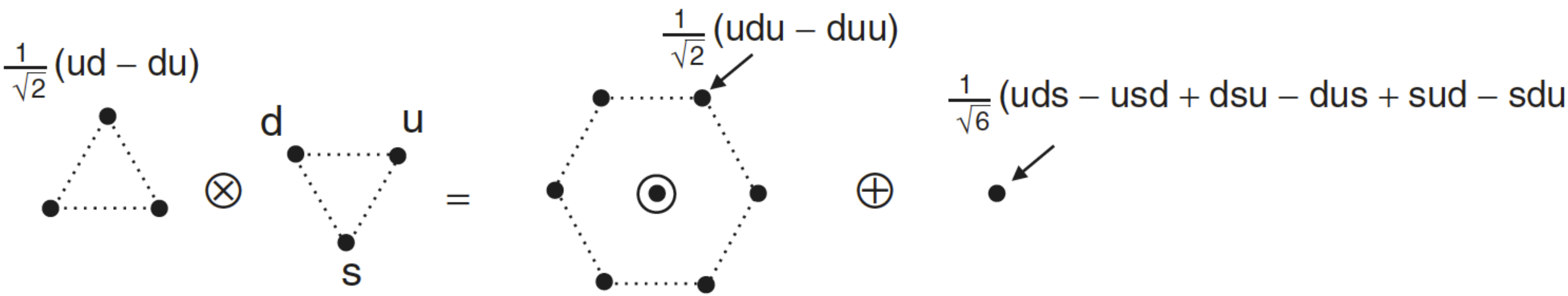
$$3 \otimes 3 = 6 \oplus \overline{3}$$

Depois multiplicamos pelo terceiro quark :





$$6 \otimes 3 = 10 \oplus 8$$



$$3 \otimes \bar{3} = 8 \oplus 1$$

$$3 \otimes 3 \otimes 3 = 3 \otimes (6 \oplus \bar{3}) = 10 \oplus 8 \oplus 8 \oplus 1$$

O singleto é dado pela combinação :

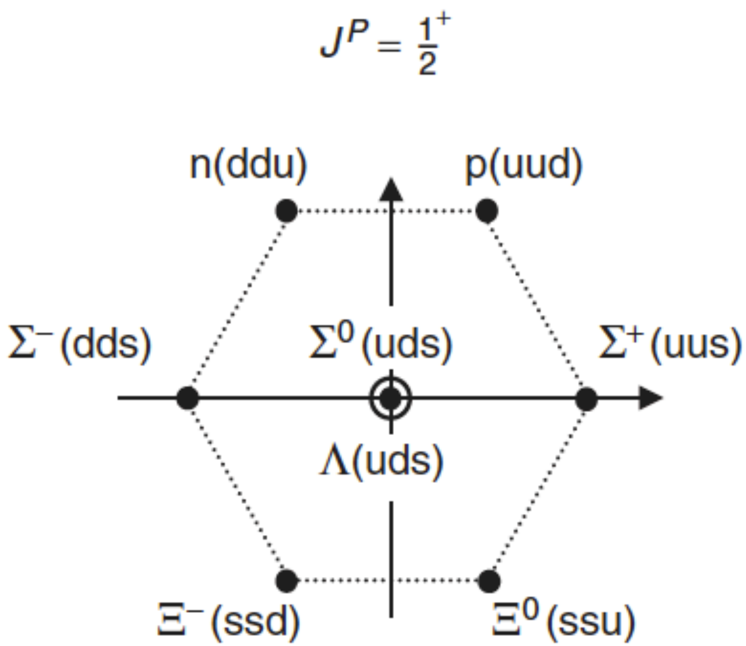
$$|\psi_S\rangle = \frac{1}{\sqrt{6}}(\mathrm{uds} - \mathrm{usd} + \mathrm{dsu} - \mathrm{dus} + \mathrm{sud} - \mathrm{sdu})$$

Verificação: usamos algum operador escada. Deve dar zero !

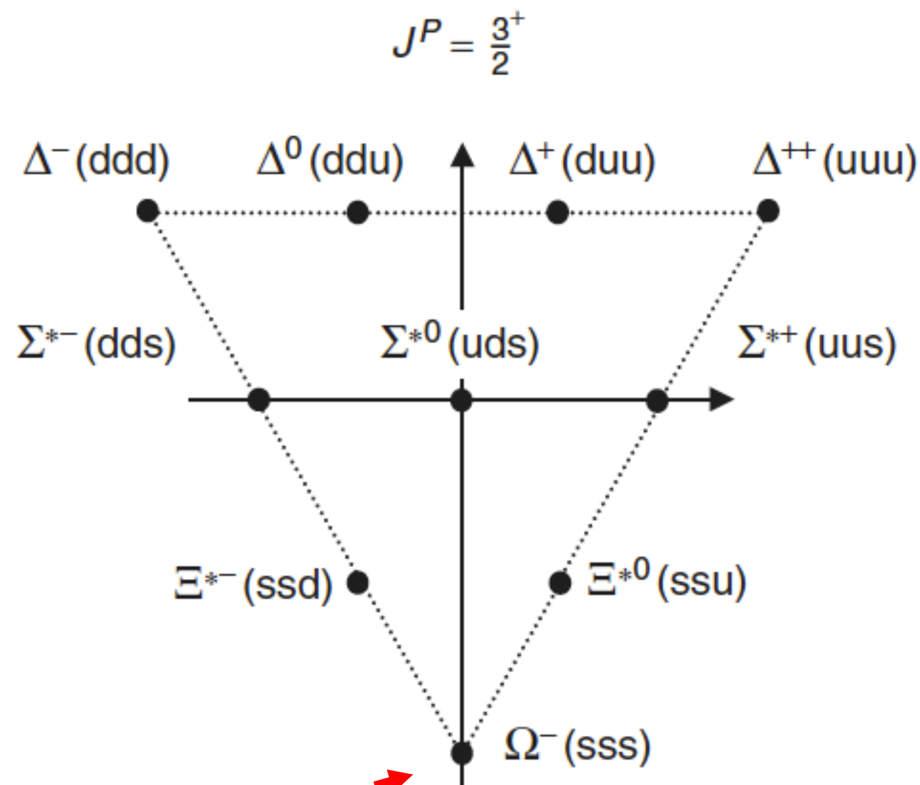
$$\begin{cases} \hat{T}_+ \mathrm{d} = +\mathrm{u} \\ \hat{T}_+ u = 0 \\ \hat{T}_+ s = 0 \end{cases}$$

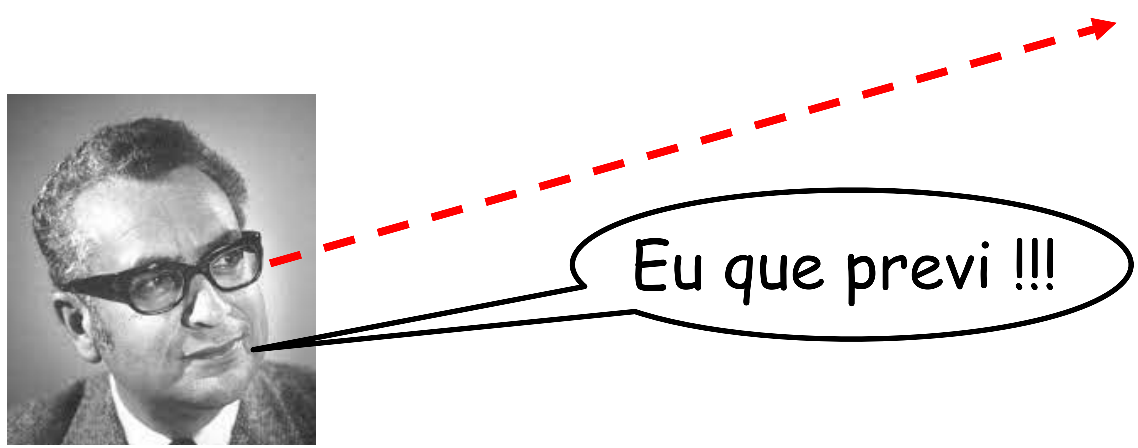
$$\hat{T}_+|\psi_S\rangle = \frac{1}{\sqrt{6}}(\mathrm{uus} - \mathrm{usu} + \mathrm{usu} - \mathrm{uus} + \mathrm{suu} - \mathrm{suu}) = 0$$

Octeto



Decupleto





Murray Gell-Mann

As massas dos barions

Table 9.2 Measured masses and number of strange quarks for the $L = 0$ light baryons.

s quarks	Octet		Decuplet	
0	p, n	940 MeV	Δ	1230 MeV
1	Σ	1190 MeV	Σ^*	1385 MeV
1	Λ	1120 MeV		
2	Ξ	1320 MeV	Ξ^*	1533 MeV
3			Ω	1670 MeV

No caso dos harmônicos esféricos, ficamos com:

$$Y_{\ell m}(\theta, \phi) \rightarrow Y_{\ell m}(\pi - \theta, \pi + \phi)$$

Que mantém seu módulo, mas pode mudar o sinal.

$$Y_{\ell m}(\pi - \theta, \pi + \phi) = \Theta_{\ell m}(\pi - \theta) e^{im(\pi + \phi)} = (-1)^m \Theta_{\ell m}(\pi - \theta) e^{im\phi}$$

As autofunções de θ são dadas pelos polinômios:

$$P_{\ell m}(\zeta) = (1 - \zeta^2)^{m/2} \frac{d^m}{d\zeta^m} P_\ell(\zeta), \qquad \zeta = \cos\theta.$$

sendo os P_ℓ conhecidos como polinômios de Legendre. Nesse caso, a operação de paridade produz uma mudança de sinal, uma vez que: $\cos(\pi - \theta) = -\cos\theta$

$$\zeta \rightarrow -\zeta \quad \text{pois} \quad \cos(\pi - \theta) = -\cos\theta.$$

Os polinômios de Legendre têm a propriedade de serem compostos de potências pares de ζ para ℓ par e potências ímpares para ℓ ímpar. Portanto: $P_\ell(-\zeta) = (-1)^\ell P_\ell(\zeta)$