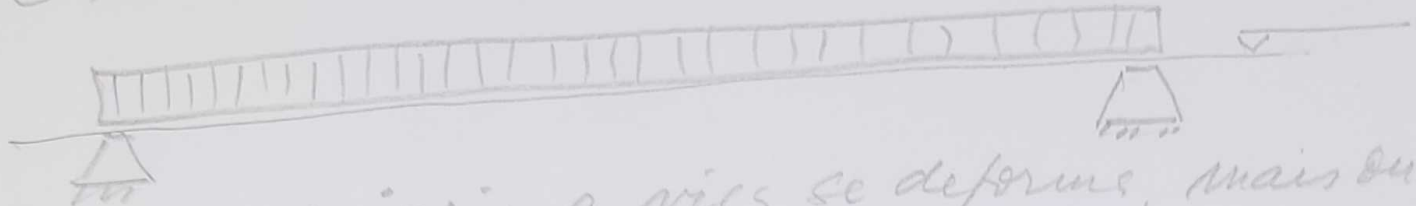


REFLITA SOBRE ESTE DESAFIO DE ENGENHARIA: ①
Uma viga está suportada por dois apoios, uma em cada extremidade:



Com o peso próprio, a viga se deforma, mais ou menos assim.



Então você monta um terceiro apoio, bem no meio:

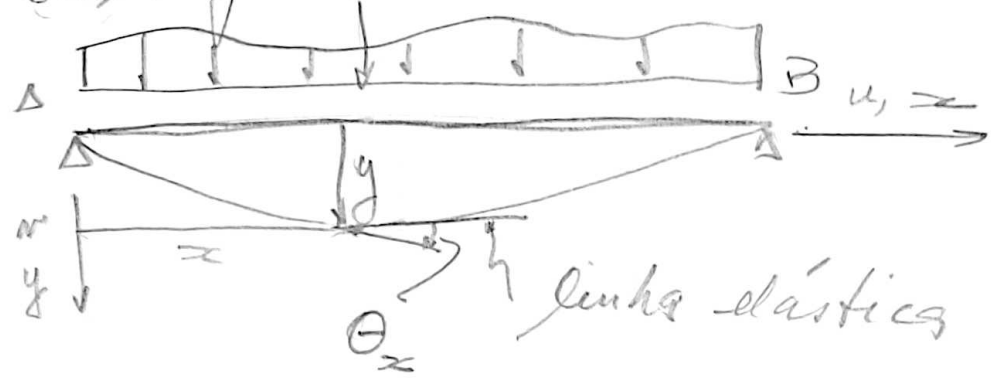


Se o peso total da viga é 10tf, como esta carga se subdividirá entre R_A , R_B e R_C ?

30/6/22 LINHA ELÁSTICA

(2)

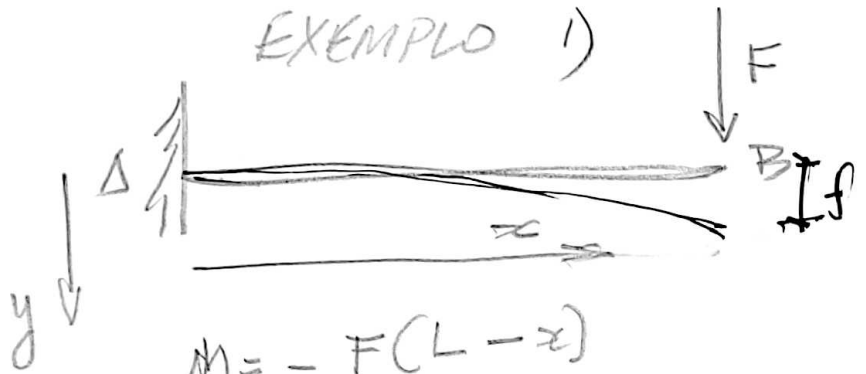
$v(x)$ = função da linha elástica



linha elástica

EXEMPLO 1)

DET. A LINHA ELÁSTICA E A FLECHA NO PONTO B



$$M = -F(L-x)$$

$$\frac{d^2v}{dx^2} = \frac{-M}{EI} = \frac{+F}{EI} \int (L-x) dx$$

$$\theta_x = \frac{dv}{dx} = \frac{+F}{EI} \left(Lx - \frac{x^2}{2} + C_1 \right)$$

$$v = \frac{+F}{EI} \left(L \frac{x^2}{2} - \frac{x^3}{6} + C_1 x + C_2 \right)$$

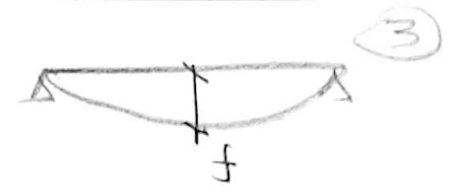
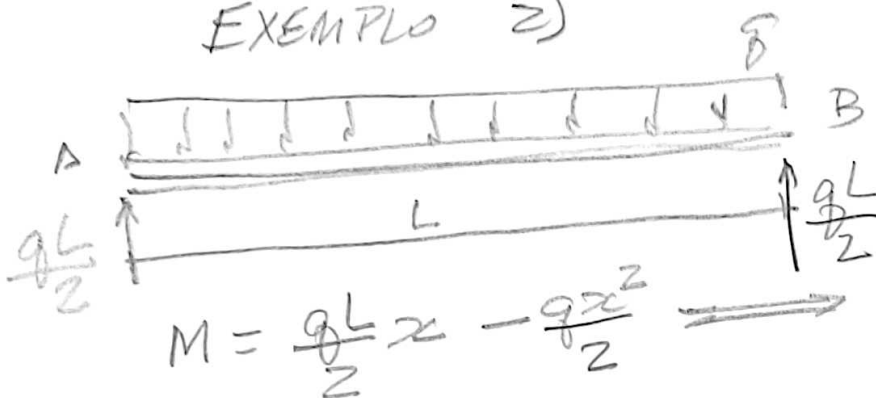
$$v_L = \frac{+FL^3}{EI} \left(\frac{1}{2} - \frac{1}{6} \right) = \frac{FL^3}{3EI} = f$$

COND. CONTORNO:

$x=0: y=0$
 $\theta=0$

$C_2=0$
 $C_1=0$

EXEMPLO 2)



$$M = \frac{qL}{2}x - \frac{qx^2}{2}$$

$$M = \frac{qL}{2} \left(x - \frac{x^2}{L} \right)$$

$$P/x = \frac{L}{2} \Rightarrow M = \frac{qL^2}{8}$$

$$v''(x) = \frac{-M}{EI} = \frac{-qL}{2EI} \left(x - \frac{x^2}{L} \right)$$

$$f = \frac{5}{384} \frac{qL^4}{EI}$$

$$\Theta(x) = v'(x) = \frac{-qL}{2EI} \left(\frac{x^2}{2} - \frac{x^3}{3L} + C_1 \right)$$

$$v(x) = \frac{-qL}{2EI} \left(\frac{x^3}{6} - \frac{x^4}{12L} + C_1 x + C_2 \right)$$

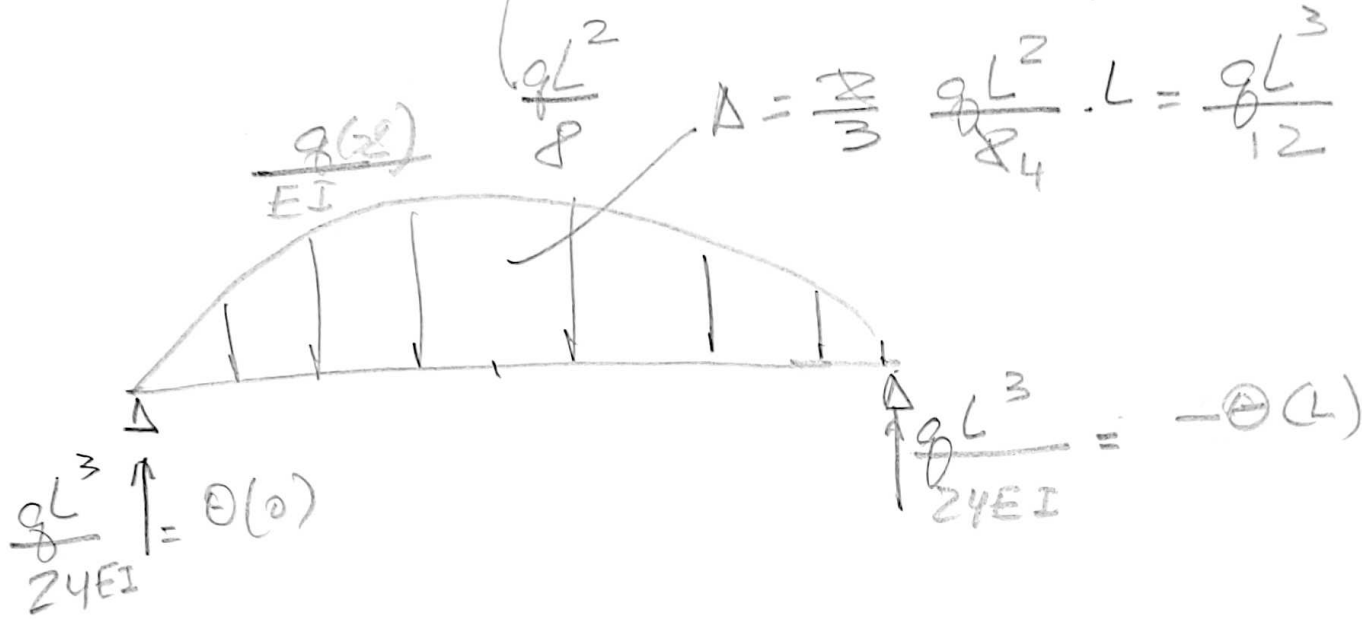
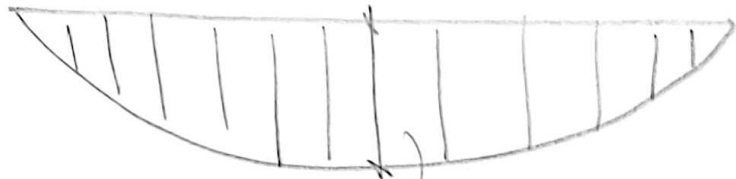
COND. CONTORNO: $\left. \begin{matrix} P/x=0 \\ x=L \end{matrix} \right\} v=0 \Rightarrow \left. \begin{matrix} C_2 = 0 \\ \frac{L^3}{12} - \frac{L^3}{12} + C_1 L = 0 \end{matrix} \right\}$

$$v(x) = \frac{-qL}{2EI} \left(\frac{x^3}{6} - \frac{x^4}{12L} - \frac{L^2 x}{12} \right)$$

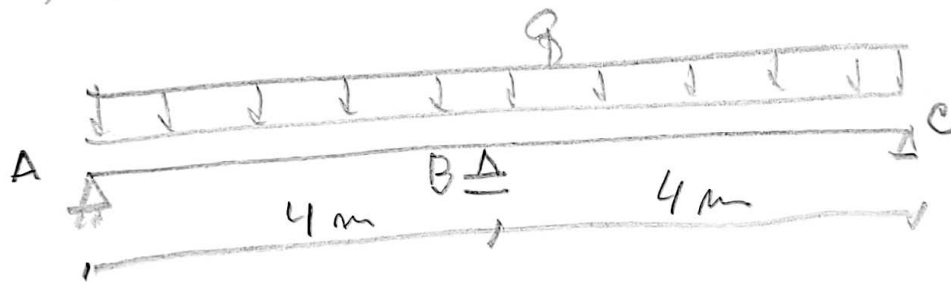
$$C_1 = -\frac{L^2}{12}$$

$$P/x = \frac{L}{2}; f = \frac{-qL}{2EI} \left(\frac{L^3}{48} - \frac{L^4}{16 \times 12L} - \frac{L^3}{24} \right) = \frac{-qL^4}{2EI} \frac{4-1-8}{16 \times 12}$$

Uma viga é algo que (4)
 transforma carregamentos
 em esforços; é uma
 dupla integração
 Que analogia pode surgir
 disto que lhe
 contei?



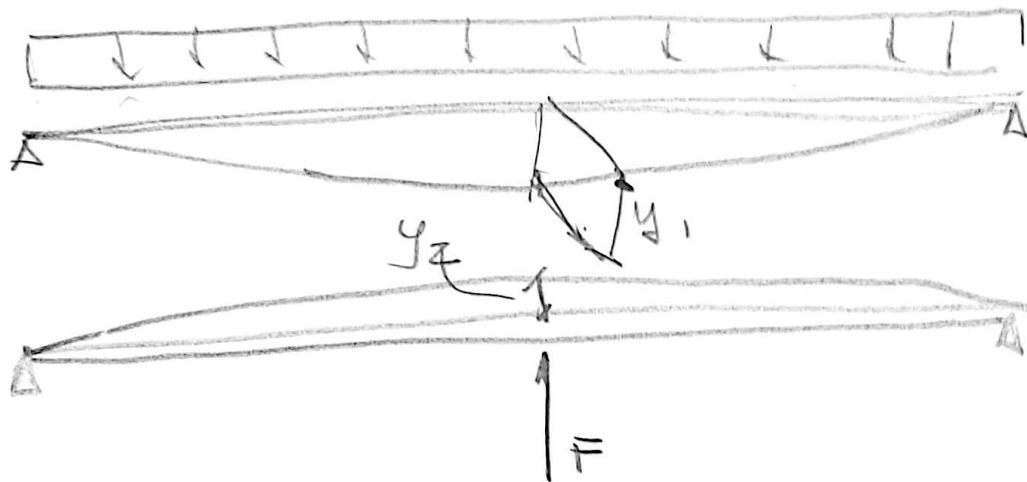
FAZER EXERCICIO PAG 34



$q = 12 \text{ kN/m}$

$E = 200 \text{ GPa}$

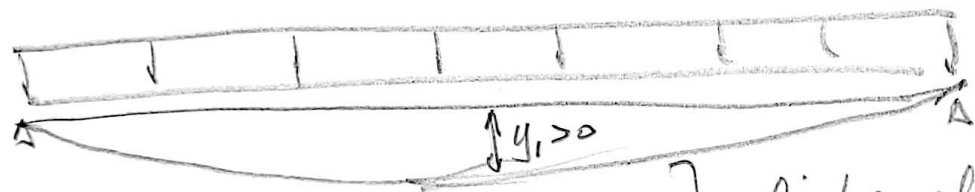
ST $h = 10 \text{ cm}$



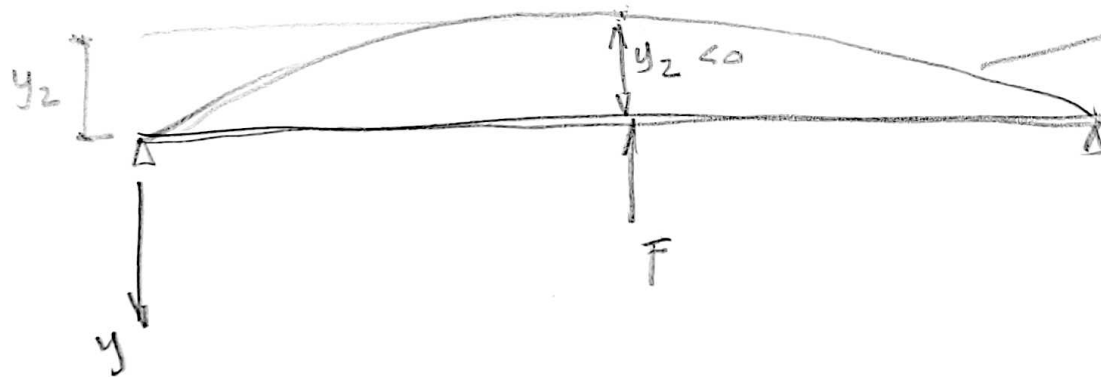
I imagine uma vig (5)
 sustentada em ambas
 as extremidades.
 Surge ela se deforma,
 e o ponto mais central
 é o que desce mais.

Você deseja que esta
 vig tenha três apoios
 todos no mesmo nível.
 Como a carga aplicada
 se distribui?

$$y_1 = \frac{5}{384} \frac{qL^4}{EI}$$



~ linhas elasticas

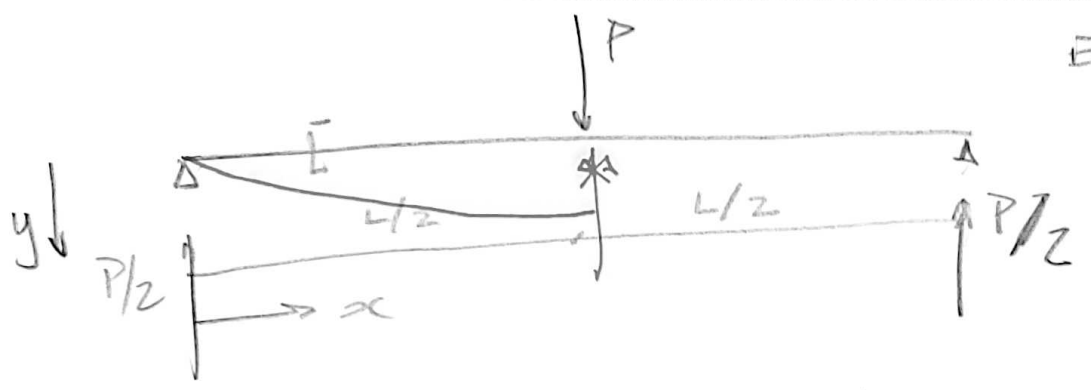


linhas elasticas

$$y_2 =$$

6

EI const



$$M = \frac{P}{2} \cdot x, \quad 0 \leq x \leq L/2$$

$$y'' = -\frac{M}{EI} = -\frac{P}{2EI} x$$

$$y' = -\frac{P}{4EI} x^2 + C_1$$

$$y = -\frac{P}{12EI} x^3 + C_1 x + C_2$$

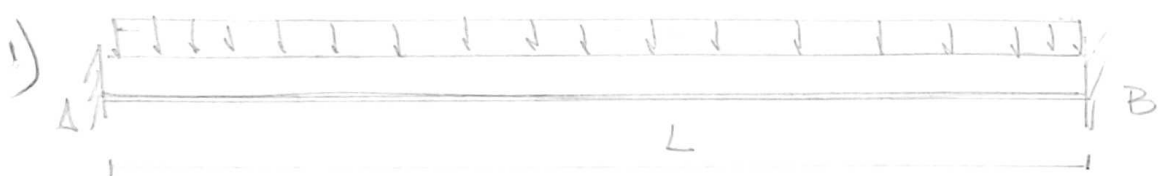
$$P / x = \frac{L}{2} \implies y' = 0 = -\frac{L}{8} + C_1 \implies C_1 = \frac{L}{8}$$

$$y_2 = \frac{P}{2} \cdot \left(\frac{L}{2}\right)^3 = \frac{PL^3}{48EI}$$

$$y = \frac{P}{2EI} \left(\frac{x^3}{6} - \frac{L^2 x}{8} \right)$$

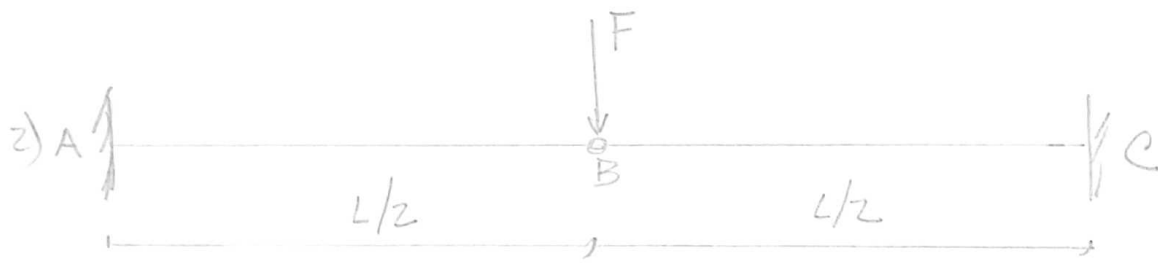
$$P / x = \frac{L}{2} \implies y = y_2 = \frac{P}{2EI} \left(\frac{L^3}{48} - \frac{L^2 \cdot L}{8 \cdot 2} \right)$$

$$y_2 = \frac{PL^3}{48EI}$$



EI const
 Det $v(x)$ linha elástica
 flecha máxima

Ex. 1: PARA ENTREGAR DURANTE A AULA



EI const
 Det $v(x)$
 flecha máxima

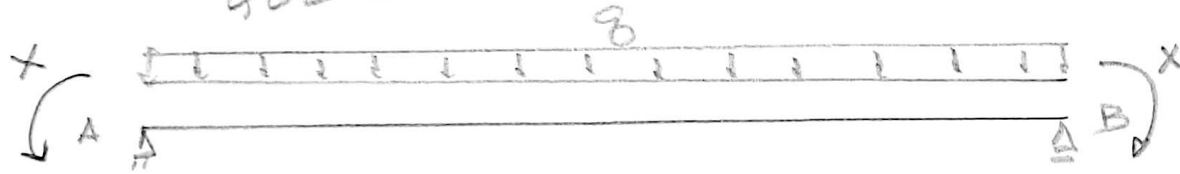


Det. a flecha no
 ponto C

Ex 2 e 3: PARA ENTREGAR ATÉ QUINTA DE MANHÃ

RESOLUÇÃO EX. 1

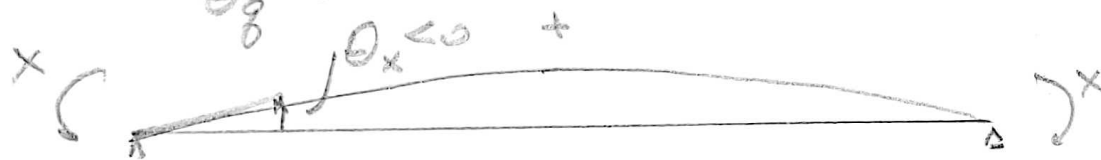
a) TRANSFORMAR A ESTRUTURA HIPERESTÁTICA EM ISOSTÁTICA, SUPONDO QUE O MOMENTO NOS ENCASTES SEJA CONHECIDO.



b) O valor correto de X é aquele que zera as rotações em Δ e B provocadas pela carga q.



of carga q

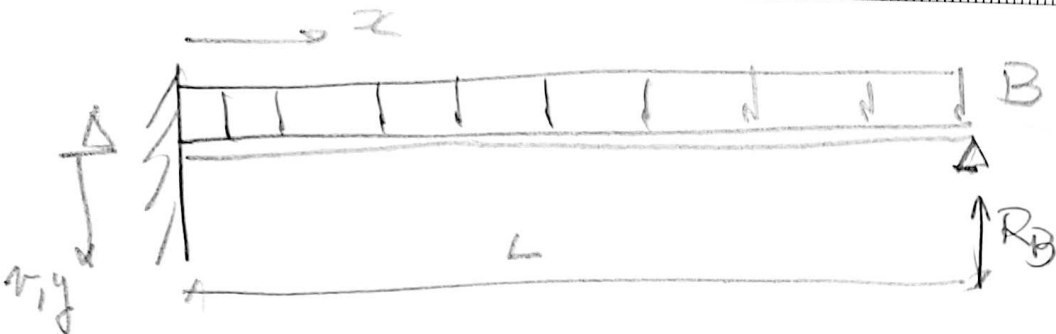


of momento X

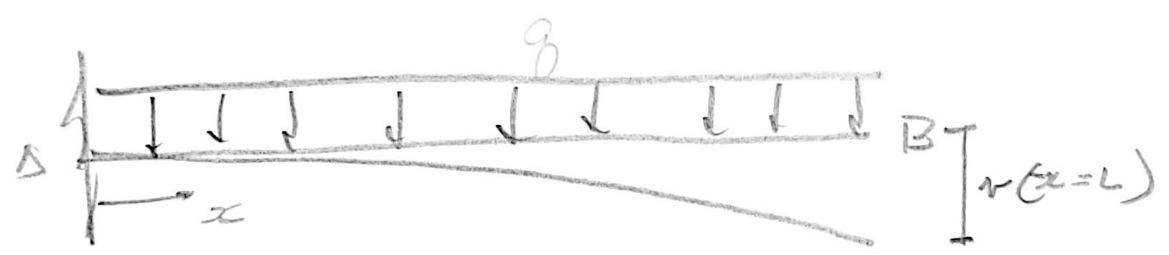
EQUAÇÃO DE COMPATIBILIDADE:

$$\theta_q + \theta_x = 0$$

9



suport conhecido



EXPRESSION DO MF:

$$M = -q \frac{(L-x)^2}{2}$$

$$v'' = \frac{-M}{EI} = \frac{-(-q)}{2EI} (L-x)^2$$

$$v' = \frac{+q}{2EI} \left(L^2 x - \frac{2Lx^2}{2} + \frac{x^3}{3} + C_1 \right)$$

Pl $x=0 \Rightarrow v'(0) = 0$
 $\therefore C_1 = 0$

logo

$$v'(x=0) = \frac{+q}{2EI} (0 - 2 + 0 + C_1) = 0$$

$$v = \frac{+q}{2EI} \left(L^2 \frac{x^2}{2} - \frac{Lx^3}{3} + \frac{x^4}{12} + C_1 x + C_2 \right)$$

$$v(x=L) = \frac{+q}{2EI} \left(\frac{L^4}{2} - \frac{L^4}{3} + \frac{L^4}{12} \right) = \frac{+qL^4}{2EI} \frac{6-4+1}{12} = \frac{+qL^4}{8EI}$$