• DISTRIBUIÇÃO BINOMIAL (DISCRETA)

DOIS <u>PARÂMETROS</u>, N E p

$$X \sim B(N,p)$$

$$\boxed{p_n \equiv \mathbb{P}(X=n) = \binom{N}{n} p^n \cdot (1-p)^{N-n}}$$

$$0 \leq p \leq 1, \quad N \in \mathbb{N}, \quad n \in \{0,1,2,\dots,N\}$$

EM GERAL, VALE O TEOREMA BINOMIAL:

$$(a+b)^m = \sum_{k=0}^{m} \binom{m}{k} \cdot a^k \cdot b^{m-k}$$

NORMALIZAÇÃO DE $\{p_n\}$:

$$\sum_{n=0}^{N} p_n = \sum_{n=0}^{N} \binom{N}{n} p^n \cdot (1-p)^{N-n} = [p+(1-p)]^N$$

$$= 1$$

$$
\begin{aligned}
\langle X \rangle &= \sum_{n=0}^{N} n . p_n \\
&= \sum_{n=0}^{N} n \left[\binom{N}{n} p^n (1-p)^{N-n} \right] \\
&= \sum_{n=0}^{N} n \left\{ \binom{N}{n} p^n . q^{N-n} \right\}_{q=1-p} \\
&= \left\{ \sum_{n=0}^{N} \binom{N}{n} (n . p^n) . q^{N-n} \right\}_{q=1-p} \\
&= \left\{ \sum_{n=0}^{N} \binom{N}{n} \left[\left(p \frac{\partial}{\partial p} \right) p^n \right] . q^{N-n} \right\}_{q=1-p} \\
&= \left\{ \left(p \frac{\partial}{\partial p} \right) \left[\sum_{n=0}^{N} \binom{N}{n} p^n . q^{N-n} \right] \right\}_{q=1-p} \\
&= \left\{ \left(p \frac{\partial}{\partial p} \right) \left[(p+q)^N \right] \right\}_{q=1-p} \\
&= \left\{ p \left[N (p+q)^{N-1} \right] \right\}_{q=1-p} \\
&= N . p
\end{aligned}
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