Introduction – Optimization techniques applied to projects of mechanical parts

PMR - 5215 – Optimization applied to projects of mechanical systems

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Presentation outline

- Introduction
- Parametric optimization
- Shape optimization
- Topology optimization (OT)
- Industrial applications of topology optimization

What is optimization concerned with?

Consider the following problem: to maximize the stiffness of an automobile car chassis?

Parameters that may be altered:

- width of reinforcers (b_1, b_2)
- moment of inertia of the reinforcers (I);
- distance between reinforcers (L_1 e L_2);
- position of the reinforcers (L_3 e L_4);
- sheet thickness at different points (h_1,h_2)
- chassis material (E);

Total: 10 parameters subject to modification

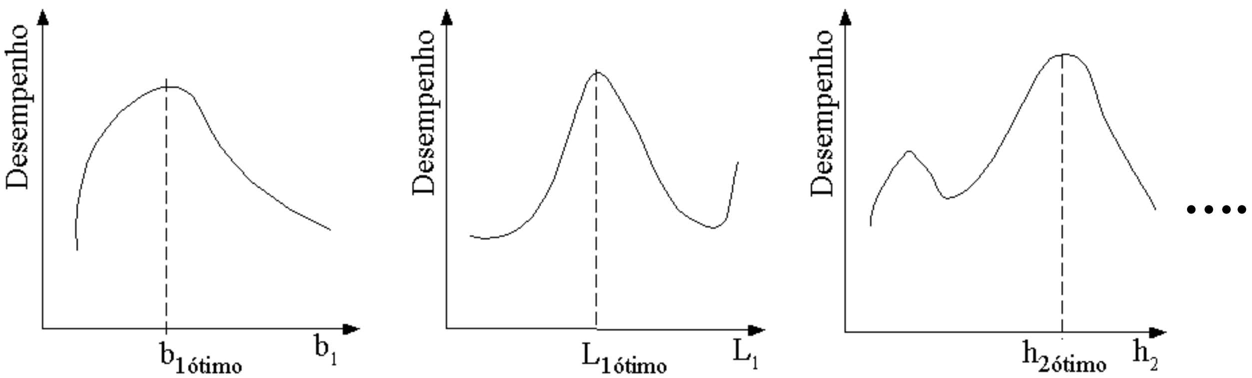
Suppose that each parameter may adopt 10 fixed values.

What is optimization concerned with?

First approach to solve the problem:

Analysis

- Execute analysis for each combination of parameters b1, b2, L1, L2, etc.;
- Plot the performance of each parameter;
- Find the combination of parameters that offers the best performance.



What is optimization concerned with?

Consequences of the analysis-based approach:

Considering only 3 parameters ➡ analysis of 10^3 combinations of parameters.
If the time spent for each analysis=0,1 s ➡ Total time: 100s

Now considering only 10 parameters ➡ analysis of 10^{10} Combinations of parameters.
If the time spent for each analysis = 10s ➡ Total time = 10^{11}s = 3200 anos!!

This approach is unfeasible for a large number of parameters!!

What is optimization concerned with?

Second approach:

Optimization (or Synthesis)

- Use of computational methods that search, in a rational way, an optimal solution;
- Renders the search for the optimal point automatic and systematic, i.e., independent of the experience of the designer;
- The time required for the solution of the previous problem would be shortened to a few hours, for example.

Introduction

Basic definition of an optimization problem

Minimize (or Maximize) **Objective Function (stiffness,**
(*project variables*) **resonance frequency, etc.)**

Subject to **Restrictions (maximum mass or volume, displacement, mechanical tension, etc...)**

Project variables (parameters subject to modification during the optimization):

- **part dimensions;**
- **curve parameters representative of the part shape;**
- **material distribution of a part;**

History of Optimization in Mechanical Engineering

Maxwell (1872)

Michell (1904)

1904–1960: There was no evolution in studies of structural optimization. Only academic problems with no practical application were treated.

1960–1970: Development of FEM: Practical problems started to be studied with optimization.

1872 1904 1960 1970

1970 1980 1990 up to now

1970–1980:

- **New algorithms emerge for optimization of nonlinear problems;**
- **Shape optimization is introduced.**

1980–1990:

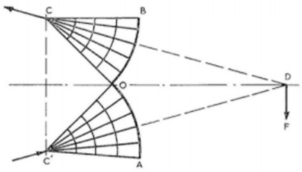
- **Commercial software become available for structural optimization;**
- **The method of topology optimization is developed (TO).**

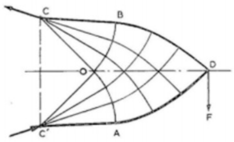
1990 up to now:

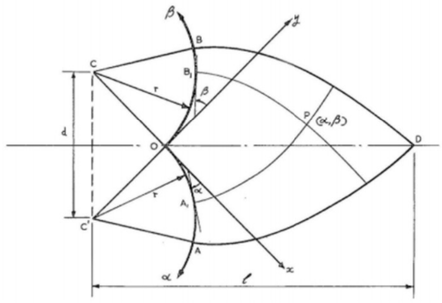
- **TO is implemented in commercial software;**
- **TO is extended to multiple areas of engineering: electrical, fluids, etc...**

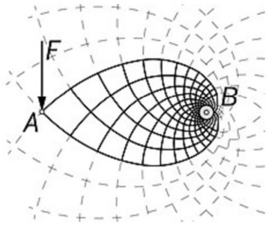
- **"The limits of economy of material in frame structures", (1904)**

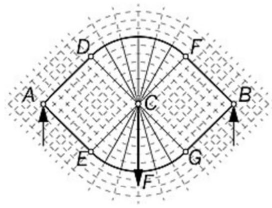
Anthony G.M. Michell
(1870-1959)

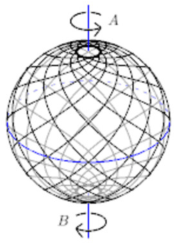












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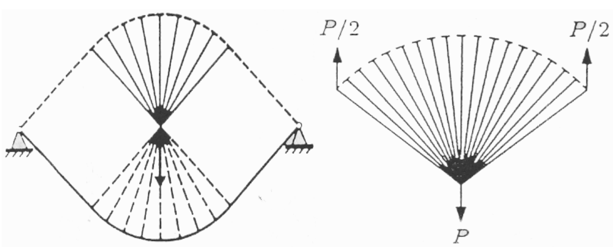
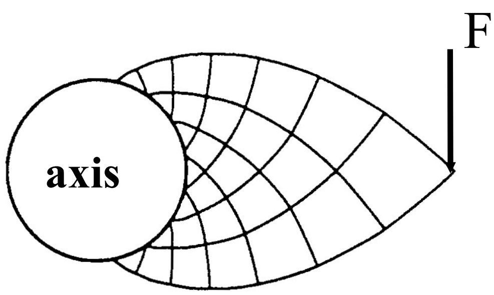
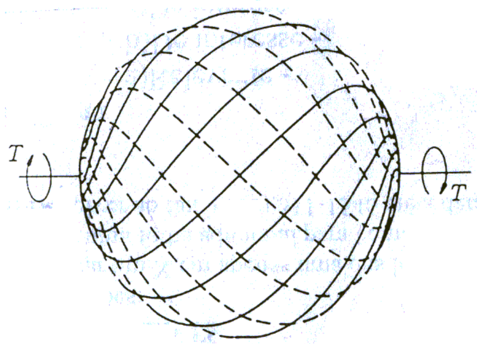
Michell (1904)



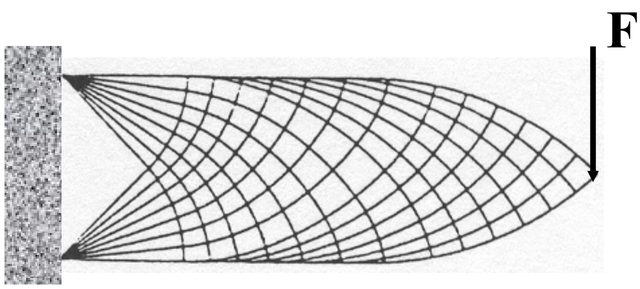
Bi-supported structure



Handle



Structure under torsion



Fixed-support structure

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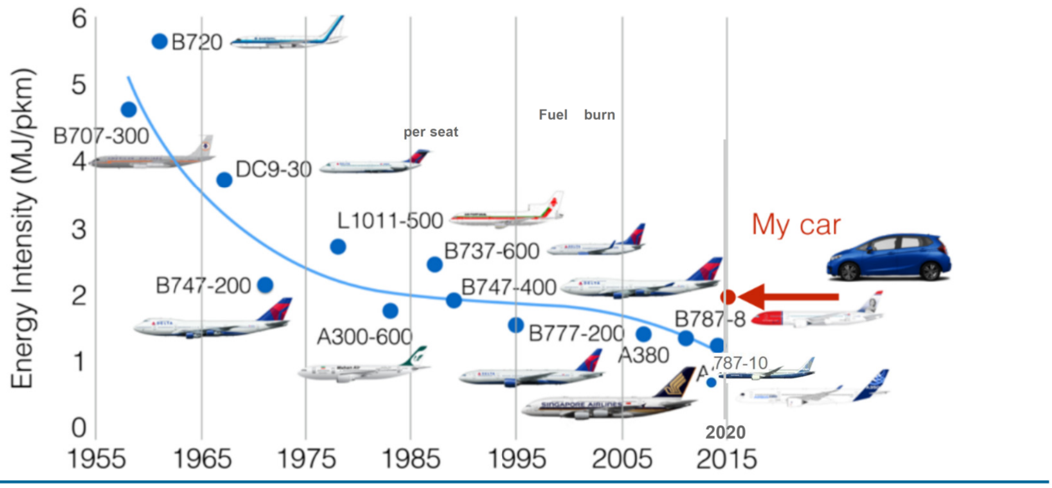
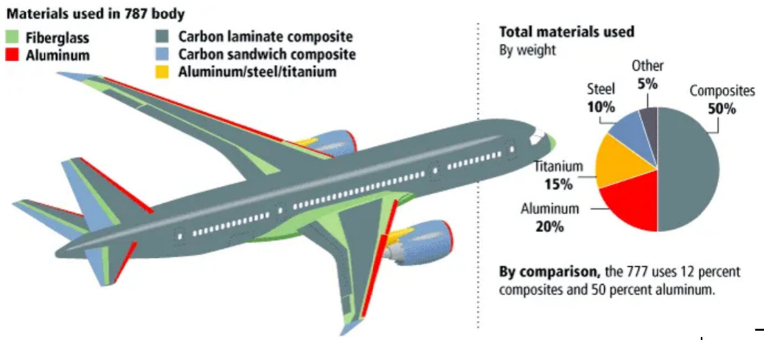
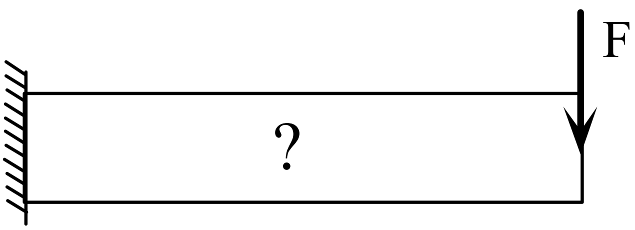
Design Optimization Impact





Available Optimization Techniques

Consider the following problem:

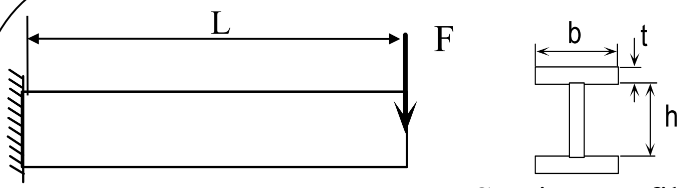


Find the structure that:

Minimizes Compliance (or maximizes stiffness)
subject to Restriction on volume of material

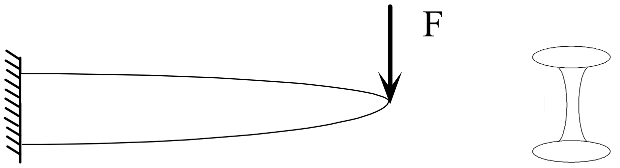
Available Optimization Techniques

Possible approaches toward the solution:



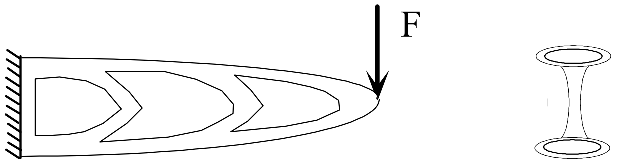
Section profile

Parametric optimization



Section profile

Shape optimization



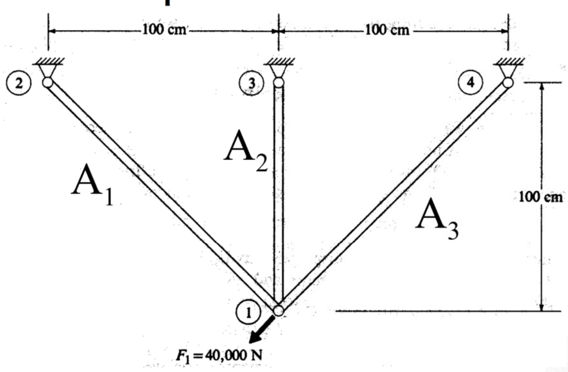
Section profile

Topology optimization

Parametric Optimization

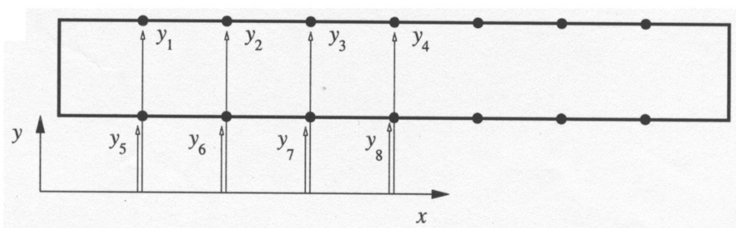
- Project variables are the dimensions or ratio of the dimensions of the part;
- It does not alter the shape of the part, only its aspect;

Examples:



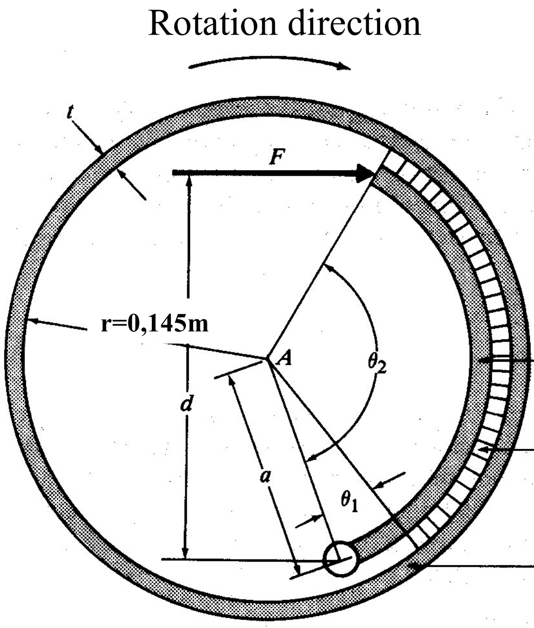
Variables are the areas A_1, A_2 and A_3.

Variables are the coordinates of the nodes



Parametric optimization

Project of an automobile breaking system



Vehicle of 2725 Kg at 90 Km/h must stop in 20s.

Min   Mass of the drum
such that

Breaking time
Parts' restrictions

Project variables: dimensions above

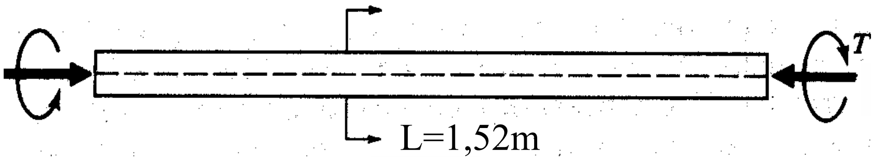
Parametric Optimization

Results obtained with an optimization software that uses finite elements:

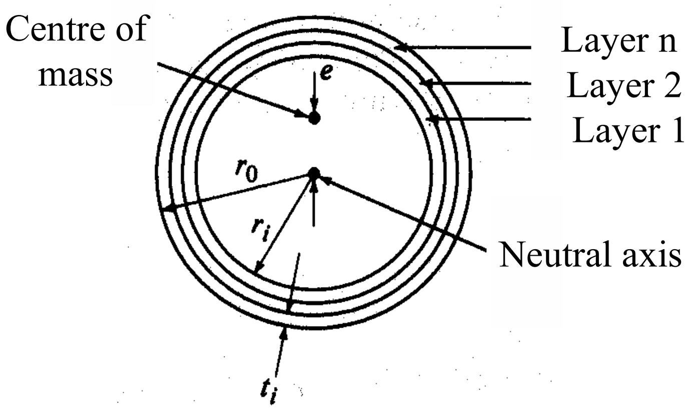
Parameters	Initial value	Optimal value
Width *b*	0.08 m	0.08 m
angle θ_2	2.10 rad	1.92 rad
Ratio *a/r*	0.75	0.755
Drum's thickness *t*	0.010 m	0.020 m
Force *F*	2815 N	2086 N
Breaking time *tm*	7.04 s	7.00 s
Maximum temperature *T*	348.7°C	229.2°C
Drum's volume *V*	754 cm^3	1590 cm^3

Parametric optimization

Project of a composite axis



Axial force and torque



Axis with 4 layers:

- 1st layer is parallel to the axis;
- 4th layer is normal to the axis;
- 2nd and 3rd layers are oriented with an angle θ and -θ relative to the axis;

Project conditions: to transmit 1000 HP at 6000 rpm with axial load of 13350 N and to transmit 1500 HP at 8000 rpm without axial load.

Parametric optimization

Min Axis mass

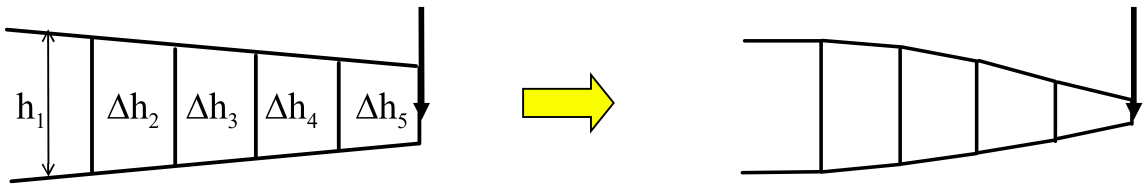
subject to Restrictions on buckling mechanical stress
Restrictions on critical frequency respose

Results obtained with an optimization software used together with an analytical model of the axis:

	Steel	Composite
Radius r_i, cm	12.7	6.91
Thickness t_1, cm	0.1258	0.3834
Thickness t_2, cm	—	0.0396
Thickness t_3, cm	—	0.0396
Thickness t_4, cm	—	0.0
Layer angle θ, degrees	—	8.6
Mass, Kg	12.03	4.90

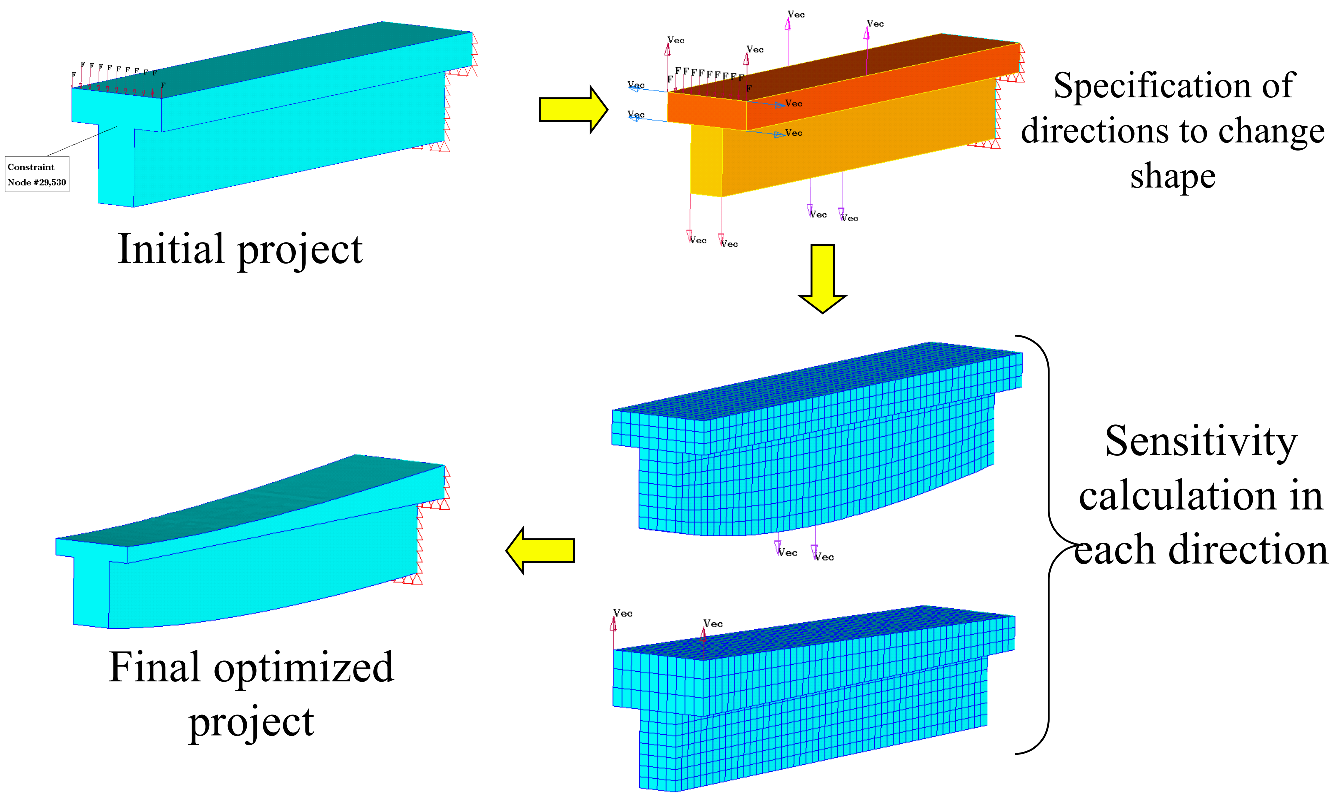
Shape optimization

- Alters only the structure's shape, and does not allow to find new 'hole' in the domain;
- Project variables are coordinated by grid nodes of FEM or curve coefficients representative of the structure's shape (e.g., 'spline' curves);
- Requires techniques for FEM grid update, because it is distorted during optimization;



Shape optimization

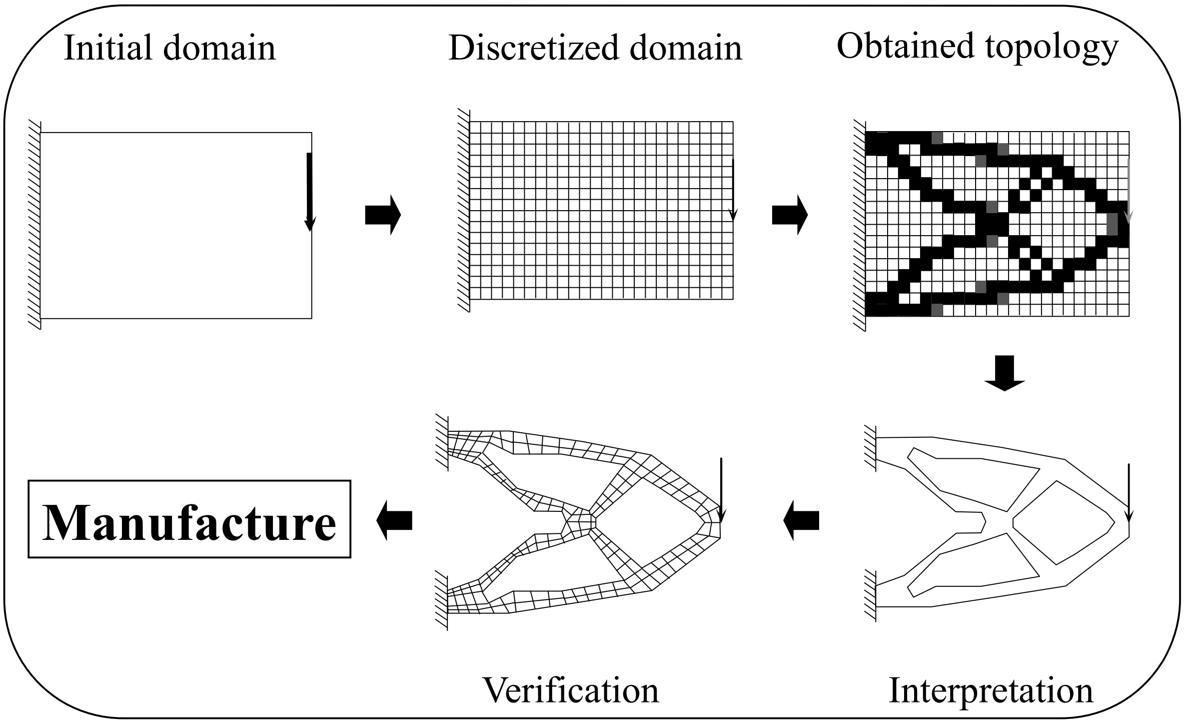
Example of procedure



Cortesia Altair Inc.

Topology Optimization (TO)

Typical procedure:



Topology Optimization (TO)

Topology optimization (TO) combines:

- Finite Element Methods (FEM);
- Optimization algorithms (mathematical programming, etc….);

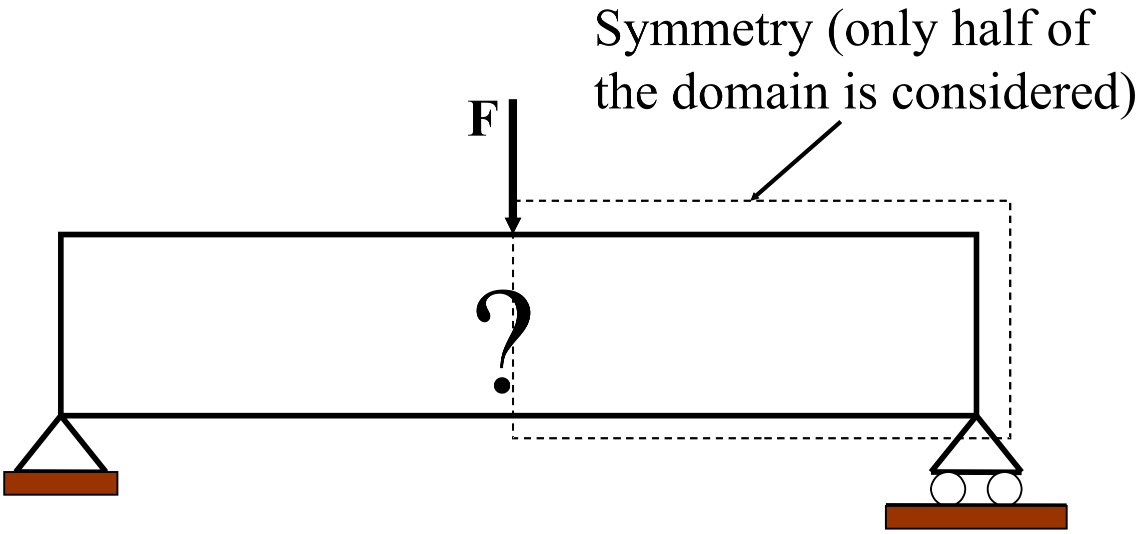
To find an optimal distribution of material for a **fixed** domain.

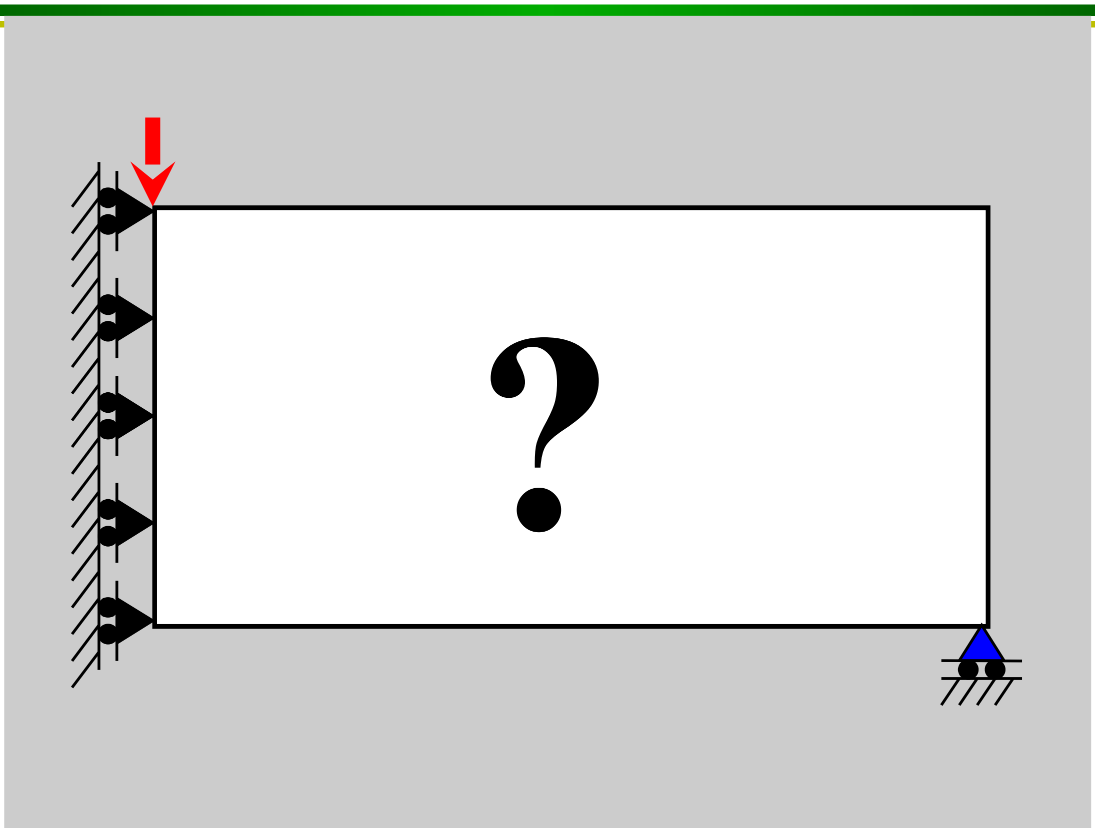
Example

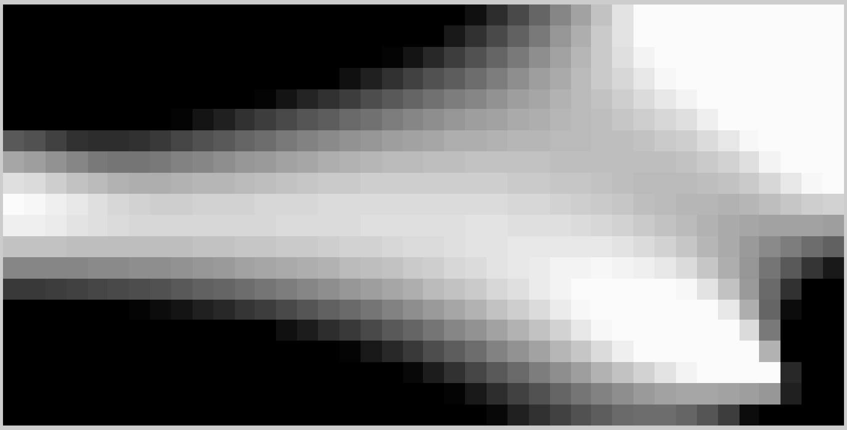
Find the structure of the domain below that:

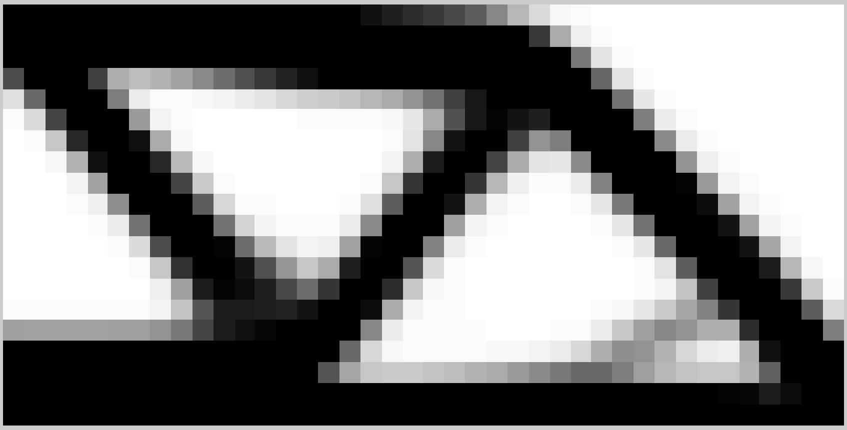
Minimizes Compliance

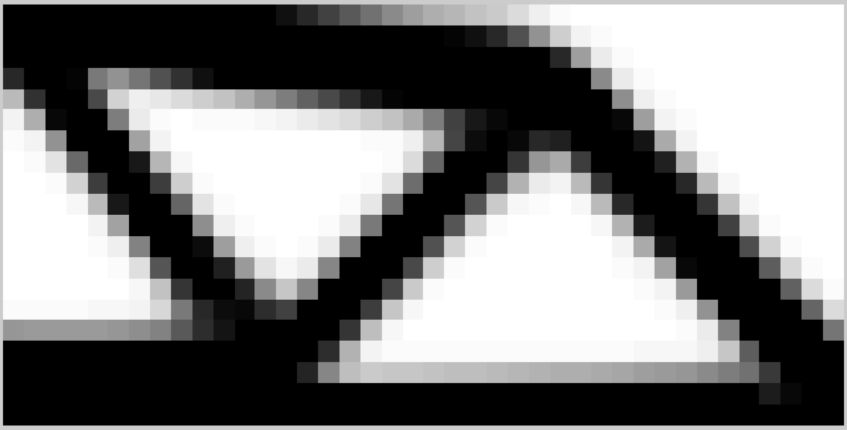
Subject to Volume restriction

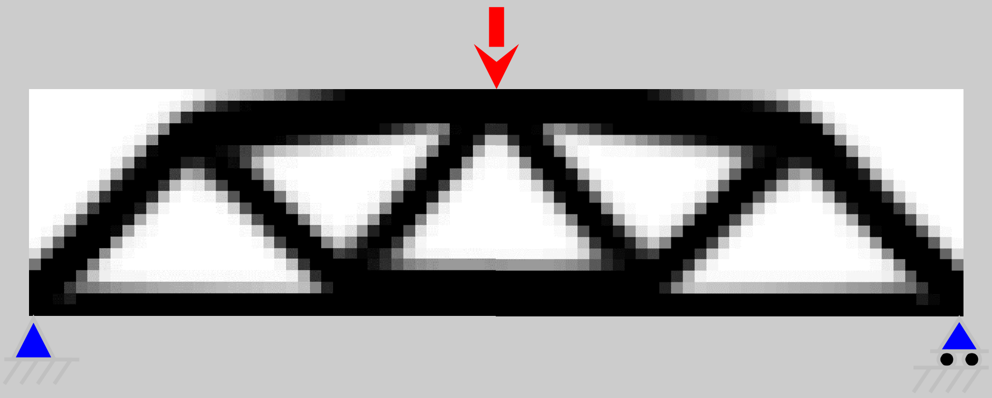








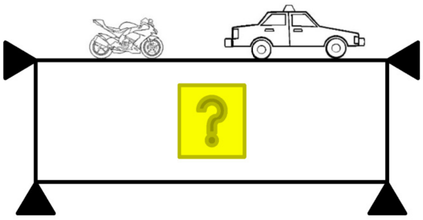


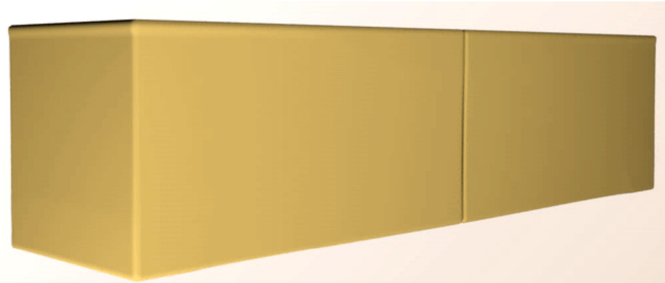


Topology Optimization (TO)

XXI century – Optimized projects

Project of a 3D bridge:





Better performance (stiffness)
Lower volume (effective use of material)

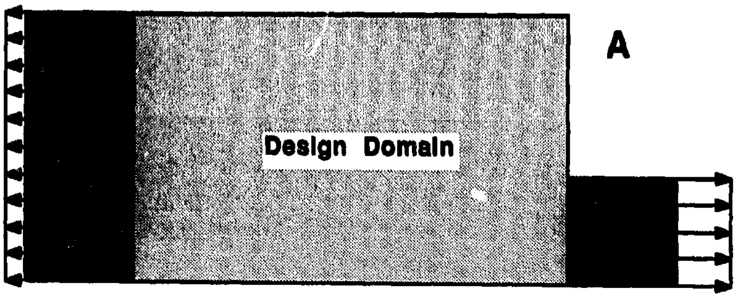
m2do.ucsd.edu/

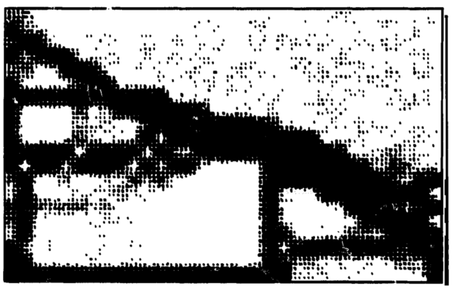
How it all started…

M. P. Bendsøe and N. Kikuchi, “Generating optimal topologies in structural design using a homogenization method”, Computer Methods in Applied Mechanics and Engineering 71, pp.197–224, 1988.

M.P. Bendsøe

Noboru Kikuchi

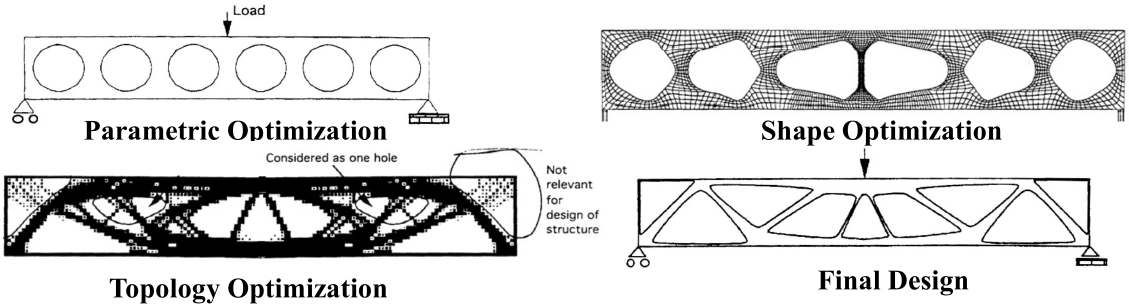




In 2018 it was celebrated 30 years!!!

Example (First Industrial Applications)

"MBB Beam - Messerschmitt-Bolkow-Blohm" – Airbus 319 (1989)



	Volume	Deflection	Max. Stress	Weight Reduction
Initial, unfeasible design	1.07	10.1	292	
Parametric optimization	1.02	10.1	248	4.7%
Shape optimization	0.95	10.1	372	11.2%
Topology optimization	0.65	10.1	227	39.2%
Final design	0.58	10.1	305	45.7%

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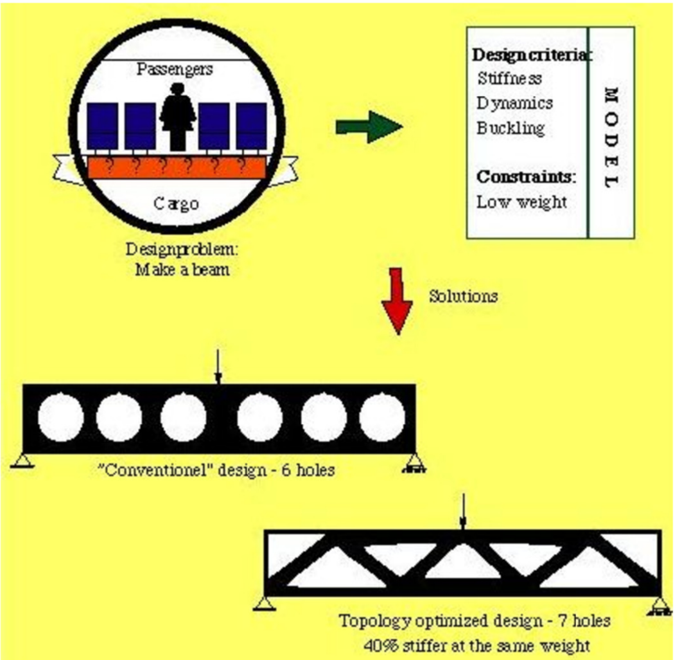
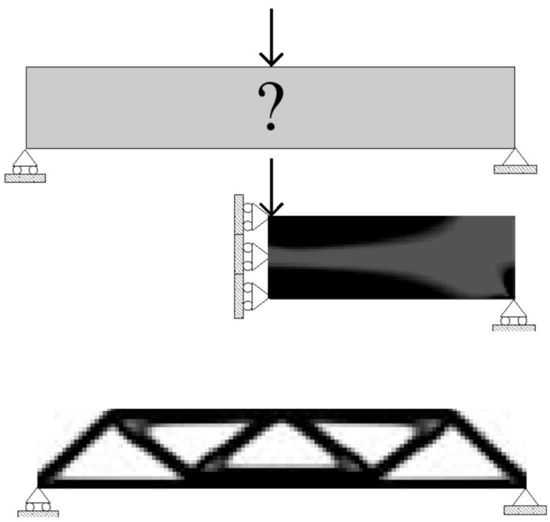
Examples

"MBB - Messerschmitt-Bolkow-Blohm" beam





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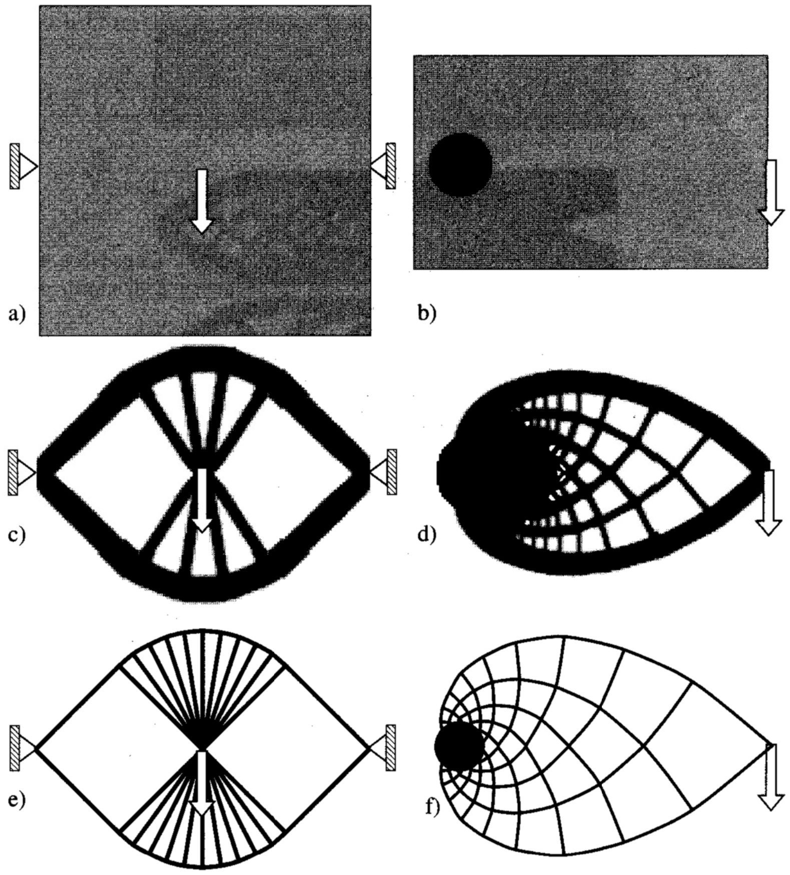
Examples

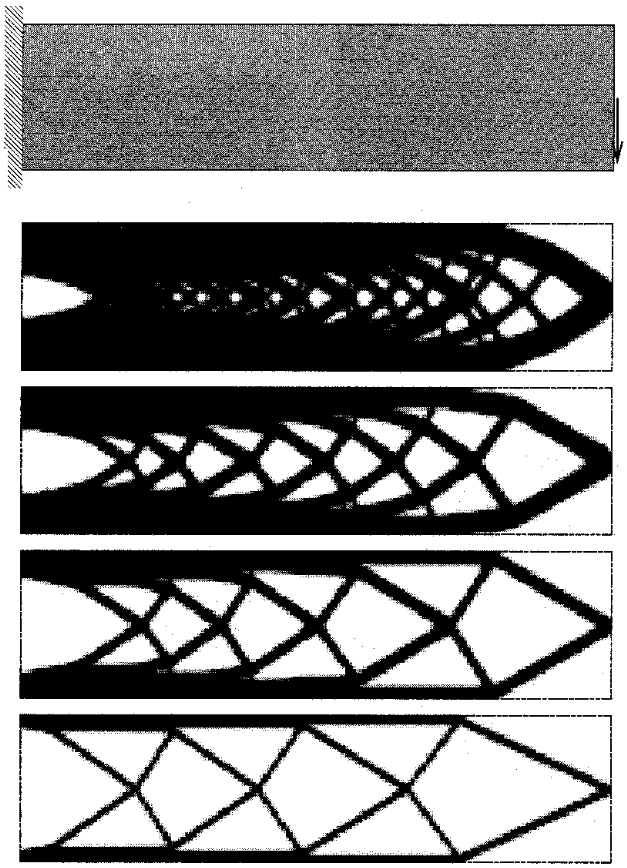


Start domain

Result of Topology Optimization

Analytical result (Michell, 1904)

Examples



Lowering the volume restriction

Fields of applications

- Structural problems in general:
  * maximization of resonance frequency (or buckling load);
  * minimization of response frequency;
  * maximization of the absorption of impact energy;
  * etc...
- Flexible Mechanisms;
- MEMS (“Microelectromechanical Systems”)
- Piezoelectric transducers (actuators, engines);
- Electromagnetic devices:
  * maximization of the ratio torque/volume in electrical engines;
  * maximization of the receptivity and emissivity in antennae;
- Projects of composite materials with desirable properties;

Check out: http://www.topopt.dtu.dk.

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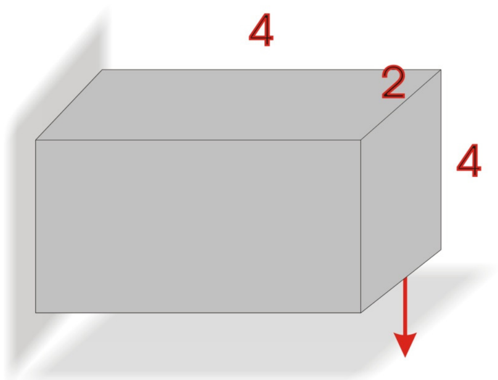
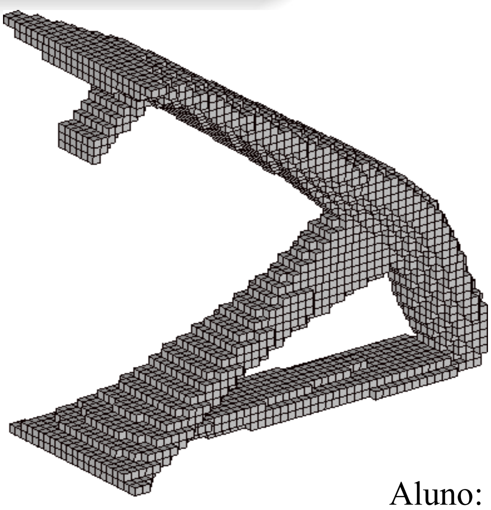
Example

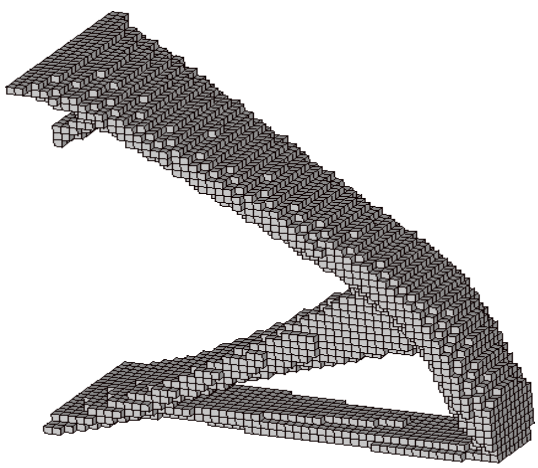


Start domain: 65.000 finite elements of the “brick” type

CPU time: 37 hours

Volume restriction: 35%





Aluno: James Edward Shooter

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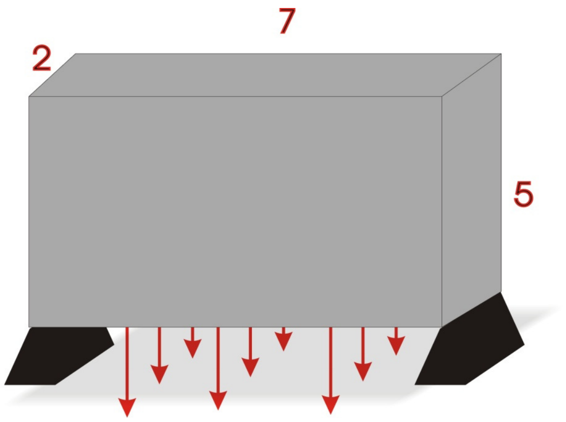
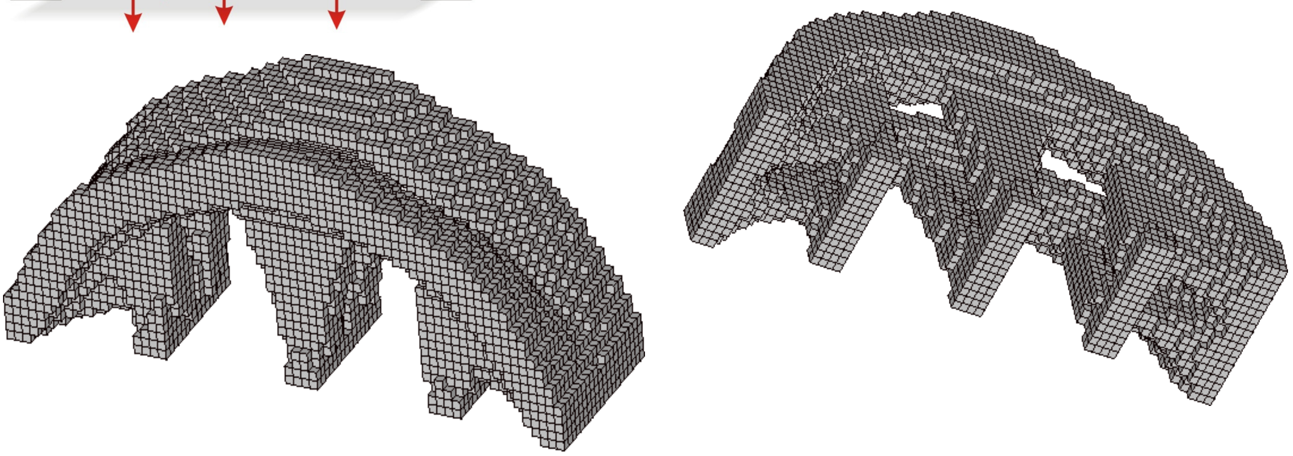
Example



Start domain: 70.000 finite elements of "brick" type

CPU time: 42 hours

Volume restriction: 35%



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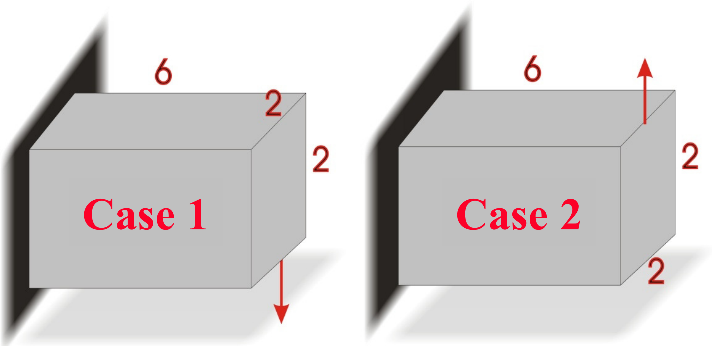
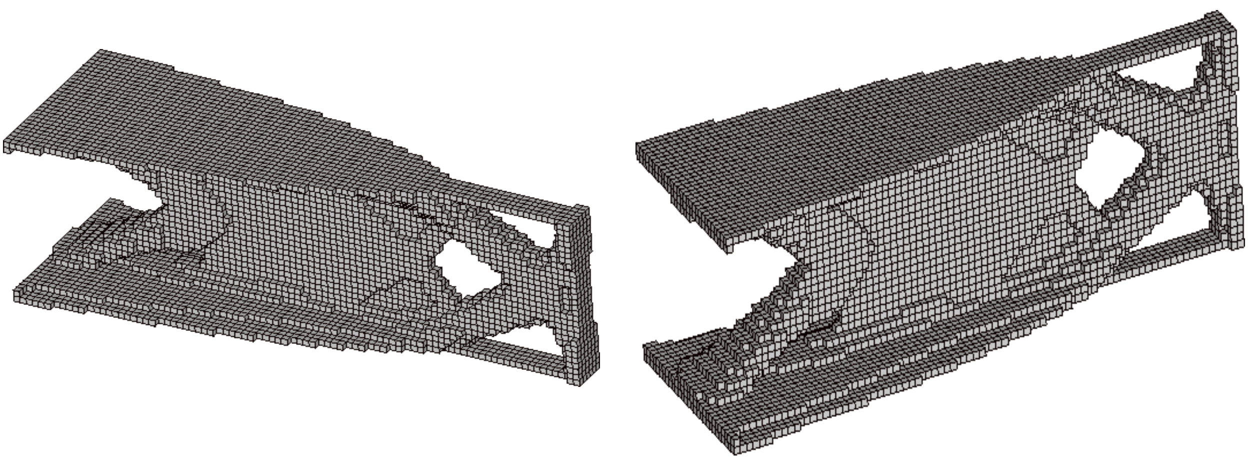
Example



Start domain: 73.000 finite elements of "brick" type

CPU type: 48 hours

Volume restriction: 35%



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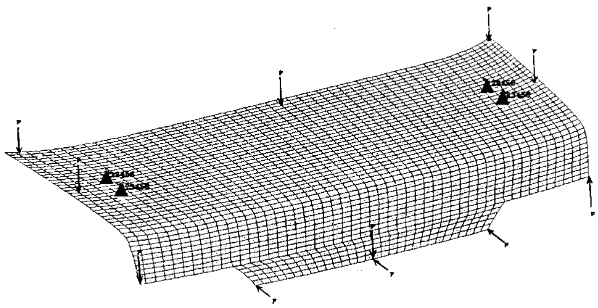
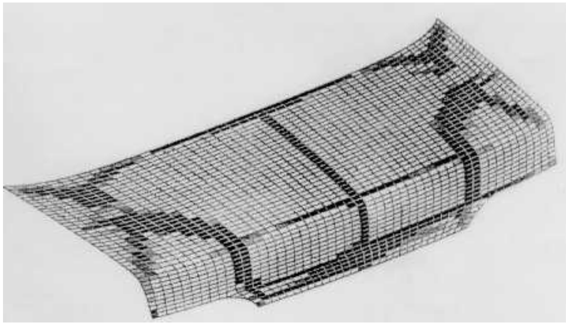
Example

Optimal sheet reinforcement with maximal ratio stiffness/volume

Automobile hood





Loads

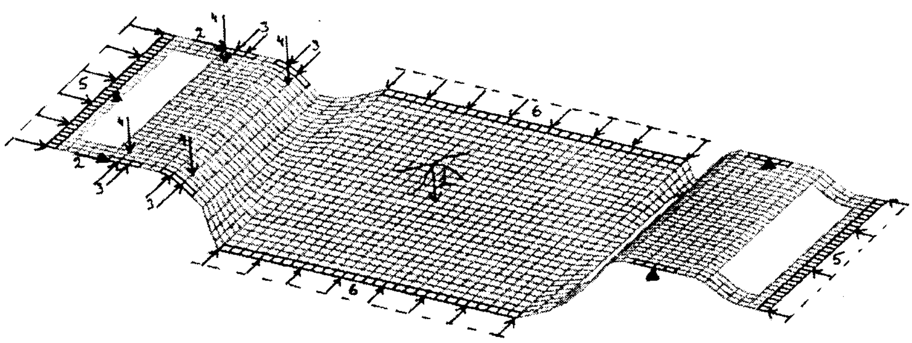
Optimized reinforcement

(Bendsoe&Kikuchi 1988)

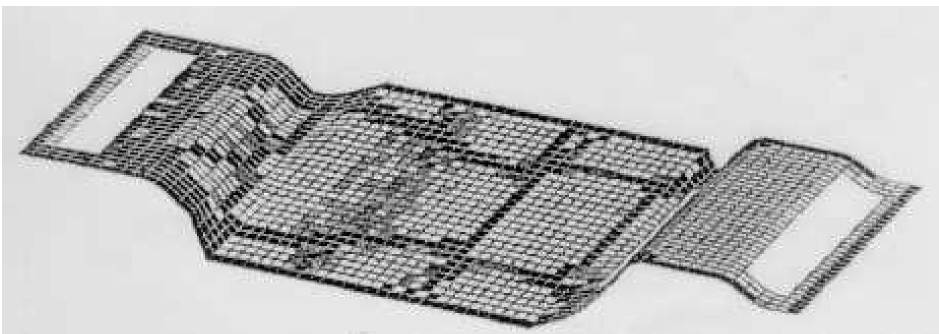
Example

Automobile chassis

Loads

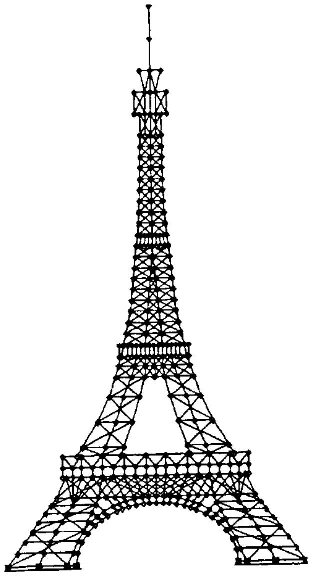


Optimized reinforcement

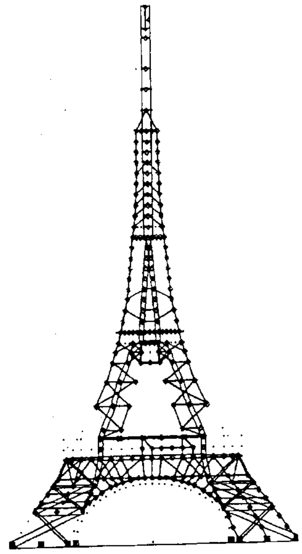


(Bendsoe&Kikuchi 1988)

OT Formulation for trusses



Initial project



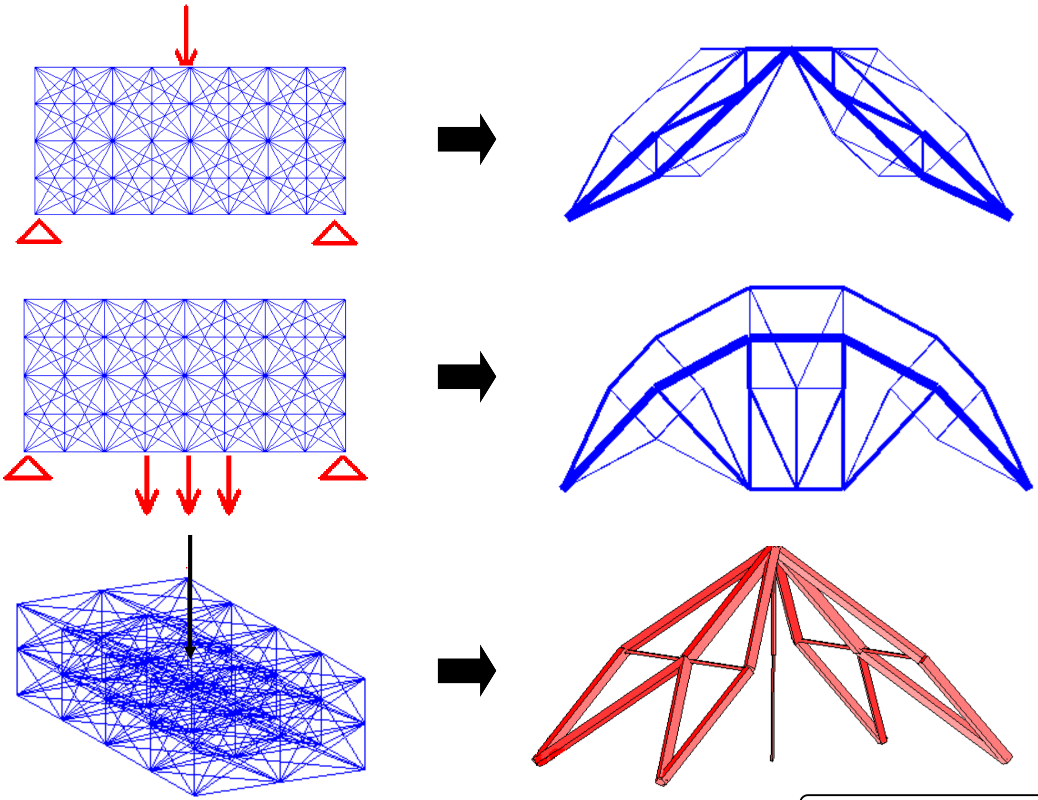
Optimized project

(Bendsoe&Kikuchi 1988)

Example results of OT application to trusses

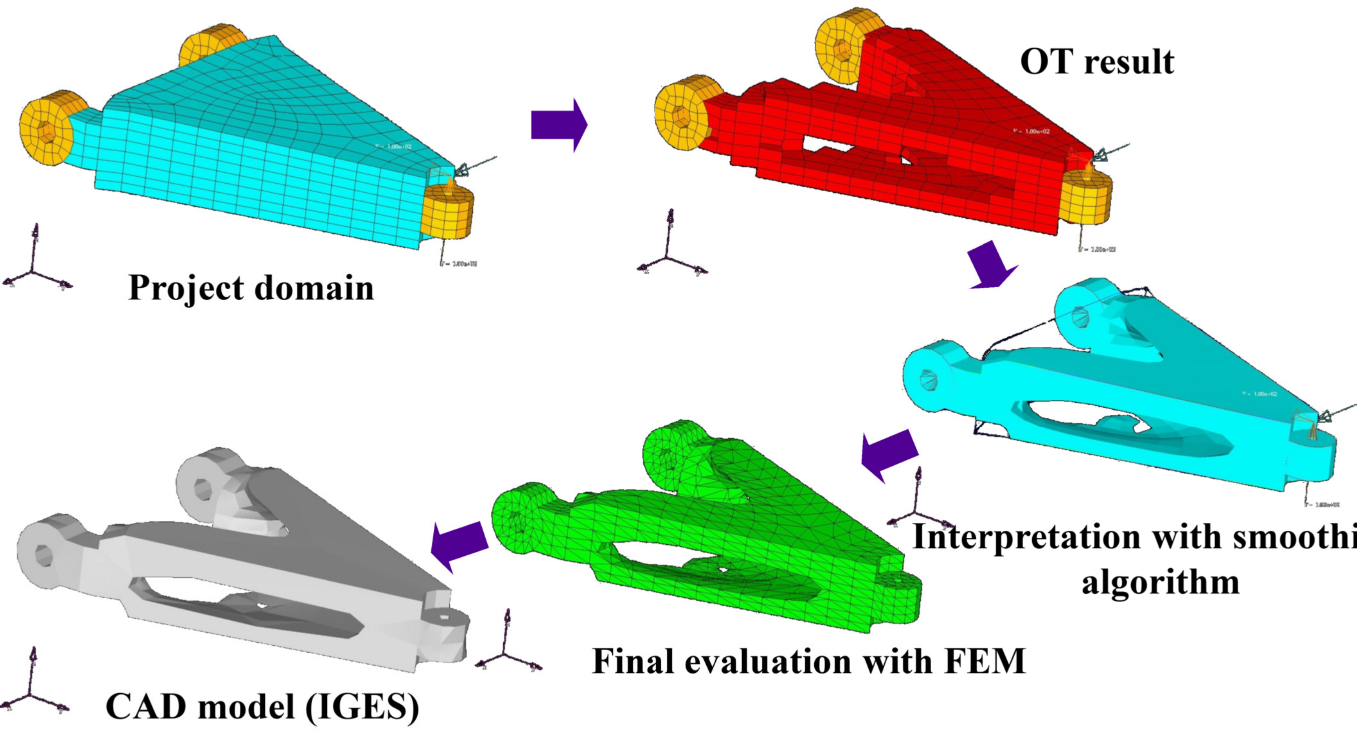
Initial project domain

OT result



Industrial applications

Example 2: Truck front suspension arm.



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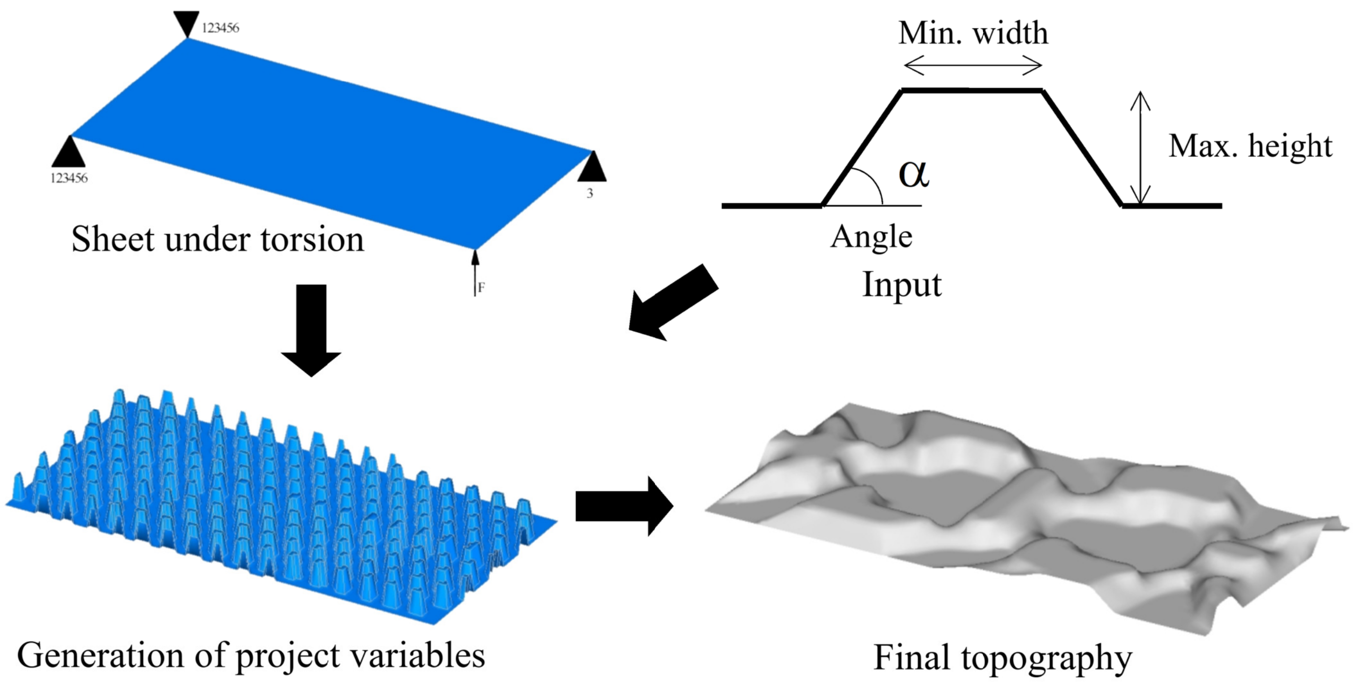
Prof. Dr. Emilio C. Nelli Silva

Topography Optimization

Objective: Find the distribution of reinforcement pattern in sheet and shell structures



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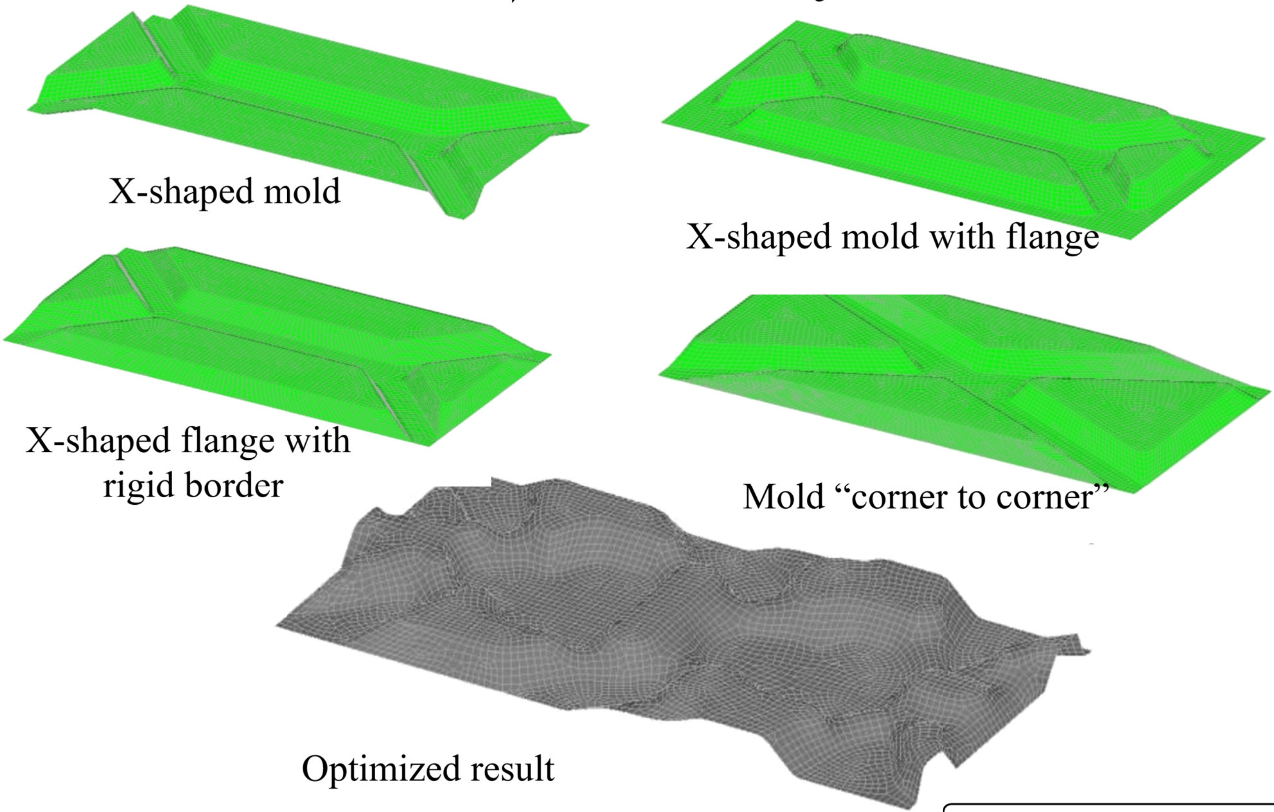
Prof. Dr. Emilio C. Nelli Silva

Topography optimization

Relevance ➡ Ex.: Sheet subject to torsion



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Topography optimization

Comparison of performance amongst the solutions

Solution	**Max. Deflection**	**Max. Stress**
Optimized	1.17	196
X-shaped	2.23	267
X-shaped with flange	4.41	644
X-shaped with rigid border	10.57	520
Corner-to-corner	6.47	434

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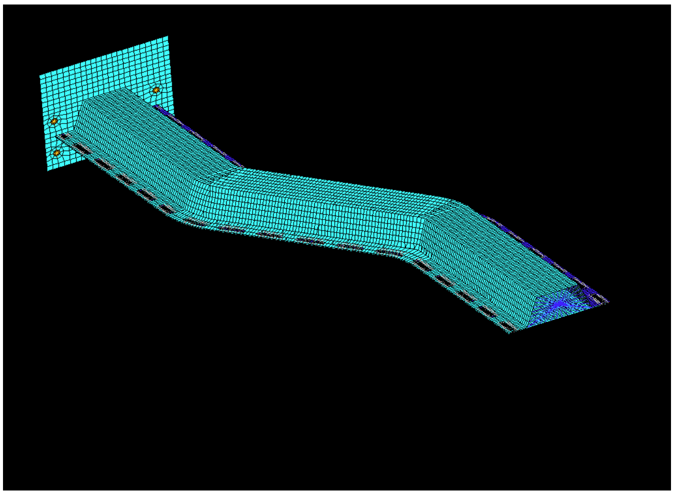
Application in optimization of welds

- The domain occupied by the weld is discretised by finite elements, in the case of continuous welding (ACM), as much as in the case of point by point (MIG)
- This domain is defined as project domain in topology optimization.

Example

- Model: fixed beam subject to flexion and torsion.
- Initial configuration: 34 MIG welds - weld domain was discretised by 260 solid elements.

Objective: Keep the same original stiffness, restricting the volume fraction of solid elements that comprise the weld in 30%.



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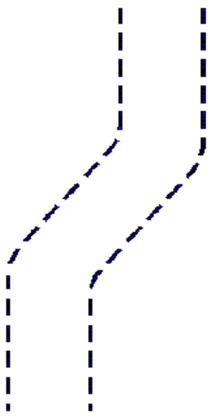
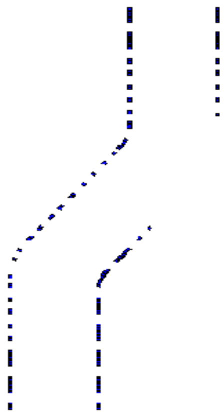
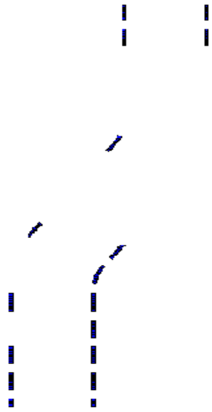
Results

Start domain	Weld topology (optimized configuration, displacement and compliance were kept)	Final "Layout" of the weld (displacement and compliance were kept)







34 welds MIG (260 elements)	Reduction in 40% the weld quantity	17 weld MIG Reduction in 50%

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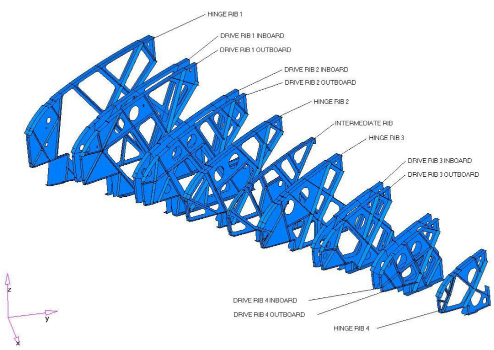
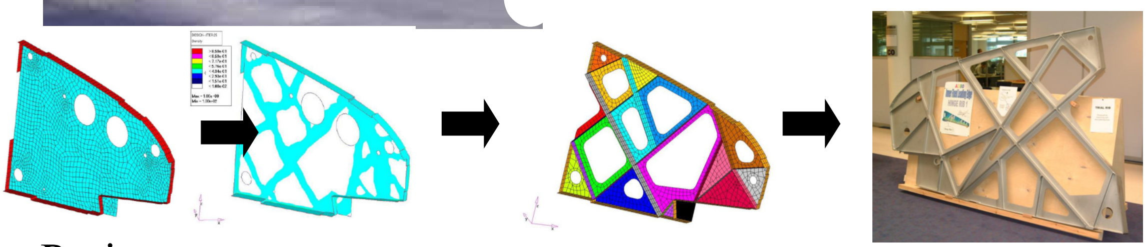
Industrial applications

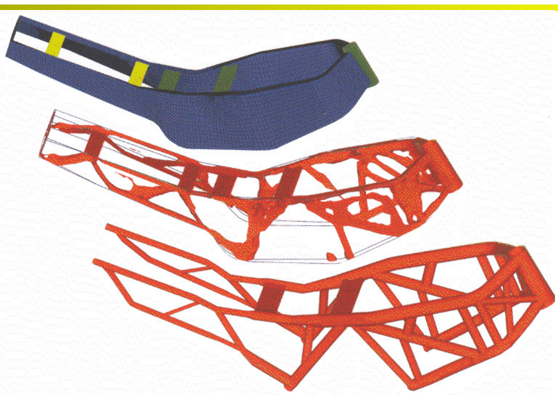
Application: Airbus A380



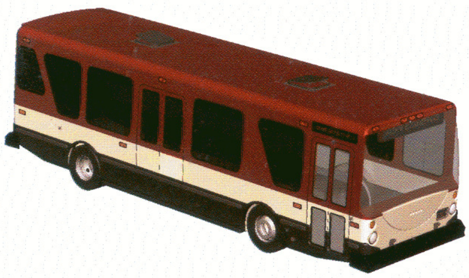


Project domain → OT solution → Verification and interpretation → Manufacturing

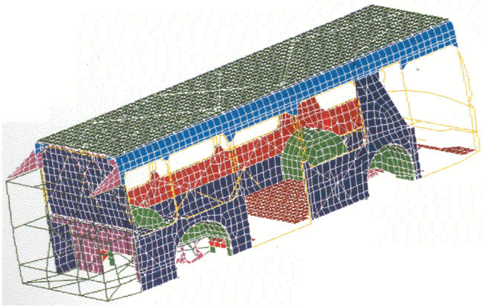
Industrial applications

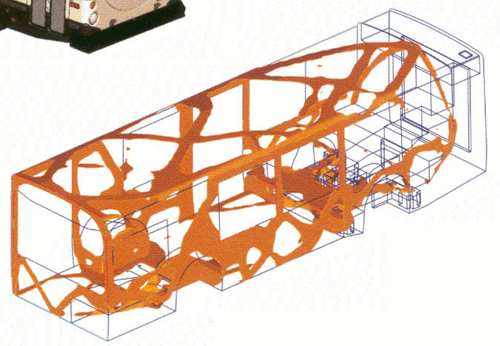


Motorcycle chassis



Bus bodywork

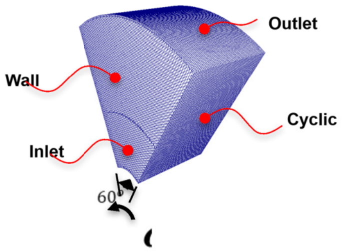


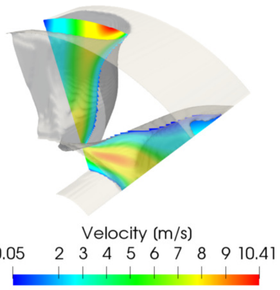


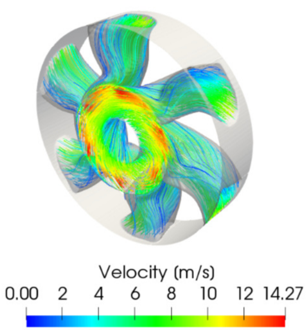
Topology Optimization of Fluids

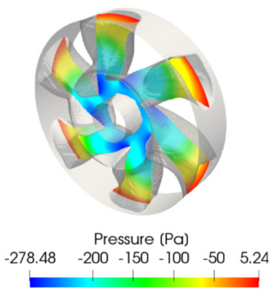
Impeller Optimization

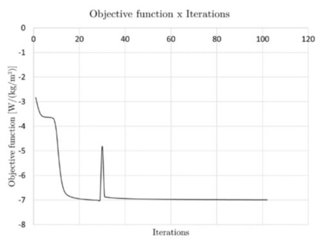
Numerical examples: 60º sector + 50% volume constraint + 1000 rpm











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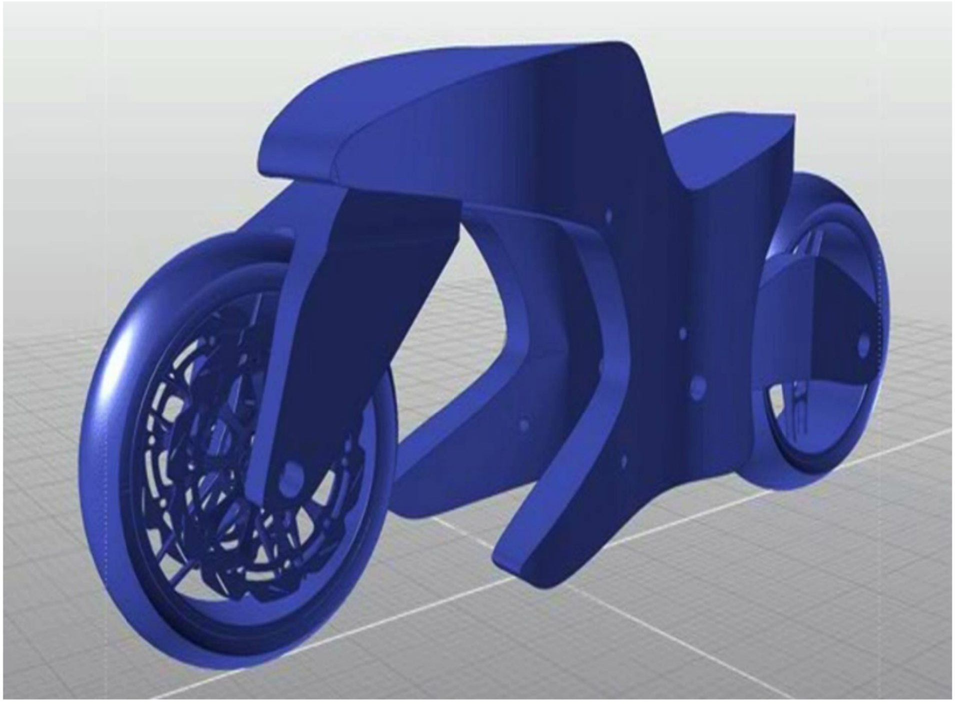
Optimized Project in Engineering

Vehicle



Large Scale Structures

Qatar Convention Center (2008)

Dubai

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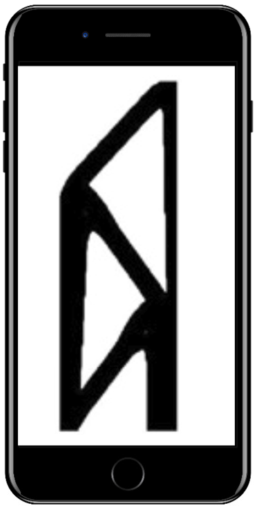
Optimized Project in Engineering

Soccer stadium

Optimized Project in Engineering

Optimization using Smartphones

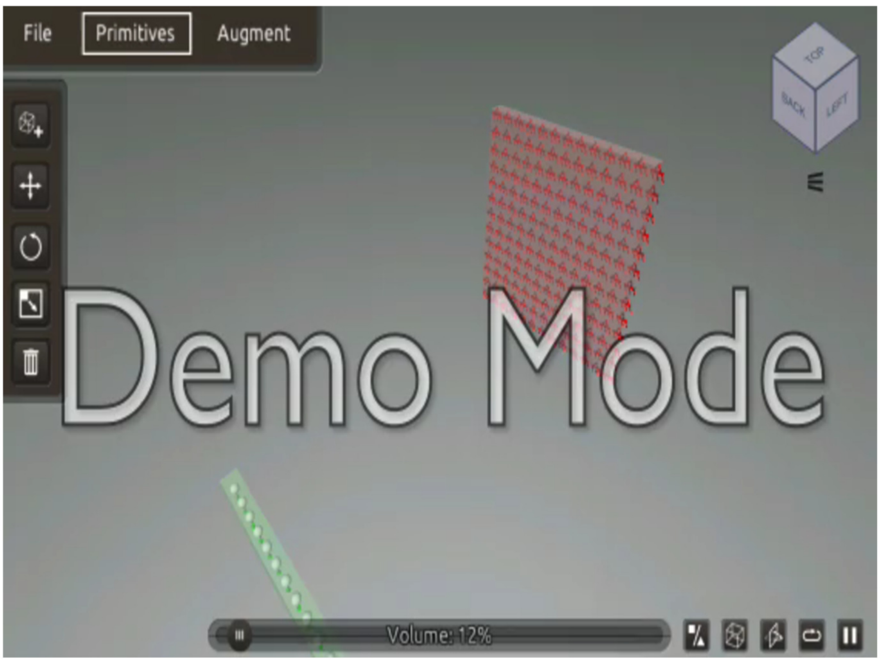
TopOpt:

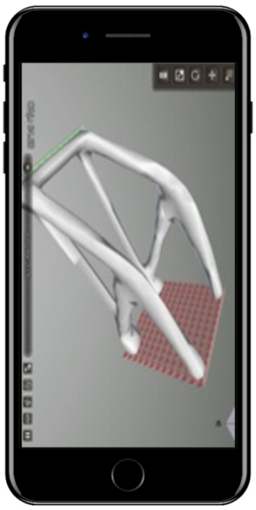


http://www.topopt.mek.dtu.dk

Optimization using Smartphones

TopOpt 3D:

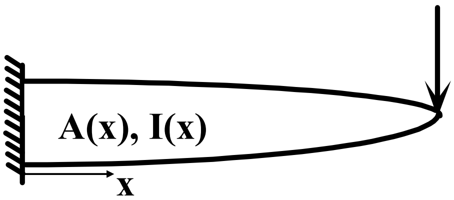


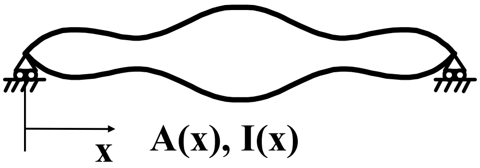


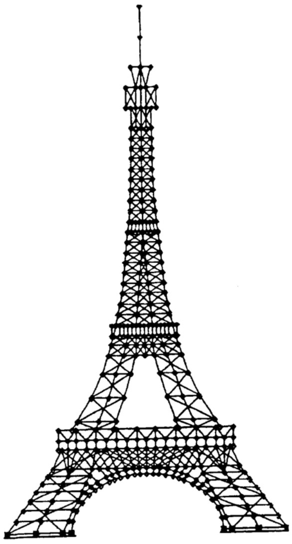
http://www.topopt.mek.dtu.dk

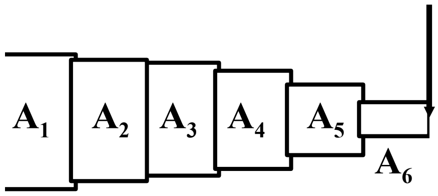
Basic definitions

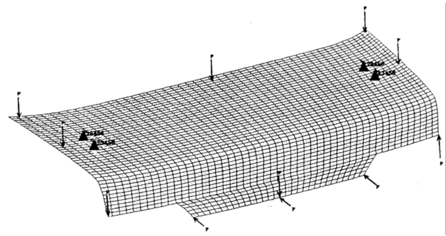
- **Classification of the Types of Structures in Optimization:**











Continuous Structures | **Discrete structures** | **Continuous cases treated as discrete**

Basic definitions

- Project variables ("Design Variables"): parameters that might be altered in the optimization procedure.

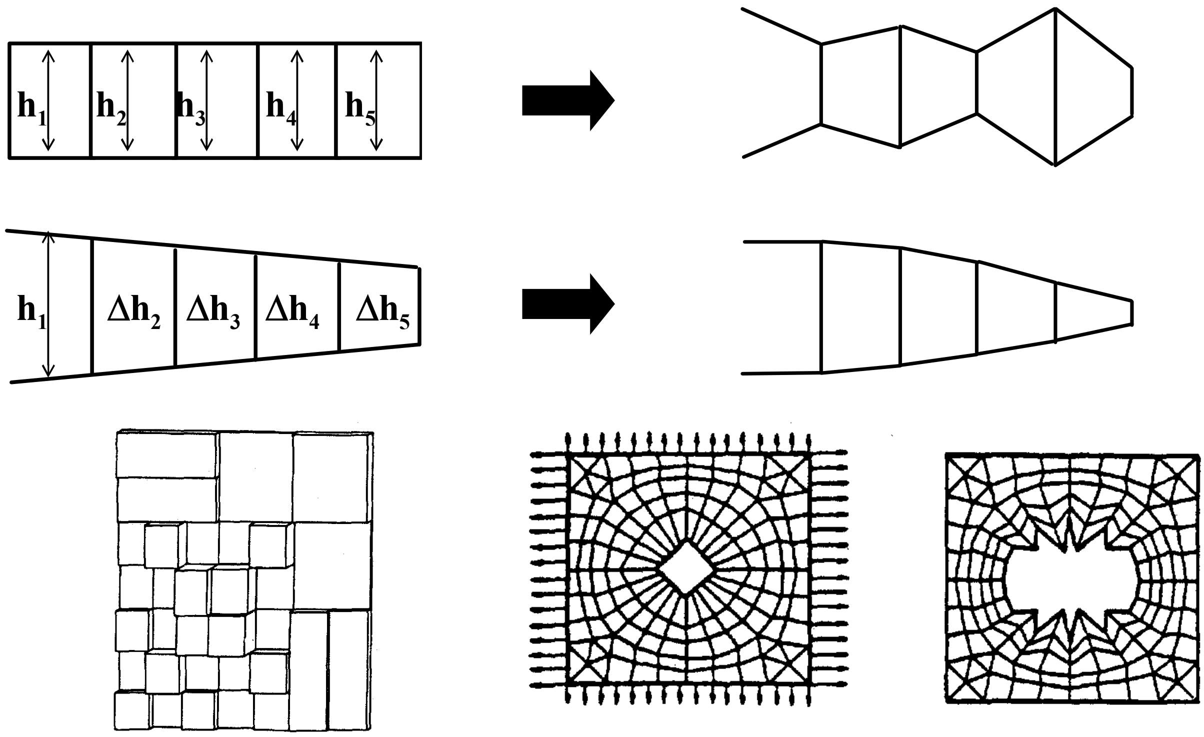
* Continuous variables (more usual)
- Distributed parameter: A(x) ➡ Area function
- Discrete parameter: $A_1, A_2, \ldots, A_n$ where A_i assume any real value.

* Discrete variables: $A_i \in \{1 \quad 1{,}5 \quad 3{,}2 \quad 5{,}7\}$ (isolated values)

The solution employs methods based on the theory of Integer Programming ➡ Complex algorithms and problems of difficult solution.

Basic definitions

The choice of project variables is critical for the success of the optimization.



Sheet

Basic definitions

- Objective function (OF): specifies what is desirable to be optimized.

Types of OF:

* Simple: max f

* Multi-criteria or Multiobjective: $\begin{cases} \max\ w_1 f_1 + w_2 f_2 \\ \max\ \dfrac{f_1}{f_2} \end{cases}$

Important: $\max f \equiv \min -f \equiv \min \dfrac{1}{f}$

$\max |x| \equiv \max x^2$; $\max k{*}f \equiv \max f$; $\max k+f \equiv \max f$

Formulation of objective function

We must be capable of expression what we want mathematically. Ex.: displacements, stiffness, resonance frequency, mechanical stress, etc...

Beware: How can we express objective functions like drivability and safety of a vehicle??

Basic definitions

- Constraints: limitations of the optimized solution.

Types:
- * Lateral: $x_{min_i} \leq x_i \leq x_{max_i}$ $i = 1..n$
- * Inequality: $g_j(\mathbf{x}) \geq 0$ $j = 1..n_g$
- * Equality: $h_k(\mathbf{x}) = 0$ $k = 1..n_e$

where: $\mathbf{x} = \{x_1, x_2, x_3, ..., x_n\}$

- With respect to $h_j(\mathbf{x}) = 0$:
  - * Complex implementation in some nonlinear optimization algorithms;
  - * In general, it may be transformed: $h_k(\mathbf{x}) \leq 0$ and $h_k(\mathbf{x}) \geq 0$

- Constraints must be normalized to avoid numerical issues:

$$g_j(\mathbf{x}) \leq g_{max_j} \Rightarrow \frac{g_j(\mathbf{x})}{g_{max_j}} \leq 1 \Rightarrow \bar{g}_j(\mathbf{x}) - 1 \leq 0$$

- One must refrain from using large n_g and n_e

Basic definitions

Constraints classification:
- * Local: mechanical stress and displacements at a point;
- * Global: material volume, stiffness and response frequency

State of an inequality constraint:
- * Active: $g_j(\mathbf{x}) = 0$
- * Inactive: $g_j(\mathbf{x}) > 0$

At the end of the optimization ➡ active constraints. Inactive constraints may be removed from the initial optimization problem.

Measure of importance of a constraint:
Lagrange Multipliers (λ_i)

If: $\lambda_i = 0 \Rightarrow$ inactive constraint (unnecessary)
$\lambda_i \neq 0 \Rightarrow$ active constraint (necessary)

} Single out important constraints

Basic definitions

Formulation of the Optimization Problem:

$$\begin{aligned} &\text{Minimize} && f(\mathbf{x}) && \\ &\text{Subject to} && g_j(\mathbf{x}) \geq 0 && j = 1..n_g \\ & && h_k(\mathbf{x}) = 0 && k = 1..n_e \end{aligned}$$

An optimization problem is linear if:

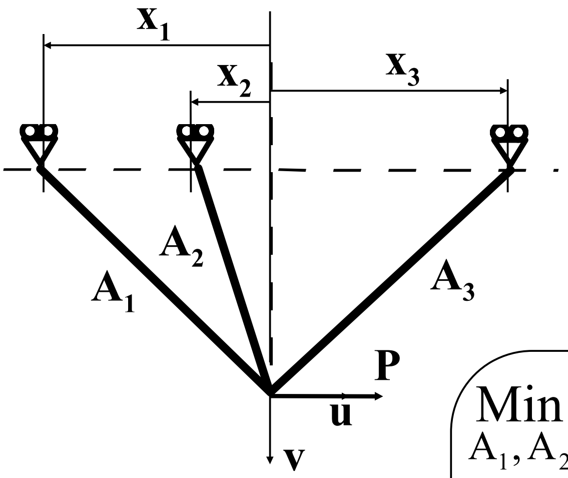
$$f(\mathbf{x}) = c_1x_1 + c_2x_2 + \ldots + c_nx_n; \qquad g_j(\mathbf{x}) = d_1x_1 + d_2x_2 + \ldots + d_nx_n;$$

and: $h_k(\mathbf{x}) = b_1x_1 + b_2x_2 + \ldots + b_nx_n$

Otherwise it is nonlinear.

If linear, it may be solved with a linear programming method.

Example



$$L_i = \sqrt{x_i^2 + 100}$$

$$\underset{A_1, A_2, A_3, x_1, x_2, x_3}{\text{Min}} \; m = 0{,}29\left(A_1L_1 + A_2L_2 + A_3L_3\right)$$

Subject to

Constraints of balance equations (Castigliano Theorem):

$$\begin{cases} k_{11}(A_i, x_i, L_i)u + k_{12}(A_i, x_i, L_i)v - 10.000 = 0 \\ k_{12}(A_i, x_i, L_i)u + k_{22}(A_i, x_i, L_i)v = 0 \end{cases}$$

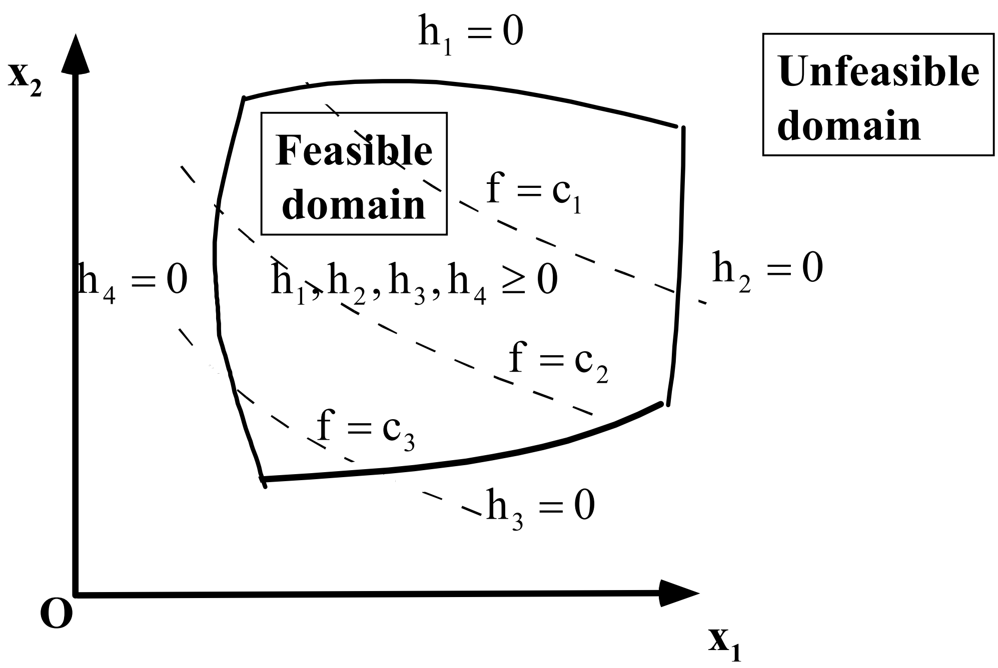
Constraints of mechanical stress:

$$\begin{cases} 30.000 - \sigma_i(u, v, x_i, L_i) \geq 0 \\ \sigma_i(u, v, x_i, L_i) + 30.000 \geq 0 \end{cases}$$

$$A_i - 0{,}1 \geq 0 \qquad i = 1,2,3$$

Basic definitions

- Feasible and unfeasible domain:



Basic definitions

- Location of the Optimal Point