Introdução à Física das Partículas Elementares

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(buscar: física das partículas elementares)

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Capítulo 7

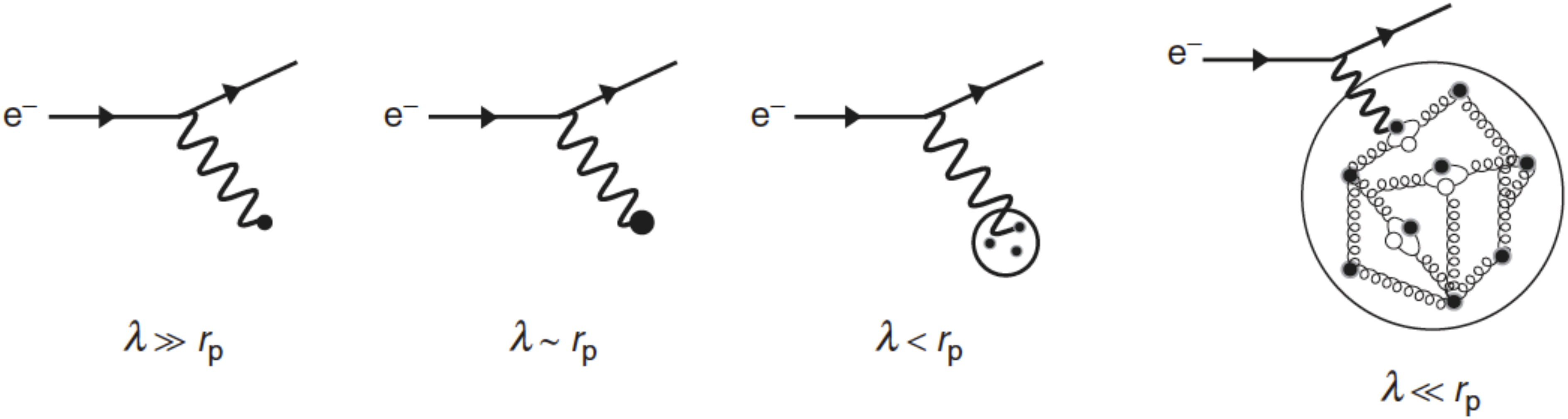
Espalhamento elétron-próton

Porque ?

Para descobrir do que e como o próton é feito !!!

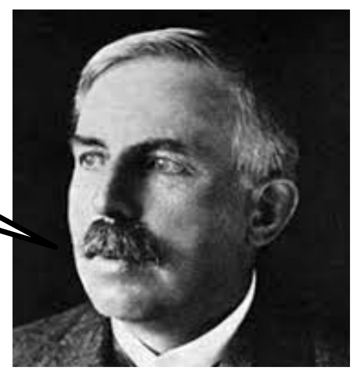
Qual é o raio do próton ?

q^μ 4-momento do fóton $\longrightarrow$ $\lambda \simeq \dfrac{1}{\sqrt{q^2}}$ comprimento de onda do fóton



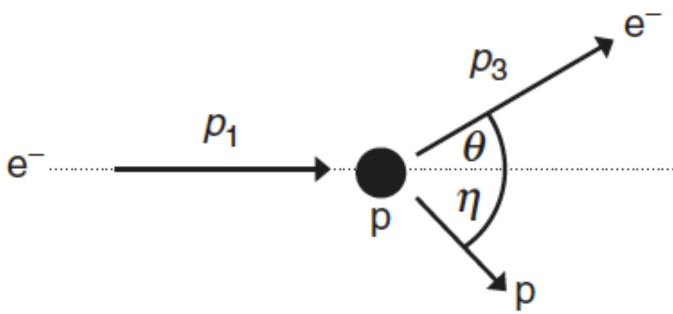
Energia

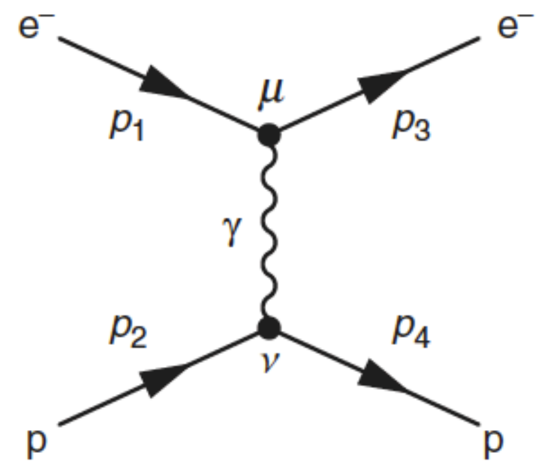
O elétron é um "microscópio" !



Rutherford

Espalhamentos Rutherford e Mott





$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2}\left[\overline{u}(p_3)\gamma^{\mu}u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_4)\gamma^{\nu}u(p_2)\right]$$

$$\boxed{\langle|\mathcal{M}_{fi}^2|\rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4(\theta/2)}\left[1 + \beta_e^2\gamma_e^2 \cos^2\frac{\theta}{2}\right]}$$

$$\langle|\mathcal{M}_{fi}^2|\rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4(\theta/2)}\left[1+\beta_e^2\gamma_e^2\cos^2\frac{\theta}{2}\right]$$

Espalhamento Rutherford $E << m_e << m_p$ $\beta_e\gamma_e \ll 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{\alpha^2}{16E_K^2 \sin^4(\theta/2)}.$$

Elétron não relativístico

Próton puntiforme parado

Espalhamento Mott $m_e \ll E \ll m_p$ $\beta_e\gamma_e >> 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)}\cos^2\frac{\theta}{2}.$$

Elétron relativístico

Próton puntiforme parado

Fator de Forma

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{\text{pt}} F(\mathbf{q}^2)$$

$F(\mathbf{q}^2)$ = Fator de forma

$$F(\mathbf{q}^2) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}\, \mathrm{d}^3\mathbf{r}$$

$\rho(\mathbf{r}')$ = Distribuição espacial de carga

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \cos^2 \frac{\theta}{2}$$

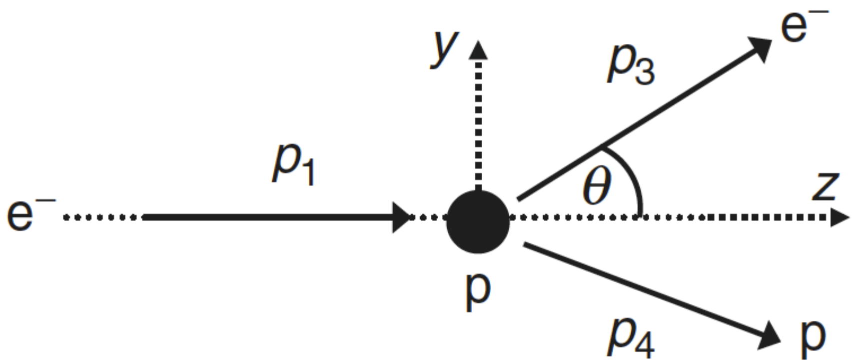
$$\Downarrow$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} \to \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \cos^2\left(\frac{\theta}{2}\right) \left|F(\mathbf{q}^2)\right|^2$$

Espalhamento Elástico Relativístico

Até aqui próton " parado" ! Aproximação boa se : $|\mathbf{q}| \ll m_p$

Caso geral:



$$\begin{cases} p_1 = (E_1, 0, 0, E_1), \\ p_2 = (m_p, 0, 0, 0), \\ p_3 = (E_3, 0, E_3 \sin\theta, E_3 \cos\theta) \\ p_4 = (E_4, \mathbf{p}_4). \end{cases}$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{8e^4}{(p_1 - p_3)^4} \left[(p_1.p_2)(p_3.p_4) + (p_1.p_4)(p_2.p_3) - m_p^2 (p_1.p_3) \right]$$

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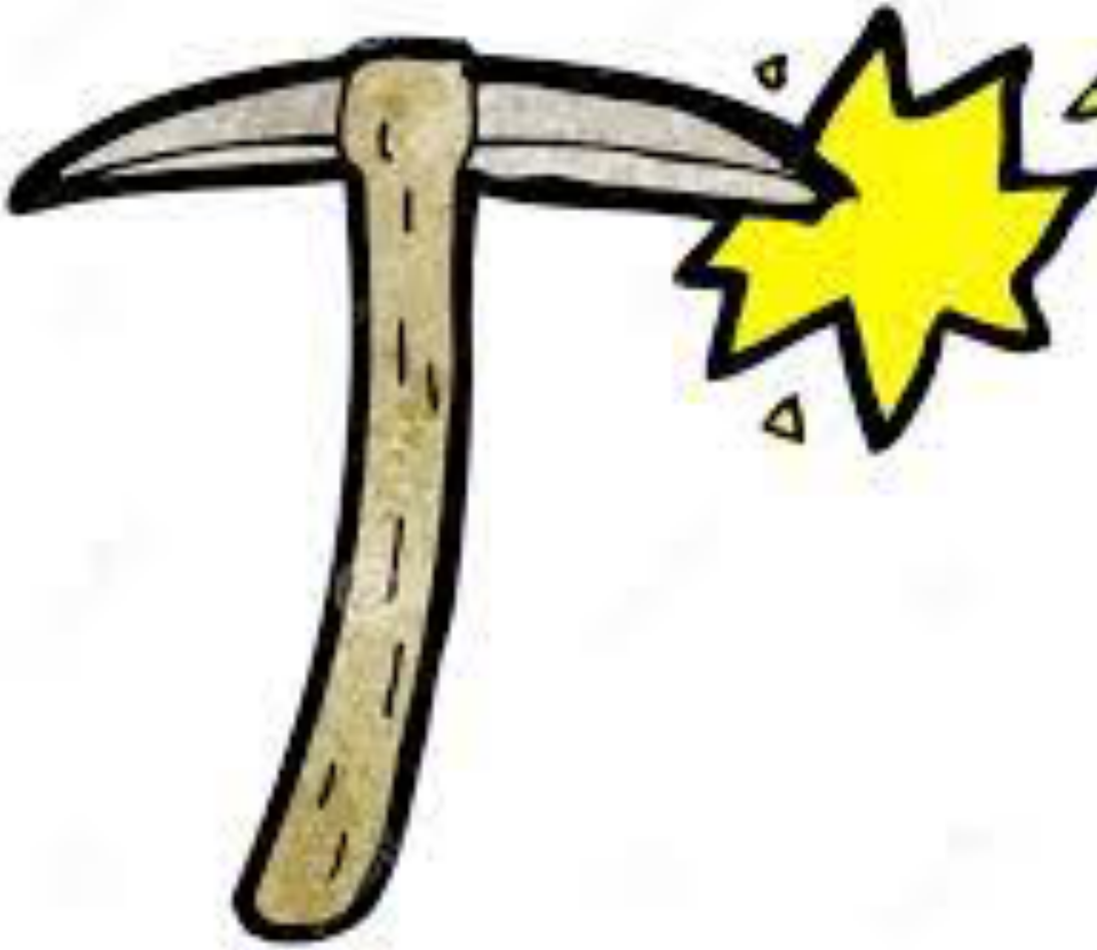
O próton final não é observado. Escrevemos a amplitude em função das variáveis do elétron. Conservação da energia e momento :

$$p_4 = p_1 + p_2 - p_3$$



$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{8e^4}{(p_1 - p_3)^4}\left[(p_1.p_2)(p_3.p_4) + (p_1.p_4)(p_2.p_3) - m_{\mathrm{p}}^2(p_1.p_3)\right]$$

$$p_1 = (E_1, 0, 0, E_1),$$
$$p_2 = (m_{\mathrm{p}}, 0, 0, 0),$$
$$p_3 = (E_3, 0, E_3 \sin\theta, E_3 \cos\theta)$$
$$p_4 = (E_4, \mathbf{p}_4).$$

Jogamos fora

$$p_1 \cdot p_1 = p_3 \cdot p_3 = m_{\mathrm{e}}^2$$

$$p_2 \cdot p_3 = E_3 m_{\mathrm{p}} \qquad p_1 \cdot p_2 = E_1 m_{\mathrm{p}} \qquad p_1 \cdot p_3 = E_1 E_3 (1 - \cos\theta).$$

Agora usando $p_4 = p_1 + p_2 - p_3$

$$p_3 \cdot p_4 = p_3 \cdot p_1 + p_3 \cdot p_2 - p_3 \cdot p_3 = E_1 E_3 (1 - \cos\theta) + E_3 m_{\mathrm{p}},$$

$$p_1 \cdot p_4 = p_1 \cdot p_1 + p_1 \cdot p_2 - p_1 \cdot p_3 = E_1 m_{\mathrm{p}} - E_1 E_3 (1 - \cos\theta),$$

$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{8e^4}{(p_1 - p_3)^4} m_p E_1 E_3 \left[(E_1 - E_3)(1 - \cos\theta) + m_p[(1 + \cos\theta)\right]$$

$$(1 - cos\,\theta) = 2\,sin^2\frac{\theta}{2} \qquad 1 + cos\,\theta = 2\,cos^2\frac{\theta}{2}$$

$$= \frac{8e^4}{(p_1 - p_3)^4} 2 m_p E_1 E_3 \left[(E_1 - E_3)\sin^2\frac{\theta}{2} + m_p \cos^2\frac{\theta}{2}\right].$$

$$q^2 = (p_1 - p_3)^2 = \cancel{p_1^2} + \cancel{p_3^2} - 2p_1 \cdot p_3 \approx -2E_1E_3(1 - \cos\theta)$$

$$q^2 = -4E_1E_3\sin^2\frac{\theta}{2}. \qquad Q^2 \equiv -q^2 = 4E_1E_3\sin^2\frac{\theta}{2} \qquad q^4 = 16\,E_1^2\,E_3^2\,sin^4\frac{\theta}{2}$$

$$\langle\mathcal{M}_{fi}\rangle = \frac{m_p\,e^4}{E_1\,E_3\,sin^4\frac{\theta}{2}}\left[(E_1 - E_3)\,sin^2\frac{\theta}{2} + m_p\,cos^2\frac{\theta}{2}\right]$$

$$\langle \mathcal{M}_{fi} \rangle = \frac{m_p\, e^4}{E_1\, E_3\, sin^4 \frac{\theta}{2}} \left[(E_1 - E_3)\, sin^2 \frac{\theta}{2} + m_p\, cos^2 \frac{\theta}{2} \right]$$

Vamos sumir com esse $(E_1 - E_3)$

$$q = p_4 - p_2 \qquad \cancel{p_4^2} = (q + p_2)^2 = q^2 + 2q \cdot p_2 + \cancel{p_2^2} \qquad p_2^2 = p_4^2 = m_\mathrm{p}^2$$

$$\left.\begin{aligned} q \cdot p_2 &= -q^2/2 \\ q \cdot p_2 &= (p_1 - p_3) \cdot p_2 = m_\mathrm{p}(E_1 - E_3) \end{aligned}\right\} \quad \boxed{E_1 - E_3 = -\frac{q^2}{2m_\mathrm{p}} = \frac{Q^2}{2m_\mathrm{p}}}$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{m_\mathrm{p}^2 e^4}{E_1 E_3 \sin^4(\theta/2)} \left[\cos^2 \frac{\theta}{2} + \frac{Q^2}{2m_\mathrm{p}^2} \sin^2 \frac{\theta}{2} \right]$$

$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{m_p^2 e^4}{E_1 E_3 \sin^4(\theta/2)}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$

Lembramos que

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{m_p E_1}\right)^2 |\mathcal{M}_{fi}|^2$$

$$e^2 = 4\pi\alpha$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)}\frac{E_3}{E_1}\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right)}$$

Próton puntiforme !

$$E_3 = \frac{E_1 m_p}{m_p + E_1(1-\cos\theta)}$$

$$Q^2 = \frac{2m_p E_1^2(1-\cos\theta)}{m_p + E_1(1-\cos\theta)}$$

A seção de choque é função apenas do ângulo teta de saída do elétron !

Nota de rodapé :

$$q^2 = (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2p_1 \cdot p_3 \approx -2E_1E_3(1 - \cos\theta),$$

$$E_1 - E_3 = -\frac{q^2}{2m_\mathrm{p}} = \frac{Q^2}{2m_\mathrm{p}},$$

$$-2m_\mathrm{p}(E_1 - E_3) = -2E_1E_3(1 - \cos\theta), \qquad \longrightarrow \qquad E_3 = \frac{E_1 m_\mathrm{p}}{m_\mathrm{p} + E_1(1 - \cos\theta)}.$$

$$Q^2 = \frac{2m_\mathrm{p}E_1^2(1 - \cos\theta)}{m_\mathrm{p} + E_1(1 - \cos\theta)}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2} \sin^2\frac{\theta}{2} \right)$$

Próton puntiforme !

Incluimos o tamanho finito próton introduzindo dois fatores de forma:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

Fórmula de Rosenbluth

$\left\{ \begin{array}{ll} G_E(Q^2) & \text{Fator de forma elétrico} \\ G_M(Q^2) & \text{Fator de forma magnético} \end{array} \right.$

It can be shown that

(Thomson feio...)

$$Q^2 \equiv -q^2 = 4E_1 E_3 \sin^2\frac{\theta}{2} \qquad \tau = \frac{Q^2}{4m_p^2}$$

Para pequenos valores de Q^2 :

$$\begin{cases} Q^2 = -q^2 = \mathbf{q}^2 - (E_1 - E_3)^2 \\ E_1 - E_3 = -\dfrac{q^2}{2m_\mathrm{p}} = \dfrac{Q^2}{2m_\mathrm{p}}, \end{cases}$$

$$Q^2\left(1 + \frac{Q^2}{4m_\mathrm{p}^2}\right) = \mathbf{q}^2$$

$$Q^2 \ll 4m_\mathrm{p}^2$$

$$Q^2 \approx \mathbf{q}^2$$

A transf. de Fourier pode ser interpretada como distribuição de carga

$$\begin{cases} G_E(Q^2) \approx G_E(\mathbf{q}^2) = \displaystyle\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})\mathrm{d}^3\mathbf{r} & G_E(0) = \displaystyle\int \rho(\mathbf{r})\,\mathrm{d}^3\mathbf{r} = 1 \\ G_M(Q^2) \approx G_M(\mathbf{q}^2) = \displaystyle\int e^{i\mathbf{q}\cdot\mathbf{r}}\mu(\mathbf{r})\mathrm{d}^3\mathbf{r} & G_M(0) = \displaystyle\int \mu(\mathbf{r})\,\mathrm{d}^3\mathbf{r} = +2.79 \end{cases}$$

2.79 ?

Momento magnético e momento magnético anômalo:

$$\vec{\mu} = 2\,\mu_N\,\vec{S} \qquad \mu_N = \frac{e\hbar}{2m_p} \qquad \boldsymbol{\mu} = \frac{q}{m}\mathbf{S},$$

O que observamos: $\boldsymbol{\mu} = 2.79\dfrac{e}{m_p}\mathbf{S}$

$\mu_N \to 2.792\,\mu_N$ Momento magnético anômalo !

Consequência de o próton ser feito de quarks

Medidas dos Fatores de Forma

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

Vamos colocar em evidência

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \left(\frac{E_3}{E_1}\right) \cos^2\frac{\theta}{2}.$$

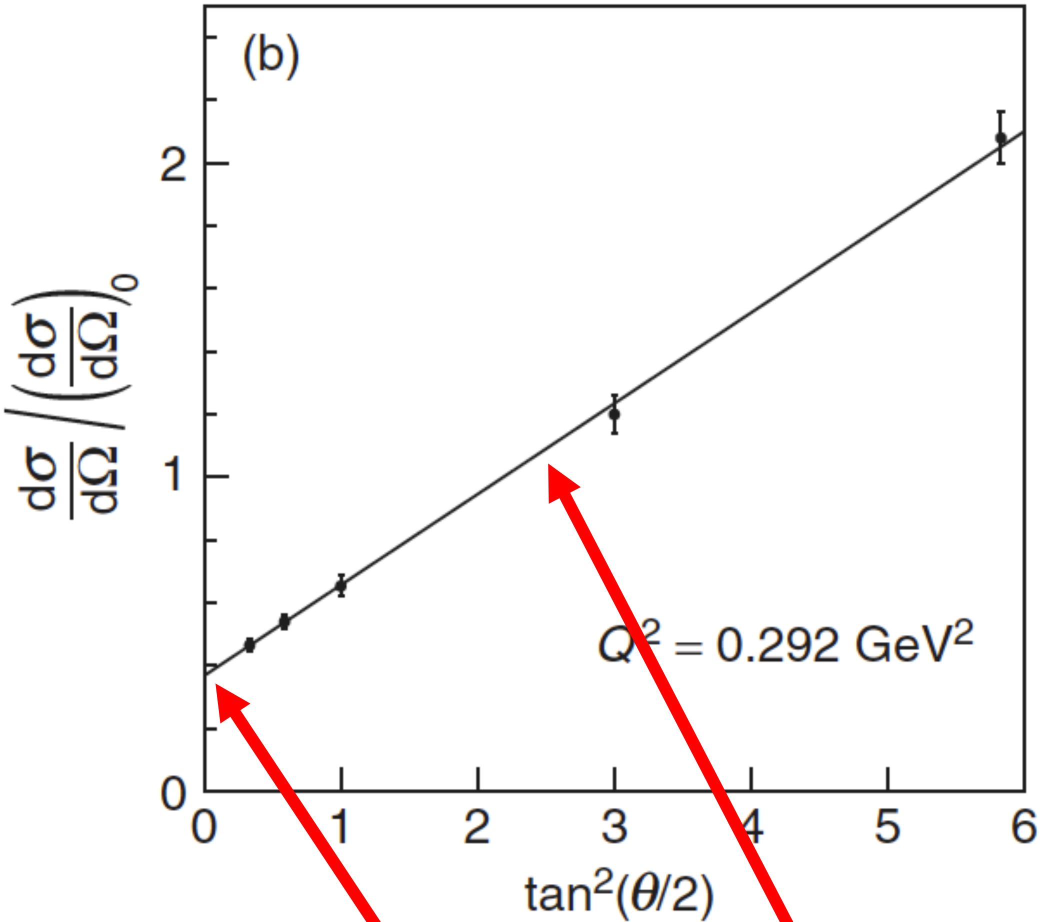
$$\frac{d\sigma}{d\Omega} = \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} + 2\tau G_M^2 \tan^2\frac{\theta}{2} \right) \cdot \left(\frac{d\sigma}{d\Omega}\right)_0$$

$$\tau = \frac{Q^2}{4m_p^2}$$

$$Q^2 \equiv -q^2 = 4E_1 E_3 \sin^2\frac{\theta}{2}$$

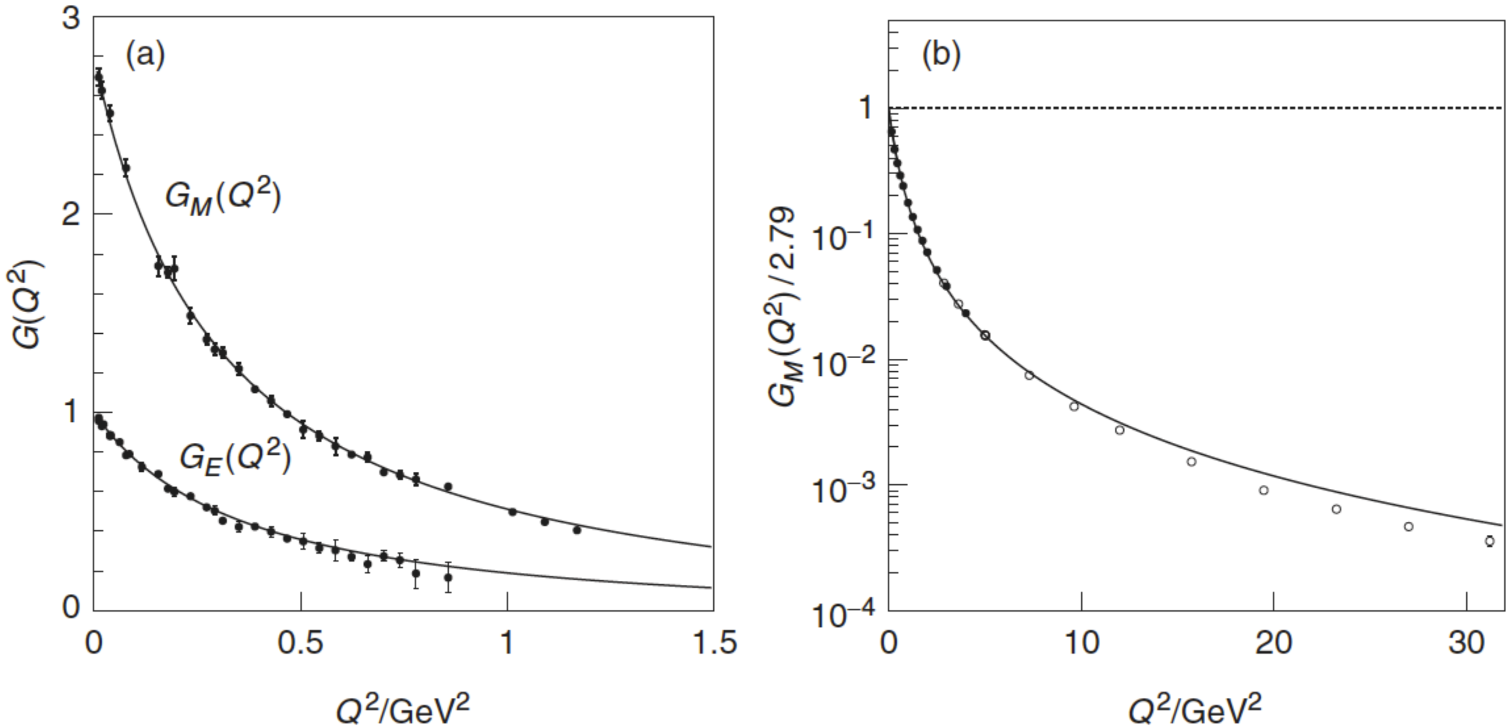
$$\tau \ll 1 \qquad \frac{d\sigma}{d\Omega} \bigg/ \left(\frac{d\sigma}{d\Omega}\right)_0 \approx G_E^2$$

$$\tau \gg 1 \qquad \frac{d\sigma}{d\Omega} \bigg/ \left(\frac{d\sigma}{d\Omega}\right)_0 \approx \left(1 + 2\tau \tan^2\frac{\theta}{2}\right) G_M^2.$$



$$\frac{d\sigma}{d\Omega} = \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right) \cdot \left(\frac{d\sigma}{d\Omega} \right)_0$$

Para Q^2 fixo determinamos G_E e G_M



$$G_M(Q^2) = 2.79 G_E(Q^2) \approx 2.79 \frac{1}{(1 + Q^2/0.71\,\mathrm{GeV}^2)^2}$$

dipolo

Fazemos a transformada de Fourier e obtemos a distribuição de carga :

$$\rho(\mathbf{r}) \approx \rho_0 e^{-\mathrm{r}/a} \qquad a \approx 0.24\,\mathrm{fm}$$

Com a distribuição de carga determinamos o raio do próton:

$$< r >= \int r \, \rho(\mathbf{r}) \, d^3\mathbf{r}$$

$$< r >= 0.879 \pm 0.005 \pm 0.006 \;\; fm$$

(em espalhamentos elétron - próton)

Em 2010 surgiu uma nova técnica baseada em espectroscopia

$$< r >= 0.84184 \pm 0.00067 \;\; fm$$

A discrepância é muito maior do que os erros: "próton radius puzzle"

O raio do próton continua em debate !

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The proton radius puzzle – 9 years later

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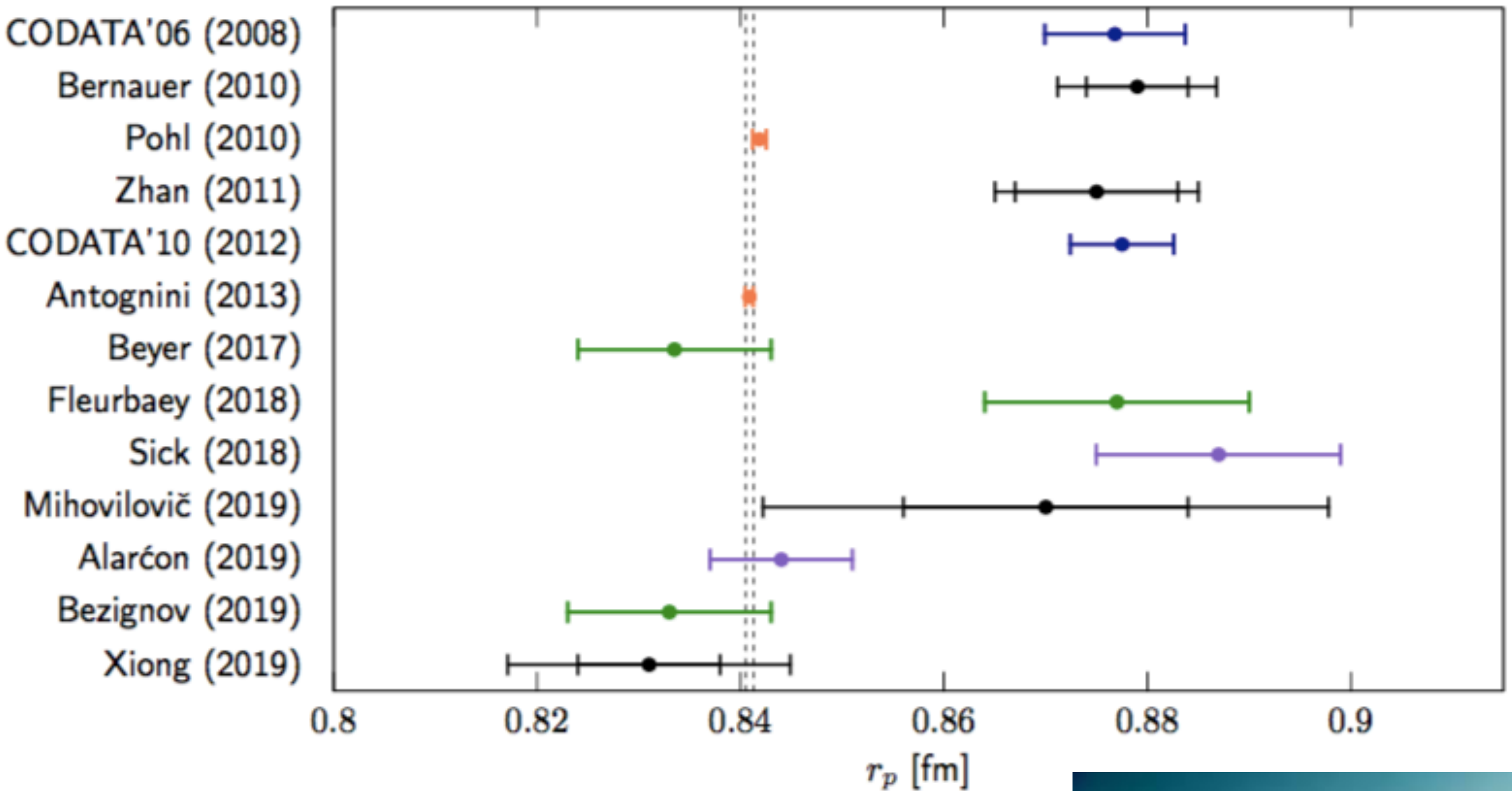
Abstract. High-precision measurements of the proton radius via scattering, electric hydrogen spectroscopy and muonic hydrogen spectroscopy do not agree on the level of more than 5 σ. This proton radius puzzle persists now for almost a decade. This paper gives a short summary over the progress in the solution of the puzzle as well as an overview over the planned experiments to finally solve this puzzle at the interface of atomic and nuclear physics.

O raio do próton continua em debate !

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