1.) f.) $\overbrace{(m+n)+p = m+(n+p)}^{(*)}$ Associativa

$X = \{p \in \mathbb{N} : (m+n)+p = m+(n+p)\}$

! Sr. Colucci ($0 \in \mathbb{N}$)

(*) $S : \mathbb{N} \to \mathbb{N}$ Inj ($S(n) = n^+$)

$\nexists\, n \in \mathbb{N}$ tq $S(n) = 0$

$0 \overset{?}{\in} X \Rightarrow (m+n)+0 = (m+n) = m+n$
$= m+(n+0)$

$0 \in X$. (Base)

HI $\Rightarrow$ PI : Suponha que $p \overset{?}{\in} X$. Queremos provar que $S(p) \in X$

$\underbrace{(m+n)}_{\in \mathbb{N}?} + \underbrace{S(p)}_{\in \mathbb{N}\,\checkmark}$

! $S(n) = n+1 \Rightarrow n + S(m) = n+m+1 = S(n+m)$

Obs: $n, m \in \mathbb{N}$ $\qquad n = \underbrace{1+1+\dots+1}_{n \text{ vez}}$

$m+n \in \mathbb{N}$

$m + \underbrace{1+1+\dots+1}_{n \text{ vez}} = S(m)+1+\underbrace{1+\dots+1}_{n-1 \text{ vz}}$

$\underbrace{S(S(m))}_{S^2(m)} + \underbrace{1+1+\dots+1}_{n-2 \text{ vz}} \Rightarrow \underbrace{m+n}_{\in \mathbb{N}} = \underbrace{S^n(m)}_{\in \mathbb{N}}$ (*)

Sabemos q/ $p \in X$ e queremos provar que $S(p) \in X$

$$(m+n) + S(p) = m + (n + S(p))$$

$$\underbrace{(m+n)}_{\in \mathbb{N}} + \underbrace{S(p)}_{\in \mathbb{N}} = S((m+n)+p)$$

$\| \; HI(p \in X)$

$m + (n+p)$

$$\Rightarrow S((m+n)+p) = S(m + (n+p)) = m + S(n+p)$$

$$= m + (n + S(p))$$

Provamos q/ $(m+n) + S(p) = m + (n + S(p))$

$S(p) \in X$

Como $\forall p \in X \Rightarrow S(p) \in X$ pelo PIF: $\boxed{X = \mathbb{N}}$ ■