Introdução à Física das Partículas Elementares

4300422

edisciplinas.if.usp.br

(buscar: física das partículas elementares)

Fernando S Navarra

navarra@if.usp.br

Guilherme Germano

guilherme.germano@usp.br

Plano do Curso

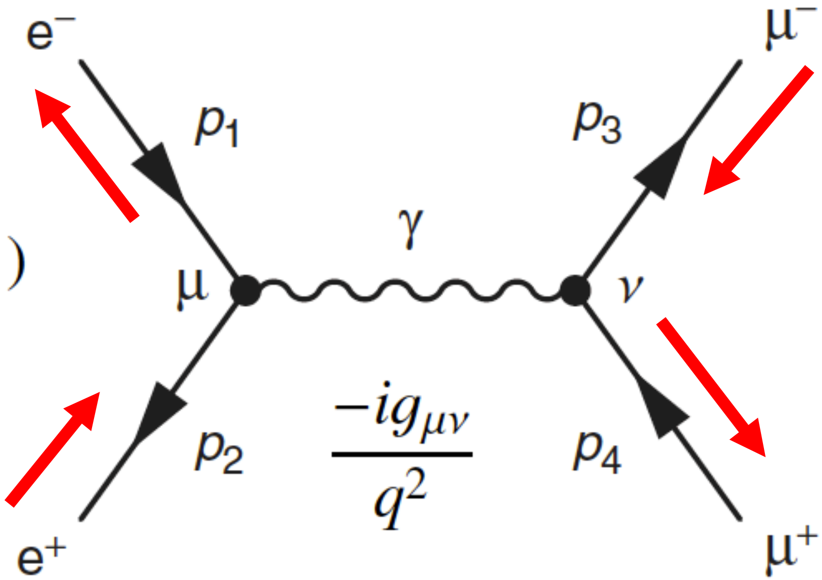
14/03	Cap. 1
16/03	Cap. 1
21/03	Cap. 2
23/03	Cap. 2
28/03	Cap. 3
30/03	Cap. 3
04/04	
06/04	
11/04	Cap. 4
13/04	Cap. 4
18/04	Cap. 4
20/04	P1
25/04	Cap. 4
27/04	Cap. 5
02/05	Cap. 6
04/05	Cap. 7
09/05	Cap. 7
11/05	Cap. 8
16/05	Cap. 8
18/05	Cap. 8
23/05	P2
25/05	Cap. 9
30/05	Cap. 9
01/06	Cap. 9
06/06	
08/06	
13/06	Cap. 10
15/06	Cap. 10
20/06	Cap. 10
22/06	Cap. 11
27/06	Cap. 11
29/06	P3
04/07	Sub

Aula 13

Capítulo 6

$$e^+e^- \rightarrow \mu^+\mu^-$$

$$\bar{v}(p_2)[ie\gamma^\mu]u(p_1)$$



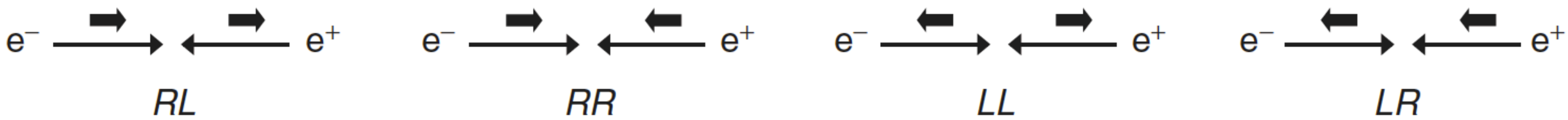
$$\bar{u}(p_3)[ie\gamma^\nu]v(p_4)$$

$$-i\mathcal{M} = [\bar{v}(p_2)\{ie\gamma^\mu\}u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\bar{u}(p_3)\{ie\gamma^\nu\}v(p_4)]$$

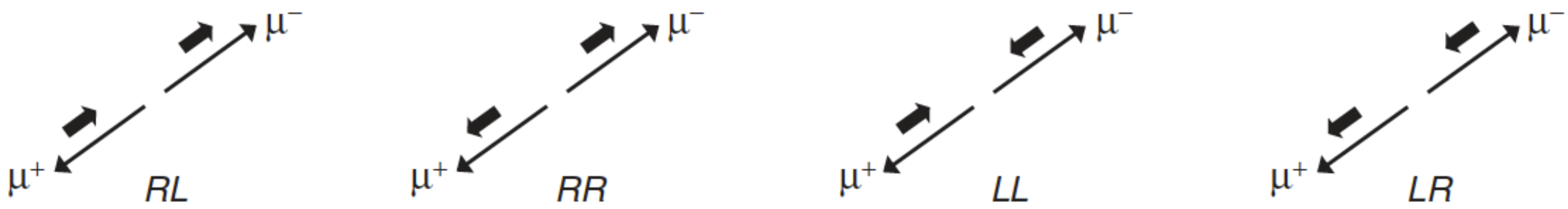
$$\mathcal{M} = -\frac{e^2}{q^2}g_{\mu\nu}\,[\bar{v}(p_2)\gamma^\mu u(p_1)][\bar{u}(p_3)\gamma^\nu v(p_4)] \quad = -\frac{e^2}{q^2}g_{\mu\nu}j_{\mathrm{e}}^\mu j_{\mu}^\nu$$

$$\boxed{j_{\mathrm{e}}^\mu = \bar{v}(p_2)\gamma^\mu u(p_1)} \qquad \boxed{j_{\mu}^\nu = \bar{u}(p_3)\gamma^\nu v(p_4)} \qquad \boxed{\mathcal{M} = -\frac{e^2}{s}j_{\mathrm{e}}\cdot j_{\mu}}$$

Helicidades no estado inicial

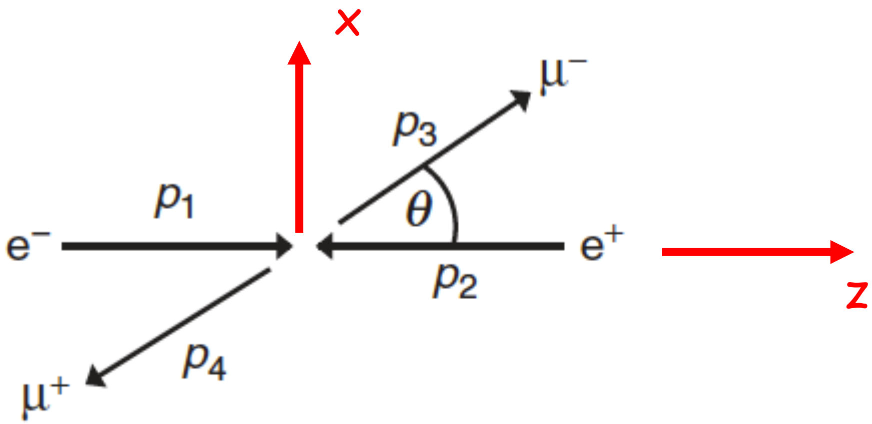


Helicidades no estado final



$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2$$

Amplitudes de Helicidade



$$p_1 = (E, 0, 0, E),$$
$$p_2 = (E, 0, 0, -E),$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta),$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta),$$

elétron :

$$u_\uparrow(p_1) = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\ u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

pósitron :

$$v_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix},\ v_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix}$$

muon

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}c\\s\\c\\s\end{pmatrix},\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix},$$

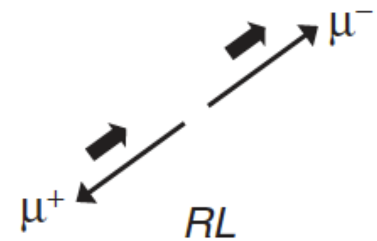
anti-muon

$$v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix}c\\s\\-c\\-s\end{pmatrix},\ v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}s\\-c\\s\\-c\end{pmatrix}$$

Para achar a amplitude precisamos calcular: $\mathcal{M} = -\frac{e^2}{s} j_e \cdot j_\mu$

Vamos calcular a corrente de mu+ e mu- : $j_\mu^\nu = \overline{u}(p_3)\gamma^\nu v(p_4)$

Vamos calcular o estado final RL : $u_\uparrow(p_3) \quad v_\downarrow(p_4)$



$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}, \qquad u_\uparrow{}^\dagger(p_3) = \sqrt{E}\,(\,c \quad s \quad c \quad s\,) \qquad v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ -c \end{pmatrix}$$

$$j_\mu^0 = \overline{u}_\uparrow(p_3)\gamma^0 v_\downarrow(p_4) = u_\uparrow{}^\dagger(p_3)\,\gamma^0\gamma^0 v_\downarrow(p_4) = u_\uparrow{}^\dagger(p_3)\,v_\downarrow(p_4)$$

$$= E(cs - sc + cs - sc) = 0,$$

Calculamos as outras componentes do quadrivetor :

$$j_{\mu}^{0} = \overline{u}_{\uparrow}(p_3)\gamma^0 v_{\downarrow}(p_4) = E(cs - sc + cs - sc) = 0,$$
$$j_{\mu}^{1} = \overline{u}_{\uparrow}(p_3)\gamma^1 v_{\downarrow}(p_4) = E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta,$$
$$j_{\mu}^{2} = \overline{u}_{\uparrow}(p_3)\gamma^2 v_{\downarrow}(p_4) = -iE(-c^2 - s^2 - c^2 - s^2) = 2iE,$$
$$j_{\mu}^{3} = \overline{u}_{\uparrow}(p_3)\gamma^3 v_{\downarrow}(p_4) = E(cs + sc + cs + sc) = 4Esc = 2E\sin\theta.$$

O quadrivetor fica : $j_{\mu,RL} = \overline{u}_{\uparrow}(p_3)\gamma^{\nu} v_{\downarrow}(p_4) = 2E(0, -\cos\theta, i, \sin\theta)$

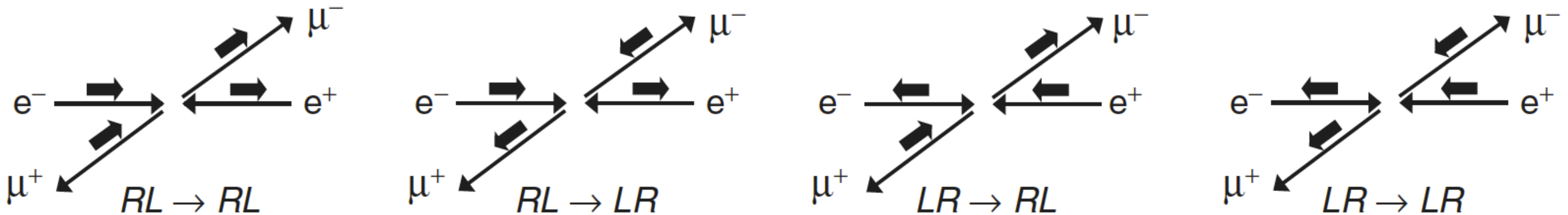
Calculando as outras combinações de helicidade temos :

$$j_{\mu,RL} = \overline{u}_{\uparrow}(p_3)\gamma^{\nu} v_{\downarrow}(p_4) = 2E(0, -\cos\theta, i, \sin\theta),$$
$$j_{\mu,RR} = \overline{u}_{\uparrow}(p_3)\gamma^{\nu} v_{\uparrow}(p_4) = (0, 0, 0, 0),$$
$$j_{\mu,LL} = \overline{u}_{\downarrow}(p_3)\gamma^{\nu} v_{\downarrow}(p_4) = (0, 0, 0, 0),$$
$$j_{\mu,LR} = \overline{u}_{\downarrow}(p_3)\gamma^{\nu} v_{\uparrow}(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

Temos que calcular as correntes do estado inicial :

$$j_{e,RL} = \bar{v}_{\downarrow}(p_2)\gamma^{\mu}u_{\uparrow}(p_1) = 2E(0,-1,-i,0),$$
$$j_{e,LR} = \bar{v}_{\uparrow}(p_2)\gamma^{\mu}u_{\downarrow}(p_1) = 2E(0,-1,i,0).$$
$$j_{e,LL} = 0$$
$$j_{e,RR} = 0$$

De 16 possibilidades apenas 4 são não nulas :



$\mathcal{M}_{RL\to RL}$ $\mathcal{M}_{RL\to LR}$ $\mathcal{M}_{LR\to RL}$ $\mathcal{M}_{LR\to LR}$

$$\mathcal{M} = -\frac{e^2}{s} j_e \cdot j_\mu$$

Vamos desenvolver o caso $\mathcal{M}_{RL\to RL}$ $\quad e^-_{\uparrow} e^+_{\downarrow} \to \mu^-_{\uparrow} \mu^+_{\downarrow}$

$$\begin{cases} j^{\mu}_{e,RL} = \bar{v}_{\downarrow}(p_2)\gamma^{\mu} u_{\uparrow}(p_1) = 2E(0,-1,-i,0), \\ j^{\nu}_{\mu,RL} = \bar{u}_{\uparrow}(p_3)\gamma^{\nu} v_{\downarrow}(p_4) = 2E(0,-\cos\theta, i, \sin\theta) \end{cases}$$

Vamos fazer o produto $j_{e,RL} \cdot j_{\mu,RL} = j_e^{\alpha}\, j_{\mu\,\alpha}$

$$\mathcal{M}_{RL\to RL} = -\frac{e^2}{s}\,[2E\ (0\,,\ -1\,,\ -i\,,\ 0\,)].[2E\ (0\,,\ cos\,\theta\,,\ -i\,,\ -sen\,\theta\,)]$$

$$= -\frac{e^2}{s}\,4E^2\ (0\ -\ cos\,\theta\ -\ 1\ +0\,) \quad = e^2\ (1\ +\ cos\,\theta)$$

$$= 4\,\pi\,\alpha\ (1\ +\ cos\,\theta) \qquad\qquad s = 4E^2$$

$$|\mathcal{M}_{RL\to RL}|^2 = (4\,\pi\,\alpha)^2\ (1\ +\ cos\,\theta)^2$$

Analogamente :

$$\begin{cases} |\mathcal{M}_{RL\to RL}|^2 = |\mathcal{M}_{LR\to LR}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2 \\ |\mathcal{M}_{RL\to LR}|^2 = |\mathcal{M}_{LR\to RL}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2 \end{cases}$$

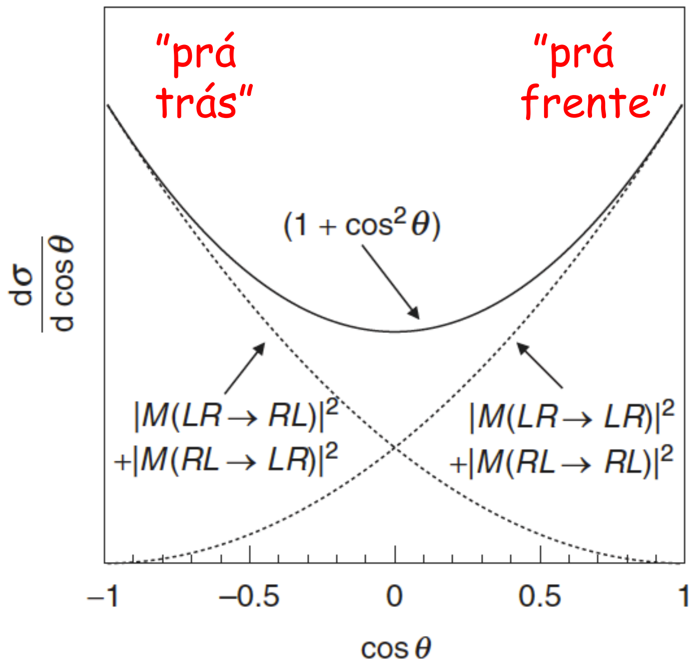
$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{1}{4}\times\left(|\mathcal{M}_{RL\to RL}|^2 + |\mathcal{M}_{RL\to LR}|^2 + |\mathcal{M}_{LR\to RL}|^2 + |\mathcal{M}^2_{LR\to LR}|\right)$$

$$= \frac{1}{4}e^4\left[2(1+\cos\theta)^2 + 2(1-\cos\theta)^2\right] \qquad = e^4(1+\cos^2\theta).$$

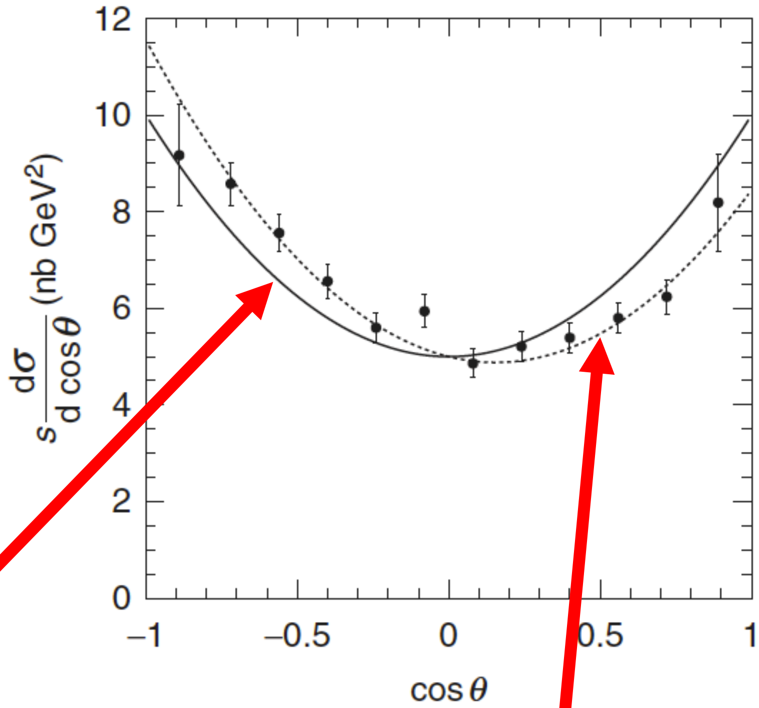
Substituimos em $\boxed{\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2}$ com $p_i^* = p_f^* = E$

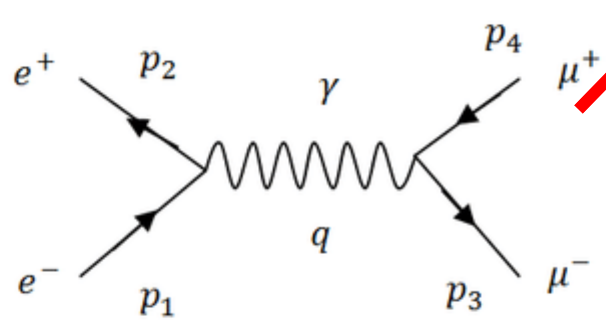
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}e^4(1+\cos^2\theta) \xrightarrow{\alpha = e^2/(4\pi)} \boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)}$$

Comparação com os dados experimentais

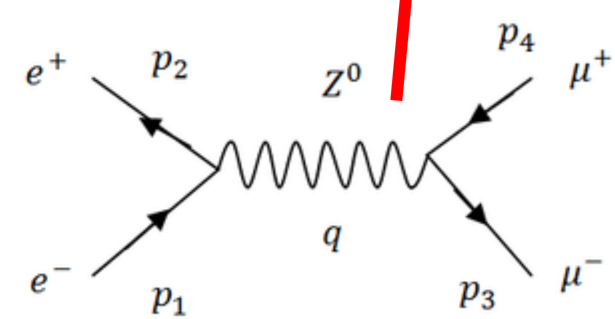


E os dados ?





(a) Photon exchange diagram (QED)



(b) Z^0 boson exchange diagram (Weak Interaction)

Seção de choque total

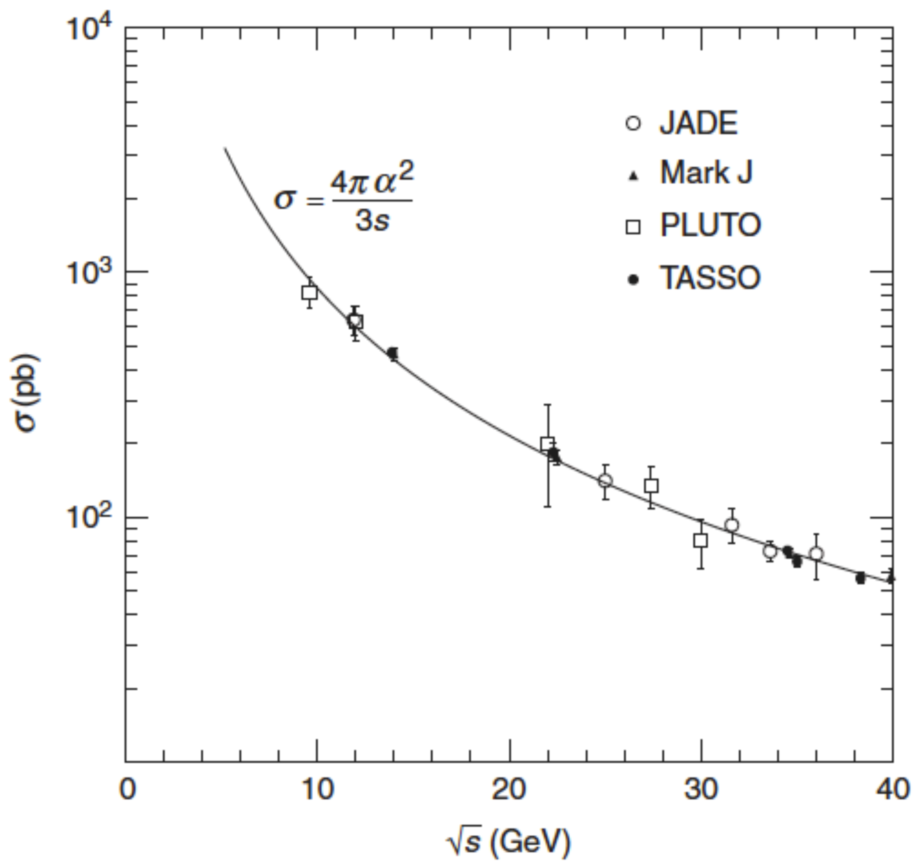
$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega$$

$$\begin{cases} \boxed{\dfrac{d\sigma}{d\Omega} = \dfrac{\alpha^2}{4s}(1+\cos^2\theta)} \\ d\Omega = d\,\phi\; d\cos\theta \end{cases}$$

$$\int (1+\cos^2\theta)\, d\Omega =$$

$$= 2\pi \int_{-1}^{+1} (1+\cos^2\theta)\, d(\cos\theta) = \frac{16\pi}{3}$$

$$\boxed{\sigma = \frac{4\pi\alpha^2}{3s}}$$

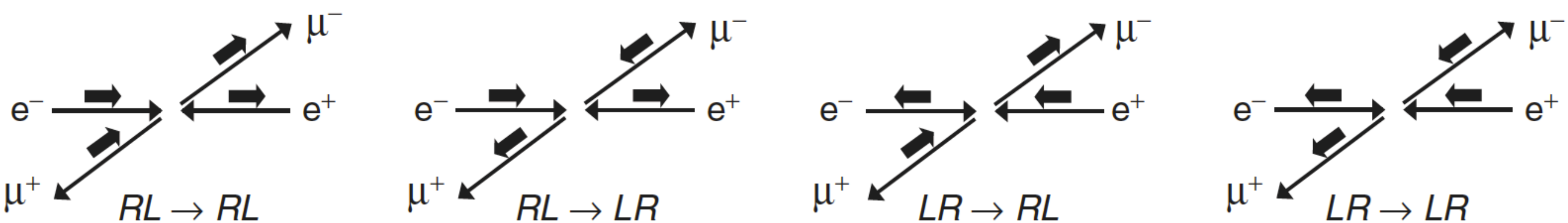


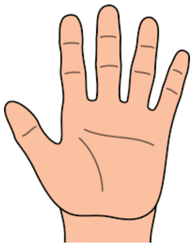
$$1\,mb = 10\,fm^2 = 10\,(10^{-15})^2\,m^2$$

$$1\,pb = 10^{-6}\,mb$$

Vamos voltar um pouco...

De 16 possibilidades apenas 4 são não nulas :





Quiralidade

Vamos definir

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

Então:

$$(\gamma^5)^2 = 1, \quad \gamma^{5\dagger} = \gamma^5, \quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5.$$

Espinores de helicidade para partículas sem massa $E \gg m$

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \; u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, \; v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}, \; v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

É fácil ver que:

$$\gamma^5 u_\uparrow = +u_\uparrow, \quad \gamma^5 u_\downarrow = -u_\downarrow, \quad \gamma^5 v_\uparrow = -v_\uparrow \quad \gamma^5 v_\downarrow = +v_\downarrow$$

Espinores de helicidade : autoestados de $\hat{h} = \frac{\hat{\Sigma}\cdot\hat{\mathbf{p}}}{2\mathrm{p}} = \frac{1}{2\mathrm{p}}\begin{pmatrix} \sigma\cdot\hat{\mathbf{p}} & 0 \\ 0 & \sigma\cdot\hat{\mathbf{p}} \end{pmatrix}$

Espinores de helicidade para partículas sem massa $E \gg m$

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \quad u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, \quad v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}, \quad v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

Autoestados quirais (autoestados de γ^5)

$$u_R \equiv N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \quad u_L \equiv N\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, \quad v_R \equiv N\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \text{ and } v_L \equiv N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}.$$

$$\gamma^5 u_R = +u_R \qquad \gamma^5 u_L = -u_L, \qquad \gamma^5 v_R = -v_R \qquad \gamma^5 v_L = +v_L.$$

$E \gg m$ $\longrightarrow$ Autoestados de helicidade = Autoestados quirais

Operadores de projeção quiral

Todo espinor de Dirac pode ser decomposto em componente R e L

Operadores de projeção
$$\begin{cases} P_R = \frac{1}{2}(1+\gamma^5) \\ P_L = \frac{1}{2}(1-\gamma^5) \end{cases}$$

$$P_R + P_L = 1, \quad P_R P_R = P_R, \quad P_L P_L = P_L \qquad P_L P_R = 0.$$

$$P_R = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad P_L = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$P_R u_R = u_R, \quad P_R u_L = 0, \quad P_R v_R = 0 \qquad P_R v_L = v_L$$

$$P_L u_R = 0, \quad P_L u_L = u_L, \quad P_L v_R = v_R \qquad P_L v_L = 0.$$

Quiralidade na QED

Todo espinor pode ser decomposto em componente direita e esquerda :

$$u = \tfrac{1}{2}(1+\gamma^5)u + \tfrac{1}{2}(1-\gamma^5)u = P_R u + P_L u$$

Vamos ver como algumas correntes dão zero! Por exemplo:

$$\bar{u}_L(p)\gamma^\mu u_R(p')$$

$$\bar{u}_L(p) = \bar{u}_L(p)\, P_R \qquad u_R(p') = P_R u_R(p')$$

$$\bar{u}_L(p)\gamma^\mu u_R(p') = \bar{u}_L(p) P_R\, \gamma^\mu P_R\, u_R(p')$$

$$P_R\gamma^\mu = \tfrac{1}{2}(1+\gamma^5)\gamma^\mu = \gamma^\mu \tfrac{1}{2}(1-\gamma^5) = \gamma^\mu P_L$$

0

$$\bar{u}_L(p)\gamma^\mu u_R(p') = \bar{u}_L(p)\gamma^\mu P_L P_R\, u_R(p') = 0,$$

Só partículas ou só antipartículas : quiralidades trocadas dão zero !

$$\bar{u}_L \gamma^\mu u_R = \bar{u}_R \gamma^\mu u_L = \bar{v}_L \gamma^\mu v_R = \bar{v}_R \gamma^\mu v_L \equiv 0$$

Mistura de partículas e antipartículas : quiralidades iguais dão zero !

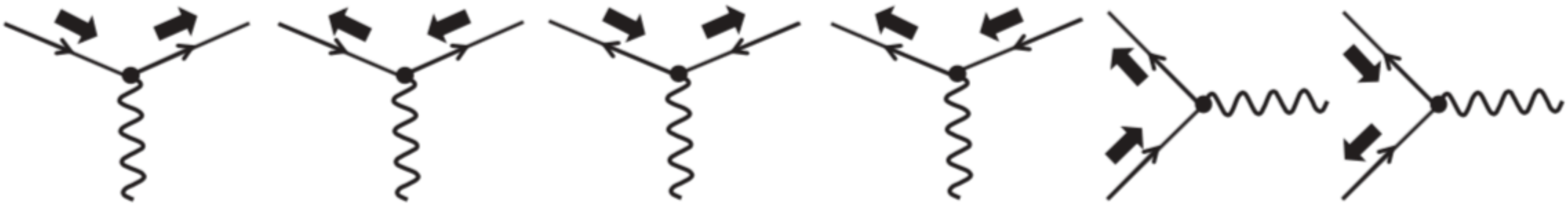
$$\bar{v}_L \gamma^\mu u_L = \bar{v}_R \gamma^\mu u_R \equiv 0$$

$$\begin{aligned}
\bar{u}_L(p) &\equiv [u_L(p)]^\dagger \gamma^0 = [P_L u_L(p)]^\dagger \gamma^0 = [\tfrac{1}{2}(1-\gamma^5)u_L(p)]^\dagger \gamma^0 \\
&= [u_L(p)]^\dagger \tfrac{1}{2}(1-\gamma^5)\gamma^0 && (\text{using } \gamma^5 = \gamma^{5\dagger}) \\
&= [u_L(p)]^\dagger \gamma^0 \tfrac{1}{2}(1+\gamma^5) && (\text{using } \gamma^0\gamma^5 = -\gamma^5\gamma^0) \\
&= \bar{u}_L(p) P_R.
\end{aligned}$$

$E >> m$

$$\bar{u}_L\gamma^\mu u_R = \bar{u}_R\gamma^\mu u_L = \bar{v}_L\gamma^\mu v_R = \bar{v}_R\gamma^\mu v_L = \bar{v}_L\gamma^\mu u_L = \bar{v}_R\gamma^\mu u_R \equiv 0$$

$$\bar{u}_\downarrow\gamma^\mu u_\uparrow = \bar{u}_\uparrow\gamma^\mu u_\downarrow = \bar{v}_\downarrow\gamma^\mu v_\uparrow = \bar{v}_\uparrow\gamma^\mu v_\downarrow = \bar{v}_\downarrow\gamma^\mu u_\downarrow = \bar{v}_\uparrow\gamma^\mu u_\uparrow = 0,$$



A helicidade (quiralidade) é conservada no vértice !

Entra e sai a mesma !

Simetria quiral !

FIM