

Introdução à Física das Partículas Elementares

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(buscar: física das partículas elementares)

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Plano do Curso

14/03	Cap. 1	25/04	Cap. 4	25/05	Cap. 9
16/03	Cap. 1	27/04	Cap. 6	30/05	Cap. 9
21/03	Cap. 2	04/05	Cap. 7	01/06	Cap. 9
23/03	Cap. 2	09/05	Cap. 7	06/06	
28/03	Cap. 3	11/05	Cap. 8	08/06	
30/03	Cap. 3	16/05	Cap. 8	13/06	Cap. 10
04/04		18/05	Cap. 8	15/06	Cap. 10
06/04		23/05	P2	20/06	Cap. 10
11/04	Cap. 4			22/06	Cap. 11
13/04	Cap. 4			27/06	Cap. 11
18/04	Cap. 4			29/06	P3
20/04	P1			04/07	Sub

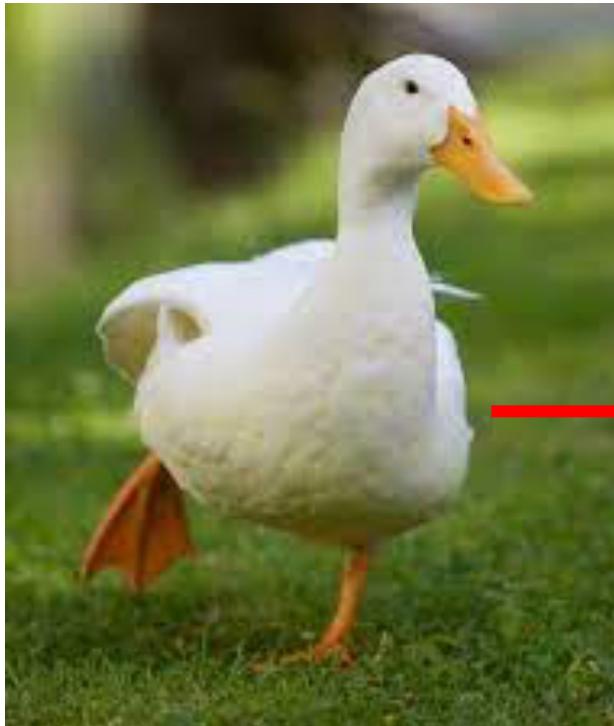
Aula 10

Capítulo 4

Spin

Helicidade

Dirac e o Spin



Pato !

Mecânica Quântica : $\frac{dO}{dt} = \frac{d}{dt}\langle\hat{O}\rangle = i\langle\psi|[\hat{H}, \hat{O}]\psi\rangle$

Se o operador comutar com o Hamiltoniano, o observável associado é conservado no tempo !

Partícula livre de Dirac $\hat{H}_D = \alpha \cdot \hat{\mathbf{p}} + \beta m$

$$[\hat{H}_D, \hat{\mathbf{L}}] = [\alpha \cdot \hat{\mathbf{p}} + \beta m, \hat{\mathbf{r}} \times \hat{\mathbf{p}}] = [\alpha \cdot \hat{\mathbf{p}}, \hat{\mathbf{r}} \times \hat{\mathbf{p}}] + [\beta m, \hat{\mathbf{r}} \times \hat{\mathbf{p}}]$$

$$[\hat{H}_D, \hat{L}_x] = [\alpha \cdot \hat{\mathbf{p}}, (\hat{\mathbf{r}} \times \hat{\mathbf{p}})_x] = [\alpha_x \hat{p}_x + \alpha_y \hat{p}_y + \alpha_z \hat{p}_z, \hat{y} \hat{p}_z - \hat{z} \hat{p}_y]$$

$$[\hat{x}, \hat{p}_x] = [\hat{y}, \hat{p}_y] = [\hat{z}, \hat{p}_z] = i$$

$$[\hat{H}_D, \hat{L}_x] = \alpha_y [\hat{p}_y, \hat{y}] \hat{p}_z - \alpha_z [\hat{p}_z, \hat{z}] \hat{p}_y = -i(\alpha_y \hat{p}_z - \alpha_z \hat{p}_y) = -i(\alpha \times \hat{\mathbf{p}})_x$$

$$[\hat{H}_D, \hat{\mathbf{L}}] = -i\alpha \times \hat{\mathbf{p}}$$

L não é conservado !!!

Vamos introduzir

$$\hat{\mathbf{S}} \equiv \frac{1}{2} \hat{\Sigma} \equiv \frac{1}{2} \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \quad (\text{candidato a spin})$$

$$\hat{\Sigma}_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \hat{\Sigma}_y = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \hat{\Sigma}_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$[\alpha_i, \hat{\Sigma}_x] = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix} - \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix} \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} = \begin{pmatrix} 0 & [\sigma_i, \sigma_x] \\ [\sigma_i, \sigma_x] & 0 \end{pmatrix}$$

Usando $[\sigma_x, \sigma_x] = 0$, $[\sigma_y, \sigma_x] = -2i\sigma_z$, $[\sigma_z, \sigma_x] = 2i\sigma_y$

$$[\alpha_x, \Sigma_x] = 0$$

$$[\alpha_y, \Sigma_x] = \begin{pmatrix} 0 & -2i\sigma_z \\ -2i\sigma_z & 0 \end{pmatrix} = -2i\alpha_z$$

$$[\alpha_z, \Sigma_x] = \begin{pmatrix} 0 & 2i\sigma_y \\ 2i\sigma_y & 0 \end{pmatrix} = 2i\alpha_y$$

$$[\hat{H}_D, \Sigma_x] = [\alpha \cdot \hat{\mathbf{p}} + \beta m, \Sigma_x]$$

$$[\beta, \hat{\Sigma}_x] = 0$$

$$[\hat{H}_D, \hat{\Sigma}_x] = [\alpha \cdot \hat{\mathbf{p}}, \hat{\Sigma}_x] = [\alpha_x \hat{p}_x + \alpha_y \hat{p}_y + \alpha_z \hat{p}_z, \hat{\Sigma}_x]$$

$$= \hat{p}_x [\alpha_x, \hat{\Sigma}_x] + \hat{p}_y [\alpha_y, \hat{\Sigma}_x] + \hat{p}_z [\alpha_z, \hat{\Sigma}_x]$$

$$[\hat{H}_D, \hat{\Sigma}_x] = -2i\hat{p}_y\alpha_z + 2i\hat{p}_z\alpha_y = 2i(\alpha \times \hat{\mathbf{p}})_x$$

Incluindo a coordenada y e z e usando $\hat{\mathbf{S}} = \frac{1}{2}\hat{\Sigma}$ \rightarrow

$$[\hat{H}_D, \hat{\mathbf{S}}] = i\alpha \times \hat{\mathbf{p}}$$



"spin" não
é conservado !

$$\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$$

$$[\hat{H}_D, \hat{\mathbf{J}}] \equiv [\hat{H}_D, \hat{\mathbf{L}} + \hat{\mathbf{S}}] = 0$$

"Momento angular
total" é conservado !

$$\hat{\mathbf{S}} = \frac{1}{2}\hat{\Sigma} = \frac{1}{2} \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \quad \longrightarrow \quad [\hat{S}_x, \hat{S}_y] = i\hat{S}_z$$

Mesma relação de comutação que o momento angular : $[\hat{L}_x, \hat{L}_y] = i\hat{L}_z$

Tem a mesma regra de quantização !

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$\hat{\mathbf{S}}^2 = \frac{1}{4}(\hat{\Sigma}_x^2 + \hat{\Sigma}_y^2 + \hat{\Sigma}_z^2) = \frac{3}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

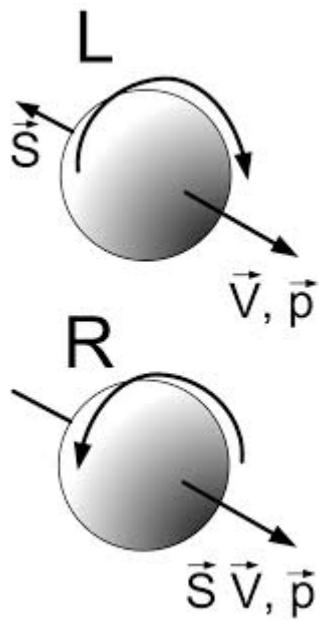


$$\hat{\mathbf{S}}^2 |s, m_s\rangle = s(s+1)|s, m_s\rangle$$

$$\hat{\mathbf{S}}^2 \psi = s(s+1)\psi = \frac{3}{4}\psi \quad \longrightarrow \quad s = \frac{1}{2}$$

Spin "sai"
da equação
de Dirac !!!

Helicidade



Partícula de mão esquerda

Partícula de mão direita

Soluções da Eq. de Dirac com momento constante $\psi_i = u_i(E, \mathbf{p})e^{i(\mathbf{p} \cdot \mathbf{x} - Et)}$

$$u_1 = N_1 \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} \quad u_2 = N_2 \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \quad u_3 = N_3 \begin{pmatrix} \frac{p_z}{E-m} \\ \frac{p_x+ip_y}{E-m} \\ 1 \\ 0 \end{pmatrix} \quad u_4 = N_4 \begin{pmatrix} \frac{p_x-ip_y}{E-m} \\ \frac{-p_z}{E-m} \\ 0 \\ 1 \end{pmatrix}$$

$$E = + \left| \sqrt{\mathbf{p}^2 + m^2} \right|$$

$$E = - \left| \sqrt{\mathbf{p}^2 + m^2} \right|$$

Em repouso $\psi = u(E, 0)e^{-iEt}$

$$\psi_1 = N \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imt}, \quad \psi_2 = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-imt}, \quad \psi_3 = N \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{+imt}, \quad \psi_4 = N \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{+imt}$$

São auto-estados de spin ?

Partículas em repouso com $u_1(E, 0)$ $u_2(E, 0)$ são autoestados de \hat{S}_z

$$\hat{S}_z = \frac{1}{2}\Sigma_z = \frac{1}{2} \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$u_1(E, 0) = N \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{S}_z u_1 = \frac{1}{2} u_1$$

Partículas em movimento com

$$u_1(p) = \sqrt{E + m} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

**não são
autoestados de
 \hat{S}_z**

Partículas viajando na direção z são autoestados de \hat{S}_z

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ \frac{\pm p}{E+m} \\ 0 \end{pmatrix}, \quad u_2 = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{\mp p}{E+m} \end{pmatrix}, \quad v_1 = N \begin{pmatrix} 0 \\ \frac{\mp p}{E+m} \\ 0 \\ 1 \end{pmatrix} \text{ and } v_2 = N \begin{pmatrix} \frac{\pm p}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$\hat{S}_z u_1(E, 0, 0, \pm p) = +\frac{1}{2} u_1(E, 0, 0, \pm p),$$

$$\hat{S}_z u_2(E, 0, 0, \pm p) = -\frac{1}{2} u_2(E, 0, 0, \pm p).$$

Porque o sinal
menos ?



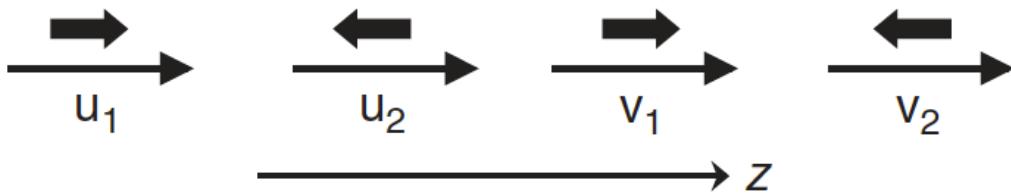
Para as antipartículas $\hat{S}_z^{(v)} = -\hat{S}_z$

Lebowski

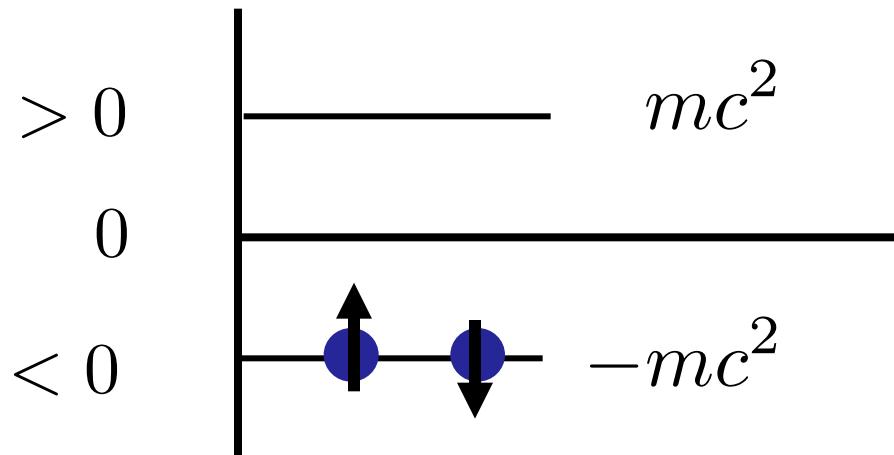
$$\hat{S}_z^{(v)} v_1(E, 0, 0, \pm p) \equiv -\hat{S}_z v_1(E, 0, 0, \pm p) = +\frac{1}{2} v_1(E, 0, 0, \pm p)$$

$$\hat{S}_z^{(v)} v_2(E, 0, 0, \pm p) \equiv -\hat{S}_z v_2(E, 0, 0, \pm p) = -\frac{1}{2} v_2(E, 0, 0, \pm p)$$

Escolhemos momento + p :



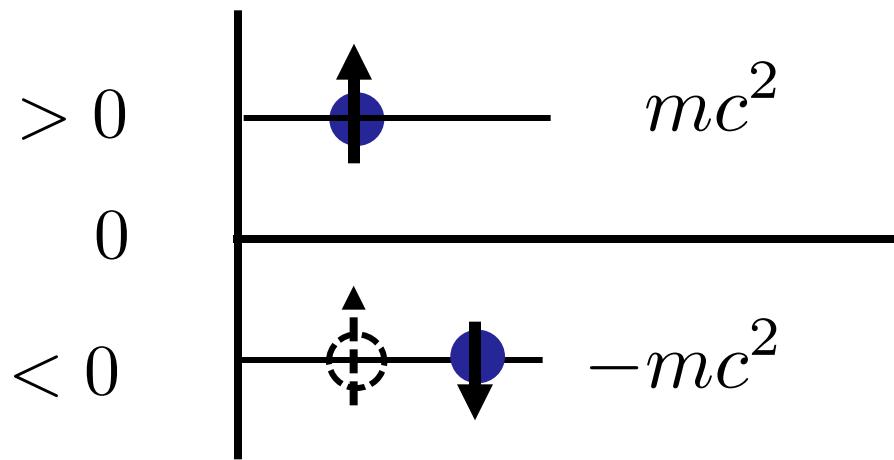
Energia



A falta do spin prá cima
é vista como excesso de
spin prá baixo !



Energia



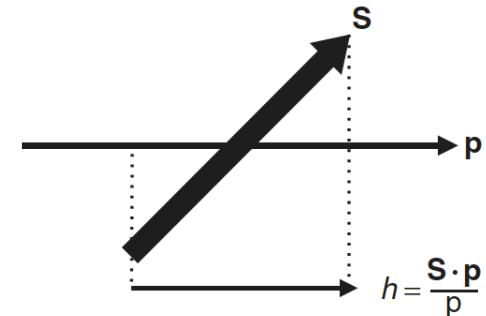
Se você tá
dizendo...



Helicidade

Componente do spin na direção de vôo

$$h \equiv \frac{\mathbf{S} \cdot \mathbf{p}}{p}$$



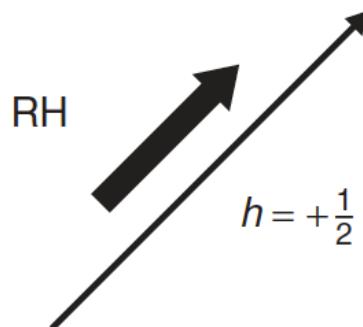
$$\hat{h} = \frac{\hat{\Sigma} \cdot \hat{\mathbf{p}}}{2p} = \frac{1}{2p} \begin{pmatrix} \sigma \cdot \hat{\mathbf{p}} & 0 \\ 0 & \sigma \cdot \hat{\mathbf{p}} \end{pmatrix}$$

$$\hat{H}_D = \alpha \cdot \hat{\mathbf{p}} + \beta m,$$

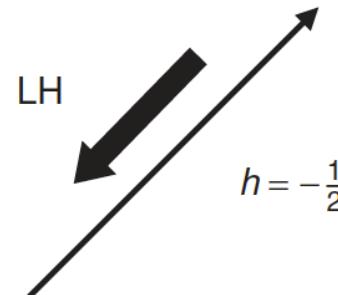
$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$$

Helicidade é conservada $[\hat{H}_D, \hat{\Sigma} \cdot \hat{\mathbf{p}}] = 0$

Dois autoestados de helicidade :



Mão direita



Mão esquerda

Autovalores de Helicidade

$$\hat{h}u = \lambda u$$

$$\frac{1}{2p} \begin{pmatrix} \sigma \cdot p & 0 \\ 0 & \sigma \cdot p \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \lambda \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$\left\{ \begin{array}{l} (\sigma \cdot p)u_A = 2p \lambda u_A \\ (\sigma \cdot p)u_B = 2p \lambda u_B \end{array} \right.$$

\longrightarrow Multiplicamos à esquerda por
 $\sigma \cdot p \times$

Lembrando que $(\sigma \cdot p)^2 = p^2$ a primeira equação fica

$$p^2 u_A = 2p \lambda (\sigma \cdot p) u_A = 4p^2 \lambda^2 u_A \longrightarrow \boxed{\lambda = \pm 1/2}$$

Lembrando que $(\sigma \cdot p)u_A = (E + m)u_B \longrightarrow u_B = 2\lambda \left(\frac{p}{E + m} \right) u_A$

Encontrando u_A logo encontramos u_B !

Autoestados de Helicidade

$$(\boldsymbol{\sigma} \cdot \mathbf{p}) u_A = 2p \lambda u_A$$

$$\mathbf{p} = (p \sin \theta \cos \phi, p \sin \theta \sin \phi, p \cos \theta) \quad (\text{esféricas})$$

$$\left\{ \begin{array}{l} \frac{1}{2p}(\boldsymbol{\sigma} \cdot \mathbf{p}) = \frac{1}{2p} \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \\ u_A = \begin{pmatrix} a \\ b \end{pmatrix} \end{array} \right.$$

$$\begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 2\lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

Escolhemos $\lambda = +1/2$ e descobrimos u_\uparrow

Resolvemos o sistema de duas equações com duas incógnitas

O espinor de mão direita

$$u_{\uparrow} = N \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi} \sin\left(\frac{\theta}{2}\right) \\ \frac{p}{E+m} \cos\left(\frac{\theta}{2}\right) \\ \frac{p}{E+m} e^{i\phi} \sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$N = \sqrt{E + m}$$

Depois escolhemos $\lambda = -1/2$ e repetimos os passos para encontrar u_{\downarrow}

$$u_{\uparrow} = \sqrt{E + m} \begin{pmatrix} c \\ se^{i\phi} \\ \frac{p}{E+m}c \\ \frac{p}{E+m}se^{i\phi} \end{pmatrix} \quad u_{\downarrow} = \sqrt{E + m} \begin{pmatrix} -s \\ ce^{i\phi} \\ \frac{p}{E+m}s \\ -\frac{p}{E+m}ce^{i\phi} \end{pmatrix}$$

$$s = \sin\left(\frac{\theta}{2}\right)$$

$$c = \cos\left(\frac{\theta}{2}\right)$$

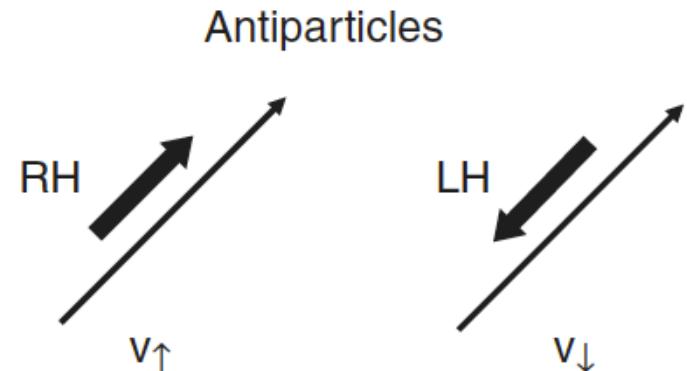
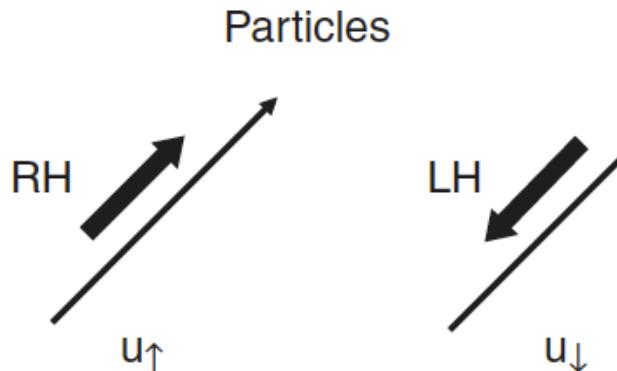
Antipartículas $\hat{\mathbf{S}}^{(v)} = -\hat{\mathbf{S}}$

$$\hat{h}u = \lambda u \quad \lambda = +1/2 \quad \rightarrow \quad \left(\frac{\Sigma \cdot p}{2p} \right) v_{\uparrow} = -\frac{1}{2} v_{\uparrow}$$

$$v_{\uparrow} = \sqrt{E+m} \begin{pmatrix} \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \quad v_{\downarrow} = \sqrt{E+m} \begin{pmatrix} \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}$$

Quando $E \gg m$

$$u_{\uparrow} \approx \sqrt{E} \begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}, \quad u_{\downarrow} \approx \sqrt{E} \begin{pmatrix} -s \\ c e^{i\phi} \\ s \\ -c e^{i\phi} \end{pmatrix}, \quad v_{\uparrow} \approx \sqrt{E} \begin{pmatrix} s \\ -c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \text{ and } v_{\downarrow} \approx \sqrt{E} \begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}$$



Capítulo 5

Propagador

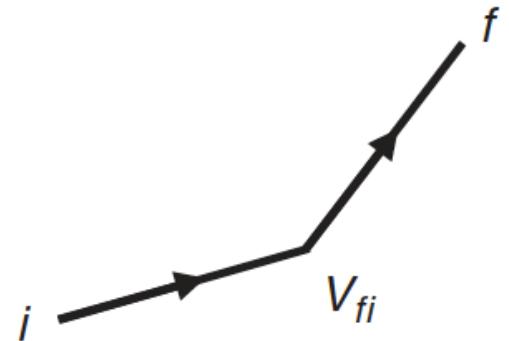
Regras de Feynman

Regra de ouro de Fermi

$$\Gamma_{fi} = 2\pi|T_{fi}|^2\rho(E_f)$$

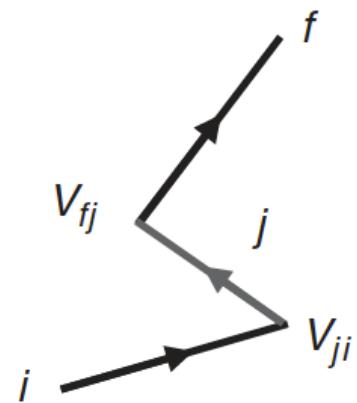
$$T_{fi} = \langle f | V | i \rangle$$

$V = H'$ Hamiltoniano de interação



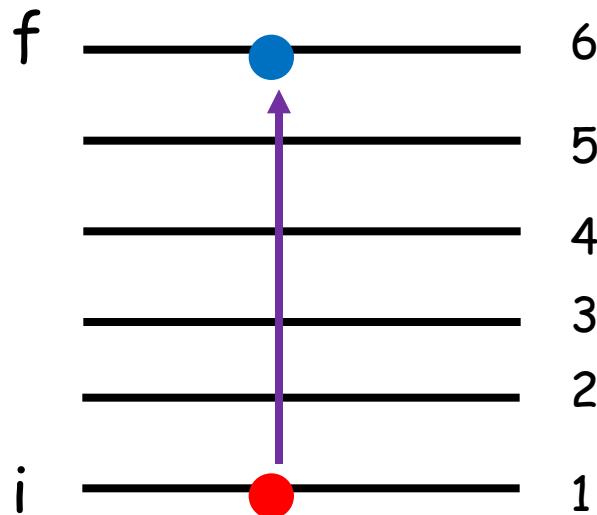
Correção de segunda ordem

$$T_{fi} = \langle f | V | i \rangle + \sum_{j \neq i} \frac{\langle f | V | j \rangle \langle j | V | i \rangle}{E_i - E_j} + \dots$$

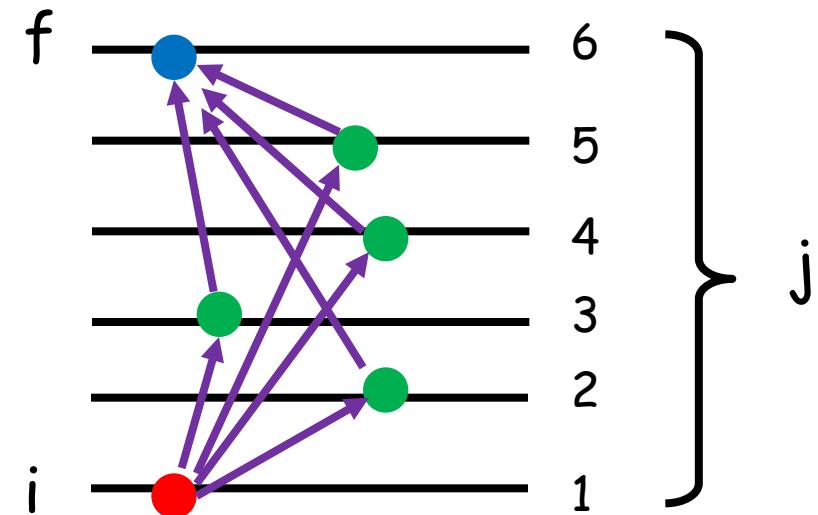


Correção de segunda ordem

Ilustração: níveis de energia



Primeira ordem



Segunda ordem

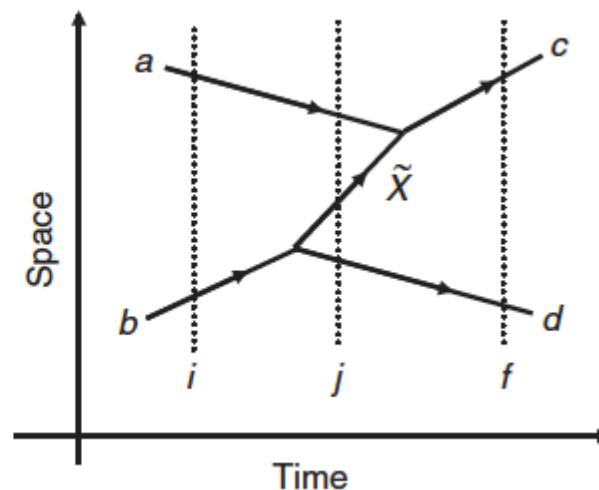
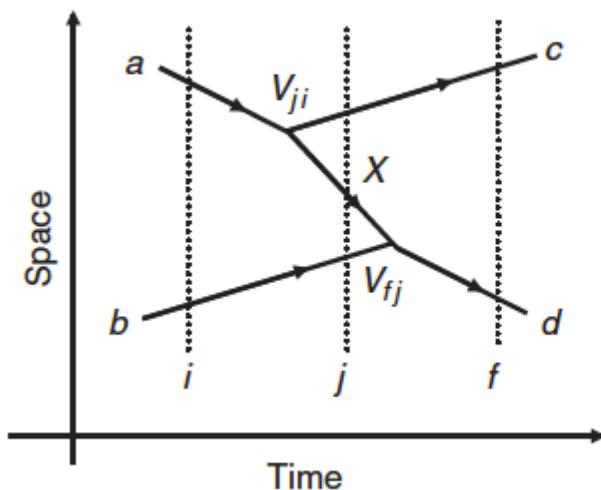


Diagrama da esquerda :

$$T_{fi}^{ab} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} = \frac{\langle d|V|X + b\rangle\langle c + X|V|a\rangle}{(E_a + E_b) - (E_c + E_X + E_b)}$$

$$V_{ji} = \langle c + X|V|a\rangle$$

$$V_{fj} = \langle d|V|X + b\rangle$$

$$V_{ji} = \mathcal{M}_{ji} \prod_k (2E_k)^{-1/2} \quad \longrightarrow$$

$$V_{ji} = \langle c + X|V|a\rangle = \frac{\mathcal{M}_{a \rightarrow c+X}}{(2E_a 2E_c 2E_X)^{1/2}}$$

$$\mathcal{M}_{a \rightarrow c+X} = g_a \quad \longrightarrow$$

(hipótese)

$$V_{ji} = \langle c + X|V|a\rangle = \frac{g_a}{(2E_a 2E_c 2E_X)^{1/2}}$$

Analogamente :

$$V_{fj} = \langle d|V|X + b\rangle = \frac{g_b}{(2E_b 2E_d 2E_X)^{1/2}}$$

$$T_{fi}^{ab} = \frac{\langle d|V|X+b\rangle\langle c+X|V|a\rangle}{(E_a + E_b) - (E_c + E_X + E_b)} = \frac{1}{2E_X} \cdot \frac{1}{(2E_a 2E_b 2E_c 2E_d)^{1/2}} \cdot \frac{g_a g_b}{(E_a - E_c - E_X)}$$

$$\mathcal{M}_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{1/2} T_{fi}^{ab} \quad \longrightarrow \quad \mathcal{M}_{fi}^{ab} = \frac{1}{2E_X} \cdot \frac{g_a g_b}{(E_a - E_c - E_X)}$$

Diagrama da direita :

$$\mathcal{M}_{fi}^{ba} = \frac{1}{2E_X} \cdot \frac{g_a g_b}{(E_b - E_d - E_X)}$$

Estado inicial e final são iguais. Então somamos as amplitudes :

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{ab} + \mathcal{M}_{fi}^{ba} = \frac{g_a g_b}{2E_X} \cdot \left(\frac{1}{E_a - E_c - E_X} + \frac{1}{E_b - E_d - E_X} \right)$$

Usando conservação da energia :

$$E_b - E_d = E_c - E_a$$

$$\mathcal{M}_{fi} = \frac{g_a g_b}{2E_X} \cdot \left(\frac{1}{E_a - E_c - E_X} - \frac{1}{E_a - E_c + E_X} \right) = \frac{g_a g_b}{(E_a - E_c)^2 - E_X^2}$$

$$\mathcal{M}_{fi} = \frac{g_a g_b}{2E_X} \cdot \left(\frac{1}{E_a - E_c - E_X} - \frac{1}{E_a - E_c + E_X} \right) = \frac{g_a g_b}{(E_a - E_c)^2 - E_X^2}$$



Usamos as definição de energia de X :

$$E_X^2 = \mathbf{p}_X^2 + m_X^2 \quad \mathbf{p}_X = (\mathbf{p}_a - \mathbf{p}_c) \quad \rightarrow \quad E_X^2 = \mathbf{p}_X^2 + m_X^2 = (\mathbf{p}_a - \mathbf{p}_c)^2 + m_X^2$$

$$\mathcal{M}_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - (\mathbf{p}_a - \mathbf{p}_c)^2 - m_X^2} = \frac{g_a g_b}{(p_a - p_c)^2 - m_X^2}$$

p_a e p_c são quadrivetores !



$$q = p_a - p_c$$

$$\mathcal{M}_{fi} = \frac{g_a g_b}{q^2 - m_X^2}$$

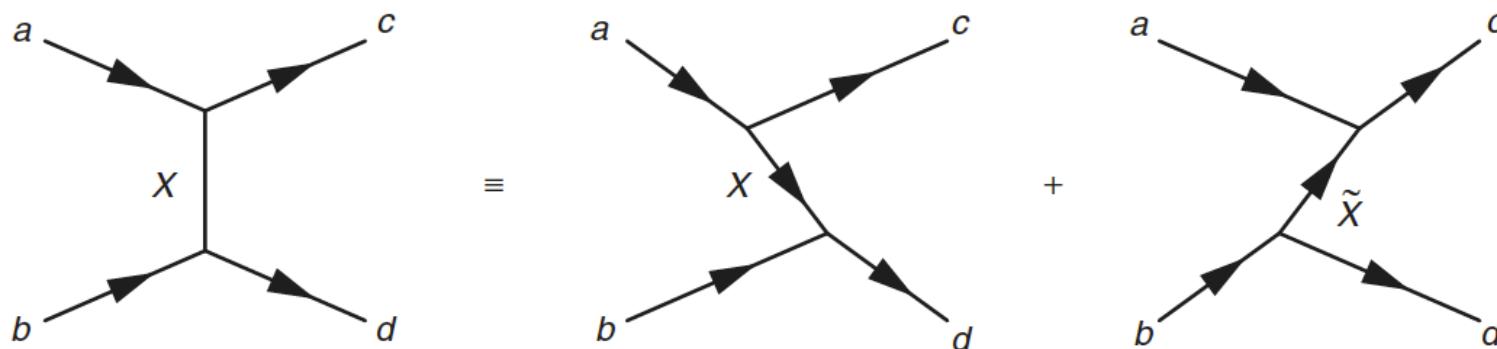
$$\frac{1}{q^2 - m_X^2}$$

é o propagador !

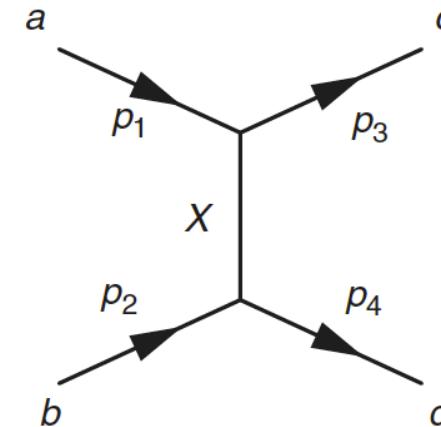
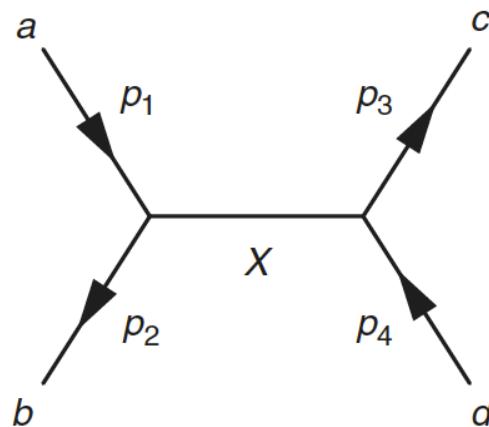


Diagrama de Feynman e partículas virtuais

Soma de todos os ordenamentos temporais = diagrama de Feynman



X é uma partícula virtual : $q = p_a - p_c = p_d - p_b$ $q^2 \neq m_X^2$



$$q = p_1 + p_2 = p_3 + p_4$$

$$q = p_1 - p_3 = p_4 - p_2$$

FIM

