MAE 5870 – Análise de Séries temporais
Lista #3
data de entrega: 24/05/2023

3.2 Let $\{w_t; t = 0, 1, \ldots\}$ be a white noise process with variance σ_w^2 and let $|\phi| < 1$ be a constant. Consider the process $x_0 = w_0$, and

$$x_t = \phi x_{t-1} + w_t, \quad t = 1, 2, \ldots .$$

We might use this method to simulate an AR(1) process from simulated white noise.

(a) Show that $x_t = \sum_{j=0}^{t} \phi^j w_{t-j}$ for any $t = 0, 1, \ldots$.
(b) Find the $\mathrm{E}(x_t)$.
(c) Show that, for $t = 0, 1, \ldots$,

$$\mathrm{var}(x_t) = \frac{\sigma_w^2}{1-\phi^2}(1 - \phi^{2(t+1)})$$

(d) Show that, for $h \geq 0$,

$$\mathrm{cov}(x_{t+h}, x_t) = \phi^h \mathrm{var}(x_t)$$

(e) Is x_t stationary?

(f) Argue that, as $t \to \infty$, the process becomes stationary, so in a sense, x_t is "asymptotically stationary."
(g) Comment on how you could use these results to simulate n observations of a stationary Gaussian AR(1) model from simulated iid N(0,1) values.
(h) Now suppose $x_0 = w_0/\sqrt{1-\phi^2}$. Is this process stationary? *Hint*: Show $\mathrm{var}(x_t)$ is constant.

3.4 Identify the following models as ARMA(p, q) models (watch out for parameter redundancy), and determine whether they are causal and/or invertible:

(a) $x_t = .80x_{t-1} - .15x_{t-2} + w_t - .30w_{t-1}$.
(b) $x_t = x_{t-1} - .50x_{t-2} + w_t - w_{t-1}$.

3.7 For the AR(2) series shown below, use the results of Example 3.10 to determine a set of difference equations that can be used to find the ACF $\rho(h)$, $h = 0, 1, \ldots$; solve for the constants in the ACF using the initial conditions. Then plot the ACF values to lag 10 (use `ARMAacf` as a check on your answers).

(a) $x_t + 1.6x_{t-1} + .64x_{t-2} = w_t$.

3.8 Verify the calculations for the autocorrelation function of an ARMA(1, 1) process given in Example 3.14. Compare the form with that of the ACF for the ARMA(1, 0) and the ARMA(0, 1) series. Plot the ACFs of the three series on the same graph for $\phi = .6$, $\theta = .9$, and comment on the diagnostic capabilities of the ACF in this case.

3.9 Generate $n = 100$ observations from each of the three models discussed in Problem 3.8. Compute the sample ACF for each model and compare it to the theoretical values. Compute the sample PACF for each of the generated series and compare the sample ACFs and PACFs with the general results given in Table 3.1.

3.10 Let x_t represent the cardiovascular mortality series (`cmort`) discussed in Example 2.2.

(a) Fit an AR(2) to x_t using linear regression as in Example 3.18.
(b) Assuming the fitted model in (a) is the true model, find the forecasts over a four-week horizon, x_{n+m}^n, for $m = 1, 2, 3, 4$, and the corresponding 95% prediction intervals.

3.18 Fit an AR(2) model to the cardiovascular mortality series (`cmort`) discussed in Example 2.2. using linear regression and using Yule–Walker.

(a) Compare the parameter estimates obtained by the two methods.
(b) Compare the estimated standard errors of the coefficients obtained by linear regression with their corresponding asymptotic approximations, as given in Property 3.10.

3.33 Fit an ARIMA(p, d, q) model to the global temperature data `globtemp` performing all of the necessary diagnostics. After deciding on an appropriate model, forecast (with limits) the next 10 years. Comment.

3.34 Fit an ARIMA(p, d, q) model to the sulfur dioxide series, `so2`, performing all of the necessary diagnostics. After deciding on an appropriate model, forecast the data into the future four time periods ahead (about one month) and calculate 95% prediction intervals for each of the four forecasts. Comment. (Sulfur dioxide is one of the pollutants monitored in the mortality study described in Example 2.2.)