PQI – 3303 – Fenômenos de Transporte III
Aula 06

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ADIMENSIONALIZAÇÃO

Expressão das equações de Fenômenos de Transporte na forma adimensionalizada

- **Números ADIMENSIONAIS (Re, Pr, Sc, Pe, Gr, Sh, Nu etc..)**
  - **Correlações semi-empíricas generalizadas**
  - **Analogia dos FT's**

- **"SCALING" Identificação das ordens de grandeza**
  - **Simplificação das equações (equações aproximadas)**
  - **Resultados semi-quantitativos**

- **CFD _Fluidodinâmica Computacional**
  - **Simplificação das equações (equações aproximadas)**
  - **Elaboração de critérios e modelos**

ADIMENSIONALIZAÇÃO

PARÂMETROS CARACTERÍSTICOS: t_0, ρ_0, v_0, L

ADIMENSIONALIZAÇÃO DAS VARIÁVEIS: $\hat{\rho} = \dfrac{\rho}{\rho_0}$; $\hat{\vec{v}} = \dfrac{\vec{v}}{v_0}$; $\hat{\vec{r}} = \dfrac{\vec{r}}{L}$; $\hat{t} = \dfrac{t}{t_0}$

$$\rho = \rho_0\,\hat{\rho} \quad ; \quad \vec{v} = v_0\,\hat{\vec{v}} \quad ; \quad \vec{r} = L\,\hat{\vec{r}} \quad ; \quad t = t_0\,\hat{t}$$

ADIMENSIONALIZAÇÃO DOS OPERADORES:

$$\hat{\text{grad}}(\;) = L\,\text{grad}(\;) \quad ; \quad \hat{\text{div}}(\;) = L\,\text{div}(\;) \quad ; \quad \hat{\text{lap}}(\;) = \hat{\text{div}}\,\hat{\text{grad}}(\;) = L^2\,\text{lap}(\;)$$

$$\frac{\partial}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \hat{t}}$$

$$\frac{\partial f}{\partial x} = \frac{\partial(\hat{f}\,f_0)}{\partial(\hat{x}\,x_0)} = \frac{f_0}{x_0}\frac{\partial \hat{f}}{\partial \hat{x}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial(\hat{x}\,x_0)}\left(\frac{\partial(\hat{f}\,f_0)}{\partial(\hat{x}\,x_0)}\right) = \frac{f_0}{{x_0}^2}\frac{\partial}{\partial \hat{x}}\left(\frac{\partial \hat{f}}{\partial \hat{x}}\right) = \frac{f_0}{{x_0}^2}\frac{\partial^2 \hat{f}}{\partial \hat{x}^2}$$

Equação da Continuidade

$$\frac{\partial \rho}{\partial t} = -\,\mathrm{div}\,\rho\,\vec{v}$$

$$\frac{\rho_0}{t_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\frac{\rho_0 v_0}{L}\,\hat{\mathrm{div}}\,\hat{\rho}\,\hat{\vec{v}} \quad \Rightarrow \quad \frac{L}{t_0 v_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\,\hat{\mathrm{div}}\,\hat{\rho}\,\hat{\vec{v}}$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\,\hat{\mathrm{div}}\,\hat{\rho}\,\hat{\vec{v}}$$

STROUHAL

$$Sr = \frac{t_0 v_0}{L}$$

t_0 : tempo para o escoamento “atingir o regime permanente” ou associado à frequência em processo oscilatório.

Equação de Conservação Generalizada

Balanço microscópico de φ:

$$\rho\frac{D\varphi}{Dt}=\frac{\partial\rho\varphi}{\partial t}+\mathrm{div}\left(\rho\varphi\vec{v}\right)=-\mathrm{div}\,\vec{j}_{\phi}+\dot{\sigma}_{\forall\Phi}$$

$$\frac{\partial\rho\varphi}{\partial t}+\mathrm{div}\left(\rho\vec{v}\varphi+\vec{j}_{\Phi}\right)=\dot{\sigma}_{\forall_{\Phi}}$$

Equação constitutiva de difusão:

$$\vec{j}_{\Phi}=-\rho\Gamma_{\varphi}\,\vec{\mathrm{grad}}\;\varphi$$

φ	Γ_{Φ}	ν/Γ_{Φ}
V	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

$$\frac{\partial\rho\varphi}{\partial t}+\mathrm{div}\,\rho\left(\vec{v}\varphi-\Gamma_{\varphi}\mathrm{grad}\;\varphi\right)=\dot{\sigma}_{\forall_{\Phi}}$$

Equação de Conservação Generalizada

$$\frac{\partial \rho\varphi}{\partial t} + \text{div}\,\rho\left(\vec{v}\varphi - \Gamma_\varphi \text{grad}\;\varphi\right) = \dot{\sigma}_{\forall_\Phi}$$

$$\rho = \rho_0\,\hat{\rho} \quad ; \quad \vec{v} = v_0\,\hat{\vec{v}} \;;\; \vec{r} = L\,\hat{\vec{r}} \;;\quad t = t_0\,\hat{t} \quad ;\quad \varphi = \hat{\varphi}\,\Delta\varphi + \varphi_0$$

$$\frac{1}{t_0}\frac{\partial \rho_0\,\hat{\rho}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)}{\partial\,\hat{t}} + \frac{1}{L}\,\hat{\text{div}}\,\hat{\rho}\left(\rho_0 v_0\,\hat{\vec{v}}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right) - \frac{\rho_0}{L}\Gamma_\varphi \hat{\text{grad}}\;\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)\right) = \dot{\sigma}_{\forall_\Phi}$$

Reagrupando-se os termos:

$$\varphi_0\frac{\rho_0 v_0}{L}\underbrace{\left[\frac{L}{t_0 v_0}\frac{\partial\,\hat{\rho}}{\partial\,\hat{t}} + \hat{\text{div}}\left(\hat{\rho}\hat{\vec{v}}\right)\right]}_{=0\;,\text{continuidade}} + \frac{\rho_0\Delta\varphi}{t_0}\frac{\partial\left(\hat{\rho}\hat{\varphi}\right)}{\partial\,\hat{t}} + \frac{\rho_0 v_0\Delta\varphi}{L}\left[\hat{\text{div}}\left(\hat{\rho}\hat{\vec{v}}\hat{\varphi}\right) - \hat{\text{div}}\,\hat{\rho}\left(\frac{\Gamma_\varphi}{v_0 L}\hat{\text{grad}}\;\hat{\varphi}\right)\right] = \dot{\sigma}_{\forall_\Phi}$$

Equação de Conservação Generalizada

$$\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\,\rho\left(\vec{v}\varphi - \Gamma_\varphi \mathrm{grad}\;\varphi\right) = \dot{\sigma}_{\forall_\Phi}$$

$$\frac{L}{v_0\, t_0}\frac{\partial\left(\hat{\rho}\hat{\varphi}\right)}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi} - \frac{\Gamma_\varphi}{v_0 L}\,\hat{\mathrm{grad}}\;\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\,\hat{\varphi}}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{\varphi} - \frac{1}{Pe}\,\hat{\mathrm{grad}}\;\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0\,\Delta\varphi\, v_0}$$

PECLET

$$Pe = \frac{v_0 L}{\Gamma_\varphi} = \frac{\nu}{\Gamma_\varphi}\frac{v_0 L}{\nu} = Re\frac{\nu}{\Gamma_\varphi}$$

$$Pe = Re\frac{\nu}{\Gamma_\varphi}$$

Equação de Conservação Generalizada

ADIMENSIONAIS

PECLET

$$Pe = Re\frac{\nu}{\Gamma_{\varphi}}$$

CONVECÇÃO / DIFUSÃO

PECLET MÁSSICO

$$Pe = Re\,Sc$$

PECLET TÉRMICO

$$Pe = Re\,Pr$$

PRANDTL

$$Pr = \frac{\nu}{\alpha}$$

$$\alpha = \frac{k}{\rho\,Cp}$$

SCHMIDT

$$Sc = \frac{\nu}{D_{AB}}$$

φ	Γ_{Φ}	ν/Γ_{Φ}
$\mathbf{V}$	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

Equação da Continuidade para espécie A:

$$\frac{L}{v_0 t_0}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial \hat{t}} + \hat{\text{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\hat{\text{grad}}\,\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow$$

$\varphi = \omega_A$ *fração mássica* $\hat{\varphi} = \hat{\omega}_A = \dfrac{\omega_A - \omega_{A0}}{\omega_{AS} - \omega_{A0}}$

Reação química

Equação Cinética:

$$\dot{\sigma}_{\forall T} = r_A \quad \left(\frac{\text{kg de A}}{m^3.s}\right)$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\text{div}}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{\omega}_A - \frac{1}{Pe}\,\hat{\text{grad}}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

Equação da Continuidade para espécie A – Escalas de tempos:

$$\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\operatorname{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{D_{AB}}{v_0 L}\,\hat{\operatorname{grad}}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\operatorname{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{D_{AB}}{v_0 L}\,\hat{\operatorname{lap}}\,\hat{\omega}_A + \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{v_0}{L}\hat{\operatorname{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{D_{AB}}{L^2}\,\hat{\operatorname{lap}}\,\hat{\omega}_A + \frac{r_A}{\rho_0\,\Delta\omega_A}$$

$$\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{1}{t_C}\hat{\operatorname{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{1}{t_D}\,\hat{\operatorname{lap}}\,\hat{\omega}_A + \frac{1}{t_R}$$

Convecção

$$t_C = \frac{L}{v_0}$$

Difusão

$$t_D = \frac{L^2}{D_{AB}}$$

Reação

$$t_R = \frac{\rho_0\,\Delta\omega_A}{r_A}$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{\omega}_A - \frac{1}{Pe}\hat{grad}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{1}{Pe} = \frac{L}{v_0}\frac{D_{AB}}{L^2} = \frac{t_C}{t_D}$$

$$\frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0} = \frac{t_C}{t_R} = Da_1$$

DAMKÖHLER 1

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{1}{Pe}\hat{grad}\,\hat{\omega}_A\right) = Da_1$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + d\hat{i}v\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{1}{Pe}\,gr\hat{a}d\;\hat{\omega}_A\right) = \frac{r_A L}{\rho_0 \Delta\omega_A v_0}$$

x Pe:

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + d\hat{i}v\,\hat{\rho}\left(Pe\;\hat{\vec{v}}\hat{\omega}_A - gr\hat{a}d\;\hat{\omega}_A\right) = \frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}}$$

Tempo de Reação:

$$t_R = \frac{\rho_0 \Delta\omega_A}{r_A}$$

Tempo de Difusão:

$$t_D = \frac{L^2}{D_{AB}}$$

$$\frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}} = \frac{t_D}{t_R} = Da_2$$

DAMKÖHLER 2

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + d\hat{i}v\,\hat{\rho}\left(Pe\;\hat{\vec{v}}\hat{\omega}_A - gr\hat{a}d\;\hat{\omega}_A\right) = Da_2$$

Reação/difusão em poros de catalisador sólido –

Módulo de Thiele = Th = $(Da_2)^{0,5}$

Coeficientes Convectivos

ADIMENSIONALIZAÇÃO

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \operatorname{div}\hat{\rho}\underbrace{\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\operatorname{gr\hat{a}d}\,\hat{\varphi}\right)} = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 \Delta\varphi v_0}$$

⇓

ESCOAMENTO + DIFUSÃO

⇓

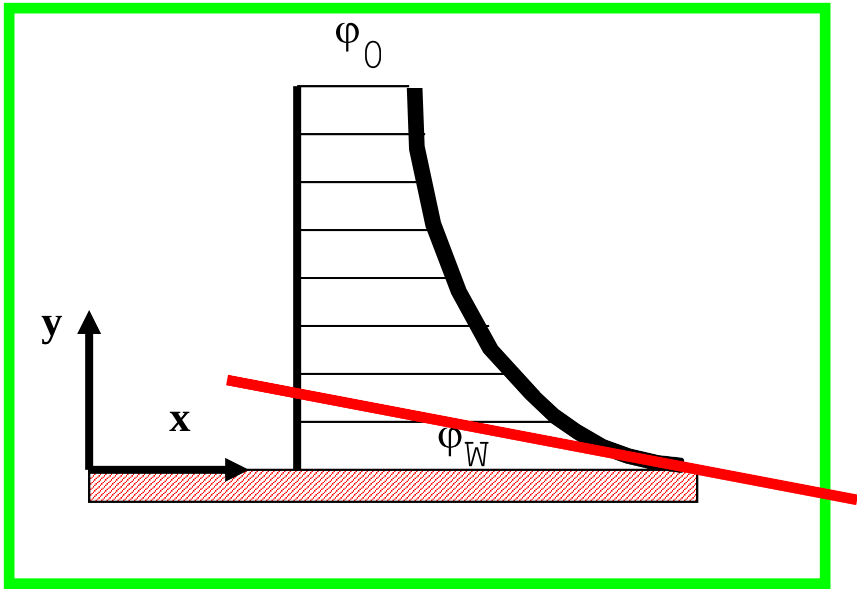
INTERFACE / PAREDE

⇓

COEFICIENTE DE CONVECÇÃO

⇓

$$\vec{j}_{\phi,\,parede} = CO_{\varphi}\left(\varphi_S - \varphi_0\right)$$

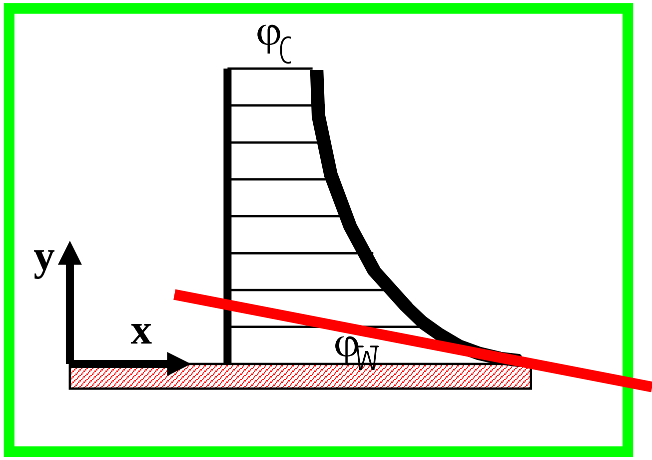


grad φ

Coeficientes Convectivos

ADIMENSIONAIS

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\underbrace{\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\mathrm{gr\hat{\vec{a}}d}\;\hat{\varphi}\right)}_{NA\ PAREDE\ \hat{\vec{v}}=0} = \frac{\dot{\sigma}_{\forall_{\Phi}}\,L}{\rho_0\,\Delta\varphi\,v_0}$$



grad φ

NUSSELT

$$Nu = \left(\mathrm{gr\hat{\vec{a}}d}\;\hat{T}\right)_{\hat{r}=0} = \frac{h\,L}{k}$$

CALOR

SHERWOOD

$$Sh = \left(\mathrm{gr\hat{\vec{a}}d}\;\hat{w}_i\right)_{\hat{r}=0} = \frac{k\,L}{D_{AB}}$$

MASSA

FATOR DE ATRITO

$$f = \frac{1}{Re}\left(\mathrm{gr\hat{\vec{a}}d}\;\hat{v}\right)_{\hat{r}=0}$$

QUANTIDADE DE MOVIMENTO

ADIMENSIONAIS – ANALOGIA de REYNOLDS
NÚMERO DE STANTON

St = NÚMERO DE STANTON

QUANTIDADE DE MOVIMENTO

$$St = \frac{f}{2}$$

FATOR DE FANNING

CALOR

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re\,Pr} = \frac{h}{\rho c_P V}$$

MASSA

$$St = \frac{Sh}{Pe} = \frac{Sh}{Re\,Sc} = \frac{k}{V}$$

ADIMENSIONAIS –ANALOGIA DE COLBURN

FATOR j

QUANTIDADE DE MOVIMENTO

$$\frac{f}{2} = j_H = j_M$$

FATOR DE FANNING

CALOR

$$j_H = \frac{Nu}{Re\,Pr^{1/3}}$$

MASSA

$$j_M = \frac{Sh}{Re\,Sc^{1/3}}$$